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RECENT DEVELOPMENTS IN FRONTIER MODELLING AND EFFICIENCY MEASUREMENT

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In this paper recent developments in the estimation of frontier functions and the measurement of efficiency are surveyed, and the potential applicability of these methods in agricultural economics is discussed. Frontier production, cost and profit functions are discussed, along with the construction of technical, allocative, scale and overall efficiency measures relative to these estimated frontiers. The two primary methods of frontier estimation, econometric and linear programming, are compared. A survey of recent applications of frontier methods in agriculture is also provided.

Introduction

What is a frontier and how can it be used to measure the efficiency of an enterprise? In this paper, a frontier refers to a bounding function. Micro-economic theory is awash with bounding functions: a production function represents the maximum output attainable from a given set of inputs; a cost function represents minimum cost, given input prices and output; a profit function represents maximal profit, given input and output prices; and so on. Yet empirical work in all fields of economics, including agricultural economics, has been dominated by ordinary least squares (OLS) regression and its variants, which fit a line of best fit through the sample data rather than over the data, as would be appropriate for a production or profit function, or under the data, as would be appropriate for a cost function.

The two main benefits of estimating frontier functions, rather than average (e.g., OLS) functions, are that: (a) estimation of an average function will provide a picture of the shape of technology of an average firm, while the estimation of a frontier function will be most heavily

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influenced by the best performing firms and hence reflect the technology they are using, and (b) the frontier function represents a best-practice technology against which the efficiency of firms within the industry can be measured. It is this second use of frontiers which has provided the greatest impetus for the estimation of frontier functions in recent years.

Before discussing efficiency further, it would be useful to clarify the definitions of the terms efficiency and productivity. These words are often used interchangeably; however they are not precisely the same things. To illustrate the distinction between the two terms, it is useful to picture a production frontier which defines the current state of technology in an industry. Firms in that industry would presently be operating either on that frontier, if they are perfectly efficient, or beneath the frontier if they are not fully efficient. Productivity improvements can be achieved in two ways. One can either improve the state of the technology by inventing new ploughs, pesticides, rotation plans, etc. This is commonly referred to as technological change and can be represented by an upward shift in the production frontier. Alternatively one can implement procedures, such as improved farmer education, to ensure farmers use the existing technology more efficiently. This would be represented by the firms operating more closely to the existing frontier. It is thus evident that productivity growth may be achieved through either technological progress or efficiency improvement, and that the policies required to address these two issues are likely to be quite different. The discussion in this paper is confined to the measurement of efficiency. Issues relating to the measurement of technological change and overall productivity growth are dealt with in other papers (e.g. see Grosskopf 1993).

If all that is required is a measure of efficiency, some people may ask: 'why bother with econometric or linear programming frontier estimators?' For example, what is wrong with using tonnes of wheat per hectare or litres of milk per cow as measures of farmer efficiency? Measures such as tonnes per hectare have a serious deficiency, in that they only consider the land input and ignore all other inputs, such as labour, machinery, fuel, fertiliser, pesticide, etc. The use of this measure in the formulation of management and policy advice is likely to result in excessive use of those inputs which are not included in the efficiency measure. Similar problems occur when other simple measures of efficiency, such as litres of milk per cow or output per unit labour, are used.

A variety of efficiency measures have been proposed which can account for more than one factor of production. The primary purpose in this paper is to outline some of these measures and to discuss how they may be calculated relative to an efficient technology, which is generally represented by some form of frontier function. A key part of this exposition is a discussion of the two primary methods of frontier estimation, namely stochastic frontiers and data envelopment analysis (DEA), which involve econometric methods and mathematical programming, respectively. The discussion also considers multiple-output technologies, and ways of accounting for a variety of behavioural objectives, such as cost minimisation and profit maximisation, through the estimation of cost and profit frontiers.

The plan of the paper is as follows. Section 2 provides a brief history of modern efficiency measurement, beginning with the seminal paper by Farrell (1957). Recent developments in frontier modelling and efficiency measurement in the econometric and mathematical programming fields, are described in sections 3 and 4, respectively. Section 5 provides a brief review of some frontiers applications in the agricultural economics literature, and the final section concludes and discusses some potential applications to Australian agriculture.

Early Literature

The purpose of this paper is to provide an introduction to the field, which is not burdened by excessive notation and technical detail. More detailed reviews include: Forsund, Lovell and Schmidt (1980), Schmidt (1986), Bauer (1990), Seiford and Thrall (1990), Lovell (1993), Greene (1993) and Ali and Seiford (1993).

This discussion of the recent history of efficiency measurement begins with Farrell (1957) who drew upon the work of Debreu (1951) and Koopmans (1951) to define a simple measure of firm efficiency which could account for multiple inputs. He proposed that the efficiency of a firm consists of two components: technical efficiency, which reflects the ability of a firm to obtain maximal output from a given set of inputs, and allocative efficiency, which reflects the ability of a firm to use the inputs in optimal proportions, given their respective prices. These two measures are then combined to provide a measure of total economic efficiency.[1]

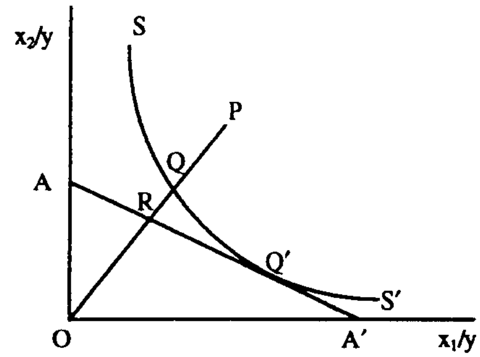
Farrell illustrated his ideas using a simple example involving firms which use two inputs (x_1 and x_2) to produce a single output (y), under an assumption of constant returns to scale.[2] Knowledge of the unit isoquant of the fully efficient firm,[3] represented by SS′ in Figure 1, permits the measurement of technical efficiency. If a given firm uses quantities of inputs, defined by the point P, to produce a unit of output, the technical efficiency of that firm is defined to be the ratio OQ/OP, which is the proportional reduction in all inputs that could theoretically be achieved without any reduction in output. Note that the point Q is technically efficient because it lies on the efficient isoquant.

[1] Some of Farrell's terminology differed from that which is used here. He used the term *price efficiency* instead of *allocative efficiency* and the term *overall efficiency* instead of *economic efficiency*. The terminology used in the present paper conforms with that which has been used most often in recent literature.

[2] Farrell also discussed the extension of his method so as to accommodate more than two inputs, multiple outputs, and non-constant returns to scale.

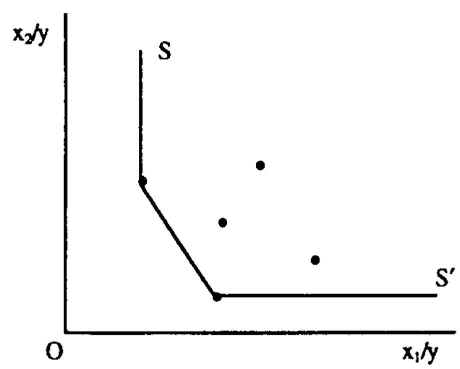
[3] The production function of the fully efficient firm is not known in practice, and thus must be estimated from observations on a sample of firms in the industry concerned. The selection of an appropriate method of estimation is the subject of considerable discussion later in this paper.

FIGURE 1
Technical and Allocative Efficiencies



If the input price ratio, represented by the line AA′ in Figure 1, is also known, allocative efficiency may also be calculated. The allocative efficiency of the firm operating at P is defined to be the ratio OR/OQ, since the distance RQ represents the reduction in production costs that would occur if production were to occur at the allocatively (and technically) efficient point Q′, instead of at the technically efficient, but allocatively inefficient, point Q. The total *economic efficiency* is defined to be the ratio OR/OP, where the distance RP can also be interpreted in terms of a cost reduction. Note that the product of technical and allocative efficiency provides the overall efficiency, (OQ/OP)(OR/OQ)=(OR/OP), and all three measures are bounded by zero and one.

FIGURE 2
Piecewise Linear Convex Isoquant



These efficiency measures assume the production function of the fully efficient firm is known. In practice this is not the case, and the efficient isoquant must be estimated from the sample data. Farrell suggested the use of either (a) a non-parametric piecewise-linear convex isoquant constructed such that no observed point should lie to the left or below it (refer to Figure 2), or (b) a parametric function, such as the Cobb-Douglas form, fitted to the data, again such that no observed point should lie to the left or below it. Farrell provided an illustration of his methods using agricultural data for the 48 continental states of the US.

The work of Farrell was subsequently adjusted and extended by a large number of authors. Aigner and Chu (1968) considered the estimation of a parametric frontier production function in input/output space. They specified a Cobb-Douglas production function (in log form) for a sample of N firms as:

$$(1) \qquad ln(y_i) = F(x_i;\beta) - u_i \qquad , i=1,2,\ldots,N$$

where y_i is the output of the i-th firm; x_i is the vector of input quantities used by the i-th firm; β is a vector of unknown parameters to be estimated; F(.) denotes an appropriate function (in this instance the Cobb-Douglas); and u_i is a non-negative variable representing inefficiency in production. The parameters of the model were estimated using linear programming, where $\sum_{i=1}^{N} u_i$ is minimised, subject to the constraints that $u_i >= 0$, $i=1,2,\ldots,N$.[4]

The ratio of observed output of the i-th firm, relative to the potential output defined by the estimated frontier, given the input vector x_i, was suggested as an estimate of the technical efficiency of the i-th firm:

$$(2) \qquad TE_i = y_i/\exp(F(x_i;\beta)) = \exp(-u_i).$$

This is an output-orientated measure as opposed to the input-oriented measure discussed above. It indicates the magnitude of the output of the i-th firm relative to the output that could be produced by the fully-efficient firm using the same input vector. The output- and input-orientated measures provide equivalent measures of technical efficiency when constant returns to scale exist, but are unequal when increasing or decreasing returns to scale are present (Fare and Lovell 1978).

Afriat (1972) specified a model similar to (1), except that the u_i were assumed to have a gamma distribution and the parameters of the model were estimated using the maximum likelihood (ML) method. Richmond (1974) noted that the parameters of Afriat's model could also be estimated using a method that has become known as corrected ordinary least squares (COLS), where the ordinary least squares (OLS) method provides unbiased estimates of the slope parameters, and the (downward biased) OLS estimator of the intercept parameter is adjusted up by the sample moments of the error distribution, obtained from the OLS residuals. Schmidt (1976) added to the discussion on ML frontiers by observing that the linear and quadratic programming estimators proposed by Aigner and Chu (1968), are ML estimators if the u_i were assumed to be distributed as exponential or half-normal random variables, respectively.

[4] Aigner and Chu (1968) also suggested the use of quadratic programming methods.

One of the primary criticisms of all of the above deterministic[5] frontier estimators is that no account is taken of the possible influence of measurement errors and other noise upon the shape and positioning of the estimated frontier, since all observed deviations from the estimated frontier are assumed to be the result of technical inefficiency. Timmer (1971) attempts to address this problem by making an adjustment to the Aigner and Chu (1968) method which involves dropping a percentage of firms closest to the estimated frontier, and re-estimating the frontier using the reduced sample. The arbitrary nature of the selection of some percentage of observations to omit, has meant, however, that Timmer's probabilistic frontier approach has not been widely followed. An alternative approach to the solution of the 'noise' problem has, however, been widely adopted. This method is the subject of the following section on stochastic frontiers.

Stochastic Frontiers

Stochastic Frontier Production Functions

Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977) independently proposed the estimation of a stochastic frontier production function, where noise is accounted for by adding a symmetric error term (v_i) to the non-negative error in (1) to provide:

$$\ln(y_i) = F(x_i;\beta) + v_i - u_i \qquad , i=1,2,\ldots,N. \tag{3}$$

The parameters of this model are estimated by ML, given suitable distributional assumptions for the error terms. Aigner, Lovell and Schmidt (1977) assume that v_i has normal distribution and u_i has either the half-normal or the exponential distribution.

This stochastic model specification not only addressed the noise problem associated with earlier (deterministic) frontiers, but also permitted the estimation of standard errors and tests of hypotheses, which were not possible with the earlier deterministic models because of the violation of certain ML regularity conditions (refer to Schmidt 1976).[6] The stochastic frontier is not, however, without problems. The main criticism is that there is no *a priori* justification for the selection of any particular distributional form for the u_i. The specification of more general distributional forms, such as the truncated-normal (Stevenson 1980) and the two-parameter gamma (Greene 1990), has partially alleviated this problem, but the

[5] The term *deterministic* is generally used to describe that group of methods which assume a parametric form for the production frontier along with a strict one-sided error term. The work of Aigner and Chu (1968), Afriat (1972) and Schmidt (1976) are examples.

[6] Greene (1980a) observed that a particular class of distributions could be assumed for the u_i which would circumvent these regularity problems. The noise criticism, however, would still remain.

resulting efficiency measures may still be sensitive to distributional assumptions.

Estimation Methods

Stochastic frontier production functions can be estimated using either the ML method or using a variant of the COLS method suggested by Richmond (1974). The COLS approach could be preferred because it is not as computationally demanding as ML which requires numerical solution of the likelihood. This distinction, however, has lessened over the past five years with the availability of software such as the LIMDEP econometrics package (Greene 1992) and the FRONTIER program (Coelli 1992, 1994), both of which automate the ML method.

The ML estimator is asymptotically more efficient than the COLS estimator, but the properties of the two estimators in finite samples cannot be analytically determined. The finite sample properties of the half-normal frontier model are investigated in a Monte Carlo experiment in Olsen, Schmidt and Waldman (1980). They observe no significant differences in the efficiencies of the two estimators and suggest that the COLS estimator may be preferred in sample sizes smaller than 400. A more recent study by Coelli (1995), involving substantially more replications, finds the ML estimator to significantly outperform the COLS estimator when the contribution of the inefficiency error to the total error term is large.[7] Given this result and the availability of automated ML routines, the ML estimator should be used in preference to the COLS estimator whenever possible.

Alternative Functional Forms

The Cobb-Douglas functional form has been most commonly used in the empirical estimation of frontier models. Its most attractive feature is its simplicity. A logarithmic transformation provides a model which is linear in the logs of the inputs and hence easily lends itself to econometric estimation. This simplicity, however, is associated with a number of restrictive properties. Most notably, returns to scale are restricted to take the same value across all firms in the sample, and elasticities of substitution are assumed equal to one.

A variety of alternative functional forms have also been used in the frontier literature. The two most popular forms are the translog (e.g., Greene 1980b) and the Zellner-Revankar generalised production function (e.g., Forsund and Hjalmarsson 1979 and Kumbhakar, Ghosh and McGukin 1991). The Zellner-Revankar form removes the returns-to-scale restrictions, while the translog form imposes no restrictions upon returns

[7] Coelli (1994b) considered 11 different values of the percentage of error due to inefficiency (λ^*) ranging from 0.0 to 1.0, in steps of 0.1, for sample sizes of N=50, 100, 400, and 800. Of these 44 cases, 14 show the mean square error of ML to be significantly smaller (at the 1% level) than that of COLS, while in only one case (λ^*=0.1, N=50) did the converse occur.

to scale or substitution possibilities, but has the drawback of being susceptible to multicollinearity and degrees of freedom problems. These problems can be avoided by jointly estimating the translog production function with the first-order conditions for profit maximisation, as suggested by Greene (1980b), but this increases the complexity of the estimation of parameters and also creates other problems which are discussed in the following section.

Dual Forms of the Technology

The discussion above concentrates upon the direct estimation of frontier production functions using single equation methods. The three main reasons for the consideration of alternative dual forms of the production technology, such as the cost or profit function, are (a) to reflect alternative behavioural objectives (such as cost minimisation), (b) to account for multiple outputs, and (c) to simultaneously predict both technical and allocative efficiency.

The direct estimation of a production function will produce biased and inconsistent estimates of the parameters if the standard behavioural objectives of either profit maximisation or cost minimisation apply. This is because the input levels are not independent of the error term and hence simultaneous equation bias results. Direct estimation of the production function can be justified if it is appropriate to assume either: (a) that the input levels are fixed and that the manager of the firm is attempting to maximise output given these input quantities; or (b) that the manager is selecting the levels of inputs and output to maximise expected (rather than actual) profit, as discussed in Zellner, Kmenta and Dreze (1966).

Given that both output prices and output quantity are rarely known with certainty when a farmer makes production decisions (such as to plant extra wheat or to buy extra sheep), the assumption of expected (rather than actual) profit maximisation is that assumption which is most commonly made in production studies involving agriculture. However, in some instances the assumption of a cost minimisation objective may be more appropriate. For example, consider the case of a dairy farm which is contracted to produce a particular level of output in a given year. In such cases it may be more appropriate to estimate a stochastic cost frontier of the form:

$$(4) \qquad \ln(c_i) = C(y_i, w_i; \alpha) + v_i + u_i \qquad , i=1,2,\ldots,N,$$

where c_i is the observed cost for the i-th firm; C(.) is a suitable functional form; w_i is a vector of (exogenous) input prices; α is a vector of unknown parameters to be estimated; u_i is a non-negative random variable reflecting cost inefficiency (which is often assumed to have a half-normal distribution); and all other variables are as defined earlier. The parameters of this equation can be estimated using standard econometric methods since the y_i and w_i are assumed to be exogenously determined. Schmidt and Lovell (1979) specify a Cobb-Douglas technology for steam-electric generating plants and show how the cost function can be estimated in a similar manner to the estimation of stochastic production frontiers using ML estimation or COLS. They also suggest the use of a ML systems estimator involving the cost function and K-1 factor demand equations, which provide more efficient estimators than the single equation estimators. This systems approach also has the advantage of explicitly accounting for allocative inefficiency, which is reflected in the error terms on the factor demand equations (which represent violations of the first-order conditions for cost minimisation).

At first glance the cost frontier approach appears to be a significant improvement. This approach accounts for exogenous output and endogenous inputs, permits the measurement of technical and allocative inefficiency, and can be extended to account for multiple outputs. However, it suffers from two serious drawbacks. First, the cost frontier approach requires input price data to be observable and to vary among firms. In many cases firms in an industry either face the same prices, or, if they do not face the same prices, the price data are difficult to collect.

Second, the Schmidt and Lovell (1979) approach to systems estimation and technical and allocative efficiency measurement is limited to the use of self-dual functional forms, such as the Cobb-Douglas. Once one specifies more flexible functional forms, such as the translog form (see Greene 1980b), which are not self-dual, a number of problems arise. The main problem is associated with selecting an appropriate way to represent the link between the allocative inefficiency errors in the input demand equations, and the allocative inefficiency error which appears in the cost function. To date, no one has solved this problem to the satisfaction of the majority, and debate continues as to how best address these issues (see Bauer 1990 and Greene 1993 for further discussion and references). If one of the existing approaches is applied (e.g., Greene 1980b or Ferrier and Lovell 1990) then criticism from some quarter is likely, and furthermore, estimation problems often arise when one tries to numerically solve the rather complicated likelihood functions that are involved. A sound approach to take (given that the cost minimising assumption is appropriate and suitable price data are available) is to estimate your cost function using the single equation ML method (which is automated in LIMDEP and FRONTIER) and use the method proposed by Kopp and Diewert (1982), and refined by Zeischang (1983), to decompose the cost efficiencies into their technical and allocative components. If the Cobb-Douglas functional form is considered appropriate, then the procedures involved simplify to those which are outlined in Schmidt and Lovell (1979).

This section has focussed on cost functions because cost minimisation is the assumption that is most often made in the dual frontier literature. Profit maximisation has also been considered by a number of authors, and may be considered to be the more appropriate assumption in many Australian agricultural industries. Examples of frontier studies which assume profit maximisation include Ali and Flinn (1989), who consider a single-equation profit frontier which can be estimated using the same methods as appropriate to production frontiers, and Kumbhakar, Ghosh and

McGukin (1991), who specify a ML systems estimator under the assumption of profit maximisation.

Panel Data

The previous discussion assumes that data on N firms, observed at one point in time, are available for use in the estimation of the frontier. If data on N firms, are observed in each of T different time periods, then this is what is known as *panel data*. Panel data have many potential advantages over a single cross-section of data in frontier estimation. It increases degrees of freedom for estimation of parameters; provides consistent estimators of firm efficiencies (given sufficiently large T); removes the necessity to make specific distributional assumptions regarding the u_i; does not require that the inefficiencies are independent of the regressors; and permits the simultaneous investigation of both technical change and technical efficiency change over time.

Pitt and Lee (1981) specified a panel data version of the Aigner, Lovell and Schmidt (1977) half-normal model:

$$\ln(y_{it}) = F(x_{it};\beta) + v_{it} - u_{it} \qquad , i=1,2,\ldots,N,\ t=1,2,\ldots,T, \tag{5}$$

which they estimate using ML estimation. This model may have degrees of freedom advantages, but otherwise is no different to the cross-sectional model. They also consider a model in which the inefficiency effects (u_{it}) are assumed to be constant through time:

$$\ln(y_{it}) = F(x_{it};\beta) + v_{it} - u_{i} \qquad , i=1,2,\ldots,N,\ t=1,2,\ldots,T, \tag{6}$$

Battese and Coelli (1988) extended this model to permit the u_i to have the more general truncated normal distribution (proposed by Stevenson 1980) and also derived panel data generalisations of the conditional inefficiency predictors of Jondrow et al (1982). Battese, Coelli and Colby (1989) further extend the model to permit unbalanced panel data. These last three models have the advantage of providing consistent estimators of the u_i, as T becomes large. However, as T becomes large the assumption that technical inefficiency is time-invariant is more difficult to justify, as one would expect managers to learn from previous experience.

Kumbhakar (1990) suggested a panel data stochastic frontier in which the inefficiency effects are permitted to vary systematically with time. His model is similar to (5) with the u_{it} assumed to have structure:

$$u_{it} = [1 + \exp(bt + ct^2)]^{-1} u_i \tag{7}$$

where u_i is assumed to have half-normal distribution and b and c are parameters to be estimated. Kumbhakar (1990) suggested that the model be estimated using ML estimation but no empirical application has yet been attempted. Battese and Coelli (1992) suggested an alternative to the Kumbhakar (1990) model where the u_{it} are assumed to be an exponential function of time, involving only one parameter, such that:

(8) $$u_{it} = \{\exp[-\eta(t-T)]\}\, u_i$$

where u_i is assumed to have truncated normal distribution and η is a parameter to be estimated. The model is illustrated in an application involving data on Indian paddy farmers. The ML estimation method and efficiency calculations have been automated in the FRONTIER program (Coelli 1994a). One advantage of these two model specifications is that the inclusion of a time trend into the production function F(.) permits the estimation of both technical change and changes in the technical inefficiencies over time.[8]

Schmidt and Sickles (1984) noted that when panel data are available there is no need to specify an explicit distribution for the inefficiency effects. They suggested estimating a model similar to (6) using the traditional panel data methods of fixed effects (dummy variables) estimation or error-components estimation, depending upon what assumptions are judged appropriate regarding the independence of the inefficiencies and the regressors.[9] The largest estimated firm intercept is then used to adjust the intercept and all other firm effects so that all firm effects are zero or negative, and thus can be used to obtain measures of the efficiencies of those firms. One criticism that can be levelled at this, and all other COLS type methods, is that the average firms are having the greatest influence on the shape of the estimated frontier, while ML estimation of stochastic frontiers allows the most efficient firms to have a greater influence upon the shape of the estimated frontier.

The Schmidt and Sickles (1984) approach has been extended by Cornwell, Schmidt and Sickles (1990) and Lee and Schmidt (1993) to account for time-varying efficiencies. Both papers suggest temporal variation which is more flexible than formulations (7) and (8) discussed above.

Determinants of Inefficiency

A number of empirical studies (e.g., Pitt and Lee 1981 and Kalirajan 1981) have investigated the determinants of technical inefficiency variation among firms in an industry by regressing the predicted inefficiencies, obtained from an estimated stochastic frontier, upon a vector of firm-specific factors, such as firm size, age and education of manager, etc. in a second-stage regression. There is, however, a significant problem with this two-stage approach. In the first stage, the inefficiency effects are assumed to be independently and identically distributed, while in the second stage they are assumed to be a function of a number of firm-specific factors which implies that they are not identically distributed.

[8] It should be kept in mind, however, that the identification of these two effects hinges upon the distributional assumptions made regarding the u_i, and that without these assumptions the two effects could not be individually identified.

[9] The fixed effects approach permits the regressors and inefficiencies to be correlated while the error components method assumes independence, as does the stochastic frontier estimation of models (5) and (6).

Recent papers by Kumbhakar, Ghosh and McGukin (1991) and Reifschneider and Stevenson (1991) noted this inconsistency and specify stochastic frontier models in which the inefficiency effects are made an explicit function of the firm specific factors, and all parameters are estimated in a single-stage ML procedure. Battese and Coelli (1995) extended this approach to accommodate panel data, which permits the simultaneous investigation of both the determinants of technical inefficiencies, along with the degree of technical efficiency change and technical change over time.

Data Envelopment Analysis (DEA)

It is evident from the discussion in section 3 that stochastic frontier methods have developed a great deal over the past two decades. During this period, a separate literature on the non-parametric mathematical programming approach to frontier estimation, known as *data envelopment analysis (DEA)*, has also been developing, almost independently of the stochastic frontier literature.

Only a small percentage of agricultural frontier applications have used the DEA approach to frontier estimation. This is, in one sense, surprising, given the popularity of mathematical programming methods in other areas of agricultural economics research during the 1960s and 1970s. However, DEA has a very large following in other professions, especially in the management science literature, and in applications to service industries where there are multiple outputs, such as banking, health, telecommunications and electricity distribution. The DEA approach suffers from the same criticism as the deterministic methods discussed in section 2, in that it takes no account of the possible influence of measurement error and other noise in the data. On the other hand, it has the advantage of removing the necessity to make arbitrary assumptions regarding the functional form of the frontier and the distributional form of the u_i.

The piecewise-linear convex hull approach to frontier estimation proposed by Farrell (1957) was considered by only a handful of papers (e.g., Sietz 1971) until a paper by Charnes, Cooper and Rhodes (1978) reformulated the approach into a mathematical programming problem and coined the term data envelopment analysis (DEA). There has since been a large number of papers which have extended and applied the DEA methodology.

Charnes, Cooper and Rhodes (1978) proposed a model which had an input orientation and assumed constant returns to scale (CRS). Subsequent papers have considered alternative sets of assumptions, such as Banker, Charnes and Cooper (1984) who proposed a variable returns to scale (VRS) model. The following discussion of DEA begins with a description of the input-orientated CRS model in section 4.1, because this model has been the one most often considered in applications to date.

The Constant Returns to Scale (CRS) Model

It is best to begin by defining some notation. Assume there is data on K inputs and M outputs on each of N firms. For the i-th firm these are represented by the vectors x_i and y_i, respectively. The KX × N input matrix, X, and the MX × N output matrix, Y, represent the data of all N firms. The purpose of DEA is to construct a non-parametric envelopment frontier over the data points such that all observed points lie on or below the production frontier. For the simple example of an industry where one output is produced using two inputs, it can be visualised as a number of intersecting planes forming a tight fitting cover over a scatter of points in three-dimensional space. Given the CRS assumption, this can also be represented by a unit isoquant in input/input space (refer to Figure 2).

The best way to introduce DEA is via the ratio form. For each firm we would like to obtain a measure of the ratio of all outputs over all inputs, such as $u'y_i/v'x_i$, where u is an M×1 vector of output weights and v is a K×1 vector of input weights. To select optimal weights (for the i-th firm) we specify the mathematical programming problem:

$$\text{(9)} \qquad \max_{u,v} (u'y_i/v'x_i),$$
$$\text{st} \quad u'y_j/v'x_j \leq 1, \; j=1,2,...,N,$$
$$u, v \geq 0.$$

This involves finding values for u and v, such that the efficiency measure of the i-th firm is maximised, subject to the constraint that all efficiency measures must be less than or equal to one. One problem with this particular ratio formulation is that it has an infinite number of solutions.[10] To avoid this one can impose the constraint $vx_i = 1$, which provides:

$$\text{(10)} \qquad \max_{\mu,\nu} (\mu'y_i),$$
$$\text{st} \quad \nu'x_i = 1,$$
$$\mu'y_j - \nu'x_j \leq 0, j = 1,2,...,N,$$
$$\mu, \nu \geq 0,$$

where the notation change from u and v to μ and ν reflects the transformation. This form is known as the *multiplier* form of the linear programming problem.

Using the duality in linear programming, one can derive an equivalent *envelopment* form of this problem:

[10] That is, if (u*,v*) is a solution, then (αu*,αv*) is another solution, etc.

$$(11)\qquad \min_{\theta,\lambda}\theta,$$

$$\text{st}\quad -y_i + Y\lambda \geq 0,$$

$$\theta x_i - X\lambda \geq 0,$$

$$\lambda \geq 0$$

where θ is a scalar and λ is a $N\times 1$ vector of constants. This envelopment form involves fewer constraints than the multiplier form, and hence is generally the preferred form to solve.[11] The value of θ obtained will be the efficiency score for the i-th firm. It will satisfy $\theta \leq 1$, with a value of 1 indicating a point on the frontier and hence a technically efficient firm, according to the Farrell (1957) definition. Note that the linear programming problem must be solved N times, once for each firm in the sample. A value of θ is then obtained for each firm.

The piecewise linear form of the non-parametric frontier in DEA can cause a few difficulties in efficiency measurement. The problem arises because of the sections of the piecewise linear frontier which run parallel to the axes (refer Figure 2) which do not occur in most parametric frontiers (refer Figure 1). To illustrate the problem, refer to Figure 3 where the firms using input combinations C and D are the two efficient firms which define the frontier, and firms A and B are inefficient firms. The Farrell (1957) measure of technical efficiency gives the efficiency of firms A and B as OA′/OA and OB′/OB, respectively. However, it is questionable as to whether the point A′ is an efficient point since one could reduce the amount of input x_2 used (by the amount CA′) and still produce the same output.[12] This is known as input slack in the literature.[13] Once one considers a multiple output situation, the diagrams are no longer as simple and the possibility of the related concept of *output slack* also occurs. Thus it could be argued that both the Farrell measure of technical efficiency (θ) and any non-zero input or output slacks should be reported to provide an accurate indication of technical efficiency of a firm in a DEA analysis.[14] Note that for the i-th firm the output slacks will be equal to zero only if $Y\lambda - y_i = 0$, while the input slacks will be equal to zero only if $\theta x_i - X\lambda = 0$ (for the given optimal values of θ and λ).

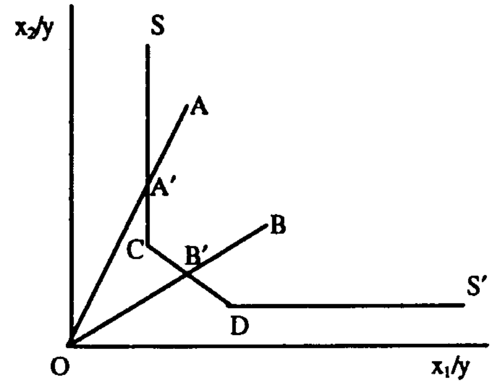
[11] The forms defined by (9) and (10) are introduced for expository purposes. They are not used again in the remainder of this paper.

[12] Note that the line segments CS and DS are parallel to the axes.

[13] Some authors use the term *input excess.*

[14] Koopman's (1951) definition of technical efficiency was stricter than the Farrell (1957) definition. The former is equivalent to stating that a firm is only technically efficient if it operates on the frontier and furthermore that all associated slacks are zero.

FIGURE 3
Efficiency Measurement and Input Slacks



In Figure 3 the input slack associated with the point A′ is CA′ of input x_2. In cases when there are more inputs and outputs than considered in this simple example, the identification of the nearest efficient frontier point (such as C), and hence the subsequent calculation of slacks, is not a trivial task. Some authors (see Ali and Seiford 1993) have suggested the solution of a second-stage linear programming problem to identify the nearest efficient frontier point, where 'nearest' is defined in terms of the minimum sum of slacks required to move from an inefficient frontier point (such as A′ in Figure 3) to an efficient frontier point (such as point C). This second stage linear programming problem may be defined by:

(12) $$\min_{\lambda,OS,IS} -(M1'OS - K1'IS),$$

$$st \quad -y_i + Y\lambda - OS = 0,$$

$$\theta x_i - X\lambda - IS = 0$$

$$\lambda \geq 0,\ OS \geq 0,\ IS \geq 0,$$

where OS is an M×1 vector of output slacks, IS is a K×1 vector of input slacks, and M1 and K1 are M×1 and K×1 vectors of ones, respectively. Note that in this second-stage linear program, θ is not a variable, its value is taken from the first-stage results. Furthermore, note that this second-stage linear program must also be solved for each of the N firms involved.[15]

[15] Many of the models which have appeared in the literature have used a single-stage linear program, involving the use of an infinitesimal, to solve this two stage problem. It has been argued that the two-stage method should be preferred, since 'attempts to solve these non-Archimedean models as a single linear program with an explicit numerical value for the infinitesimal frequently creates computational inaccuracies and leads to erroneous results.' (Ali and Sieford 1993).

One major problem associated with the above second-stage approach is that it is not invariant to units of measurement. The alteration of the units of measurement, say for a fertiliser input from kilograms to tonnes (while leaving other units of measurement unchanged), could result in the identification of different 'nearest' efficient boundary points and hence different slack measures.[16] As a result of this problem, many studies simply solve the first-stage linear program for the values of the Farrell technical efficiency measures (θ) for each firm and ignore the slacks completely. We believe the best approach is to use this first-stage linear program which does not explicitly account for slacks (e.g. equation 11), and then report both θ and the residual slacks. This approach has two advantages over the two-stage approach. It involves less programming and it also avoids the units of measurement problem.[17]

The Variable Returns to Scale (VRS) Model

Given that many industries are not perfectly competitive, the CRS assumption is often not appropriate. Banker, Charnes and Cooper (1984) suggested an extension of the CRS DEA model to account for variable returns to scale (VRS) situations. The CRS linear programming problem can be easily modified to account for VRS by adding the convexity constraint: $N1'\lambda = 1$ to (11) to provide:

$$(13) \quad \min_{\theta,\lambda} \theta,$$
$$\text{st} \quad -y_i + Y\lambda \geq 0,$$
$$\theta x_i - \lambda \geq 0,$$
$$N1'\lambda = 1$$
$$\lambda \geq 0,$$

where N1 is an N×1 vector of ones. This approach forms a convex hull of intersecting planes which envelope the data points more tightly than the CRS conical hull and thus provides technical efficiency scores which are less than or equal to those obtained using the CRS model.

Output-Orientated Models

In the preceding input-orientated models, the method sought to identify technical inefficiency as a proportional reduction in input usage. This

[16] Charnes, Cooper, Rousseau and Semple (1987) suggest a units invariant model where the unit worth of a slack is made inversely proportional to the quantity of that input or output used by the i-th firm. This does solve the immediate problem, but does create another, in that there is no obvious reason for the slacks to be weighted in this way.

[17] Note that the technical efficiency measures obtained from this approach will be identical to those obtained from the two-stage approach. It is only the slack measures which may differ slightly.

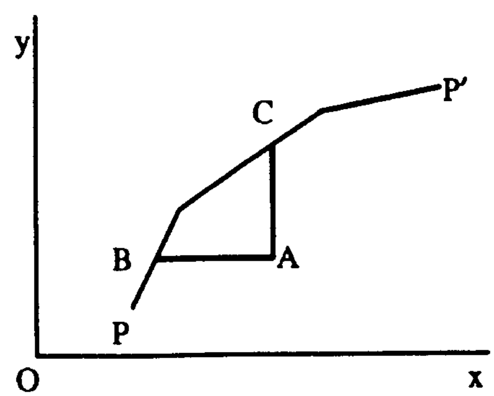
corresponds to Farrell's input-based measure of technical inefficiency. As discussed in section 3, it is also possible to measure technical inefficiency as a proportional increase in output production. The two measures provide the same value under CRS but are unequal when VRS is assumed. Given that linear programming cannot suffer from such statistical problems as simultaneous equation bias, the choice of an appropriate orientation is not as crucial. In many studies the analysts have tended to select input-orientated models because many firms have particular orders to fill and hence the input quantities appear to be the primary decision variables, although this argument may not be as strong in agriculture as it is in manufacturing and service industries.

The output-orientated models are very similar to their input-orientated counterparts. Consider the example of the following output-orientated, VRS model:

$$(14)\qquad \max_{\phi,\lambda} \phi, \quad \text{st} \quad -y_i + Y\lambda \geq 0, \quad \phi x_i - X\lambda \geq 0, \quad N1'\lambda = 1 \quad \lambda \geq 0$$

where ϕ is the proportional increase in outputs that could be achieved by the i-th firm. An output-oriented CRS model is defined in a similar way, but is not presented here for brevity.

FIGURE 4
Output- and Input-based Efficiency Measures



One point that should be made is that the output- and input-orientated models will estimate exactly the same frontier and therefore, by definition, identify the same set of firms as being efficient. It is only the efficiency measures associated with the inefficient firms that may differ between the two methods. The two types of measures may be illustrated using the simple example depicted in Figure 4, where output (y) is on the vertical axis and input (x) on the horizontal axis, and the production frontier is depicted by PP'. For the inefficient firm operating at A, the distance AB is associated with input-based technical inefficiency, while AC is associated with output-based technical inefficiency.

Other Variants and Extensions

Possible extensions to the above models include: replacement of the piecewise linear frontier with a piecewise log-linear or piecewise Cobb-Douglas frontier (Charnes, Cooper, Seiford and Stutz 1982, 1983); incorporation of a cost minimisation behavioural objective[18] (e.g., Ferrier and Lovell 1990 and Chavas and Aliber 1993); consideration of a stochastic element into the DEA (Sengupta 1990); inclusion of categorical and environmental variables in the analysis (Banker and Morey 1986a,b); and use of panel data and the Malmquist index approach to investigate technical change and technical efficiency change (Fare, Grosskopf, Norris and Zhang 1994). For a more complete list of possible extensions and variants, the reader is advised to consult Seiford and Thrall (1990) and Ali and Seiford (1993).

Applications to Agriculture

The purpose of this brief section is to identify a few studies which illustrate the application of various frontier techniques to agriculture. This does not involve a comprehensive survey of frontier applications to agriculture. This has already been partially completed by the survey of applications of parametric production frontiers to agricultural industries by Battese (1992), and the survey of applications of frontier methods to developing country agriculture by Bravo-Ureta and Pinheiro (1993).

There appear to be only a few applications of frontier models to Australian agriculture. The only two that could be found were a study by Battese and Corra (1977), which involved an application of deterministic and stochastic frontiers to Eastern Australian broad acre agriculture; and the analysis of Battese and Coelli (1988), which involved an application of a panel data stochastic frontier to the dairy industries of NSW and Victoria.

There have been a vast number of applications of frontier methodologies to agricultural data in other countries around the world. A selection of studies from the last ten years are listed in Table 1, along with brief descriptions of the industries analysed and the methods used. This list is

[18] The construction of both DEA production and cost frontiers will permit the measurement of technical and economic efficiencies, and hence the calculation of allocative efficiencies as well.

[19] Entries in Table 1 are derived from the survey papers by Battese (1992) and Bravo-Ureta and Pinheiro (1993) and a recent search conducted by the author.

designed to indicate the breadth of analyses that have been conducted. It is by no means an exhaustive list.[19]

Of the 38 papers listed in Table 1, only three involve DEA (non-parametric linear programming) while the remainder involve the construction of a variety of parametric frontiers.[20] Of the latter, 7 papers have estimated deterministic frontiers; 24 have estimated stochastic frontiers; and four have estimated both deterministic and stochastic frontiers. The deterministic frontiers are those discussed in section 2, which specify a parametric structure and assume that all deviations from the frontier are due to inefficiency. The majority of applications of the deterministic frontiers are limited to the first half of Table 1 (where the papers are ordered by year of observation). The lack of recent applications of deterministic frontier models is most likely a consequence of researchers becoming more aware of the deficiencies of deterministic frontiers and the advantages of stochastic frontiers. Lovell (1993, p. 21) explained his objection to deterministic frontiers by noting that the approach 'combines the bad features of the econometric and programming approaches to frontier construction: it is deterministic and parametric'. That is, deterministic frontiers may be criticised for not accounting for measurement error and other noise, and also for imposing a particular functional form upon the technology.

The stochastic frontier model has been, by far, the most popular, with 28 of the 38 papers in Table 1 involving applications of this model. The majority of these studies have involved the estimation of a single-equation production function using cross-sectional data. Exceptions to this are noted in the table, with nine papers considering panel data models; two estimating a profit frontier; and three estimating a system of equations involving a production function and input demand equations derived from the first-order conditions for profit maximisation.

A large number of different agricultural industries are mentioned in Table 1. The most common frontier applications appear to be rice production, with 11 papers, and the dairy industry, with seven papers. The attention that rice has received is most likely a consequence of its vital importance to the food supplies of so many countries around the world, while the attention given to dairy industries is more probably a consequence of recent debate surrounding the high degree of regulation that it attracts in many developed countries. Other industries mentioned in Table 1 include maize, wheat, and rubber. However, the largest group of studies involve multi-product farming with 15 analyses of this type listed in Table 1. Applications from a total of 15 different countries are listed, ranging from developed countries, such as the USA, England and Australia, to small developing countries, such as Guatemala, Paraguay and Nepal. A researcher who wishes to conduct a frontier study of a particular agricultural

[20] The high percentage of parametric papers in Table 1 could be partly a consequence of the journals I have searched in, but I believe it is an accurate depiction of the dominance of parametric methods in the agricultural economics literature.

enterprise should be able to identify at least a few papers from the above list, which would be relevant to the application involved.

TABLE 1
Applications of Frontiers to Agriculture 1985–1994

Authors	Year	Industry	Methodology
Belbase and Grabowski	1985	Nepalese farms	deterministic
Dawson	1985	farms in North West England	deterministic
Fare, Grabowski and Grosskopf	1985	Filipino farms	DEA
Rawlins	1985	Jamaican farms	stochastic
Ray	1985	Indian farms	DEA
Bravo-Ureta	1986	New England (US) dairy farms	deterministic
Huang, Tang and Bagi	1986	Indian farms	stochastic profit frontier
Kalirajan and Shand	1986	Malay rice farms	stochastic
Phillips and Marble	1986	Guatemalan maize farms	stochastic
Taylor and Shonkwiler	1986	Brazilian farms	deterministic and stochastic
Taylor, Drummond and Gomes	1986	Brazilian farms	deterministic
Aly, Belbase, Grabowski and Kraft	1987	Illinios (US) grain farms	deterministic
Ekanayake	1987	Sri Lankan rice farms	deterministic and stochastic
Ekanayake and Jayasuriya	1987	Sri Lankan rice farms	deterministic and stochastic
Tauer and Belbase	1987	New York (US) dairy farms	deterministic
Battese and Coelli	1988	NSW and Victorian dairy farms	stochastic panel
Ali and Flinn	1989	Punjab (Pakistan) rice farms	stochastic profit frontier
Bailey, Biswas, Kumbhakar and Schulthies	1989	Ecuadorian dairy farms	stochastic
Battese, Coelli and Colby	1989	Indian farms	stochastic panel
Dawson and Lingard	1989	Filipino rice farms	stochastic

Authors	Year	Industry	Methodology
Kalirajan	1989	Filipino rice farms	stochastic
Kalirajan and Shand	1989	Indian rice farms	stochastic panel
Kumbhakar, Biswas and Bailey	1989	Utah (US) dairy farms	stochastic system
Ali and Chaudry	1990	Punjab (Pakistan) farms	deterministic
Bravo-Ureta and Rieger	1990	North Eastern US dairy farms	deterministic and stochastic
Bravo-Ureta and Rieger	1991	North Eastern US dairy farms	stochastic
Dawson, Lingard and Woodford	1991	Filipino rice farms	stochastic
Kalirajan	1991	Filipino rice farms	stochastic panel
Kumbhakar, Ghosh and McGukin	1991	US dairy farms	stochastic system
Squires and Tabor	1991	Indonesian farms	stochastic
Battese and Coelli	1992	Indian paddy farms	stochastic panel
Battese and Tessema	1993	Indian farms	stochastic panel
Battese, Malik and Broca	1993	Pakistani Wheat farms	stochastic panel
Chavas and Aliber	1993	Wisconsin (US) farms	DEA
Tran, Coelli and Fleming	1993	Vietnamese rubber farms	stochastic panel
Bravo-Ureta and Evenson	1994	peasant farms in Paraguay	stochastic
Kumbhakar	1994	farms in West Bengal (India)	stochastic system
Battese and Coelli	1995	Indian paddy farms	stochastic panel

Conclusions

The main conclusion of this paper is that none of the proposed methods of measuring efficiency relative to an estimated frontier is perfect. However, they all provide substantially better measures of efficiency than simple partial measures, such as output per unit of labour or land. Given these qualifications, one frequently asked question is: which method of frontier estimation — stochastic frontier or DEA — should one use? The answer to this question often depends upon the application being considered. If one is using farm level data where measurement error, missing variables (e.g., data on an input is not available or not suitably measured), weather, etc. are likely to play a significant role, then the assumption that all deviations from the frontier are due to inefficiency, which is made by DEA, may be a brave assumption. Hence the stochastic frontier method is recommended for use in most agricultural applications. This method also has the added advantage of permitting the conduct of statistical tests of hypotheses regarding the production structure and the degree of inefficiency. There are instances, however, where the above mentioned factors are not likely to have great influence (e.g., poultry and pig farming, abattoirs and grain silos), and hence DEA could also be used. Furthermore, in instances where production involves more than one product, and the construction of an aggregate measure of output is difficult, DEA may be more attractive than estimating a multi-product cost or profit frontier, especially if price data are difficult to obtain.

As with all forms of empirical modelling, a frontier study can suffer from a variety of possible pitfalls, such as: the possibility that omitted or poorly measured inputs may influence technical efficiency measures; the possibility that unaccounted for environmental factors, such as soil quality or topography, may also influence technical efficiency measures; the possibility that poorly measured price variables (e.g., transport costs not properly accounted for) may influence allocative inefficiency measures; and the use of data from a single season to measure efficiency may result in some farmers being labelled as inefficient, because of low stocking rates, when over a longer time they may be shown to be more efficient because of their more conservative approach. This last issue points toward an interesting area of possible research. Many past analyses of farm efficiency have only involved efficiency measures derived from data from a single season. The development of a methodology to suitably account for the issues of risk aversion and multi-season efficiency would be a valuable contribution to the frontier literature.

There are a variety of policy issues which could be investigated using frontier methods. For example: identifying the influence of pollution controls upon efficiency in feedlots, abattoirs and irrigation farms; measuring the effect of salinity and soil degradation upon farm efficiency; measuring the influence of farm size upon efficiency; and investigating the effect of recent reforms upon various agricultural sectors, such as the dairy industry, using 'before' and 'after' data. The method could also be used to determine the extent to which the utilisation of agricultural extension advice may improve farmer efficiency.

Australian agriculture is faced with declining world commodity prices, increased competition from both subsidised and non-subsidised overseas industries, and declining expenditure on agricultural research. A suitable rate of productivity growth is required in order to remain competitive. However, to attain this without continuing to rely upon significant advances in technology from agricultural research, requires that industry uses the existing technology more efficiently. Frontier functions and efficiency measurement can assist in this endeavour.

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