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Regulating a Monopolist with Unknown Demand
and Cost Functions

by

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Abstract

We consider the design of regulatory policy when the regulator is imperfectly informed about both the firm's cost function and the demand function it faces. To some extent, the optimal policy is the natural "combination" of the policies when there is uncertainty about demand alone or cost alone. But important qualitative differences also exist. For example, prices may optimally be set *below* marginal cost when there is uncertainty about both cost and demand, although prices will never fall below marginal cost when there is only a single source of uncertainty, whether it pertains to cost or demand.

1. Introduction.

In most regulatory settings, the regulator is faced with a myriad of uncertainties regarding the attributes of the firm he is regulating. For example, the regulator generally has imperfect knowledge of both the firm's demand and cost structure. Demand is determined both by the attributes (e.g., quality) of the goods and services provided, and by the characteristics of consumers; and the firm will generally have better information than the regulator about both of these determinants of demand.¹ Similarly, because of its experience in the industry, the firm will often have better information about production costs.

The design of regulatory policy in the presence of asymmetric technological information has been analyzed in papers by Baron and Myerson [1982], Sappington [1983], Rochet [1984], and Laffont and Tirole [1986], among others. Optimal regulation in the presence of asymmetric demand information has been studied by Riordan [1984] and Lewis and Sappington [1987]. These two strands of the literature reveal that the nature of the asymmetry in knowledge is critical in determining optimal regulatory policy.

When costs are unknown (but demand is known) to the regulator, significant pricing flexibility is generally delegated to the perfectly-informed firm. Although the delegation results in rents for the firm, social welfare is enhanced when the firm employs its superior technological information to choose prices which track the movement of efficient prices. The regulator also finds it advantageous to induce prices in excess of realized marginal cost in order to limit the firm's rents.²

When demand is unknown (but costs are known) to the regulator, the optimal regulatory policy is very different. In this case, when marginal costs of production increase with output, the firm commands no rents from its superior information, and efficient prices will always be charged. Although pricing authority is delegated to the firm in this setting, the firm can be

induced to employ its superior knowledge of demand in the social interest. On the other hand, when marginal costs decline with output, the regulator will not delegate any pricing authority to the firm. Instead, the regulator will optimally rely on his own imperfect knowledge of demand to set a single price.³

In practice, the regulator is faced with uncertainty about demand and cost simultaneously. It seems important, therefore, to examine the optimal regulatory policy when the regulator is imperfectly informed about both the firm's cost function and the demand function it faces. The purpose of the present inquiry is to do just that. Specifically, we examine the extent to which pricing decisions are delegated to the firm when costs and demand are unknown to the regulator. We also explore the inefficiencies that result, and consider the pattern of rents that accrue to the firm because of its superior knowledge. Our focus is on a comparison of the properties of the optimal regulatory policy when uncertainty is two-dimensional with the properties of the optimal policies when the relevant uncertainty is uni-dimensional.

We find a number of similarities between the optimal regulatory policies with one and two sources of uncertainty. For example, recall that when only costs are unknown to the regulator, prices are induced in excess of marginal cost. We find that when the regulator is uncertain about both demand and costs, price is set in excess of *expected* marginal cost. In addition, we find that regulated prices optimally increase with both demand and cost realizations, just as when there is a single source of uncertainty. We also demonstrate that in the limiting cases where one of the two sources of uncertainty becomes inconsequential, the optimal regulatory policy converges to the relevant policy with only a single source of uncertainty.

Despite these similarities, we find that important qualitative differences are introduced by the second source of uncertainty. Foremost among these differences is the fact that prices may optimally be set *below* realized marginal cost. Prices will tend to be set below marginal cost for

smaller demand realizations. Differences also arise in the pattern of rents received by the firm, and in the extent to which pricing decisions are delegated to the firm. The firm's rents will generally be minimized for the highest cost realization and for intermediate levels of demand. The pricing flexibility afforded the firm will tend to be reduced when marginal costs decline with output.

The analysis proceeds as follows. The basic elements of the model under consideration are outlined in the next section, and a formal statement of the regulator's problem is also presented. In addition, the benchmark solutions to the regulator's problem, where the uncertainty pertains to demand or to cost, but not both, are discussed. In section 3, our basic solution technique (which borrows heavily from Laffont, Maskin and Rochet [1987] and McAfee and McMillan [1987]) is described. There, the two-dimensional problem is effectively translated into a uni-dimensional problem, which facilitates its solution. In section 4, the solution to the regulator's problem is derived and interpreted, and compared to the benchmark solutions recorded in section 2. Concluding remarks are contained in section 5.

2. Description of the Model.

The regulatory environment under consideration is the following. The cost of producing output, q , is given by

$$\tilde{C}(q, c) = C(q) + \delta^C c q,$$

where $c > 0$ is a random parameter that shifts the position of the cost function, and where $\delta^C \geq 0$ is a constant. By varying δ^C , we can examine the effects of increasing cost uncertainty on the optimal regulatory policy. Demand for the firm's product, $q = Q(p, \theta)$, is given by

$$Q(p, \theta) = h(p) + \delta^D \theta,$$

where θ is a random parameter that shifts the position of the demand function, and where $\delta^D \geq 0$ is a constant. By varying δ^D , we can examine the effects of increasing demand

uncertainty.

The critical information asymmetry is captured by the random variables c and θ . Their joint probability density function is $f(\theta, c)$, which is assumed to have strictly positive support in the set $D = [\underline{\theta}, \bar{\theta}] \times [\underline{c}, \bar{c}]$. The realizations of these random variables are known to the firm but not to the regulator.⁴ The demand function, cost function, and density function $f(\theta, c)$ are all common knowledge.

The demand and cost functions are assumed to have the usual properties. In particular, more output is demanded the lower the regulated price, i.e., $Q_p(p, \theta) = h'(p) < 0$, where subscripts denote partial derivatives. Also, costs increase with output, i.e., $\tilde{C}_q(q, c) \equiv C'(q) - \partial^D c > 0$. The sign of $C_{qq}(\cdot)$ is not restricted; but we do require that $[1 - C''(\cdot)Q_p(\cdot)] > 0$ when $\tilde{C}_{qq}(\cdot) < 0$. In words, demand curves are assumed to be more steeply sloped than the firm's marginal cost curve when the latter is downward sloping. This convention is introduced to avoid characterization of local minima rather than maxima.

For simplicity, we also assume that demand is sufficiently great and costs are sufficiently low for all possible realizations of c and θ that the demand price exceeds marginal cost at zero output. Furthermore, we focus in the text on the region of the state space, $D' \subseteq D$, for which output is strictly positive under the optimal regulatory policy, i.e., $D' \equiv \{(\theta, c) \mid Q(p(\theta, c), \theta) > 0\}$. The modifications needed when no output is produced are outlined in footnotes.

In our model, the regulator is endowed with the power to establish a unit price for the firm's output and to specify a transfer payment, T , from consumers to the firm. The transfer payment might be thought of as an access charge, or as the fixed fee associated with a two-part tariff.⁵ The regulator can observe (and therefore enforce) the regulated price, but the firm's sales are presumed prohibitively costly to monitor. However, the regulator can still ensure the firm fulfills

its mandate to serve all demand at the established price. He does so by allowing customers to register grievances if they are refused service at the regulated price (or if they are charged a higher price), and punishing the firm for such transgressions.

To sum up, the timing in the model is the following. First, the firm (alone) observes the realizations of ϵ and θ . Then, based upon his prior beliefs (as captured in $f(\theta, \epsilon)$), the regulator designs the regulatory policy to maximize expected consumers' surplus, in the usual Stackelberg fashion.⁶ Next, the firm observes the terms of the announced policy, which is a "menu" of two-part tariffs; then the firm chooses one of the tariffs from the menu. Finally, production occurs to serve all demand at the established price.⁷ Note that the firm's participation in the regulated environment is voluntary, and it can always choose to shut down and earn its reservation profit level (normalized here at zero) rather than choose one of the pricing options stipulated by the regulator.

The regulator's problem [RP] can be stated formally as follows.

$$\text{Maximize}_{p, T} \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{\epsilon}}^{\bar{\epsilon}} \left[V(Q(p, \theta)) - \bar{C}(Q(p, \theta), \epsilon) - \pi(\theta, \epsilon) \right] f(\theta, \epsilon) d\epsilon d\theta$$

subject to:

$$\pi(\theta, \epsilon) \equiv \bar{\pi}(\theta, \epsilon | \theta, \epsilon) \geq 0 \quad \forall (\theta, \epsilon) \in D, \text{ and} \quad (\text{IR})$$

$$\pi(\theta, \epsilon) \geq \pi(\hat{\theta}, \hat{\epsilon} | \theta, \epsilon) \quad \forall (\hat{\theta}, \hat{\epsilon}), (\theta, \epsilon) \in D \quad (\text{IC})$$

where

$$V(Q(p, \theta)) \equiv \int_0^{Q(p, \theta)} \bar{p} dp, \text{ and}$$

$$\bar{\pi}(\hat{\theta}, \hat{\epsilon} | \theta, \epsilon) \equiv p(\hat{\theta}, \hat{\epsilon}) Q(p(\hat{\theta}, \hat{\epsilon}), \theta) - \bar{C}(Q(p(\hat{\theta}, \hat{\epsilon}), \theta), \epsilon) + T(\hat{\theta}, \hat{\epsilon}).$$

In the statement of [RP], $p(\theta, \epsilon)$ and $T(\theta, \epsilon)$ are the price and transfer payment that will be

implemented when the firm's private information is (θ, c) . $Q(p(\theta, c), \theta)$ is the corresponding production level. The incentive compatibility constraints (IC) ensure that when its private information is (θ, c) , the firm prefers the $\{p(\theta, c), T(\theta, c)\}$ two-part tariff to any other tariff in the menu offered by the regulator. The individual rationality constraints [IR] guarantee the firm at least the level of (extranormal) profit it could earn by refusing to implement any of the two-part tariffs allowed by the regulator.

Before proceeding to characterize the solution to [RP] in general, we record the properties of the solution in the two limiting cases of no demand uncertainty (Lemma 1) and no cost uncertainty (Lemmas 2 and 3).

Lemma 1. (Baron and Myerson [1982].) When $\delta^D = 0$, the solution to [RP] has $\forall \theta \in [\underline{\theta}, \bar{\theta}]$:

- (i) $p(\theta, c) = \tilde{C}_Q(Q(p(\theta, c), \theta), c)$;
- (ii) $p(\theta, c) > \tilde{C}_Q(Q(p(\theta, c), \theta), c) \quad \forall c > \underline{c}$;
- (iii) $p_c(\theta, c) \geq 0 \quad \forall c \in [\underline{c}, \bar{c}]$;
- (iv) $\pi(\theta, \bar{c}) = 0$; and
- (v) $\pi(\theta, c) > 0 \quad \forall c < \bar{c}$.

Lemma 1 reports that when the regulator is uncertain only about the firm's cost function, he will induce the firm to price above marginal cost for all cost realizations other than the lowest. It also states that the firm will earn rents from its private technological information under these circumstances whenever $c < \bar{c}$. Also, the optimal regulated price will be greater the higher the realization of the random cost parameter, c .⁸

Now consider the case of asymmetric demand information.

Lemma 2. (Lewis and Sappington [1987].) When $\delta^C = 0$ and $C''(q) \geq 0 \quad \forall q \geq 0$, the solution to [RP] has $\forall (\theta, c) \in D$:

- (i) $p(\theta, c) = \tilde{C}_Q(Q(p(\theta, c), \theta), c)$; and
- (ii) $\pi(\theta, c) = 0$.

Thus, when the position of the demand curve is the only source of uncertainty for the regulator and marginal costs increase with output, the information asymmetry is inconsequential for the regulator.⁹ He can ensure efficient prices and extract all rents from the firm. A very different conclusion arises, though, when marginal production costs decline with output. Here, the regulator relies solely on his own imperfect knowledge of demand to establish a single price for all realizations of θ .

Lemma 3. (Lewis and Sappington [1987].) When $\delta^C = 0$ and $C''(q) < 0 \quad \forall q \geq 0$, then the solution to [RP] has $\forall c \in [c, \bar{c}]$:

- (i) $p(\hat{\theta}, c) = \hat{p}$, a constant $\forall \theta \in [\hat{\theta}, \bar{\theta}]$;
- (ii) $\pi(\hat{\theta}, c) = 0$ where $\hat{p} = \tilde{C}_Q(Q(\hat{p}, \hat{\theta}), c)$; and
- (iii) $\pi(\theta, c) > 0$ for $\theta = \hat{\theta}$.

With these benchmark solutions in mind, we now begin to characterize the general solution to [RP] by introducing a reformulation of the problem in section 3.

3. Reformulation of the Problem.

To solve [RP], it is helpful to rewrite the problem as a single-dimensional maximization problem. To do so, we first construct iso-price loci in (θ, c) space. Each locus is the set of all points (θ, c) for which the same price, $p(\theta, c)$, will be charged. These loci are characterized in Lemma 4. The proofs of all results that are not presented in the text can be found in the Appendix.

Lemma 4. Let $\Pi(p, \theta, c) = pQ(p, \theta) - \tilde{C}(Q(p, \theta), c) + T$. Then at the solution to [RP], the

slope (λ) of an iso-price locus in (θ, c) space is given by:

$$\lambda \equiv \left. \frac{dc}{d\theta} \right|_{dp=0} = - \frac{\Pi_p(p, \theta, c)}{\Pi_{p,c}(p, \theta, c)} = \frac{\delta^D [1 - C''(Q(p, \theta)) h'(p)]}{\delta^C h'(p)} < 0. \quad \bullet$$

Notice that if $C'''(\cdot) = 0$, each iso-price locus is a straight line segment. To simplify the ensuing analysis without altering substantively the basic conclusions (with the exception of Lemma 5), we presume $C'''(\cdot) = 0 \quad \forall q \geq 0$.¹⁰

Representative iso-price loci are drawn in Figure 1. These loci enable a reduction in the dimensionality of the problem from two to one by introducing a new variable, ε , which is defined by

$$r(\varepsilon) = p(\theta, \varepsilon) = p(\theta, c). \quad (3.1)$$

As is apparent from Figure 1, ε is the intercept on the vertical (c) axis of the identified iso-price line. Thus, the relation between c and ε is given by:

$$c = \varepsilon + [\theta - \underline{\theta}] \lambda. \quad (3.2)$$

Notice that since $\lambda < 0$, ε increases with both c and θ . Hence, somewhat loosely, higher values of ε correspond to settings where an appropriately weighted combination of demand and costs are higher. In terms of Figure 1, higher values of ε ($\varepsilon_2 > \varepsilon_1$) are identified with iso-price lines further out in a north-easterly direction.

Now, consider how this change of variables allows us to rewrite [RP]. First note that the local representation of the incentive compatibility constraints (LIC) state that $\pi_c(\cdot) |_{c=\bar{c}} = 0$.

Therefore, $\pi_c(\theta, c) = \pi_c(\theta, \bar{c} | \theta, c) |_{c=\bar{c}} = -\delta^C Q(p(\theta, \bar{c}), \theta)$. Consequently,

$$\pi(\theta, c) = \pi(\theta, \bar{c}) - \delta^C \int_{\bar{c}}^c Q(p(\theta, \bar{c}), \theta) d\bar{c}. \quad (3.3)$$

Substituting (3.3) into the maximand in [RP] and integrating by parts yields the following

restatement:

$$\text{Maximize}_{p, T} \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{c}}^{\bar{c}} \left[V(Q(p, \theta)) - \tilde{C}(Q(p, \theta), c) - \delta^C Q(p, \theta) \frac{F(\theta, c)}{f(\theta, c)} - \Pi(\theta, \bar{c}) \right] f(\theta, c) dc d\theta, \quad (3.4)$$

where $F(\theta, c) = \int_{\underline{c}}^c f(\theta, \tilde{c}) d\tilde{c}.$

Thus, $F(\theta, c)$ represents the cumulative distribution function for c , conditional on θ .

Now, changing variables from c to s , reversing the order of integration, and using (3.1) and (3.4), [RP] can be rewritten as the following problem, [RP]':

$$\text{Maximize}_{r(s)} L(r(s))$$

$$\text{subject to: } (IR), (IC)$$

$$\text{where } L(\cdot) \equiv \int_{\underline{\theta}}^{\bar{\theta}} \int_{\underline{c}}^{\bar{c}} \left[V(Q(r(s), \theta)) - C(Q(r(s), \theta), c(s, \theta)) - \delta^C Q(r(s), \theta) \frac{F(\theta, c(s, \theta))}{f(\theta, c(s, \theta))} \right] f(\theta, c(s, \theta)) d\theta ds; \quad \text{and} \quad (3.5)$$

where $c(s, \theta) = s - \lambda [\theta - \underline{\theta}]$, from (3.2).

The importance of this restatement of [RP] as [RP]' is that the necessary conditions for a solution to [RP]' can be obtained simply from pointwise maximization of $L(r(s))$ with respect to prices, $r(s)$.¹¹ Most importantly, even though [RP]' incorporates only the local representation of the incentive compatibility conditions (LIC), their global counterparts (IC) will also be satisfied provided $C'''(q) = 0 \quad \forall q \geq 0$. This conclusion is recorded formally as Lemma 5.

Lemma 5. Suppose that $C'''(q) = 0 \quad \forall q \geq 0$, and that a candidate solution to [RP]' satisfies (LIC), the local representation of the incentive compatibility constraints. Then the

candidate solution also satisfies (IC), the global incentive compatibility constraints.

Lemma 5 is instrumental in characterizing the optimal regulatory policy, which is the subject of section 4.

4. Properties of the Optimal Regulatory Policy.

With the reformulation given in section 3, the solution to [RP] can now be described. The main characterization results are presented in Propositions 1 and 2. Corollaries 1-5 and Lemmas 6 and 7 highlight the major implications of our findings, and serve to compare our conclusions with those cited in Lemmas 1-3. As noted above, our focus in the text is on the region of the state space, $D' \subseteq D$, in which output by the firm is strictly positive.

Before stating Proposition 1, the following definition is helpful.

$$\text{Definition. } k'(\varepsilon) \equiv \frac{\int_{\underline{\theta}}^{\bar{\theta}} \tilde{C}_Q(Q(r(\varepsilon), \theta), c(\varepsilon, \theta)) f(\theta, c(\varepsilon, \theta)) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} f(\theta, c(\varepsilon, \theta)) d\theta}.$$

In words, $k'(\varepsilon)$ is the expected marginal cost along the iso-price line with intercept ε .

Proposition 1. For $(\theta, c) \in D'$, the solution to [RP]' is given by:

$$r(\varepsilon) = k'(\varepsilon) + \varepsilon^C \frac{\int_{\underline{\theta}}^{\bar{\theta}} F(\theta, c(\varepsilon, \theta)) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} f(\theta, c(\varepsilon, \theta)) d\theta}, \quad (4.1)$$

provided $r'(\varepsilon) \geq 0 \quad \forall \varepsilon \geq c$.¹²

Proof. (4.1) defines necessary conditions for pointwise maximization of the objective function in [RP]'. It is obtained by differentiating $L(r(\varepsilon))$ (defined in (3.5)) with respect to $r(\varepsilon)$. From Lemma 5, these conditions are also sufficient when $C'''(q) = 0 \quad \forall q \geq 0$ provided $r'(\varepsilon) \geq 0$. ■

The monotonicity condition ($r'(\varepsilon) \geq 0$) in Proposition 1 states that the prices given by (4.1) must increase with ε . An obvious sufficient condition is that both terms on the right hand side of (4.1) increase with ε . The second term is a generalization of the hazard rate that is usually assumed to increase with c in models where only the realization of c is unknown to the regulator. (See, for example, Baron and Myerson [1982].) The requirements for monotonicity may be more restrictive here since the expected marginal cost term, $k'(\varepsilon)$, is also involved. Nevertheless, in the example we present below, it is straightforward to verify that $r'(\varepsilon) \geq 0 \quad \forall \varepsilon \geq c$. We will return in Proposition 2 to examine the solution to [RP] when the monotonicity condition is not satisfied in the solution given by (4.1).

Corollary 1. If the conditions of Proposition 1 hold and $c^C > 0$, then $r(\varepsilon) > k'(\varepsilon) \quad \forall \varepsilon > c$.

Corollary 1 points out a similarity between: (1) the optimal regulatory policy when the regulator is uncertain about both demand and cost; and (2) the corresponding policy with no demand uncertainty. (Recall Lemma 1.) To prevent the firm from exaggerating costs in both settings, the regulator will optimally set prices in excess of expected marginal cost. Though some surplus is sacrificed by doing so, there is a decrease in expected rents for the firm that is more than offsetting. The output reduction that results from the price increase reduces the difference in total production costs between a firm with low costs and one with high costs. Thus, the incentives for cost exaggeration are mitigated.

Another similarity between the solution to [RP] and the optimal regulatory policy when there is only a single source of uncertainty is evident in Corollary 2.

Corollary 2. Under the conditions of Proposition 1, $p(\mu, c) = \bar{C}_Q(Q(p(\mu, c), \theta), c)$.

Proof. Let $\tilde{\theta}(\varepsilon) = \text{minimum } \{\bar{\theta}, \theta^I(\varepsilon)\}$, where $\theta^I(\varepsilon) =$ the θ -intercept of the iso- $r(\varepsilon)$ loci.

Thus, $\int_{\underline{\theta}}^{\bar{\theta}} f(\theta, c(\varepsilon, \theta)) d\mu = \int_{\underline{\theta}}^{\tilde{\theta}(\varepsilon)} f(\theta, c(\varepsilon, \theta)) d\mu$. Now, taking the limit as $\varepsilon \rightarrow c$ of the last term in

(4.1), and using l'Hopital's rule, we have:

$$\lim_{\epsilon \rightarrow \underline{\epsilon}} \frac{\int_{\underline{\theta}}^{\bar{\theta}} F(\theta, c(\epsilon, \theta)) d\theta}{\int_{\underline{\theta}}^{\bar{\theta}} f(\theta, c(\epsilon, \theta)) d\theta} = \frac{\lim_{\epsilon \rightarrow \underline{\epsilon}} \frac{d}{d\epsilon} \int_{\underline{\theta}}^{\bar{\theta}(\epsilon)} F(\theta, c(\epsilon, \theta)) d\theta}{\lim_{\epsilon \rightarrow \underline{\epsilon}} \frac{d}{d\epsilon} \int_{\underline{\theta}}^{\bar{\theta}(\epsilon)} f(\theta, c(\epsilon, \theta)) d\theta} = \frac{\bar{\theta}'(\underline{\epsilon}) \int_{\underline{\theta}}^{\bar{\theta}(\underline{\epsilon})} f(\theta, \underline{c}) d\theta}{\bar{\theta}'(\underline{\epsilon}) \int_{\underline{\theta}}^{\bar{\theta}(\underline{\epsilon})} f(\theta, \underline{c}) d\theta} = 0. \quad \blacksquare$$

Corollary 2 reports that output will be supplied at the efficient level for the smallest demand and cost realization. The absence of distortions reflects the fact that there is no incentive for the firm to simultaneously understate costs, and claim that demand is at its lowest level.

However, the firm may be tempted to simultaneously exaggerate costs and understate demand. To mitigate such incentives, the regulator may find it optimal to establish a unit price *below* realized marginal cost. This will be true, for instance, in the following example.

Example.

Demand: $Q(p, \theta) = a + \theta - p$

Cost: $\tilde{C}(q, c) = K - [c_0 + c] q$

Beliefs: $f(\theta, c) = 1 \quad \forall \theta \in [0, 1], c \in [0, 1]$.

The key features of the example are the linear demand function, constant marginal cost, and uniform density. These relatively simple functional forms facilitate explicit calculation of the optimal regulatory policy. In keeping with our simplifying assumption about positive production levels, we presume that a is sufficiently large relative to K and c_0 that output will be strictly positive even for the smallest realizations of θ .

As the proof of Corollary 3 makes clear, there will be realizations of θ and c for which the optimal regulated price is strictly less than marginal cost in the solution to [RP] for the Example. This region is depicted in Figure 2.

Corollary 3. The solution to [RP] may have $p(\theta, c) < \tilde{C}_Q(Q(p(\theta, c), \theta), c)$ for some $(\theta, c) \in D$, even when $\tilde{C}_{qq}(q, c) \geq 0 \quad \forall q \geq 0$.

Proof. From Lemma 4, the slope (λ) of iso-price loci in the Example is given by $\lambda = -\frac{\delta^D}{\delta^C}$. If $\delta^D = \delta^C = 1$, then it follows from Proposition 1 that for $\varepsilon \leq 1$, $r(\varepsilon) = c_0 + \frac{\varepsilon}{2} [1 + \frac{1}{2\sqrt{2}}]$. Hence, we will have $r(\varepsilon) < c_0 + c$ provided $c > \frac{\varepsilon}{2} [1 + \frac{1}{2\sqrt{2}}]$ for $\varepsilon \leq 1$.

Similar calculations reveal that for $\varepsilon \in (1, 2]$, $r(\varepsilon) < c_0 + c$ provided $c \in (0, 1)$ and $c > \frac{\varepsilon}{2} + \frac{1 - .5[2-\varepsilon]^2}{\sqrt{2}[2-\varepsilon]}$. Straightforward but tedious algebra then reveals that the remainder of the state space can be divided as in Figure 2. ■

Corollary 3 may be surprising because it reports a conclusion that is qualitatively different from those reported in Lemmas 1 and 2. When only costs are unknown to the regulator, the regulated price optimally exceeds marginal cost. When only demand is unknown to the regulator and marginal costs increase with output, the optimal regulated price is equal to realized marginal cost. But when both demand and cost are unknown, the regulated price will generally be *below* marginal cost in some ranges. Thus, the conjunction of two sources of uncertainty results in distortions that neither source alone would provide.

The reported distortion may be optimal to reduce the incentives for the firm to understate demand.¹³ Recall from the discussion on page 11 that the regulator can discourage cost exaggeration by setting the regulated price (p) in excess of realized marginal cost, and lowering the associated transfer payment (T) to offset any increase in revenues resulting from the price increase. Note, however, that if T is lowered to offset the revenue increase based upon a belief that the quantity demanded is low, then the "offsetting" changes in p and T can enhance the

firm's profits when demand is actually high. Consequently, in an attempt to control cost exaggeration in this manner, the regulator may actually invite the understatement of demand.

When price is set below marginal cost and the transfer payment raised, the opposite effect results. Now, the "offsetting" variations in p and T reduce the firm's actual profits by a greater amount the higher is realized demand, because the loss from sales at a price below cost is greater the larger the number of units sold. Thus, by setting price below marginal cost when low demand realizations are reported, the regulator can limit the firm's gains from understating demand.

Whether price is set above, below, or equal to marginal cost, therefore, depends upon the probability and likely consequences of demand and cost misrepresentation. As is the case illustrated in Figure 2, price is more likely to be set below marginal cost when θ is low, *ceteris paribus*, because the smaller is θ , the more likely are demand realizations in excess of θ thought to be, and thus the more likely it is that demand understatement must be deterred.

Two other features of the optimal regulatory policy warrant brief mention. First, the regulated price must not decrease as θ or c increase. Thus, the higher are costs or demand, the higher the regulated price.

Lemma 6. In the solution to [RP], $p_1(\theta, c) \geq 0$ and $p_c(\theta, c) \geq 0 \quad \forall (\theta, c) \in D$.

This conclusion points to another similarity between the optimal regulatory policies when there are one and two sources of uncertainty. Recall from Lemmas 1-3 that the regulated price must increase with the realization of the single random parameter, whether it affects the demand or the cost function. When there are two unknown parameters, one affecting demand and the other costs, the optimal price increases with the realization of each.

The second feature of the optimal regulatory policy that remains to be analyzed is the pattern

of rents that accrue to the firm. The pattern is readily characterized when marginal costs decline with output. In this case, the difference between price and marginal cost increases the higher the realization of the demand parameter, θ .

Lemma 7. Suppose $C''(q) \leq 0 \quad \forall q \geq 0$. Then

$$\frac{d}{d\theta} \left\{ p(\theta, c) - \tilde{C}_Q(Q(p(\theta, c), \theta), c) \right\} \geq 0 \quad \forall (\theta, c) \in D \text{ in the solution to [RP]}.$$

Proof.
$$\frac{d}{d\theta} \left\{ p(\theta, c) - \tilde{C}_Q(Q(p(\theta, c), \theta), c) \right\}$$

$$= p_\theta(\cdot) \left[1 - C''(\cdot) Q_p(\cdot) \right] - C''(\cdot) Q_\theta(\cdot) \geq 0 \quad \text{for } C''(\cdot) \leq 0. \quad \blacksquare$$

This observation is instrumental in showing that the state space can be divided into regions where price exceeds, is equal to, and falls short of marginal cost, as in Figure 2. Given this division, it is not difficult to demonstrate that the firm's profits are minimized at $(\theta^0, \bar{c})$, where costs are at their highest level, and where price and marginal cost are equated.

Corollary 4. Suppose $C''(q) \leq 0 \quad \forall q \geq 0$, and θ^0 is defined by $p(\theta^0, \bar{c}) = \tilde{C}_Q(Q(p(\theta^0, \bar{c}), \theta^0), \bar{c})$. Then in the solution to [RP], $\pi(\theta^0, \bar{c}) = 0$.

Proof. The local incentive compatibility (IC) constraints imply

$$\pi_\theta(\theta, c) = \delta^D \left[p_\theta(\cdot, c) - \tilde{C}_Q(Q(\cdot, c), c) \right], \quad \text{and} \quad (4.2)$$

$$\pi_c(\theta, c) = -\delta^C Q(p(\cdot, c), \theta). \quad (4.3)$$

Therefore, the firm's profits decrease with c and increase with θ when price exceeds marginal cost. Profits also increase as θ falls when price is less than marginal cost. Using Lemma 7, the implicit function theorem (e.g., Takayama [1985]) implies that there exists a function $\hat{\theta}(c) \quad \forall c \in [c, \bar{c}]$ such that $p(\theta, c) = \tilde{C}_Q(Q(p(\theta, c), \theta), c)$ where θ is evaluated $\hat{\theta}(c)$. Hence, the

state space can be divided as shown in Figure 2, and profits are minimized at $(\bar{\theta}(\bar{c}), \bar{c}) \equiv (\theta^0, \bar{c})$. ■

Corollary 4, then, reflects the natural combination of Lemmas 1 and 3. As in Lemma 1, profits are minimized for the highest cost realization. And, as in Lemma 3, the individual rationality constraint generally binds for an intermediate demand realization, rather than one of the extreme realizations. Note, though, that with both demand and cost uncertainty present, the individual rationality constraint will generally not bind for all (θ, c) realizations, even when marginal production costs are constant. Thus, a divergence from Lemma 2 arises.¹⁴

When marginal costs increase with output, a complete characterization of the pattern of rents for the firm is more problematic. In this case, the difference between price and marginal cost need not increase with θ , so the neat division of the state space shown in Figure 2 is not guaranteed. Nevertheless, (4.2) and (4.3) continue to hold, so the firm's profits are minimized at $\bar{c}$, and they increase with θ when price exceeds marginal cost, and otherwise fall with θ .¹⁵

What remains is to determine the solution to [RP] when the right hand side of equation (4.1) is not everywhere increasing in ε . This concern is particularly relevant when marginal production costs decline with output. In this case, it is straightforward to demonstrate that for δ^C sufficiently small, the right hand side of (4.1) decreases as ε increases, so the relevant monotonicity condition will be violated.

In general, then, the sign of $r'(\varepsilon)$ defined by (4.1) is indeterminant. In particular, there may be intervals over which this "unconstrained" price schedule, $p^*(\varepsilon) = r(\varepsilon)$ defined by (4.1), is declining, as illustrated in Figure 3. Figure 3 also illustrates the price schedule, $p^*(\varepsilon)$, that maximizes $L(\cdot)$ in equation (3.5) subject to the constraint that $p^*(\varepsilon) \geq \forall \varepsilon \geq \underline{\varepsilon}$. $p^*(\varepsilon)$ has the following properties: (1) whenever $p^*(\varepsilon)$ is strictly increasing, it coincides with the unconstrained solution from (4.1), $p^*(\varepsilon)$; (2) $p^*(\varepsilon)$ is constant whenever the unconstrained solution, $p^*(\varepsilon)$, is

declining; and (3) over intervals where $p^*(\varepsilon)$ is constant, its level is chosen to maximize expected social returns. This third property reflects the natural counterpart to the pointwise maximization that underlies Proposition 1; but when pointwise maximization would call for the price to fall as ε increases (which violates the incentive compatibility (IC) constraints in [RP]), the regulator must resort to selecting the best level of p^* over the relevant interval.

These conclusions are summarized in Proposition 2.

Proposition 2. Let $p^*(\varepsilon)$ represent the price schedule, $r(\varepsilon)$, that solves (4.1). Then the solution to [RP]', $p^*(\varepsilon)$, has the following properties:

- (1) $p^*(\varepsilon) = p^*(\varepsilon)$ whenever $p^{**}(\varepsilon) > 0$;
- (2) $p^*(\varepsilon)$ is constant over intervals where $p^{**}(\varepsilon) < 0$; and
- (3) If $p^*(\varepsilon)$ is constant over the interval $(\varepsilon_1, \varepsilon_2)$, then

$$\int_{\varepsilon_1}^{\varepsilon_2} [p^*(\varepsilon) - \tilde{C}_Q(Q(p^*(\varepsilon), \theta), c(\varepsilon, \theta)) - \delta^C \frac{F(\theta, c(\varepsilon, \theta))}{f(\theta, c(\varepsilon, \theta))} Q_p(p^*(\varepsilon), \theta) f(\theta, c(\varepsilon, \theta))] d\theta = 0.$$

A corollary of Proposition 2 is that when marginal production costs are known to decline with output, and when the firm has private information about both costs and demand, some delegation of pricing authority to the firm may optimally be restored. In other words, unlike the case where the only uncertainty is about demand (as summarized in Lemma 3), the regulator will not necessarily rely solely on his own prior beliefs to set a single price. Instead, the menu of two-part tariffs offered to the firm may consist of more than a singleton.

As noted above, $p^{**}(\varepsilon)$ will be everywhere declining as cost uncertainty becomes negligible when marginal production costs decline with output. Thus, in this case, $p^*(\varepsilon)$ will be everywhere constant, as Lemma 3 would predict. The solution to [RP] will also converge to the solution reported in Lemma 2 as δ^C approaches zero when marginal costs increase with output. Note

from Lemma 4 that iso-price lines become vertical as $\delta^C \rightarrow 0$; and from (4.1) price is set equal to expected marginal cost along the iso-price line. With vertical lines, $k'(\varepsilon) = \tilde{C}_Q(Q(r(\varepsilon), \theta), c(\varepsilon, \theta))$, so price is established at the level of marginal cost.

Similarly, as $\delta^D \rightarrow 0$, the solution to [RP] approaches the solution identified in Lemma 1. In this case, iso-price lines become horizontal (from Lemma 4), indicating that the optimal price depends only on the realization of c , and not the realization of θ . And with price set in excess of marginal cost $\forall c > \underline{c}$ as indicated by (4.1), the same qualitative distortions identified in properties (i) and (ii) of Lemma 1 emerge.¹⁶ These observations are summarized in Corollary 5.

Corollary 5. (i) As $\delta^D \rightarrow 0$, the properties of the solution to [RP] approach those recorded in Lemma 1. (ii) When $C_{qq}(q) \geq 0 \quad \forall q \geq 0$, the properties of the solution to [RP] approach those recorded in Lemma 2 as $\delta^C \rightarrow 0$. (iii) When $C_{qq}(q) < 0 \quad \forall q \geq 0$, the properties of the solution to [RP] approach those recorded in Lemma 3 as $\delta^C \rightarrow 0$.

This concludes our discussion of the major properties of the solution to [RP]. A brief recapitulation and some thoughts on related lines of inquiry are offered in section 5.

5. Conclusions.

We have characterized the optimal regulatory policy when the regulator is uncertain about both the firm's demand function and its cost function. Some of our major findings can be summarized as follows. First, the optimal regulated price will be higher the higher are the firm's costs and the more favorable is demand. Second, distortions from efficient production levels will generally be introduced to control the firm's rents. Full efficiency will only occur in the special case where cost uncertainty is negligible and marginal costs of production are everywhere increasing with output. More generally, regulated prices will optimally be established in excess of expected marginal cost, but prices may either exceed or fall short of realized marginal cost.

Prices will tend to be below marginal cost for small realizations of demand, and above marginal cost for large demand realizations.

Third, the regulated firm will generally be afforded considerable flexibility in setting prices. Decision-making is thereby delegated to the party with the superior information about demand and costs. The flexibility afforded the firm will be less pronounced when marginal costs decline with output. Fourth, the firm's rents are minimized for the highest cost realization and, generally, for an intermediate realization of demand, where price and marginal cost are equated. Consequently, if the firm is shut down, production will generally be terminated for intermediate demand realizations, and in regions where the firm would otherwise be supplying output at efficient levels.

The two-dimensional uncertainty we considered concerned the position of the firm's demand and cost functions. Other characterizations of multidimensional cost and demand uncertainty are possible, of course. For example, the regulator might know the level of the constant marginal cost of production, but be uncertain about both the requisite level of fixed costs of production and the realized demand curve. It is not difficult to show that in this case, the conclusions of Lemma 2 hold, so that efficient prices are instituted and the firm earns no rents. Thus, the price distortions reported in Corollaries 1 and 3, for example, do not arise when the cost and demand uncertainties are "separable", i.e., when there is no uncertainty about variable production costs.

We note in closing that although our analysis assumes the regulator has less precise knowledge of the regulatory environment than he is often assumed to have, the regulator is not completely uninformed by any means. By reducing uncertainty about demand and cost to a single dimension in this Bayesian setting, the regulator is implicitly endowed with significant, detailed knowledge of the environment. An important line of research that remains is to examine the properties of optimal regulatory policy when the regulator (and perhaps the firm, as well) has

even more limited knowledge of the regulatory environment.

Footnotes

1. To illustrate, consider the following conclusion of the California Public Utility Commission's Policy and Planning Division, in their report "Risk, Return and Ratemaking" (October 1, 1986): "The commission must evaluate ... arguments with minimal information about a customer's economic situation. Utilities have greater manpower and closer contact with customers, and can probably make better judgements about the conditions customers face."
2. For details, see Baron and Myerson [1982]
3. For details, see Lewis and Sappington [1987].
4. Obviously, our characterization of the two-dimensional information asymmetry is not the most general. The two sources of uncertainty we consider pertain to the positions, but not the shapes, of the demand and cost functions. Though more generality might be desirable, it comes at the expense of considerable mathematical complexity. In particular, the solution generated by considering only the local representation of the incentive compatibility constraints will not necessarily satisfy the global incentive compatibility constraints. (See Lemma 5 below.)
5. In this interpretation, we assume that demand is unaffected by the fixed-fee component of the two-part tariff.
6. It would be straightforward, though notationally cumbersome, to extend the analysis to allow the regulator to maximize a weighted average of consumers' surplus and profits. Our qualitative conclusions would be unaltered by this extension, provided the weight the regulator places on consumers' surplus exceeds the corresponding weight on profits.

7. The revelation principle (e.g., Myerson [1979]) establishes the optimality of this representation of the regulator's problem.
8. For simplicity, Lemmas 1-3 (like most of the ensuing analysis) abstract from shutdown considerations. Baron and Myerson [1982] show that under the conditions of Lemma 1, the firm may be shut down with probability one for $c \in [\underline{c}, \bar{c}]$ where $\bar{c} \in [\underline{c}, \bar{c}]$ even though total surplus is strictly positive from continued operation at $p(c) = c \quad \forall c \in [\underline{c}, \bar{c}]$. Rochet [1984] shows that when the regulator is uncertain about both the realized marginal cost and fixed cost of production, the optimal shutdown rule may involve randomization. We do not consider random shutdown rules here.
9. Lewis and Sappington [1987] actually show that this conclusion holds more generally. In particular, the information asymmetry need not apply only to the position of the demand curve for the asymmetry to be inconsequential for the regulator.
10. If $C'''(q) = 0 \quad \forall q \geq 0$, then equation (3.2), below, must be replaced by $c = \varepsilon + \int_{\varepsilon}^{\theta} \lambda(p, \tilde{\theta}) d\tilde{\theta}$, where p identifies the price under consideration. The ensuing calculations can then be carried out for the case of non-linear iso-price loci. The major qualitative change that results when $C'''(\cdot) \neq 0$ is that the incentive compatibility constraints do not necessarily hold globally when they are satisfied locally. (See Lemma 5 below.)
11. The term $\pi(\theta, \bar{c})$ is not reflected in $L(r(\varepsilon))$ because it does not contain the critical price variable, $r(\varepsilon)$.
12. The possibility exists that for sufficiently small demand realizations, no output will be demanded at the optimal regulated price. To account for this possibility, the θ in the lower

limits of the integrals in (4.1) would be replaced by $\theta^m(\varepsilon)$, which is the maximum value of θ for which $Q(r(\varepsilon), \theta) = 0$ in the solution to [RP]' for the identified price, $r(\varepsilon)$.

Furthermore, it is not difficult to show that as ε approaches infinity, the last term on the right hand side of (4.1) does also. Therefore, the firm will not produce in a neighborhood of $(\bar{\theta}, \bar{c})$ if there is a finite price at which demand falls to zero $\forall \theta$.

13. Lemma 2 reveals (and Lewis and Sappington [1987] explain in more detail) that the firm has no incentive to understate demand when costs are known to the regulator.
14. Note that the optimal regulated price may induce no demand for certain extreme realizations of θ (recall footnote 12). However, the firm may still receive rents even though it does not produce.
15. Corollary 4 has some interesting implications for shutdown. Recall that shutdown of the firm may be imposed for some realizations of demand and cost because shutdown can be an effective means of limiting the firm's rents. With θ^0 interior, shutdown of the firm will be particularly costly for two reasons. First, the regulator cannot terminate operations for the smallest demand realizations (where shutdown might be socially desirable) without cutting off production for intermediate levels of demand also. Second, shutdown will occur in regions where output would otherwise be supplied efficiently. Hence, shutdown of the firm may be quite costly; and so the firm may be allowed to produce under circumstances for which shutdown would be directed by the regulator if he shared the firm's private demand and cost information.
16. Actually, a stronger result is available. It is straightforward to demonstrate that as $\delta^C \rightarrow 0$, the magnitude of the price distortions in the solution to [RP] approaches that identified by Baron and Myerson [1982].

Figure 1. Iso-price loci, $r(\epsilon)$.

Figure 2. Price - marginal cost margins for the Example.

$$\textcircled{A} = \left\{ (\theta, c) \in D \mid p(\theta, c) < \bar{C}_Q(Q(p(\theta, c), \theta), c) \right\}$$

$$\textcircled{B} = \left\{ (\theta, c) \in D \mid p(\theta, c) > \bar{C}_Q(Q(p(\theta, c), \theta), c) \right\}$$

$$\tilde{\theta}(c) = D - \textcircled{A} \cup \textcircled{B}$$

Figure 3. Constrained, $p^*(\varepsilon)$, and unconstrained, $p^u(\varepsilon)$, prices.

Appendix

Proof of Lemmas 4 and 6.

The local incentive compatibility conditions require

$$\tilde{\pi}_1(\theta, c | \theta, c) = \tilde{\pi}_2(\theta, c | \theta, c) = 0, \quad (\text{A1})$$

where $\tilde{\pi}_i(\cdot)$ represents the partial derivative of $\tilde{\pi}(\cdot)$ with respect to its i th argument, $i = 1, \dots, 4$.

They also require:

$$\tilde{\pi}_{11}(\theta, c | \theta, c) \leq 0 \text{ and } \tilde{\pi}_{22}(\cdot) \leq 0. \quad (\text{A2})$$

Since (A1) must hold $\forall (\theta, c) \in D$, we have

$$\tilde{\pi}_{11}(\cdot) - \tilde{\pi}_{13}(\cdot) = 0. \quad (\text{A3})$$

Hence, since $\tilde{\pi}_{13}(\cdot) = \Pi_{p^*}(\cdot) p^*(\cdot)$, we have from (A3)

$$p^*(\cdot) = - \frac{\tilde{\pi}_{11}(\cdot)}{\Pi_{p^*}(\cdot)} \geq 0. \quad (\text{A4})$$

The inequality in (A4) follows from (A2) and the fact that $\Pi_{p^*}(\cdot) = \delta^D [1 - \tilde{C}_{QQ}(\cdot) h'(p)] > 0$ for $\delta^D > 0$.

Repeating these same arguments reveals that

$$p_c(\cdot) = - \frac{\tilde{\pi}_{22}(\cdot)}{\Pi_{p_c}(\cdot)} \geq 0, \quad (\text{A5})$$

where the inequality follows from (A2) and the fact that $\Pi_{p_c}(\cdot) = -\delta^C h'(p) > 0$ for $\delta^C > 0$.

Finally the local incentive compatibility conditions also require:

$$\frac{d}{dc} \tilde{\pi}_1(\cdot) = \frac{d}{d\theta} \tilde{\pi}_2(\cdot) = 0.$$

These conditions imply

$$\Pi_{p_c}(\cdot) p_s(\cdot) = \tilde{\pi}_{14}(\cdot) = \tilde{\pi}_{23}(\cdot) = \Pi_{p_s}(\cdot) p_c(\cdot). \quad (\text{A6})$$

(A6) implies that the slope of an iso-price locus is given by:

$$\left. \frac{dc}{d\theta} \right|_{dp=0} = - \frac{p_s(\cdot)}{p_c(\cdot)} = - \frac{\Pi_{p_s}(\cdot)}{\Pi_{p_c}(\cdot)}.$$

The last equality in the statement of Lemma 4 follows immediately from straightforward differentiation of $\Pi(\cdot)$. ■

Proof of Lemma 5.

Our problem satisfies the generalized single crossing property of McAfee and McMillan [1987], and so Lemma 5 follows from their analysis. However, to keep our analysis self-contained and to suggest an alternative proof technique, we proceed as follows.

We need to show that

$$\tilde{\pi}(\theta, c | \theta, c) \geq \tilde{\pi}(\hat{\theta}, \hat{c} | \theta, c) \quad \forall (\theta, c), (\hat{\theta}, \hat{c}) \in D, \quad (\text{A7})$$

given that the incentive compatibility constraints hold locally.

To begin, define:

$$\theta^\alpha = \alpha \hat{\theta} + [1 - \alpha] \theta, \text{ and}$$

$$c^\alpha = \alpha \hat{c} + [1 - \alpha] c.$$

Then we have

$$\tilde{\pi}(\hat{\theta}, \hat{c} | \theta, c) = \tilde{\pi}(\theta, c | \theta, c) + \int_0^1 \frac{\partial \tilde{\pi}(\theta^\alpha, c^\alpha | \theta, c)}{\partial \alpha} d\alpha, \quad (\text{A8})$$

where

$$\frac{\partial \tilde{\pi}(\theta^0, c^0 | \theta, c)}{\partial \alpha} = \tilde{\pi}_1(\theta^0, c^0 | \theta, c) [\hat{\theta} - \theta] + \tilde{\pi}_2(\theta^0, c^0 | \theta, c) [\hat{c} - c] , \quad (A9)$$

again letting $\tilde{\pi}_i(\cdot)$ represent the partial derivative of $\tilde{\pi}(\cdot)$ with respect to its i th argument.

Note that

$$\tilde{\pi}_1(\theta^0, c^0 | \theta, c) = \tilde{\pi}_1(\theta^0, c^0 | \theta^\lambda, c^\lambda) + \int_0^1 \frac{\partial}{\partial \lambda} [\tilde{\pi}_1(\theta^\lambda, c^\lambda | \theta^\lambda, c^\lambda)] d\lambda , \quad \text{and} \quad (A10)$$

$$\tilde{\pi}_2(\theta^0, c^0 | \theta, c) = \tilde{\pi}_2(\theta^0, c^0 | \theta^\lambda, c^\lambda) + \int_0^1 \frac{\partial}{\partial \lambda} [\tilde{\pi}_2(\theta^\lambda, c^\lambda | \theta^\lambda, c^\lambda)] d\lambda , \quad (A11)$$

where $\theta^\lambda = \lambda \theta + [1 - \lambda] \theta^0$,

$c^\lambda = \lambda c + [1 - \lambda] c^0$,

$$\frac{\partial}{\partial \lambda} [\tilde{\pi}_1(\theta^\lambda, c^\lambda | \theta^\lambda, c^\lambda)] = \tilde{\pi}_{13}(\cdot) [\theta - \theta^0] + \tilde{\pi}_{14}(\cdot) [c - c^0] , \quad \text{and} \quad (A12)$$

$$\frac{\partial}{\partial \lambda} [\tilde{\pi}_2(\theta^\lambda, c^\lambda | \theta^\lambda, c^\lambda)] = \tilde{\pi}_{23}(\cdot) [\theta - \theta^0] + \tilde{\pi}_{24}(\cdot) [c - c^0] . \quad (A13)$$

Also recall from the definitions of $\tilde{\pi}(\hat{\cdot}, \hat{c} | \theta, c)$ and $\Pi(p, \theta, c) \equiv pQ(p, \theta) - \bar{C}(Q(p, \theta), c) - T$ that $\tilde{\pi}_{13}(\cdot) = \Pi_{p\theta}(\cdot) p_\theta(\cdot)$, $\tilde{\pi}_{14}(\cdot) = \Pi_{pc}(\cdot) p_\theta(\cdot)$, $\tilde{\pi}_{23}(\cdot) = \Pi_{p\theta}(\cdot) p_c(\cdot)$, and $\tilde{\pi}_{24}(\cdot) = \Pi_{pc}(\cdot) p_c(\cdot)$.

Substituting these four relationships into (A12) and (A13), (A10) and (A11) can be rewritten as:

$$\tilde{\pi}_1(\theta^0, c^0 | \theta, c) = \int_0^1 \{ \Pi_{p\theta} p_\theta [\theta - \theta^0] + \Pi_{pc} p_\theta [c - c^0] \} d\lambda , \quad \text{and} \quad (A14)$$

$$\tilde{\pi}_2(\theta^0, c^0 | \theta, c) = \int_0^1 \{ \Pi_{p\theta} p_c [\theta - \theta^0] + \Pi_{pc} p_c [c - c^0] \} d\lambda , \quad (A15)$$

where we have made use of the fact that

$$\tilde{\pi}_1(\theta^0, c^0 | \theta^0, c^0) = \tilde{\pi}_2(\theta^0, c^0 | \theta^0, c^0) = 0$$

from the local incentive compatibility conditions. Notice that with $C'''(q) = 0 \quad \forall q \geq 0$, $\Pi_{p\theta}(\cdot)$ and $\Pi_{pc}(\cdot)$ depend only on the regulated price, p , and thus only on the reported realizations of θ

and c .

Now, substituting (A14) and (A15) into (A9), we have from (A8):

$$\begin{aligned} \hat{\pi}(\hat{\theta}, \hat{c} | \theta, c) &= \hat{\pi}(\theta, c | \theta, c) + \int_0^1 \int_0^1 \{ \Pi_{p^*} p^* [\theta - \theta^0] + \Pi_{p_c} p_c [c - c^0] \} [\hat{\theta} - \theta] d\lambda d\alpha \\ &+ \int_0^1 \int_0^1 \{ \Pi_{p^*} p^* [\theta - \theta^0] + \Pi_{p_c} p_c [c - c^0] \} [\hat{c} - c] d\lambda d\alpha. \end{aligned} \quad (A16)$$

As the proof of Lemma 4 reveals, the local incentive compatibility conditions (LIC) imply

$$\Pi_{p^*} p_c = \Pi_{p_c} p^*. \quad (A17)$$

Using (A17), (A16) can be rewritten as:

$$\hat{\pi}(\hat{\theta}, \hat{c} | \theta, c) = \hat{\pi}(\theta, c | \theta, c) + \int_0^1 \int_0^1 \{ [\hat{\theta} - \theta] \Pi_{p^*} + [\hat{c} - c] \Pi_{p_c} \} \{ [\theta - \theta^0] p^* + [c - c^0] p_c \} d\lambda d\alpha. \quad (A18)$$

Finally, since $\Pi_{p_c} = \Pi_{p^*} p_c / p^*$ from (A17), and using the facts that $\theta - \theta^0 = -\alpha [\hat{\theta} - \theta]$ and $c - c^0 = -\alpha [\hat{c} - c]$, (A18) can be rewritten as:

$$\hat{\pi}(\hat{\theta}, \hat{c} | \theta, c) = \hat{\pi}(\theta, c | \theta, c) - \alpha \int_0^1 \int_0^1 \frac{\Pi_{p^*}}{p^*} \{ [\hat{\theta} - \theta] p^* + [\hat{c} - c] p_c \}^2 d\lambda d\alpha. \quad (A19)$$

Therefore, since $\Pi_{p^*}/p^* > 0$ when $\delta^C > 0$ from the (LIC) conditions (recall the proof of Lemma 4), (A19) guarantees that (A7) holds. ■

Proof of Proposition 2.

The regulator's problem is to

$$\text{Maximize}_{r(\varepsilon)} L(r(\varepsilon))$$

$$\text{subject to: } r'(\varepsilon) \geq 0$$

where $L(\cdot)$ is as defined in (3.5).

Letting $r'(\varepsilon)$ be the control variable, the Hamiltonian, $H(\varepsilon)$, associated with the regulator's problem is:

$$H(\varepsilon) = V(r(\varepsilon)) + \xi r'(\varepsilon) + \lambda r'(\varepsilon), \quad (\text{A20})$$

where ξ is the costate variable, λ is the multiplier associated with the constraint that $r'(\varepsilon) \geq 0$,

and $L(r(\varepsilon)) \equiv \int_{\varepsilon}^{\infty} V(r(\varepsilon)) d\varepsilon$. The state variable is $p^*(\varepsilon) \equiv r(\varepsilon) = r(\varepsilon) + \int_{\varepsilon}^{\infty} r'(\varepsilon) d\varepsilon$. Necessary

conditions for a solution to the regulator's problem include:

$$\frac{\partial H(\cdot)}{\partial r'(\varepsilon)} = \xi(\varepsilon) + \lambda(\varepsilon) = 0; \quad (\text{A21})$$

$$\xi'(\varepsilon) = -\frac{\partial H(\cdot)}{\partial r(\varepsilon)} = -\frac{\partial V(r(\varepsilon))}{\partial r(\varepsilon)}; \text{ and} \quad (\text{A22})$$

$$\lambda(\varepsilon) r'(\varepsilon) = 0. \quad (\text{A23})$$

If $r'(\varepsilon) > 0$, then $\lambda = 0$ from (A23), and thus $\xi(\varepsilon) = 0$ from (A21). Hence, $\xi'(\varepsilon) = 0$, and condition (1) of the proposition follows from (4.1) and from (A22).

If $p^{**}(\varepsilon) < 0$, then we cannot have $r'(\varepsilon) = p^{**}(\varepsilon)$. But if $r'(\varepsilon) > 0$, then $r'(\varepsilon) = p^*(\varepsilon)$, from the preceding argument. Hence, $r'(\varepsilon) = 0$, proving condition (2) of the proposition.

Now suppose the constraint that $r'(\varepsilon) \geq 0$ is binding over some interior interval $(\varepsilon_1, \varepsilon_2)$ where $0 < \varepsilon_1 < \varepsilon_2 < \bar{\varepsilon} \equiv \bar{c} - \lambda[\bar{\theta} - \underline{\theta}]$. Then with $\lambda(\varepsilon_1) = \lambda(\varepsilon_2) = 0$, $\xi(\varepsilon_1) = \xi(\varepsilon_2) = 0$ from (A21). Hence, from (A22)

$$\xi(\varepsilon_2) = \xi(\varepsilon_1) - \int_{\varepsilon_1}^{\varepsilon_2} \xi'(\varepsilon) d\varepsilon = 0 - \int_{\varepsilon_1}^{\varepsilon_2} \frac{\partial V(r(\varepsilon))}{\partial r(\varepsilon)} d\varepsilon;$$

or $\int_{\varepsilon_1}^{\varepsilon_2} \frac{\partial V(r(\varepsilon))}{\partial r(\varepsilon)} d\varepsilon = 0$, which proves condition (3) of the proposition for cases where $\varepsilon_1 > \varepsilon$ and

$\epsilon_2 < \bar{\epsilon}$. The proof for other cases is analogous. ■

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