THE REPRESENTATION OF RISK IN ECONOMETRIC MODELS OF SUPPLY: SOME OBSERVATIONS

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The high correlation between many of the measures used to represent risk in econometric models of supply is demonstrated. A case is made for a very simple measure, the moving range, to be used to represent risk in these models.

The only sure thing in this world is the past, but all we have to work with is the future.

—Auguste Detoeuf

Introduction

There has been increasing interest in recent years in the inclusion of variables representing risk in econometric studies of supply of agricultural commodities. Authors such as Behrman (1968), Freebairn (1973), Just (1974), Freebairn and Rausser (1975), Lin (1977), Ryan (1977), Traill (1978), Reeves (1980), Brennan (1980), Wilson, Arthur and Whittaker (1980) and Harrison (1981) have included variables representing risk in their econometric models. There is no consensus as to how variables representing riskiness in production or prices should be formulated in econometric studies. The methods used have varied from simple measures of instability to complex variables requiring complicated estimation procedures.

In this paper the purpose is to demonstrate that a simple measure of risk (the moving range) can be used by those constructing econometric models to represent risk. This variable is simple to calculate and does not require any complex estimation procedure. It can be used by any researcher using simple methods of estimation.

The variables used in the studies listed above have represented risk in prices, production or incomes. As a simplification, the discussion in this paper is restricted to price risk, although some of the implications may be applicable to other sources of risk.

Representation of Risk

Price risk is the variability associated with an estimate of the expected price. Such unobservable variability has to be represented by some approximation, and an observation on risk in a particular period has been approximated in various ways in econometric models. The expected or anticipated risk can be formed by a weighted sum of past observations of risk, estimated in a distributed lag formulation.

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The means by which an observation on price risk has been represented can be categorised broadly into:

(a) the recorded variability or instability over recent periods; and

(b) the extent to which this variability was not expected.

The first category is based on the assertion that risk is directly related to the recorded instability or variability of prices in recent periods. This involves the implicit assumptions that perceived risk is equated with or directly related to variability, and that present riskiness is related to riskiness in the recent past. Freebairn (1973) used the range of recent prices as a 'crude and arbitrary' specification of variability, and noted that 'a more appropriate measure would be a moving variance estimate' (p. 58, footnote 11). The use of a moving variance, a moving standard deviation or a moving weighted standard deviation are all means of trying to capture aspects of this recent variability in a 'more appropriate' manner.

The second category of measures of risk is based on the assertion that risk is some function of the difference between the expected price and the actual price. Thus, variability which is expected does not induce any reaction through risk. In models in which the expected price is formed using distributed lags on past prices, either a maximum likelihood (as in Just 1974 and Harrison 1981) or an iterative procedure (as in Traill 1978 and Brennan 1980) has to be used. These more complex estimation procedures can make it difficult for researchers using limited resources for computation or those only familiar with basic techniques, such as regression procedures using OLS, to test whether a variable representing risk should be included in their models.

Some analysts have modified these variables to allow for the posssibility that farmers perceive a situation as being risky only when there is a high probability of low prices, so that a given variability at high prices represents a lower risk than the same variability at low prices (as in Wilson et al. 1980). Another possibility is that farmers are concerned only with 'down-side' risk, that is, the risk of prices falling (as in Traill 1978).

The manner in which risk is included in relation to the risk of competing products has also differed between the various studies. The competition between products is often incorporated into models by the use of relative prices. However, it is inappropriate to use measures of the variability of the relative price as measures of the relative risk. The variability in the relative price may result equally from fluctuations in either price, and would not reflect the relative variability of the price of one product in relation to the price of the other. Measures which have been used to represent the relative variability are some of the above variables in a ratio (for example, the standard deviation of one product's prices divided by the standard deviation of the prices of the competing product, as in Behrman 1968) and the ratio of covariance to variance (as in Ryan 1977).

Choice Among Variables to Represent Risk

There are three major drawbacks with the approach which defines risk in terms of the difference between the expected price and the actual price. First, the result depends critically on the formulation of the expected price. This involves the question of whether price expectations are formed from past prices and, if so, what length and shape of lag is appropriate. Second, the approach requires a more complex estimation procedure where the expected price is formed from a distributed lag on past prices. Third, problems can arise when price variables enter the model as a ratio.

Traill (1978) and Brennan (1980) compared a number of different variables representing price risk, including some which defined risk as the difference between the expected price and the actual price, and some which were based simply on recent variability of prices. Although the former group of variables had greater theoretical appeal, neither found any superiority in terms of explanatory power for the more complex variables.

Based on these findings, perhaps little if anything would be lost in terms of accuracy by using the simpler approach, but much can be gained through the simplicity and ease of the approach.

Within the category of simple measures of recent variability, the moving range and the moving standard deviation are highly correlated if both are calculated over the same short period. The maximum value of the standard deviation of a set of n observations with a given range is independent of the number of observations if n is even, and equals half the range. If n is odd, the maximum value of the standard deviation is $(1/2n)(n^2-1)^{0.5}$ times the range. The minimum value of the standard deviation of a set of n observations with a given range is equal to $1/(2n)^{0.5}$ times the range.[1] Thus, for a small number of observations, say $n \leq 4$, the ratio of the standard deviation to the range is narrowly confined. There will be a tendency for the standard deviation to be distributed about some point between its maximum and minimum values. Therefore, where n is small, the moving range will be closely correlated with the moving standard deviation.

An Empirical Example

For purposes of clarification and illustration, some measures of risk have been calculated for the annual price of wheat for the period 1948-49 through 1977-78. The wheat price used is the annual pool payment to growers per tonne of bulk wheat (BAE 1981). The measures calculated are as follows:

(a) moving range (3 periods);
(b) moving standard deviation (3 periods);
(c) moving range (4 periods);
(d) moving standard deviation (4 periods);

[1] This is because the variance will be maximised for a given range when the observations are clustered at each end of the range, and minimised for a given range when all observations except the two outlying values lie at the mean. Thus, for example:

if $n=2$, $S=0.500$ of range;
if $n=3$, S lies between 0.408 and 0.471 of range;
if $n=4$, S lies between 0.354 and 0.500 of range;
if $n=5$, S lies between 0.316 and 0.490 of range; and
if $n=8$, S lies between 0.250 and 0.500 of range.

These calculations have been made using the formula for the population. If the formula for the sample variance is used, as is likely with a statistical program, the formulae are slightly different but the close relationship still holds.

(e) magnitude of difference between expected and actual prices (naive expectations);
(f) magnitude of difference between expected and actual prices (adaptive expectations); and
(g) magnitude of difference between expected and actual prices (Almon lags).[2]

The coefficients used for the last two variables relate to Victoria (Brennan 1980). The expected risk in a particular period can be formed by a distributed lag on past observations of risk, if required. Observations of each of the measures of risk are shown in Table 1.

TABLE 1

Comparison of Measures of Risk in Wheat Prices ($/t)

Year	Payments to growers	Range (3 yrs)	S.D. (3 yrs)	Range (4 yrs)	S.D. (4 yrs)	$Wt = \lvert P^*_{t-1} - P_{t-1} \rvert$ Naive	Adaptive[a]	Almon[b]
1948-49	41.46	25.41	10.86	34.87	12.75	19.51	31.47	28.60
1949-50	47.79	19.51	7.99	25.41	9.55	11.12	13.61	9.03
1950-51	46.35	11.12	4.55	19.51	7.31	6.33	16.61	8.37
1951-52	52.34	6.33	2.71	11.12	3.96	1.44	11.24	1.97
1952-53	55.03	5.99	2.55	10.88	3.88	5.99	14.28	2.89
1953-54	44.33	8.68	3.63	8.68	3.48	2.69	13.32	8.02
1954-55	44.00	10.70	4.54	10.70	4.34	10.70	0.87	5.84
1955-56	44.09	11.03	5.12	11.03	4.86	0.33	1.78	9.24
1956-57	45.97	0.33	0.14	11.03	4.72	0.09	2.18	4.99
1957-58	47.75	1.97	0.91	1.97	0.80	1.88	0.42	0.35
1958-59	48.38	3.66	1.49	3.75	1.54	1.78	1.03	3.26
1959-60	49.41	2.41	1.02	4.29	1.67	0.63	1.64	3.01
1960-61	50.09	1.66	0.68	3.44	1.25	1.03	2.16	2.80
1961-62	53.06	1.71	0.70	2.34	0.90	0.68	2.43	2.18
1962-63	51.23	3.65	1.58	4.68	1.74	2.97	4.88	4.16
1963-64	50.43	2.97	1.22	3.65	1.38	1.83	2.15	1.09
1964-65	49.58	2.63	1.10	2.97	1.15	0.80	1.10	0.64
1965-66	51.81	1.65	0.67	3.48	1.29	0.85	0.12	1.81
1966-67	52.04	2.23	0.92	2.23	0.84	2.23	1.97	0.78
1967-68	54.07	2.46	1.11	2.46	1.01	0.23	1.63	1.43
1968-69	45.46	2.26	1.02	4.49	1.59	2.03	3.18	3.16
1969-70	43.85	8.61	3.67	8.61	3.23	8.61	6.17	6.43
1970-71	46.82	10.22	4.49	10.22	4.30	1.61	6.62	7.44
1971-72	48.75	2.97	1.21	10.22	3.91	2.97	2.34	2.22
1972-73	49.54	4.90	2.02	4.90	1.80	1.93	0.10	1.78
1973-74	110.07	2.72	1.14	5.69	2.19	0.79	0.89	3.50
1974-75	108.40	61.32	28.72	63.25	26.74	60.53	61.42	62.72
1975-76	99.24	60.53	28.15	61.32	30.05	1.67	47.26	47.53
1976-77	82.85	10.83	4.76	60.53	24.75	9.16	28.11	19.93
1977-78	89.92	25.55	10.56	27.22	10.80	16.39	5.38	12.44

[a] Based on model in which the coefficient of expectation is 0.174.
[b] Based on model in which coefficients of lagged prices are as follows: 1.246 P_{t-1}, 1.868 P_{t-2}, 1.868 P_{t-3}, 1.246 P_{t-4} (sum of weights = 6.228).

[2] Expected prices are defined as follows:

naive: $P^*_t = P_{t-1}$;
adaptive: $P^*_t = P^*_{t-1} + \delta(P_{t-1} - P^*_{t-1})$; and
Almon lags: $P^*_t = w_1 P_{t-1} + w_2 P_{t-2} + \ldots + w_n P_{t-n}$,
where the w_i's lie along a polynomial.

TABLE 2

Correlation Coefficients Between Measures of Risk (1948-49 through 1977-78)

Risk measure	Range 3 yrs	S.D. 3 yrs	Range 4 yrs	S.D. 4 yrs	W_t naive	W_t adaptive	W_t Almon
Moving range (3 years)	1.000						
Moving standard deviation (3 yrs)	0.999	1.000					
Moving range (4 years)	0.858	0.854	1.000				
Moving standard deviation (4 yrs)	0.880	0.879	0.993	1.000			
W_t (naive expectations)	0.710	0.707	0.650	0.607	1.000		
W_t (adaptive expectations)	0.899	0.902	0.914	0.912	0.736	1.000	
W_t (Almon lag)	0.955	0.960	0.900	0.910	0.774	0.954	1.000

The various measures of risk are highly correlated (Table 2). The correlation coefficient between the moving range and the moving standard deviation is 0.999 calculated over three years and 0.993 calculated over four years. Thus, they are almost perfectly correlated when both are calculated over the same short period, following the relationship discussed in the previous section. The correlation coefficient between the moving range calculated over three years and over four years is 0.858, while that of the standard deviation over the two time periods is 0.880.

The general similarity between the measures based on the difference between the expected and the actual price and those based on recorded variability is also apparent. For example, the correlation coefficients in Table 2 of the moving range over three years and the more complex variables are 0.710 (naive expectations), 0.899 (adaptive expectations) and 0.955 (expectations based on Almon lags).

Conclusion

Researchers who have compared different variables to represent risk in prices in econometric models of supply have found no superiority for the more complex variables over the simpler variables. The more complex variables often require complex procedures for estimation beyond the reach of those who are using simple models or simple techniques. Comparisons of the different variables show high correlation between the simple and some complex variables. In view of these findings, it is apparent that there is little to be lost by those interested in testing for the presence of risk if the simple measures are used.

There is an algebraic relationship between the moving range and the moving standard deviation of a series of numbers. In view of this relationship, the moving range and the moving standard deviation of prices over recent periods (say, not more than four periods) will be highly correlated. The choice between them to represent risk is immaterial, provided the period is short. Thus, what seemed a crude measure to Freebairn is very closely correlated with other measures and so is more useful than Freebairn allowed.

An appropriate measure for researchers to use to test for the presence of risk would be the moving range over three or four periods. It is easy to calculate and easy to manipulate in the context of model experimentation. Where a measure of relative risk is required, the relative range would be an appropriate measure. Where it is desirable to test whether farmers react more to risk at lower prices than at higher prices, the range divided by the price can be used.

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