RISK ATTITUDES AMONGST AUSTRALIAN FARMERS: REPLY

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Drynan (1981) has raised a number of issues concerning the validity of the methodology used in our recent article (Bond and Wonder 1980). In particular, it is implied that a mean-variance (E-V) derivation of the expected utility hypothesis leads to axiomatic inconsistencies and that certain important assumptions have been left unstated. In addition, a number of comments are made concerning the numerical accuracy of our results. Drynan's dissatisfaction with the E-V approach leads him to propose an alternative estimation methodology, one which is based on the properties of the absolute risk aversion function, $r(X)$.

In this reply, we take the opportunity to provide a more complete exposition of the fundamental properties and assumptions of our E-V estimation framework. First, we investigate the indifference curve properties of our certainty-equivalence expression in relation to issues raised concerning the independence axiom and the variability of risk coefficients. Second, we discuss Drynan's proposal to go beyond the E-V framework. Finally, we consider the questions of numerical accuracy.

Indifference Curves and the Independence Axiom

The essential features of our estimation methodology are contained in the specification of the certainty equivalent (given in our equation 9) which was:

$$x_o = x^* + \tfrac{1}{2} V[x]u''(x^*)/u'(x^*), \qquad (1)$$

where
- x_o = certainty equivalent of the random variable x;
- x^* = expected value of x;
- $V[x]$ = the variance of x; and
- $u''(x^*)/u'(x^*)$ = the ratio of the second and first derivatives of the utility function evaluated at the point x^*.

From this equation can be derived the combinations of x^* and $V[x]$ that give the same level of expected utility. The locus of these combinations may be termed an indifference curve in E-V space. As with conventional indifference curve analysis, any movement along the indifference curve defines the rate at which x^* can be substituted for $V[x]$

* The authors would like to thank J. C. Quiggin for his helpful comments on an earlier draft of this reply and provision of a proof concerning violation of the independence axiom.

in such a way that expected utility remains constant. Whether or not this rate of trade-off is constant depends entirely on the curvature properties of the indifference curves. These are derived as follows. Taking the total differential of equation (1) with respect to its arguments (x^*, $V[x]$) and setting the result equal to zero provides an equation for the indifference curve. Note that, in carrying out this differentiation, $u''(x^*)/u'(x^*) = f(x^*)$ is a function of x^* and we have assumed $u'''(x^*) = 0$.

The slope of the indifference locus at the point (x^*, $V[x]$) is therefore given by:

(2) $$[dx^*/dV[x]]_u = (-\tfrac{1}{2}a)/(1 - \tfrac{1}{2}V[x]a^2),$$

where $a = u''(x^*)/u'(x^*)$ for notational convenience.

From the definition of a risk-averse decision maker, the slope of the indifference locus will be positive in E-V space as long as $V[x] < 2/a^2$.

The curvature of the indifference locus is determined by differentiating equation (2) with respect to $V[x]$. In carrying out this differentiation, we bear in mind that, along a given indifference curve, x^* is itself a function of $V[x]$. Thus:

(3) $$[d^2x^*/dV[x]^2]_u = (-1/b^2)(\tfrac{1}{2}a^3/b),$$

where $b = (1 - \tfrac{1}{2}V[x]a^2)$.

From equation (3), we may note that downward convexity in the indifference locus for a risk-averse individual requires that $V[x] < 2/a^2$. This restriction is the same as that noted above for a positive slope under risk aversion. We may interpret this condition as guaranteeing a positive, increasing marginal gain in $Eu(x)$ with an increase in x^* as long as the variance is kept within a bound determined by $2/a^2$. This result stems from the expression for the certainty equivalent, equation (1), in which an increase in x^* is accompanied simultaneously by an offsetting factor equal to $\partial a/\partial x^*(\tfrac{1}{2}V[x])$. If the increase in x_o occasioned by an increase in x^* is not to be 'swamped' by this offsetting process, then $\partial a/\partial x^*(\tfrac{1}{2}V[x])$ must be less than 1 (the coefficient of x^* in equation (1)). This can occur only when $V[x] < 2/a^2$.

More generally, however, this limit on the value of $V[x]$ corresponds to the same constraints placed on a quadratic utility function over the range $[0, x]$ such that marginal utility always remains positive. That is, for the familiar quadratic function $U(x) = dx - ex^2$, risk aversion is defined only up to $x < d/2e$ and it can be shown that when $V[x] = 2/a^2$, $x = d/2e$.

This result establishes the essential quadratic nature of our estimation procedure and there are two important properties (or restrictions) which follow from this. We note from equation (2) that, as $V[x]$ approaches zero, the slope of the indifference locus approaches a constant value of $-\tfrac{1}{2}a$. Thus, the indifference curves intersect the x^* axis at a positive slope under risk aversion. Secondly, we can investigate the change in the slope of the indifference curves as x^* increases by differentiating (2) with respect to x^* and holding $V[x]$ constant. The result of this is that the slope of the indifference curves is everywhere increasing in x^* for $V[x] < 2/a^2$. This is the case of increasing absolute risk aversion.

In summary, therefore, our estimation methodology allows for nonlinearity in the trade-off between x^* and $V[x]$ over a range of values of x for which marginal utility is positive. If we examine the arguments used by Drynan to criticise our methods, we see from the example provided in his footnote (1), that neither of these properties have been recognised. First, Drynan has stipulated that the trade-off between x^* and $V[x]$ takes a constant value of -1, which is equivalent to an assumption of linear indifference curves. It is evident from the results presented above that the only circumstance under which linear indifference curves can validly exist is when they have zero slope, i.e. under risk neutrality. More generally, however, once it is recognised that the ratio $u''/x^*/u'(x^*)$ is itself a function of x^*, a constant trade-off between x^* and $V[x]$ (and hence, linear indifference curves) must be regarded as meaningless for all utility functions in which the second derivative takes non-zero values.[1] We shall return to this point subsequently.

The second unsatisfactory aspect of Drynan's criticism is that, in constructing the example in his footnote (1), he has forced the decision maker into a position of negative marginal utility. There are two ways of seeing this. One approach is to note that, if risk 1 of his example is equivalent to risk 2, then with equal probabilities, $0.5u(25)+0.5u(35) = 0.5u(5)+0.5u(5)$. Clearly, for $u(x)$ increasing in all $x>0$, such a result is meaningless. The alternative approach is to interpret the constant trade-off value of -1 in our certainty-equivalence expression (equation 1) above. From this, it may be deduced that Drynan's decision maker obeys the rule $\tfrac{1}{2}u''(x^*)/u'(x^*) = -1$ for all x. As noted above, however, the requirements of positive marginal utility in E-V analysis are such that $V[x] < 2/a^2$ where $a = u''(x^*)/(u'(x^*))$. That is, for Drynan's decision maker, risky prospects with a variance greater than 0.5 are inadmissible. Clearly, risk 1 of Drynan's example has a variance considerably in excess of this value and, hence, a violation of the transitivity axiom has resulted. For these reasons, we do not consider Drynan's criticisms of our techniques to be valid.

[1] It can also be shown that linear indifference curves with non-zero slope necessarily lead to a violation of the independence axiom in an E-V decision model. If x_A is equivalent to x_B and if x_C is any other risky prospect, define $x_X = \alpha x_A + (1-\alpha)x_C$ and $x_Y = \alpha x_B + (1-\alpha)x_C$. A linear trade-off would imply that, for some k, x_A is equivalent to x_B only if:

(i) $$E[x_A] - E[x_B] = k(V[x_B] - V[x_A]).$$

Given that:

$$\begin{aligned} E[x_X] &= \alpha E[x_A] + (1-\alpha)\ E[x_C]; \\ E[x_Y] &= \alpha E[x_B] + (1-\alpha)\ E[x_C]; \\ V[x_X] &= \alpha V[x_A] + (1-\alpha)\ V[x_C] + \alpha(1-\alpha)(E[x_A] + E[x_C]^2; \text{ and} \\ V[x_Y] &= \alpha V[x_B] + (1-\alpha)\ V[x_C] + \alpha(1-\alpha)(E[x_B] + E[x_C]^2. \end{aligned}$$

It can be shown that, if (i) holds, then:

(ii) $$E[x_X] - E[x_Y] = k(\mathrm{V}[x_Y] - V[x_X]),$$

only when $k=0$. (As a simple illustration of this result, take $x_C = 0$ with probability 1 for abitrary x_A, x_B.) In other words, satisfaction of the independence axiom requires that the value of k varies according to the risk being considered in all cases of non-risk neutral behaviour.

It may be noted at this stage that the results obtained in our study were well within the limit required for positive marginal utility. By use of equation (1) above, it can be shown that the requirement $V[x] < 2/a^2$ is equivalent to $\pi < (V[x]/2)^{1/2}$. A comparison of the variance values in Figure 1 with the mean values of π reported in Table 3 confirms that, for all risks in Response Codes 1 and 2, this criterion was satisfied.

The risk coefficients ϕ, A and γ used in our study are derived in a very simple manner from the certainty equivalent expression (equation 1). Accordingly they reflect the underlying nonlinear indifference properties given above and because of this are not invariant to the size of the risky prospect that is being evaluated. Drynan seems to imply that there is something inherently wrong with risk coefficients that vary with the level of risk yet, as we have already established, such variation derives entirely from the fact that the ratio $u''(x^*)/u'(x^*)$ varies with the level of risk. To impose a fixed non-zero value on a risk coefficient over a wide range of risks is equivalent to assuming that $u''(x^*)/u'(x^*)$ is stationary and it is only in the unique circumstance where we are comparing risks with identical expected payoffs that such an assumption is justified.

Beyond the E-V Framework

The mathematical link between the expected utility hypothesis and the E-V model we employed is contained in equation (7) of our original article. The procedure used to obtain this equation involves the expansion of both $u(x)$ and $u(x^* - \pi)$, both at the point x^* and then equating $Eu(x)$ with $u(x^* - \pi)$. The resultant E-V expression is obtained by expanding $u(x)$ up to its second derivative and $u(x^* - \pi)$ up to only its first derivative. The limit on the latter expansion is based entirely on the sheer complexity of obtaining solutions for π in polynomials of degree $n = 2$ or greater.

We may, however, consider the prospects of expanding $u(x)$ beyond its second derivative. By taking $u(x)$ to its third derivative and maintaining the limited expansion on $u(x^* - \pi)$, we obtain

(4) $$x_o = x^* + \tfrac{1}{2} V[x]u''(x^*)/u'(x^*) + 1/6u'''(x^*)/u'(x^*).\ T[x]$$

where $T[x]$ = the third central moment of $u(x)$.

This expression for the certainty equivalent of x may be compared with equation (1) herein. Equation (4) has the advantage that, if $u'''(x)$ and $T[x]$ take non-zero values, then they will be reflected in the underlying properties of the indifference loci. Presumably, they would also enhance the accuracy with which the modelled preference structure approximates the true preference structure.

A second feature of equation (4) is that it contains two unknowns, these being the ratios $u''(x^*)/u'(x^*)$ and $u'''(x^*)/u'(x^*)$. In general, it will be possible to obtain these ratios through use of questioning procedures that have been outlined by Anderson et al. (1977, pp. 70-6). However, unlike the questionnaire used in our study, these procedures have been developed for the purpose of eliciting utility functions. While questions that check for inconsistency in response are included in the latter methods, the series of questions presented to subjects in our study did not include such a device (for reasons outlined in our original article). Moreover, estimation of utility functions from our data base would be limited by the availability of only four certainty equivalents for each response code. Consequently, we would not recommend that our data be used for the purpose of utility function estimation.

Drynan's desire to go beyond the E-V framework is to be applauded. In particular, the use of $r(X)$ to measure risk attitudes may embody an individual's attitude towards risky prospects with respect to third and higher derivatives of the utility function. However, Drynan suggests that a useful starting point is to assume a particular form of absolute risk aversion function, $r(X)$. Such a strategy would appear to undermine the usefulness of $r(X)$ as it imposes specific functional forms on an individual's attitude towards risk. Instead, it is proposed that it would be more appropriate to use statistical criteria to determine the form of the utility function and $r(X)$.

Comments on Results

Whilst we disagree with Drynan in regard to allegations concerning inconsistencies and undefined assumptions in our analysis, we are grateful that he has identified several errors in Table 3 of our article. First, he has identified a typographical error concerning the significance of the estimated values of γ and A for Risk 4, Response Code 1 and Risk 1, Response Code 2, respectively. Both of these estimated parameters are significantly different from zero at the 5 per cent level of significance. Second, the estimated values of ϕ, A and γ for Risk 4, Response Code 2 were reported incorrectly. The correct estimates of ϕ, A and γ are -0.13, -9.77×10^{-5} and -0.16, respectively. Third, the coefficients of variation (CVs) reported in Table 3 should, as Drynan suggests, be constant (apart from sign) for a given risk. However, because the computation routine used for the analysis does not generate coefficients of variation, these were calculated manually using rounded estimates of means and standard deviations obtained from the computer. Consequently, the calculated CVs differed marginally for a given risk but it was considered inappropriate to standardise these to a single value as the true value of the latter is unknown. An additional error in Table 3 was the reported CV of γ for Risk 1, Response Code 1. Its value is -15.24 rather than 15.24 as reported.

Regarding the purpose of examining the discrimination properties of the estimated risk coefficients between risks, Drynan has indicated that we have not been clear with respect to the usefulness of this exercise. Moreover, he has suggested that 'for an explicit risk attitude model, the less discrimination provided by the risk coefficient the better in that, the risk coefficient is less affected by the risk itself' (p. 74). Such an argument requires clarification.

The major reason for investigating the discrimination properties of estimated risk parameters between risks is to obtain some evidence concerning the suitability of a particular median value or some other measure of central tendency for programming applications over an array of risky prospects. If a user of a programming model, which incorporates risk through formulations such as our original equations (11), (12) or (15), wishes to employ one estimated median value of ϕ, A or γ

for all risks under examination, it is proposed that, whilst this may be an acceptable practice for some objective functions, it may be a poor assumption for others. Drynan has also suggested that: 'Variation in the distribution between risks (for an estimated risk coefficient) would imply that the risk coefficient for some (perhaps all), individuals was not constant, and that the corresponding model did not apply for them' (p. 74). This view seems to equate applicability of objective functions in programming models with absence of discrepancies between median values of estimated risk coefficients between risks.

It may be true that programming analysts wish to employ an objective function, the risk coefficient of which does not vary for different risky prospects, but variations in measures of central tendency between risks does not mean necessarily that a particular model is unsuitable or inapplicable to the population under study. Indeed, for a given set of risky prospects where an individual displays a variety of risk attitudes (because of differing $u''(x)^*/u'(x)^*$ and varying values of $V[x]$), the estimated range of median values for the estimated risk parameters may differ significantly. Such results are indicative of the caution required before using a single risk coefficient, rather than forming a basis for selection of an objective function of superior empirical applicability. Hence, selection will depend on a range of criteria (including ease of computation) that cannot be discussed here, but which have been described elsewhere by, amongst others, Anderson et al. (1977).

References

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