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DYNAMICS OF HOUSING INVESTMENT IN A
PERFECT FORESIGHT MACRO MODEL

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Expectations of capital gains (losses) have been recognized as important determinants of the demand for capital assets and, in particular, housing. Within the context of a macro model, we study the dynamic paths of housing investment along which expectations of capital gains are fulfilled.

We concentrate on housing as an example of capital accumulation primarily for expositional ease. The flow of new housing is small relative to the existing stock and housing is a relatively homogeneous capital good so that the "supply of investment" approach may be legitimately employed. This allows a clear separation between the production of new housing and the demand for the existing stock.

The analysis is restricted to dynamic paths along which changes in the price of housing are perfectly anticipated. The basic logic of this approach is to examine what types of price paths are possible with fully rational actors. If behavior is circumscribed by the assumption of rationality, then testable econometric propositions are implied. On the other hand, if rational behavior does not delimit the price paths, then we know that a wide class of behavior is consistent with rational behavior and there is no necessity, a priori, to introduce other assumptions into our models. The case where expectations of capital gains are formed adaptively has been analyzed by Foley and Sidrauski [1971].

Within a macro model, it is possible to trace the effects of macro policies on the rate of housing investment; for example, the short and long run effects of a government purchase of a fraction of the housing stock from the public may be analyzed. This example and others will be discussed below. Fiscal policy instruments will be chosen to keep the price of money constant in order to concentrate on the effects of anticipated and unanticipated disturbances that influence housing investment. This simplifies the interactions between the production of new housing and the requirements of portfolio equilibrium. A related dynamic analysis of capital accumulation is presented in Abel [1977]. In his model, the demand for capital is derived from an explicit model of intertemporal profit maximizing behavior by firms but abstracts from portfolio consideration.

The first section of the paper details the requirements of asset, goods and rental market equilibrium in addition to presenting the dynamic equations. We show how fiscal policy can be manipulated to simplify the analysis of perfect foresight paths for housing investment.

The second section investigates housing investment following unanticipated shocks and the announcement of future disturbances. The effects of typical macro policies are also outlined. In particular, we analyze a government purchase of part of the housing stock and an increase in the steady-state government deficit.

## II. The Macro Model[1]

We will develop the macro model by examining equilibrium in three markets: rental, assets and goods markets and specifying dynamic relations governing the growth of nominal money and housing.

### A. The Rental Market

At any point of time, the stock of existing housing, $H_t$, is given. Let the services be proportional to the existing stock and units chosen so that one unit of housing provides one unit of service. The demand for housing services $S^d(r,\alpha)$ is a decreasing function of the rental rate, r, and an increasing function of a shift parameter $\alpha$ representing tastes. In equilibrium, the rental rate adjusts to equilibrate the market for services. Thus,

$$r = r(H,\alpha) \qquad r_1<0 \qquad r_2>0 \tag{1}$$

B. The Assets Market

There are two assets in the model, housing and money. The price of consumption goods will be taken to be the numeraire and $P_h$ and $P_m$ will be the price of a unit of housing and money respectively. Demand for assets will be a function of the real rates of return on assets and wealth. The instantaneous expected real rate on housing is the rental yield plus expected capital gains while the anticipated rate of deflation or anticipated increase in the price of money is the anticipated yield on money.

We can write the asset market equilibrium conditions as:

$$P_h H = H^d\left[\frac{r(H,\alpha)}{P_h} + \frac{\dot{P}_h^{\varepsilon}}{P_m}, \frac{\dot{P}_m^{\varepsilon}}{P_m}, P_h H + P_m M\right] \qquad (2)$$

$$H_1^d > 0 \qquad H_2^d < 0 \qquad 1 > H_3^d > 0$$

$$P_m M = M^d\left[\frac{r(H,\alpha)}{P_h} + \frac{\dot{P}_h^{\varepsilon}}{P_h}, \frac{\dot{P}_m^{\varepsilon}}{P_m}, P_h H + P_m M\right] \qquad (3)$$

$$M_1^d < 0 \qquad M_2^d > 0 \qquad 1 > M_3^d > 0$$

By Walras Law of Stocks, total asset demand must sum to total wealth so that there is only one independent asset market equilibrium condition. Our assumptions on assets demands are sufficient to insure that along a locus of points on which the asset market clears $\frac{\partial P_h}{\partial P_m} > 0$. An increase in the price of capital creates excess supply by increasing supply and lowering the rate of return. The price of money must rise to increase wealth to restore equilibrium.

C. Goods Market

New housing, NH, and consumption goods, C, are produced by a two sector technology. An increase in the relative price of housing increases new housing production and decreases production of consumption goods.

Private demand for consumption goods depends on both wealth and disposable income. disposable income equals national income (evaluated in consumption goods) plus the deficit minus government spending. Government purchases only consumption goods denoted by g. The equilibrium condition for the goods market is:

$$C^{s}(P_h) = C^{d}(P_m M + P_h H,\ P_h NH + C + d - g) + g \tag{4}$$

$$C_1^d > 0 \qquad 1 > C_2^d > 0 \qquad C^{s'}(P_h) < 0$$

An increase in the price of housing creates excess demand in the goods market. As the price of housing increases, supply decreases while both wealth and disposable income rise. To restore equilibrium in the goods market, the price of money must fall to reduce the real value of nominal wealth. Thus, along a locus of points where the consumption market clears, $\frac{\partial P_H}{\partial P_m} < 0$.

At any moment of time, equations (2) and (4), the asset and goods market equilibrium determine the price of housing and price of money as illustrated in Figure 1.

There are two fiscal policy instruments in the model, the deficit and the level of spending. In the standard manner, increases in either d or g are expansionary and require, for any price of housing, a lower price of money. Thus, both policies will shift the goods market locus to the left. It is important to note that we can set the deficit at any level we choose and still shift the goods market locus to maintain any price of money we desire. In particular, it will be convenient to analyze policies when balanced budget fiscal policy keeps $P_m$ constant.

D. Dynamics

The stock of housing and the outstanding stock of money evolve over time.

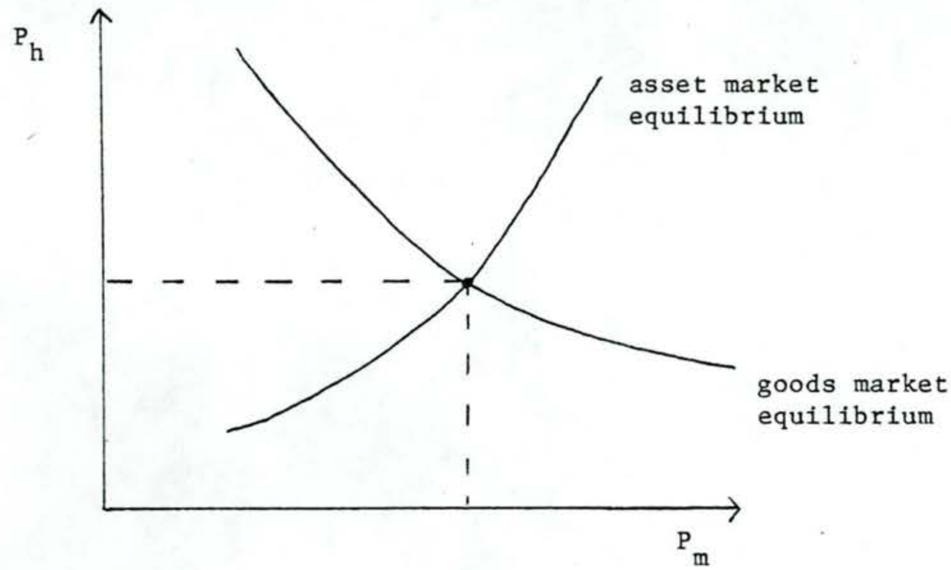


Figure 1

New production of housing minus depreciation equals the change in the existing stock of housing. Assuming exponential depreciation we have,

$$H = NH(P_H) - \delta H \quad (5)$$

Government deficits must be financed in our model by new issues of money so that:

$$P_m\dot{M} = d \quad (6)$$

As H and M evolve over time, equilibrium values of $P_m$ and $P_h$ will change.

For the exercises below, fiscal policy will be set to maintain a balanced budget and a constant price of money. This implies that there is no new money creation and no capital gains or losses on the existing money stock. The public is assumed to know these policies will be in effect so $M_t = \bar{M}$ and $(\frac{\dot{P}^{\varepsilon}_m}{P_m}) = 0$.

With these assumptions, it is possible to analyze the system by examining the equations for asset market equilibrium and housing accumulation. In Figure 2, we plot the loci along which the housing stock is constant and the loci on which there are no capital gains in $P_h$-H space. The $\dot{H}=0$ locus is positively sloped because at a higher stock of housing depreciation is higher and to maintain the existing stock the price of housing must rise to induce more production of new housing. The $\dot{P}_h=0$ locus is simply the demand curve for housing when there are no anticipated gains or losses and is downward sloping.

Above the $\dot{H}=0$ locus, new production exceeds depreciation so that the housing stock is increasing. Above the $\dot{P}_h=0$ locus, housing prices must be expected to rise to maintain equilibrium in the face of a higher price of housing. The dynamic system exhibits saddle-point instability with one stable node AC. For any level of the housing stock, unless the price of housing lies on AC it will

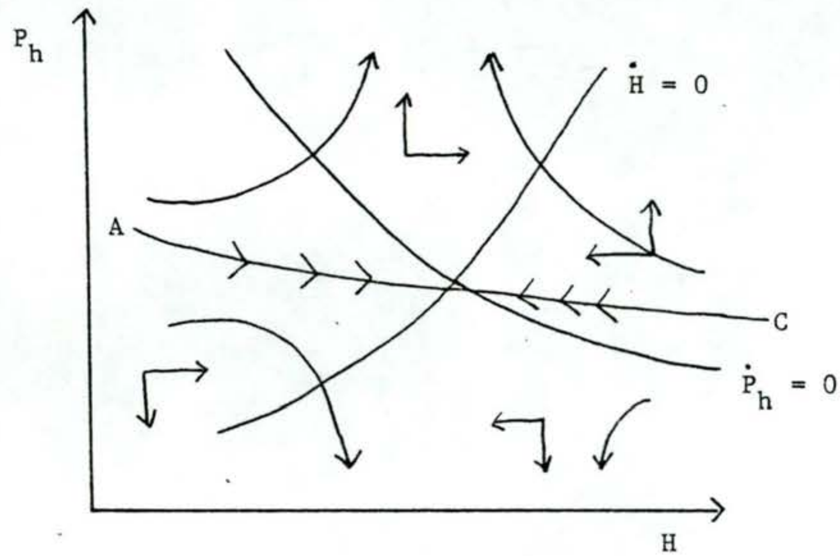


Figure 2

either become infinite or reach zero.

We can rule out paths along which the price goes to zero by assuming that the demand for housing will become infinite as $P_h \to 0$. Since capital losses must stop when $P_h=0$, there will be unsatisfied demand at that point so that perfect foresight investors will know that these cannot be equilibrium paths. If we assume that there is a lower bound to current consumption demand, then paths along which $P_h \to \infty$ can also be eliminated. For as the price of housing grows without bound the supply of consumption goods approaches zero. If consumption demand if bounded from below then eventually excess demand will appear in the goods market so that paths along which the relative price of housing approaches infinity cannot be equilibrium paths. We, therefore, can restrict ourselves to dynamic trajectories along the downward sloping stable node.

The speed of convergence to the new equilibrium depends on four factors. Convergence is faster when the $\dot{H}=0$ curve becomes flatter. This can be caused by a lower depreciation rate or a greater responsiveness of new housing production to the price of housing. Convergence will also be faster when $\dot{P}_h=0$ locus becomes steeper. The locus will tend to be steeper the greater is the excess supply caused by an increase in the stock of housing and the smaller is the excess supply created by an increase in the price of housing. Derivation of these results and the equation for the stable node are in the Appendix.

## II. Unanticipated and Anticipated Shocks and Macro Policies

The model will first be employed to analyze the effect of an autonomous increase in the demand for housing services. Two cases will be examined: an unanticipated and anticipated increase in demand.

An unanticipated increase in the demand for housing services will lead to an upwards shift in the $\dot{P}_h=0$ locus. In Figure 3, we picture the increase when the economy was originally in a steady-state with $P_h=P_h^*$ and $H=H^*$.

Following the unanticipated increase in demand, the price of housing immediately jumps to the point A along the new stable node at the existing housing stock $H^*$. Over time, the price of housing falls and the housing stock rise to the new equilibrium at C. Along the perfect foresight path, portfolio holders know that the price of housing will be falling and expect capital losses. If wealth holders ignored capital losses, the price would jump to the higher price B to clear the asset market, but they subsequently would suffer unanticipated losses. An econometrician who ignored anticipated capital losses would <u>overestimate</u> the price sensitivity of the demand for housing as an asset.

The time path for the price of housing and the capital stock is quite different when the increase in the demand for housing services is anticipated. Suppose at t=0 that economic actors become aware that the demand for housing services will increase at t=s. In Figure 4, the shift upwards in the $\dot{P}_h=0$ locus will occur at time s. As the news is announced, the price of housing jumps to point A and prices and the housing stock begin to rise before the increase in demand becomes effective. At time s, the price of housing and the housing stock will meet the stable node at B and then converge to the new long run equilibrium at point C. Before the shift in demand, prices rise; after the shift in demand, prices fall. The price initially jumps to point A so that at time t=s there are no jumps in the price of housing. If there were jumps at that time there would be anticipated infinite instantaneous capital gains or losses which are inconsistent with the demand shock having been perfectly anticipated. A naive econometrician ignoring capital asset changes would find an upwards sloping demand

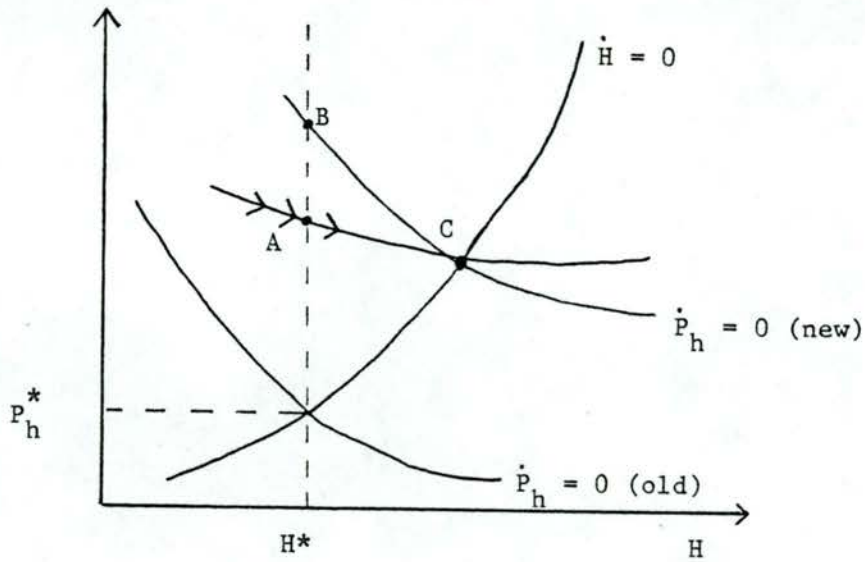


Figure 3

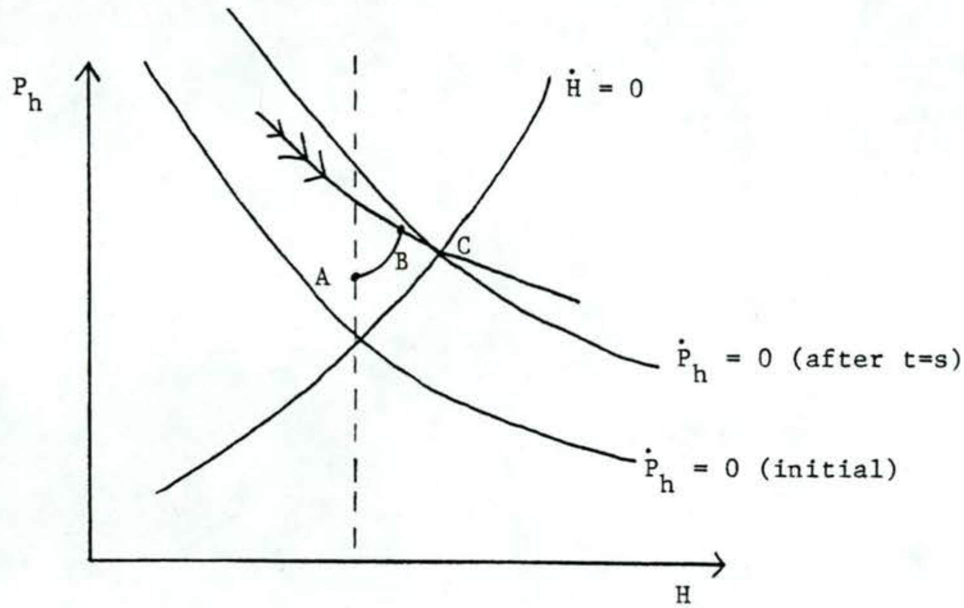


Figure 4

curve from A to B and would overestimate the price sensitivity from B to C.

Government policies can be analyzed in a similar fashion. Suppose the government increases taxes temporarily, neutralizes the effect on the market for goods by cutting spending and taxes together, and accumulates a surplus.[2] The government then uses the accumulated surplus to purchase a fraction of the housing stock from the public and restores fiscal policy variables to their original levels. The government rents out its housing so the supply of rental property remains constant.

The public now finds itself with unchanged total wealth but an excess demand for housing. This shifts up the $\dot{P}_h=0$ locus. If this change was unanticipated then prices of housing and the housing stock behave as in our previous unanticipated shock with the price of housing jumping initially and then falling. The anticipation of this policy would lead the economy to behave as in Figure 4. In either case, both the long run stock and price of housing will be higher while aggregate consumption and rental rates will be lower.

To investigate the impact of other government policies, it is convenient to change the macro model to a growth model. Let population grow at rate n and interpret all demand functions and variables in per capita terms. Only two equations change, the dynamic equations of motion. In per capita terms, the equations now become:

$$\dot{H} = NH(P_h) - (\delta + n)H \qquad (5')$$

$$\dot{P}_m M = d - nMP_M \qquad (6')$$

The interpretation of these modified equations is straightforward. Housing per capita will increase only if new production is sufficient to replace the depreciated stock and give each new person the same housing as the existing population.

Real balances per capita will be constant if new money creations via deficit financing is just sufficient to give each new entrant to the economy the existing per capita real balances.

These modifications allow us to analyze situations in which the government maintains a steady state deficit. An increase in the steady state deficit will increase the stock of real balances per person until $d = nMP_m$.

Suppose the government announces a policy to increase the steady state deficit but use balanced budget fiscal policy to keep the price of money constant. With the price of money held constant, the increase in real balances per capita shifts the $\dot{P}_h=0$ locus until a new steady state is reached with higher per capita real balances. Figure 5 shows the old and new long run equilibrium in per capita terms. The $\dot{P}_h=0$ locus shifts up gradually as real balances per capita increase. Since the policy has been announced in advance, the price of housing can only jump initially and then must follow a continuous path. As the locus shifts, the price of housing always lies above the stable nodes and increases to the new long run equilibrium.

Because capital gains are anticipated the price trajectory lies above the $\dot{H}=0$ locus. Expectations of capital gains increase demand so that the existing stock of housing will only be willingly held at a higher price. If actors were myopic, the price of housing would be along the $\dot{H}=0$ locus.

## Summary

Within the context of a macro model we investigated dynamic paths of housing investment along which agents perfectly forecast prices of the housing. Time paths for investment and prices depend crucially on whether shocks or policies are anticipated or unanticipated. Following an unanticipated increase in rental

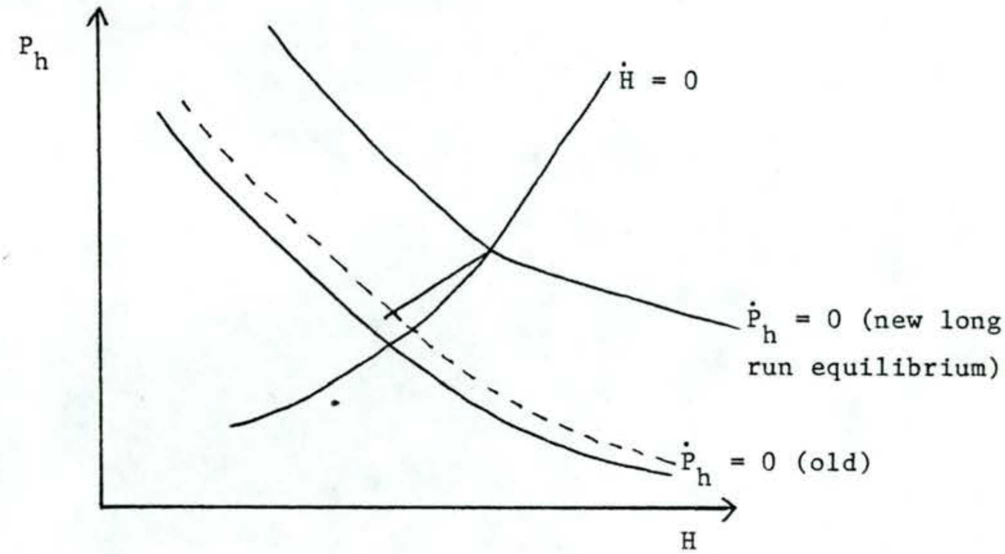


Figure 5

demand, the price of housing will jump initially and then fall over time. Anticipated increases in rental demand will lead to increases in the housing stock occurring before demand actually shifts while the price of housing rises before the shift and falls after the demand shift.

By specifying housing investment within a macro model, it was possible to trace the effects of government macroeconomic policies on investment dynamics. The analysis can be extended to a general analysis of capital accumulation; at that point the supply of investment approach should be altered to incorporate explicit demand for capital by firms along the lines of Abel [1977].

There are two implications for empirical work stemming from the analysis. First, the short run dynamics of the housing market depend critically on whether shifts in exogenous or policy variables are anticipated or unanticipated. We would expect, for example, that both forecasts and actual values of exogenous variables should enter an estimated equation with different coefficients. Second, the specification of anticipated capital gain or losses is critical. Simply ignoring or imposing a constant expectation formula may cause serious biases in estimation or forecasting.

Footnotes

1. The basic macro model stems from Foley and Sidrauski [1971].

2. Although $d<0$, private holding of money remain constant and the government accumulates the money rather than retiring money issues.

References

Abel, A. "Investment Theory: An Integration of Two Approaches," November 1977, mimeo.

Foley, D. and Sidrauski, M., Monetary and Fiscal Policy in a Growing Economy, MacMillan, 1971.

## Appendix

The asset equilibrium and stock of housing equation are two differential equations in $P_h$ and H.

1. $P_h H = H^d[\frac{r(H,\alpha)}{P_h} + \frac{\dot{P}_h}{P_h}, \frac{\dot{P}_m}{P_m}, P_h H + P_m M]$

2. $H = NH(P_h) - \delta H$

As long as $H_1^d \neq 0$, we can linearize the system:

$$\begin{bmatrix} \dot{P}_h \\ \dot{H} \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} P_h - P_h^* \\ H - H^* \end{bmatrix}$$

Where:

$$a = \frac{P_h\{H(1-H_3) + \frac{H_1 r}{P_h^2}\}}{H_1} > 0 \qquad c = \frac{\partial NH}{\partial P_h} > 0$$

$$b = \frac{P_h\{P_h(1-H_3) - \frac{H_1 \frac{\partial r}{\partial H}}{P_h}\}}{H_1} > 0 \qquad d = -\delta < 0$$

The characteristic roots are $\lambda_{1,2} = \frac{(a+d) \pm \sqrt{(a-d)^2 + 4bc}}{2}$

The pattern of signs guarantee that the roots are real and because the determinant is negative they have opposite signs. Along the stable node, the positive root must vanish so the equations of motion are:

$$P_h(t) = c_1 z_1 e^{\lambda_1 t}$$

$$H(t) = c_1 z_2 e^{\lambda_1 t}$$