THE DEMAND FOR DOMESTICALLY-PRODUCED SAWN TIMBER: AN APPLICATION OF THE DIEWERT COST FUNCTION

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The Australian demand for domestically-produced sawn timber is investigated by considering its major use—as an input into residential construction. Using a cost function approach, a system of equations is derived expressing quantities demanded in terms of relative prices. Cross-price elasticities are estimated and the falling input-output ratio of timber in residential construction is analysed by decomposing the change in this ratio into price, outputs and taste/technology effects. A major finding is that, while substitution of timber for other inputs has been encouraged through relative price movements, this effect has been more than offset by taste and technology trends away from timber usage.

Introduction

Attempts to quantify the factors determining the Australian demand for domestically-produced sawn timber (hereafter demand for domestic timber) have led to somewhat contradictory results. Ferguson (1973, 1979) found this demand to be relatively price elastic, but the BAE (1977) found it to be relatively price inelastic. The results from earlier studies are also conflicting (e.g. Mead 1966 and McKillop 1967).

The major proportion (about 60 per cent) of the supply of domestic timber is used as an input for the residential construction industry and any analysis of the demand for domestic timber should take account explicitly of the derived nature of that demand. While Ferguson (1973) recognised this, he did not account adequately for the effects of substitutes and complements in his analysis.

In this paper an attempt is made to analyse the demand for domestic timber as one of several major inputs in residential construction. The production technology is specified by a cost function which is used (by Shephard's 1953 Duality Theorem) to derive a system of equations relating the demand for a particular input to the relative prices of other inputs. An advantage with this approach is that it enables the nature and strengths of substitution and complementarity to be identified. In principle, the own-price elasticities of the inputs can also be estimated, although for reasons that will be discussed subsequently, these estimates will be of somewhat dubious value.

The last two decades have seen very substantial growth in the residential construction industry and it might, therefore, be expected that consumption of domestically-produced timber would also have grown. In fact, as can be seen from Figure 1, consumption has remained relatively static. This may be due to one of various configurations of movements in the demand and supply functions for domestic timber. One possibility is that there has been negligible growth in the demand for domestic timber because of rapid increases in its relative price leading to substitution of other inputs for timber (see, for example, Hanson and Wilson 1960, Leslie 1963, Ferguson 1973 and Muncey 1978). One of the major aims in this study is to investigate the role that other input prices have played in the demand for domestic timber. The method employed is well suited to addressing this particular question. The relative strengths of substitution and complementarity, scale effects and changes in tastes and technology can be related mathematically to changes in relative prices, enabling movements of input-output ratios to be disaggregated into identifiable components.

Theoretical Considerations

One approach to the quantification of derived demand is to postulate a production function which is specified up to a set of unknown parameters and then estimated. The quantities of inputs required to produce a given level of output with minimum cost (prices of inputs being predetermined) are obtained by minimising the cost function, subject to the production function constraint. That is:

(1) $y = f(x_1, x_2, \ldots, x_m)$ is a production function,
where y = output; and
x_i = input quantities.

The cost function C is given by:

(2) $$C = \sum_{i=1}^{m} p_i x_i,$$

where p_i = predetermined input prices.

A Lagrangian constrained minimisation of the cost function produces $m+1$ equations of the form:

(3) $$(\partial f/\partial x_i)/p_i = \lambda^{-1}, \text{ for } i = 1, 2, \ldots, m; \text{ and}$$
$$f(x_1, x_2, \ldots, x_m) - y = 0.$$

This set of equations is solved simultaneously to obtain the optimal 'bundle' of inputs, x_i^*, in the form:

(4) $$x_i^* = x_i^*(p_1, p_2, \ldots, p_m; y), \text{ for } i = 1, 2, \ldots, m,$$

where x_i^* = derived demand for input i.
This simple formulation disguises the fact that equation (3) is likely to be highly nonlinear, and the solution very complicated algebraically.

An alternative approach to obtaining the derived demand is to use the Shephard Duality Theorem. Shephard (1953, 1970) has shown that the production function and minimum cost function are two equivalent descriptions of the production process. In particular, suppose some function $C(p;y)$ satisfies the following conditions[1]:

(a) $C(p;y)$ is a positive real valued function for all positive p and y;
(b) $C(p;y)$ is a non-decreasing function in p and y;
(c) $C(p;y)$ is linear homogeneous in p; and
(d) $C(p;y)$ is a concave function in p.

[1] For convenience we will use p to represent the price vector $p_1, p_2, \ldots, p_m$.

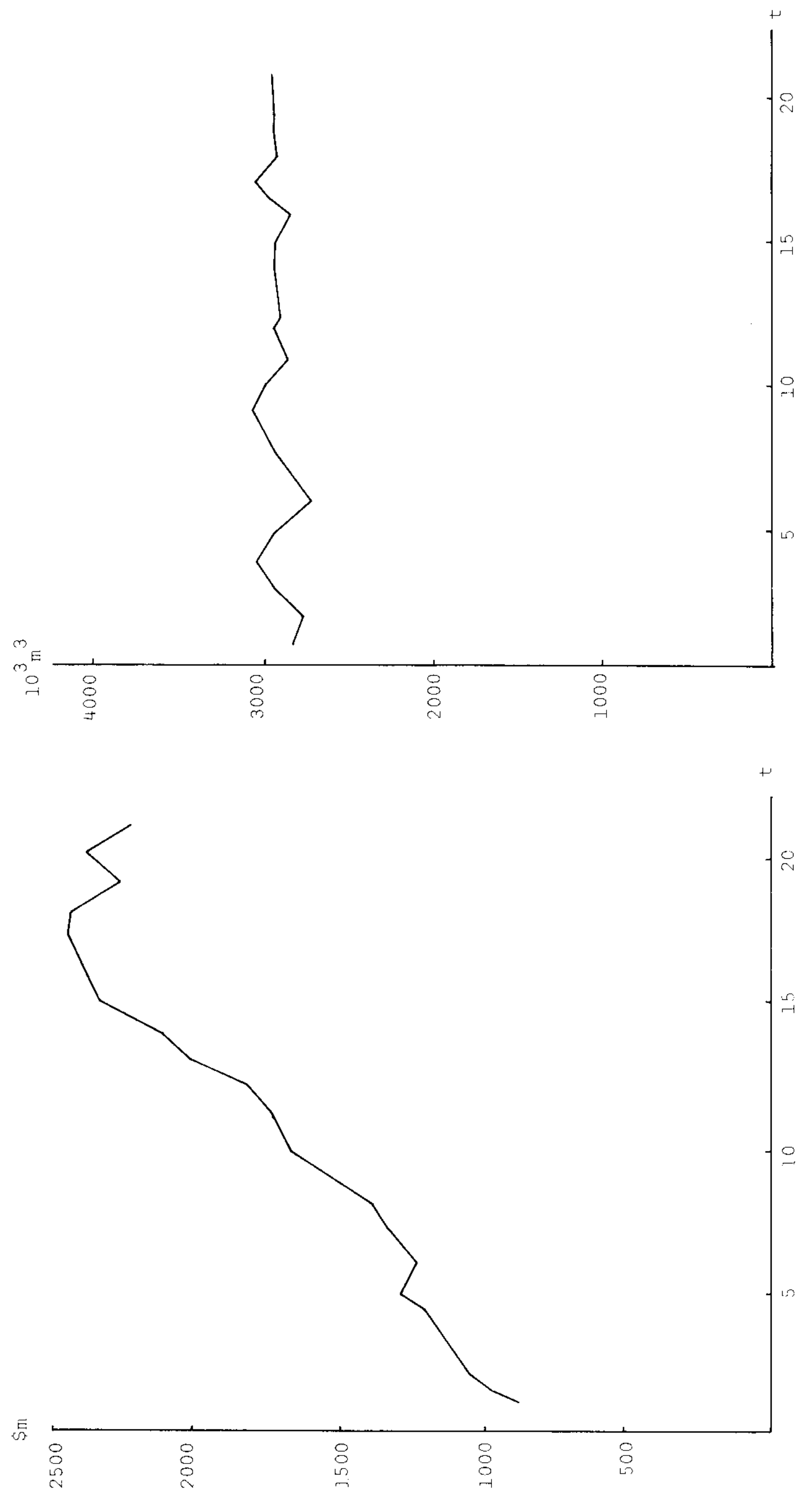

FIGURE 1—Output of house building and consumption of sawn timber, 1956-57 through 1976-77.

Then $C(p;y)$ is a minimum cost function obtainable from some feasible production function. Furthermore, the cost-minimising bundle of inputs is obtained by partial differentiation with respect to price. That is:

$$x_i^* = \partial C/\partial p_i \text{ for } i = 1, 2, \ldots, m. \tag{5}$$

Thus, if the starting point is a cost function satisfying the above conditions, the optimal inputs are obtained simply by partial differentiation, and the Lagrange minimisation and subsequent simultaneous equation solution are avoided.

Application of this theory requires that a cost function be postulated, and we have adopted an elaboration of the Diewert (1971) function which is a quadratic in the square root of prices and directly proportional to output. That is:

$$C(p;y) = y \sum_{i=1}^{m} \sum_{j=1}^{m} \beta_{ij}(p_i p_j)^{0.5}, \tag{6}$$

with $\beta_{ij} = \beta_{ji}$.[2]

With this cost function, the demand for input i is given by:

$$x_i^* = \partial C/\partial p_i = y \sum_{j=1}^{m} \beta_{ij}(p_j/p_i)^{0.5}; \tag{7}$$

and the input-output ratio, $a_i = x_i^*/y$, by:

$$a_i = \sum_{j=1}^{m} \beta_{ij}(p_j/p_i)^{0.5}. \tag{8}$$

Note that formulation (6) of the cost function has the following implications:

(a) $C(p;y)$ is linear homogeneous in prices, as required in condition (c) above;
(b) the input-output ratio a_i is a function of relative prices, and thus incorporates explicitly substitutability of factors;
(c) the input-output ratio is independent of the level of output, implying constant returns to scale; and
(d) x_i^* and a_i are linear functions of the unknown parameters β_{ij}, and can, therefore, be estimated using linear regression techniques.

Following Parks (1971) and Woodland (1975), equation (8) can be made more general by including two extra terms, to give a_i in the form:

$$a_i = \sum_{j=1}^{m} \beta_{ij}(p_j/p_i)^{0.5} + \beta_{i,m+1} y + \beta_{i,m+2} t. \tag{9}$$

The input-output ratio now depends on the level of output y and non-constant returns to scale will hold unless $\beta_{i,m+1} = 0$. The second additional term involving time (t) may be used as a proxy for changes in technology and taste.

Cross-price and own-price elasticities of demand may be calculated from equation (9) as:

$$\epsilon_{ij} = (\partial x_i/\partial p_j)(p_j/x_i) = \beta_{ij}(p_j/p_i)^{0.5}/2a_i, \tag{10}$$

and

[2] This constraint on the β's is necessary to ensure that:

$$\partial^2 C/\partial p_i \partial p_j = \partial^2 C/\partial p_j \partial p_i.$$

(11) $$\epsilon_{ii} = (\partial x_i / \partial p_i)(p_i / x_i) = -\sum_{j \neq i} \epsilon_{ij}.$$

Provided it can be assumed that the proportion of total usage in each end-use category is constant over time, there is no need to disaggregate total usage into its various components. In equation (10), both β_{ij} and a_i would be multiplied by the same constant, leaving ϵ_{ij} unchanged. This justifies our use of total domestic timber production, rather than total domestic timber used in residential construction, as a dependent variable.

Estimation of the parameters of equation (9) requires data on the *prices* of the inputs. In many applications, however, only *price indexes* are available. In order to modify the theory to include this situation, it is assumed that the actual price of any input and the associated price index are directly proportional. That is:

(12) $$p_i^* = p_i / \alpha_i,$$

where p_i^* is the price index corresponding to the actual price p_i; and α_i is the constant of proportionality.

Substituting equation (12) into equation (9), we obtain:

$$a_i = \sum_{j=1}^{m} \beta_{ij} \alpha_j^{0.5} \alpha_i^{-0.5} (p_j^* / p_i^*)^{0.5} + \beta_{i,m+1} y + \beta_{i,m+2} t,$$

and on multiplying through by α_i,

(13) $$\alpha_i a_i = \sum_{j=1}^{m} \beta_{ij}^* (p_j^* / p_i^*)^{0.5} + \beta_{i,m+1}^* y + \beta_{i,m+2}^* t,$$

where $\beta_{ij}^* = \alpha_i^{0.5} \alpha_j^{0.5} \beta_{ij}$;

(14) $$\beta_{i,m+1}^* = \alpha_i \beta_{i,m+1}; \text{ and}$$
$$\beta_{i,m+2}^* = \alpha_i \beta_{i,m+2}.$$

Thus, when price indexes rather than actual prices are available, the appropriate system is equation (13), which is equivalent to equation (9) if the dependent variables a_i are weighted by the factors α_i.

In most applications, inputs are aggregated into groups and the α_i must be interpreted with this in mind. From equation (12),

(15) $$\alpha_i = p_i / p_i^* = p_i x_i / p_i^* x_i = V_i / p_i^* x_i,$$

where V_i is the monetary value of the i-th input group.

Equation (15) is taken as the definition of α_i and it is assumed to be approximately constant through time. From equation (14), it is clear that as $\beta_{ij} = \beta_{ji}$, then $\beta_{ij}^* = \beta_{ji}^*$. Also,

(16) $$[\beta_{ij}^* (p_j^* / p_i^*)^{0.5}] / (\alpha_i a_i) = [\beta_{ij} (p_j / p_i)^{0.5}] / a_i,$$

and, by equation (10), the elasticities will be the same whether computed from equations (9) or (13).

Estimation

The model

It would be possible to estimate equation (9) separately for each input group. However, there are gains in efficiency from estimating the equations jointly as a system. These gains come from two sources:

(a) the across-equation restrictions $\beta_{ij} = \beta_{ji}$ can be incorporated explicitly, reducing substantially the number of parameters to be estimated (e.g. with six inputs there is a reduction from 48 to 33 parameters); and
(b) estimation as a system allows advantage to be taken of any correlations that may exist between the disturbances of different equations.

The system of equations (9) was estimated using time-series data, assuming six ($m = 6$) major input factors for the residential construction industry. The prices p_i will refer to price indexes and, following the discussion in the previous section, the input-output ratios will be defined by $a_i = \alpha_i x_i^*/y$. The index i identifies the inputs as follows:

$i = 1$: domestically-produced sawn timber;
$i = 2$: imported sawn timber;
$i = 3$: bricks and tiles;
$i = 4$: cement products;
$i = 5$: plaster products; and
$i = 6$: labour.

In order to estimate the system using T observations and taking account of the linear constraints $\beta_{ji} = \beta_{ij}$, the following vectors are defined:

ι is a T-vector of ones;

$$\tau = [1, 2, \ldots, T]';$$
$$a_i = [a_{i1}, a_{i2}, \ldots, a_{iT}]';$$
$$a = [a'_1, a'_2, \ldots, a'_6]';$$
$$y = [y_1, y_2, \ldots, y_T]';$$
$$\beta_i = [\beta_{ii}, \beta_{i,i+1}, \ldots, \beta_{i6}, \beta_{i7}, \beta_{i8}]';$$
$$\beta = [\beta'_1, \beta'_2, \ldots, \beta'_6]'; \text{ and}$$
$$X_{ij} = [(p_{i1}/p_{j1})^{0.5}, (p_{i2}/p_{j2})^{0.5}, \ldots, (p_{iT}/p_{jT})^{0.5}]'.$$

Matrices X_1, X_2, X_3, X_4 and X_5 are defined (using zero to denote the null vector of length T) as follows:

$$X_1 = \begin{bmatrix} \iota & X_{21} & X_{31} & X_{41} & X_{51} & X_{61} & y & \tau \\ 0 & X_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_{13} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_{14} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_{15} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & X_{16} & 0 & 0 \end{bmatrix},$$

$$X_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \iota & X_{32} & X_{42} & X_{52} & X_{62} & y & \tau \\ 0 & X_{23} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & X_{24} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & X_{25} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_{26} & 0 & 0 \end{bmatrix},$$

$$X_3 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \iota & X_{43} & X_{53} & X_{63} & y & \tau \\ 0 & X_{34} & 0 & 0 & 0 & 0 \\ 0 & 0 & X_{35} & 0 & 0 & 0 \\ 0 & 0 & 0 & X_{36} & 0 & 0 \end{bmatrix}, \ldots, X_6 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \iota & y & \tau \end{bmatrix}$$

Introducing a disturbance vector u given by:

$u = [u'_1, u'_2, \ldots, u'_6]'$,

where $u_i = [u_{i1}, u_{i2}, \ldots, u_{iT}]$,

the system of equations (9) can be written in the form:

(17) $$a = X\beta + u,$$

where $X = [X_1, X_2, X_3, X_4, X_5, X_6]$.

In order to allow for the possibility that disturbances may be correlated across equations and autocorrelated, it was postulated initially that u was generated by a vector first-order autoregressive process. That is:

(18) $$u_{it} = \sum_{j=1}^{6} \rho_{ij} u_{j,t-1} + \epsilon_{it} \text{ for } i = 1, 2, \ldots, 6 \text{ and } t = 2, 3, \ldots, T,$$

where $E(\epsilon_{it}) = 0$;

$E(\epsilon_{it}\epsilon_{js}) = \sigma_{ij}\delta_{ts}$, with $\sigma_{ij} = \sigma_{ji}$; and

δ denotes the Kronecker delta function.

The quite general model (equation 18) includes three simpler models as special cases:

(a) if $\beta_{ij} = 0$ when $j \neq i$, then autocorrelation is confined to within equations, but there may be contemporaneous correlation across equations;
(b) if $\beta_{ij} = 0$ for all i and j, there is contemporaneous correlation across equations, but no autocorrelation; and
(c) if $\beta_{ij} = 0$ for all i and j and, in addition, $\sigma_{ij} = k_i\delta_{ij}$, the disturbances are not correlated across equations.

Following other authors (see, for example, Kmenta and Gilbert 1970) we will assume (a) to be true and use the data to determine whether the model can be further specialised to (b) or (c).

Data requirements and problems

It is clear from equations (9) and the following discussion, that estimation of the unknown parameters, β_{ij}, requires the following data in each time period:

(a) quantities consumed of all inputs (x_i^*) and quantity of output (y);
(b) price indexes (p_i) for all inputs; and
(c) the constant weighting factors α_i.

The quantity data were available for the 21 years from 1956-57 through 1976-77. However, price data could only be obtained for the period

1966-67 through 1976-77, and the missing observations were predicted using the 'related series' approach originally proposed by Friedman (1962).

It was found that, over the period for which price data were available, the price indexes were very highly correlated with the wholesale price index of materials used in house building (WPI)—a series available over the whole 21-year period (ABS 1982). For each price index, a regression relationship relating it to the WPI was estimated and used to predict values over the period 1956-57 through 1965-66.

A different kind of problem arose in connection with the plaster products quantity series. Prior to 1965-66, the data were for fibrous plaster sheets only. The discontinuity in this series is quite apparent, as shown in Figure 2. In order to partially overcome this deficiency, a log-linear model was fitted to the plaster series over the period 1966-67 through 1976-77, which was then used to extrapolate prior to 1965-66, as shown in Figure 2.

Using these two data constructions, 21 annual observations were available for estimation. These observations are tabulated in Table A.1 of the Appendix. When the system was constructed to incorporate the 15 constraints $\beta_{ij}=\beta_{ji}$ $(j \neq i)$, there were 126 observations available to estimate the 33 parameters β_{ij} $(i, j = 1, 2, \ldots, 6; j \geq i)$ and β_{i7}, β_{i8} $(i = 1, 2, \ldots, 6)$.

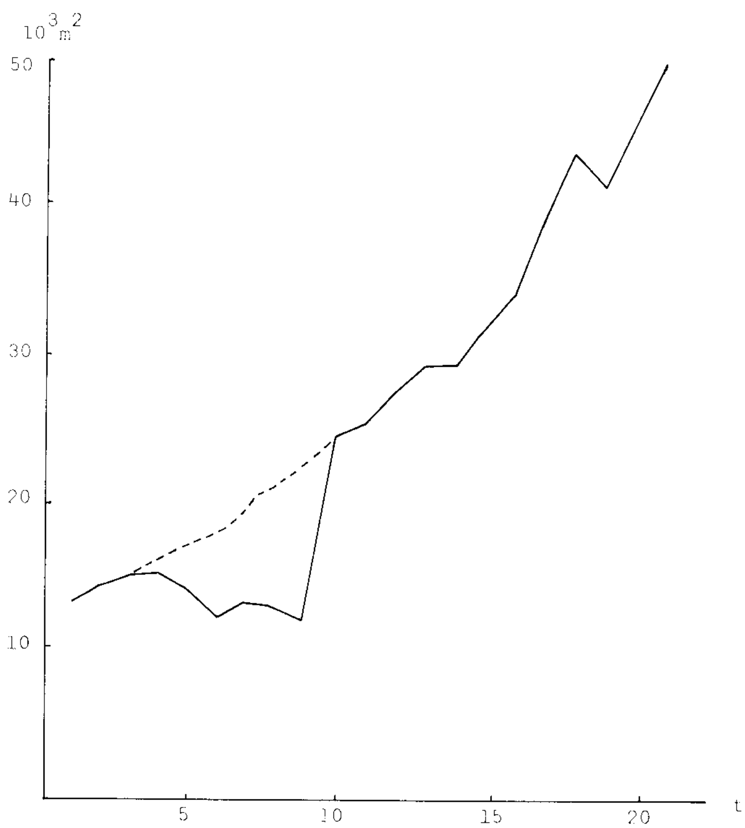

FIGURE 2—Quantity of plaster products, 1956-57 through 1976-77 ($t = 1, 21$).

From equation (15), the weighting factors α_i are defined by $\alpha_i = V_i/p_i^* x_i$. Observations on V_i were not available for every time period. Using the available observations the computed values of α_i showed only small variation, confirming the reasonableness of using weighting factors which are constant over time. The mean value was taken as the weighting factor, with the following results:

$$\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_6) = (0.51, 0.38, 0.50, 0.18, 7.9, 0.035).$$

Estimation results

As a first step, the system was estimated using OLS. The results of this regression are shown in Table 1. The residuals were examined to find an appropriate model for the disturbances within the framework of equation (18). Following Parks (1967), the residuals $\hat{u}_{it}$ were regressed on $\hat{u}_{i,\,t-1}$ ($i = 1, 2, \ldots, 6$; $t = 2, 3, \ldots, 21$) to estimate the parameters ρ_{ii}. The results are shown in Table 2. On the basis of these estimates, it appears that there is no significant autocorrelation present. That is, we may assume $\rho_{ij} = 0$ for all i and j.

The contemporaneous correlations between residuals from different equations are shown in Table 3. These highly significant correlations give a clear indication that estimation by OLS would be very inefficient. The appropriate estimation technique is Zellner's (1962) Seemingly Unrelated Regressions (ZSUR). We therefore estimate β, defined in equation (17) by:

$$\hat{\beta} = [X'(\hat{\Sigma}^{-1} \otimes I_T)X]^{-1} X'(\hat{\Sigma}^{-1} \otimes I_T)a, \tag{19}$$

where $(\hat{\Sigma})_{ij} = \hat{\sigma}_{ij}$;

I_T is the unit matrix of dimension T; and

$\otimes$ denotes the Kronecker product.

The results of the ZSUR estimation are also shown in Table 1. The comparison of the estimated standard errors from OLS and ZSUR estimation demonstrates the considerable gains in efficiency obtained by utilising the across-equations residual correlations.

Discussion of Results

The economic implications of the estimated model will now be discussed under four headings, taking as our estimated demand equation for domestic timber (with standard errors in parentheses):

$$\begin{aligned} a_{1t} = \underset{(47.1)}{336} &- \underset{(18.0)}{54.2}(p_{2t}/p_{1t})^{0.5} - \underset{(36.7)}{85.5}(p_{3t}/p_{1t})^{0.5} + \underset{(24.3)}{23.3}(p_{4t}/p_{1t})^{0.5} \\ &- \underset{(34.6)}{185}(p_{5t}/p_{1t}) + \underset{(36.2)}{174}(p_{6t}/p_{1t}) - \underset{(0.771)}{0.591}y_t - \underset{(1.05)}{8.58}t. \end{aligned} \tag{21}$$

Returns to scale

Referring to Table 1, it can be seen that for each of the six equations the coefficient of output $y_t(\beta_{i7})$ is not significantly different from zero. This suggests that the scale effect is very weak and constant returns to scale may exist in the building industry.

TABLE 1

Parameter Estimates of Equation System (9) with Linear Constraints[a]

Input	1. Domestic timber	2. Imported timber	3. Bricks	4. Cement	5. Plaster	6. Labour	7. Output	8. Time
Ordinary Least Squares (OLS)								
1. Domestic timber	392**	−73.4	−85.8	33.7	−279*	245**	−1.38	−9.50**
	(184)	(57.2)	(221)	(152)	(119)	(77)	(1.57)	(2.28)
2. Imported timber		71.3	17.8	−21.5	40.6	−6.1	−0.82	0.47
		(42.9)	(58.5)	(55.0)	(49.5)	(43.4)	(1.37)	(1.48)
3. Bricks			27.7	−37.5	70.4	59.3	−1.18	0.84
			(498)	(241)	(201)	(82.6)	(1.69)	(2.47)
4. Cement				−53.3	61.3	68.2	−0.73	0.14
				(229)	(129)	(80.2)	(1.62)	(2.31)
5. Plaster					40.7	79.6	−0.68	0.81
					(126)	(67.3)	(1.71)	(2.40)
6. Labour						−29.4	−4.79**	−1.10
						(93.9)	(1.44)	(2.02)
Zellner's Seemingly Unrelated Regressions (ZSUR)								
1. Domestic timber	336**	−54.2**	−85.5**	23.3	−185**	174**	−0.60	−8.58**
	(47)	(18.0)	(36.7)	(24.3)	(35)	(36)	(0.77)	(1.05)
2. Imported timber		64.0**	23.2**	−9.34	5.33	−8.21	0.50	−0.67**
		(9.9)	(8.4)	(8.17)	(14.7)	(7.11)	(0.46)	(0.26)
3. Bricks			55.8	−19.8	61.9	4.36	−0.28	0.69**
			(74.6)	(32.3)	(36.4)	(7.91)	(0.49)	(0.34)
4. Cement				−47.5	78.0**	13.9	−0.25	0.53
				(27.8)	(20.3)	(16.0)	(0.31)	(0.42)
5. Plaster					−16.7	60.4*	0.90	0.76
					(33.6)	(27.4)	(0.67)	(0.92)
6. Labour						165**	−1.37	−6.10*
						(62)	(2.56)	(2.38)

* significant at the 5 per cent level.
** significant at the 1 per cent level.
[a] $\beta_{ij} = \beta_{ji} (i, j = 1, 2, \ldots, 6)$

TABLE 2

Estimation of the Parameters ρ_{ii}

i	$\hat{\rho}_{ii}$	Standard error	t-ratio
1	−0.03	0.18	−0.16
2	0.19	0.17	1.07
3	0.20	0.20	0.97
4	0.20	0.20	1.04
5	0.37	0.20	1.85
6	−0.29	0.21	−1.40

Price effects: substitution and complementarity

The signs of the estimated parameters are taken to indicate substitutes and complements. When $\beta_{ij} > 0$, inputs i and j are regarded as substitutes and when $\beta_{ij} < 0$ they are regarded as complements. These relationships are summarised in Table 4.

In relation to the demand for domestic timber, two features are worthy of note. First, apart from the relationship between domestic timber and cement products, the substitution/complementarity relationships appear to be clearly defined. Second, domestically-produced timber and imported timber appear to be complements. This is probably because of physical differences between the two inputs. Domestic timber is predominantly broad-leaved (hardwood) whereas imported timber is basically coniferous (softwood).

The strengths of the substitution/complementarity effects can be measured by the cross-price elasticities, calculated from equation (10). We will be concerned only with domestic timber relationships. The elasticities will vary with time and have been tabulated in Table A.2 in the Appendix. We reproduce in Table 5 results for three years from the sample period to highlight the main features.

As the $\hat{\beta}_{ij}$ are (with the exception of the cement coefficient $\hat{\beta}_{14}$) significant at the one per cent level, we can infer[3] that the corresponding elasticities are also significant at (approximately) this level. Clearly, the strength of complementarity between domestic timber and plaster and that of substitution between domestic timber and labour are the dominant price effects. As discussed later, this feature is important in analysing reasons for the static consumption of domestic timber.

Own-price elasticity of domestic timber

The own-price elasticity of an input has been defined in equation (11) as:

$$\epsilon_{ii} = -\sum_{j \neq i} \epsilon_{ij}.$$

[3] For fixed prices, var (ϵ_{ijt}) is proportional to var (β_{ij}/a_{it}). Using an asymptotic approximation (see Theil 1971, p. 374) it can be shown that:

$$\text{var}(\hat{\beta}_{ij}/a_{it}) \approx [\text{var}(\hat{\beta}_{ij})/a^2{}_{it}][1\text{-}2\rho h\hat{\beta}_{ij}/a_it + h^2\hat{\beta}^2{}_{ij}/a^2_{it}],$$

where $h = [\text{var}(a_{it})/\text{var}(\hat{\beta}_{ij})]^{0.5}$; and

ρ is the correlation between $\hat{\beta}_{ij}$ and a_{it}.

Estimates of h show it to be of smaller order than one, and hence:

$$\text{var}(\hat{\beta}_{ij}/a_{it}) \approx [\text{var}(\hat{\beta}_{ij})]/a^2_{it}.$$

TABLE 3

Contemporaneous Correlations Between Residuals of Equations i and *j* ($i, j = 1, 2, \ldots, 6$)

i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$	$j=6$
1	1.00					
2	0.76**	1.00				
3	0.79**	0.74**	1.00			
4	0.80**	0.67**	0.90**	1.00		
5	0.46*	0.49*	0.75**	0.68**	1.00	
6	0.46*	0.33	0.32	0.26	0.30	1.00

* significant at the 5 per cent level.
** significant at the 1 per cent level.

Using this formula for the three years shown in Table 5, we computed the own-price elasticity of demand for domestic timber to be 0.41, 0.65 and 0.48 for the years 1956-57, 1966-67 and 1976-77, respectively. For each of these years, and indeed for all years of the sample period, these elasticities are *positive*—a result which absolutely conflicts with previous studies and *a priori* expectations. We believe that the reason for this is inherent in the method of computing elasticities with models of the type expressed by equation (9). The own-price elasticity is calculated as the negative sum of the cross elasticities with respect to all other inputs. Thus, if variables are omitted from the model, the own-price elasticity will be biased. If those variables are substitutes ($\epsilon_{ij} > 0$) the own-price elasticity would be *positively* biased. In our case, two inputs have been omitted due to lack of data. These two inputs—steel products and wood panel products—would be expected to be substitutes and, if included, would reduce the algebraic value of the own-price elasticity of demand for domestic timber.

It is legitimate to enquire whether the omission of these variables would bias the $\hat{\beta}_{ij}$ to such an extent that the cross-price elasticities would also be unreliable. The correlations[4] between the regressor variables corresponding to the price coefficients $\beta_{12}, \beta_{13} \ldots, \beta_{16}$ were small in every case (of the order of 0.25). Therefore, it seems reasonable to assume that correlations between omitted price variables and included price variables would also be low. Any bias in the estimated price coefficients $\hat{\beta}_{ij}$ (due to the omission of relevant variables) should thus be negligible, and the cross-price elasticities (where they are statistically significant) should be reliable.

In an attempt to overcome this problem with computing own-price elasticities, we searched for proxy variables to represent the missing variables. It was found that each of the variables $p_i^{0.5}$ ($i = 1, 2, \ldots, 6$) could be represented quite adequately by a cubic equation in t. The $\bar{R}^2$s varied from 0.96 to 0.99. Assuming, then, that a cubic equation in t would also be an appropriate representation of the missing variables, we were led to the following equation:

[4] The correlations referred to here are *not* the correlations between relative prices (square-rooted), for example between $(p_2/p_1)^{0.5}$ and $(p_3/p_1)^{0.5}$, but rather between columns of the matrix X defined earlier.

TABLE 4

Substitution (S) and Complementary (C) Relationships Among Inputs

Input	Domestic timber	Imported timber	Bricks	Cement	Plaster	Labour
Domestic timber	X	C**	C**	?	C**	S**
Imported timber		X	S**	?	?	?
Bricks			X	?	S*	?
Cement				X	S**	?
Plaster					X	S**
Labour						X

? not statistically significant.
* significant at the 5 per cent level.
** significant at the 1 per cent level.

$$(22) \qquad a_{it} = \beta_{ii} + p_i^{-0.5}[\gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3] + \beta_{i7} y_t + \beta_{i8} t.$$

The own-price elasticities can be calculated directly from this equation as:

$$\epsilon_{ii} = (p_i/x_i)(\partial x_i/\partial p_i),$$
$$= (p_i/a_i)(\partial a_i/\partial p_i),$$

that is:

$$(23) \qquad \epsilon_{ii} = [-p_i^{0.5}/2a_i][\gamma_0 + \gamma_1 t + \gamma_2 t^2 + \gamma_3 t^3].$$

For the domestic timber input:

$$\underset{(52.4)}{a_{1t} = 382} + p_{1t}^{0.5}\ [\underset{(493)}{-1450} + \underset{(48.7)}{60.2t} - \underset{(2.7)}{0.79t^2} + \underset{(0.11)}{0.17t^3}]$$
$$-\underset{(0.91)}{4.29y_t} - \underset{(3.73)}{13.0t}.$$
$$\bar{R}^2 = 0.9911.$$

Using these parameter estimates with equation (23), the own-price elasticities for domestic timber were computed and are tabulated in Table A.3 in the Appendix. The elasticities for the years 1956-57, 1966-67 and 1976-77 were 0.40, 0.34 and −0.40, respectively. These results, though a slight improvement on the earlier figures, are (apart from the last few years) still positive and remain a disappointing aspect of the study.

TABLE 5

Cross-Price Elasticities of Domestic Timber with Other Inputs

Year	Imported timber	Bricks	Cement	Plaster	Labour
1956-57	−0.15	−0.23	0.06	−0.51	0.42
1966-67	−0.28	−0.44	0.12	−0.95	0.90
1976-77	−0.38	−0.51	0.14	−0.97	1.25

In order to arrive at some general assessment of the own-price elasticity of demand for domestic timber, we can use some general principles of derived demand (see, for example, Lipsey et al. 1981, p. 353). Timber is highly substitutable with other inputs and forms a very substantial proportion of the total cost of residential building.[5] These two facts lead us to believe that the demand for timber is relatively price elastic, as maintained by Ferguson (1973, 1979). However, against this it must be recognised that demand for the end product, housing, appears to be inelastic (see Powell 1966 and Al-Tayeb 1982).

If we accept Ferguson's (1979) estimate of the own-price elasticity of demand for sawn timber of about -1.5, the estimated cross-price elasticities imply that the sum of the cross-price elasticities of the omitted inputs (steel products and wood panel products) is about 1.8. In view of the magnitudes of the estimated cross-price elasticities, together with the likelihood that the omitted inputs are, in the main, substitutes, a sum of 1.8 seems plausible. Thus, our estimates of the cross-price elasticities seem to be consistent with a relatively price-elastic demand for timber.

Time-path of the input-output ratio

A major concern of the timber industry is that, over a period in which there has been great expansion in residential building, the consumption of timber has remained static. That is, the input-output ratio, a_1, has declined steadily over time. Equation (9) can be used to analyse this decline. Differentiating equation (9) with respect to time, and using the definition of cross-price elasticities in equation (10), we obtain:

$$(25) \quad (da_{it}/dt)/a_{it} = \sum_{j \neq i} \epsilon_{ijt} d[\ln(p_{jt}/p_{it})]/dt + \beta_{i,m+1}(dy_t/dt)/a_{it} + \beta_{i,m+2}/a_{it}.$$

The time-rate of change of the input-output ratio (expressed as a proportion of that ratio) is thus given as the sum of three components:

(a) $\Sigma_{j \neq i} \epsilon_{ijt} d/dt \ln[p_{jt}/p_{it}]$ – a term which we call the 'total price effect' (the individual terms in this summation are essentially the product of a cross-price elasticity and a relative price movement; e.g. if input j is a substitute for timber ($\epsilon_{ijt} > 0$) and if the price of timber is rising relative to the price of this input ($d[\ln(p_{jt}/p_{it})]/dt < 0$) the partial effect from input j is negative, indicating that over time it will tend to replace timber);

(b) $\beta_{i,m+1}(dy_t/dt)/a_{it}$ – an output effect, related to returns to scale; and

(c) $\beta_{i,m+2}(1/a_{it})$ – a time effect, which can be interpreted in terms of technological and taste changes.

The signs and relative magnitudes of these three components could indicate the reason for the falling input-output ratio.

Quantification of the price effect would be greatly facilitated if it could be assumed that $d[\ln(p_{jt}/p_{it})]/dt$ is constant over time. This is equivalent to the assumption that $\ln[p_{jt}/p_{it}]$ is linearly related to time. Carrying out regressions of the form:

$$(26) \quad \ln[p_{jt}/p_{it}] = \beta_0 + \beta_1 t,$$

[5] The Australian Bureau of Statistics uses the percentage weighting 36.16 for the input group 'timber, board and joinery' when constructing the price index of materials used in house building (ABS 1982).

we obtained:

$$\ln(p_{2t}/p_{1t}) = \underset{(0.034)}{0.083} + \underset{(0.0027)}{0.0021}\, t \quad (R^2 = 0.03);$$

$$\ln(p_{3t}/p_{1t}) = \underset{(0.013)}{0.077} - \underset{(0.0010)}{0.0081}\, t \quad (R^2 = 0.75);$$

$$\ln(p_{4t}/p_{1t}) = \underset{(0.014)}{0.060} - \underset{(0.0011)}{0.0067}\, t \quad (R^2 = 0.65);$$

$$\ln(p_{5t}/p_{1t}) = \underset{(0.28)}{0.21} - \underset{(0.002)}{0.023}\, t \quad (R^2 = 0.85); \text{ and}$$

$$\ln(p_{6t}/p_{1t}) = \underset{(0.02)}{-0.17} + \underset{(0.002)}{0.017}\, t \quad (R_2 = 0.80).$$

Thus, apart from the case $j = 2$ (in which $\ln(p_{jt}/p_{it})$ does not have any trend), models of the form of equation (26) appear to be quite satisfactory. We therefore estimate the terms $d[\ln(p_{jt}/p_{it})]/dt$ for $i = 1$ and $j = 2, 3, \ldots, 6$ by zero, -0.0081, -0.0067, -0.023 and 0.017, respectively.

Finally, if y_t is approximately linearly related to time, then dy_t/dt could also be taken to be constant. Regressing y_t on t, we obtained:

$$y_t = \underset{(0.66)}{8.64} + \underset{(0.05)}{0.799}\, t \quad (\mathrm{R}^2 = 0.92),$$

yielding an estimate of dy/dt of 0.799. Using the cross-price elasticity estimates already calculated (see Tables 5 and A.2), we can now quantify the different components. We demonstrate using 1966-67 values, for which $a_1 = \alpha_1 x_1^*/y = 97$.

Price Effect:

Imported timber	$-0.28 \times 0.00 = 0.00$	
Bricks	$-0.44 \times -0.0081 = 3.6 \times 10^{-3}$	
Cement	$0.12 \times -0.0067 = -0.80 \times 10^{-3}$	
Plaster	$-0.95 \times -0.0233 = 22 \times 10^{-3}$	
Labour	$0.89 \times 0.0170 = 15 \times 10^{-3}$	
Total price effect	$= 40 \times 10^{-3}$.	

Output Effect: $(-0.59 \times 0.799/97) = -5 \times 10^{-3}$.

Time Effect: $(-0.08578/0.9921) = -88 \times 10^{-3}$. Thus, for 1966-67:

$$(da_1/dt)/a_1 = (40 - 5 - 88) \times 10^{-3} = -53 \times 10^{-3}.$$

This disaggregation of $(da_1/dt)/a_1$ into components reveals some interesting features. First, the overall price effect is strongly positive, indicating that substitution and complementarity have worked in favour of domestic timber over the sample period. This is, in turn, clearly due to the strong complementarity with plaster products, whose price has steadily declined relative to timber, and the strong substitution with labour whose price has risen relative to timber. Second, the output (or scale) effect is very weak, as would be expected if constant returns to scale hold. Third, the dominant negative effect (working against consumption of timber) arises from the taste/technology component. This is strong enough to outweigh the price effect. We surmise that it may be due to an increasing proportion of flat construction in the building industry. The results of similar calculations for years 1956-57 and 1976-77 show exactly the same features. Together with the above figures, these results are shown in Table 6. As an interesting confirmation of the model we note that if da_1/dt is estimated (by multiplying the first and last columns in Table 6), we obtain -5.4, -5.2 and -5.3 for the three years 1956-57, 1966-67 and 1976-77, respectively. These estimates of da_1/dt vary so little across 20 years that a model in which a_1 is linearly related to time should be appropriate. This regression yielded:

$$\hat{a}_{it} = \underset{(5.2)}{165} - \underset{(0.4)}{5.2}t \quad (R^2 = 0.89).$$

The estimated slope of -5.2 is quite consistent with those obtained from Table 6.

TABLE 6

Relative Magnitudes of Price Effect (P.E.), Output Effect (O.E.) and Time Effect (T.E.)

Year	a_1	P.E.	O.E.	T.E.	Total
1956-57	187	20×10^{-3}	-3×10^{-3}	-46×10^{-3}	-29×10^{-3}
1966-67	97.3	40×10^{-3}	-5×10^{-3}	-88×10^{-3}	-53×10^{-3}
1976-77	79.4	47×10^{-3}	-6×10^{-3}	-108×10^{-3}	-67×10^{-3}

Uses and Disadvantages of the Method

The approach used in this study of the demand for domestic timber has been to regard it as an input into residential construction. Such an approach focuses on the interrelationships that exist with other inputs, and involves system estimation as opposed to single equation estimation. This has advantages and disadvantages. An important advantage in the system approach is that it enables across-equation constraints and correlations between disturbances of the system to be used to increase the efficiency of estimation. This helps overcome the problem of short time series mentioned by Ferguson (1979). The method would be appropriate whenever the variable of interest is demanded primarily as an input of production and its use is concentrated in a few well-defined areas.

One of the implicit assumptions of the model is that prices are exogenously determined. In view of the circumstances just described (i.e. concentration in usage) this assumption is open to question. The authors' view is that simultaneity bias in the present application is slight because quantities demanded probably adjust to price changes more rapidly than prices adjust to changes in demand.

The main disadvantage of the method is that the own-price elasticity, being estimated by the sum of the cross-price elasticities, will be biased unless *all* the inputs have been included in the model. For practical reasons this will usually be impossible. First, it is unlikely that data will be available on every input. Second, even if these data are available, the size of the system is likely to become too large to be manageable. Suppose we denote by K_1 the number of inputs and by K_2 the number of other variables not subject to across-equation constraints. Then the total number of parameters to be estimated is K, given by $K_1(K_1+1)/2 + K_1K_2$. This number becomes very large for a production process with several inputs. For example, if there are 10 inputs and two other variables (in our study these are y_t and t), $K=75$. Thus, in practice, this approach should be used in conjunction with more conventional methods in order to obtain satisfactory estimates of the own-price elasticity.

Summary and Conclusions

A system of equations has been estimated in which the input-output ratios of six of the major inputs to the residential construction industry are expressed in terms of relative prices. By concentrating on the demand for domestic timber it has been possible to estimate cross-price elasticities, scale and taste/technology effects. With models of the kind used in this analysis, own-price elasticities are found indirectly as the sum of all cross-price elasticities. Inevitably, as all inputs can never be included, estimates of own-price elasticities will be biased.

The falling input-output ratio for domestic timber has been analysed by disaggregating its time-rate of change and the evidence suggests that price movements over the last two decades have worked strongly in favour of timber. However, this favourable price effect has been more than offset by unfavourable changes in taste and technology. The main changes in this area are as follows:

(a) an increase in the proportion of flats being constructed, from about 3 per cent in 1953-54 to 30 per cent in 1973-74 (BAE 1977), probably leading to a reduction in the demand for timber—it has been estimated that the average volume of timber used in flat construction is only 5.7 cubic metres compared with 18.5 cubic metres in houses (Senate Standing Committee on Trade and Commerce 1981); and

(b) changes in technology have resulted in much greater use of concrete floors, a decrease in the use of timber for exterior cladding and building techniques designed to economise on the use of timber (Wymond 1973).

In these circumstances, the industry will need to find ways of reversing the taste/technology trend if the demand for timber in the building industry is to be boosted.

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APPENDIX TABLES

TABLE A.1

Quantities[a] and Price Indexes[b] of Material Inputs, and Output for the Building Industry in Australia

Year	Domestic timber		Imported timber		Bricks		Cement		Plaster			Labour		Output adjusted
	Quantity	Price	Quantity	Price	Quantity	Price	Quantity	Price	Quantity		Price	Quantity	Price	
	10^3m^3	index	10^3m^3	index	10^6	index	10^3	index	10^3m^2		index	10^3man h	index	$\$m \times 10^2$
1956-57	3336	88.1	758	96.1	804	90.3	2124	89.8	13.500	(12.586)	94.1	113 852	70.3	8.927
1957-58	3264	81.1	736	92.9	876	84.7	2244	83.9	14.520	(12.754)	90.6	109 616	71.3	10.487
1958-59	3444	78.2	718	84.2	936	82.3	2436	81.4	15.216	(15.469)	89.2	119 937	72.7	11.240
1959-60	3588	81.5	867	97.2	1032	85.0	2580	84.2	15.444	(16.754)	90.8	132 628	77.4	11.738
1960-61	3432	81.8	922	104.9	1056	85.2	2796	84.5	14.256	(17.241)	91.0	119 614	80.5	12.903
1961-62	3180	81.8	699	87.9	996	85.2	2748	84.5	12.732	(19.135)	91.0	129 612	82.5	12.410
1962-63	3324	85.0	738	93.9	1056	87.8	2880	87.2	13.140	(21.264)	92.5	135 693	83.3	13.178
1963-64	3432	95.3	858	100.0	1236	96.1	3252	95.9	13.152	(21.201)	97.6	145 760	85.6	13.852
1964-65	3564	98.8	918	104.5	1344	99.1	3672	99.0	11.976	(23.013)	99.4	157 835	90.5	15.436
1965-66	3504	99.2	813	101.4	1356	99.4	3612	99.3	24.816		99.6	152 570	93.1	16.747
1966-67	3372	100.0	826	100.0	1356	100.0	3720	100.0	25.584		100.0	150 847	100.0	17.334
1967-68	3432	103.0	913	104.0	1440	103.6	3864	102.2	27.432		101.7	160 091	103.8	18.113
1968-69	3432	108.6	968	118.2	1609	107.8	4140	105.4	29.532		103.0	171 482	113.4	19.969
1969-70	3432	113.5	968	132.8	1692	112.4	4500	108.4	29.892		105.1	172 087	119.1	21.271
1970-71	3432	118.5	975	129.5	1668	118.0	4680	117.6	32.112		109.4	171 580	124.6	23.190
1971-72	3360	124.8	958	130.0	1740	124.5	4884	126.1	34.320		116.9	170 079	142.0	23.551
1972-73	3540	137.0	1142	147.9	1872	130.7	5040	133.5	39.792		118.7	180 847	153.9	24.193
1973-74	3456	169.1	1322	218.1	2028	146.8	5328	145.9	44.532		122.2	180 057	176.4	24.072
1974-75	3432	203.5	887	239.0	1716	180.3	5088	179.4	42.012		147.8	142 873	242.0	22.209
1975-76	3456	226.2	1003	251.3	1836	205.1	5004	211.1	46.080		167.7	142 400	283.0	23.478
1976-77	3456	254.1	1148	322.5	2064	227.8	5040	238.4	50.508		178.8	131 200	326.8	21.767

[a] The quantities in parentheses for the plaster quantity series are estimated. These figures were used in the analysis.
[b] Price indices for the years 1956-57 through 1965-66 are predictions.

TABLE A.2

Timber Cross-Price Elasticities

Year	Imported timber	Bricks	Cement	Plaster	Labour
1956-57	-0.15	-0.23	0.06	-0.51	0.42
1957-58	-0.19	-0.28	0.08	-0.63	0.53
1958-59	-0.18	-0.29	0.08	-0.64	0.55
1959-60	-0.19	-0.29	0.08	-0.64	0.56
1960-61	-0.23	-0.33	0.09	-0.73	0.65
1961-62	-0.23	-0.36	0.10	-0.80	0.72
1962-63	-0.23	-0.35	0.09	-0.76	0.68
1963-64	-0.22	-0.35	0.09	-0.75	0.67
1964-65	-0.24	-0.37	0.10	-0.80	0.72
1965-66	-0.26	-0.41	0.11	-0.88	0.81
1966-67	-0.28	-0.44	0.12	-0.95	0.90
1967-68	-0.29	-0.45	0.12	-0.97	0.92
1968-69	-0.33	-0.50	0.13	-1.05	1.04
1969-70	-0.36	-0.53	0.14	-1.10	1.11
1970-71	-0.38	-0.58	0.16	-1.20	1.21
1971-72	-0.39	-0.60	0.16	-1.25	1.30
1972-73	-0.38	-0.57	0.16	-1.17	1.26
1973-74	-0.43	-0.56	0.15	-1.09	1.24
1974-75	-0.38	-0.52	0.14	-1.02	1.23
1975-76	-0.39	-0.55	0.15	-1.08	1.33
1976-77	-0.38	-0.51	0.14	-0.97	1.25

TABLE A.3

Alternative Timber Own-Price Elasticities[a]

Year	Elasticity	Standard error	*t*-ratio
1956-57	0.40	0.13	3.10
1957-58	0.48	0.15	3.20
1958-59	0.47	0.14	3.30
1959-60	0.44	0.13	3.40
1960-61	0.48	0.14	3.50
1961-62	0.47	0.13	3.50
1962-63	0.43	0.12	3.50
1963-64	0.39	0.11	3.50
1964-65	0.37	0.11	3.30
1965-66	0.36	0.12	3.00
1966-67	0.34	0.13	2.70
1967-68	0.29	0.13	2.20
1968-69	0.24	0.15	1.60
1969-70	0.18	0.17	1.00
1970-71	0.10	0.19	0.50
1971-72	0.01	0.22	0.02
1972-73	-0.09	0.23	-0.41
1973-74	-0.19	0.24	-0.80
1974-75	-0.25	0.23	-1.10
1975-76	-0.35	0.26	-1.30
1976-77	-0.40	0.26	-1.50

[a] Estimates based on equation (23).