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AN APPROACH TO THE ECONOMETRIC ESTIMATION OF ATTITUDES TO RISK IN AGRICULTURE: COMMENT

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Bardsley and Harris (1987) test empirically the effects of changes in deterministic wealth and random income on the measure partial risk aversion. The paper, which is otherwise well written, failed to impose the relationship between the two effects and estimated them independently. Consequently, inconsistent estimates were derived for the elasticities of the measure of partial risk aversion with respect to wealth and to income. The purpose of this comment is to derive that relationship theoretically and to point out the resulting inconsistencies in the original paper.

The Relationships

Let the measures of absolute, relative, and partial risk aversion be denoted by A, R, and P, respectively. The measure of partial risk aversion is related to the measure of absolute risk aversion as follows:

$$P(W_0,\pi) = A(W_0 + \pi)\pi, \tag{1}$$

where W_0 denotes nonstochastic initial wealth and π is the level of random income. Differentiating both sides with respect to W_0 gives

$$\frac{\partial P}{\partial W_0} = A'\pi\,. \tag{2}$$

Hence, decreasing absolute risk aversion implies decreasing partial risk aversion with respect to initial wealth. The measure of partial risk aversion is also related to the measure of relative risk aversion. Its relationship is given by:

$$P(W_0, \pi) = R(W_0 + \pi)\frac{\pi}{W_0 + \pi}\,. \tag{3}$$

Differentiating both sides with respect to π yields

$$\frac{\partial P}{\partial \pi} = R'\frac{\pi}{W_0 + \pi} + R\frac{W_0}{(W_0 + \pi)^2}\,. \tag{4}$$

Hence, increasing relative risk aversion implies increasing partial risk aversion with respect to income. However, the opposite does not hold:

it is possible to have increasing partial risk aversion with respect to income and decreasing relative risk aversion, simultaneously.

Information concerning the behavior of the measures of absolute and relative risk aversion, when the latter is increasing, is sufficient to determine the behavior of the measure of partial risk aversion, but not vice-versa. Next, it will be shown that the elasticity of the measure of absolute risk aversion with respect to wealth (initial wealth plus income) contains information sufficient to determine the behavior of both measures of absolute and relative risk aversion. The elasticity of the measure of absolute risk aversion with respect to wealth is defined as

$$\varepsilon_W^A \equiv A' \frac{W}{A}. \tag{5}$$

Assuming risk aversion, this elasticity is negative (zero, positive) when the measure of absolute risk aversion is decreasing (constant, increasing). In addition, because $R' = A'W + A > 0$ if and only if $\varepsilon_W^A > -1$, the measure of relative risk aversion is increasing (decreasing) when the elasticity of absolute risk aversion is greater (smaller) than minus one. It follows that $0 > \varepsilon_W^A > -1$ is equivalent to decreasing absolute risk aversion and increasing relative risk aversion. Note that when $\varepsilon_W^A = -1$ the measure is relative risk aversion is constant.

It follows from the above analysis that the elasticity of the measure of absolute risk aversion with respect to wealth contains enough information to determine the behavior of all three measures of risk aversion. Moreover, the elasticities of the measure of partial risk aversion with respect to both income and initial wealth are functions of the elasticity of the measure of absolute risk aversion. Writing equation (2) in elasticity terms and using equations (1) and (5) gives:

$$\varepsilon_{W_0}^P = \frac{W_0}{P} A'\pi = \varepsilon_W^A \frac{W_o}{W} \tag{6}$$

that is, the elasticity of the measure of partial risk aversion with respect to initial wealth equals the elasticity of the measure of absolute risk aversion with respect to wealth multiplied by the share of non-stochastic wealth in total wealth. Similarly, writing equation (4) in elasticity terms and using (i) the relationship between R and A, namely $R = AW$, and (ii) equations (1) and (5) gives:

$$\varepsilon_\pi^P = \frac{\pi}{P}\left[R' \frac{\pi}{W} + R \frac{W_0}{W^2}\right] = \varepsilon_W^A \frac{\pi}{W} + 1 \tag{7}$$

that is, the elasticity of the measure of partial risk aversion with respect to income equals the elasticity of the measure of absolute risk aversion

with respect to wealth times the share of the random income in total wealth plus one.

Combining equations (6) and (7) gives:

$$\varepsilon_{\pi}^{P} = \varepsilon_{W_0}^{p} \frac{\pi}{W_0} + 1 \tag{8}$$

which is the explicit relationship between the two elasticities estimated by Bardsley and Harris.

The Inconsistencies

The analysis above indicates that when estimating the effects of changes in wealth and/or income on the measures of risk aversion, only one parameter (elasticity) should be estimated. Attempts to estimate more than one parameter without restricting the relationships between them may result in inconsistent estimates. This is the case in the Bardsley and Harris paper. They made independent estimates of the elasticities of the measure of partial risk aversion with respect to income and wealth in three different zones. The estimated elasticities were in the range of 0.129 to 0.194 with respect to income and –0.312 to –0.642 with respect to wealth. The relationship given in (8) was used to check the credibility of Bardsley and Harris' results for the three zones. In the pastoral zone, the ratio of random income and initial wealth at the median points is 0.0492. Using this, relationship (8) and their –0.471 estimate of the elasticity of partial risk aversion with respect to wealth yields 0.976 as a calculated value for the elasticity of partial risk aversion with respect to income. However, the estimated value for the same elasticity is 0.194. Similar calculation for the wheat-sheep zone, in which $\frac{\pi}{W_0} = 0.0636$ and $\varepsilon_{W_0}^{P} = -0.642$, yields 0.959 as a calculated value for ε_{π}^{P}, while the estimated value for the same elasticity is 0.182. In the high rainfall zone, $\frac{\pi}{W_0} = 0.0349$ and $\varepsilon_{W_0}^{P} = -0.312$ result in a calculated value for ε_{π}^{P} of 0.989. However, the estimated value for the same elasticity is 0.129.

Thus, an inconsistency exists in Bardsley and Harris' estimation of the elasticities of the measure of partial risk aversion with respect to wealth and to income. This inconsistency results from failing during the estimation to restrict the relationship between the two elasticities, as implied by the analysis presented above.

Concluding Comments

Many of the comparative static results concerning behavior under uncertainty are based on the properties of the absolute, relative and partial measures of risk aversion. In particular, the effects of changes

in initial wealth and/or random income on the three measures are of special importance. While economic theory provides some intuition on the direction of these effects, there is not much empirical evidence supporting that intuition. This note shows that a single parameter — the elasticity of the measure of absolute risk aversion with respect to wealth — contains information sufficient to determine the behavior of all the three measures with respect to changes in wealth and/or income. Therefore, attempts to make independent estimates, say, of changes in wealth and income on the measure of partial risk aversion may lead to inconsistent estimates.

Reference

Bardsley, P. and Harris, M. (1987), 'An Approach to the Econometric Estimation of Attitudes to Risk in Agriculture', *The Australian Journal of Agricultural Economics* 31, 112-126.