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INNOVATION-INDUCED CHANGES IN THE RATE OF PROFIT IN THE VON NEUMANN MODEL

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1. Introduction

A classical problem of Marxian economics has been to investigate the effect of technical innovation upon the equilibrium profit rate. Marx (1966) surmised that, if the real wage remained constant, the technical changes which capitalists introduced would have a "tendency" to lower the rate of profit. It has been shown this is not the case: briefly, viable technical changes cause the wage-profit rate frontier to move outwards, and therfore raise the equilibrium profit rate at constant real wages. This result was rigorously demonstrated by Okishio (1961) in a linear, Leontief model of an economy. More recently, the question has been investigated by this author (Roemer 1977, 1978).

Formal discussions of the falling (or rising) rate of profit have been limited to simple Leontief models. For a treatment of the question in which the existence of fixed capital, differential turnover times, and joint products are fully taken into account, it is natural to ask what happens to the rate of profit consequent upon technical change in a von Neumann model of an economy, a model capable of handling these more general specifications of production. This is the purpose of the present paper.

Let a von Neumann economy be specified (B,A), where B is the n x m matrix of outputs and A is the n x m input matrix. The i^{th} column of B or A specifies the vector of outputs or inputs produced or used from unit opertation of the i^{th} process. (There are n goods and m processess.) We abstract away from the question of labor by assuming that labor's requirements are already embodied in the matrix A. (Hence (B,A) is a model of commodities produced entirely by commodities.) Since we shall assume the real wage is fixed, this is an appropriate abstraction. We say the semi-positive vector p is a price vector associated with profit factor ρ if:

$p \geq 0, \quad \rho > 0$ and $pB \leqq \rho pA$.

Under the assumptions that $A \geq 0$, $B \geq 0$, $A^i \geq 0$ and $B_j \geq 0$ (where A^i is the i^{th} column of A and B_j is the j^{th} row of B), it is well-known that there exists a minimal positive value ρ_{min} with respect to which a semipositive price vector exists.[1] (See Gale (1960).) If we write $\rho_{min} = 1 + \pi_{min}$, then π_{min} can be thought of as the minimum possible profit rate which the economy can sustain, or what Morishima (1974) calls the guaranteed profit rate.

Suppose the economy is sustaining equilibrium prices p at ρ_{min}. A technical innovation is a new pair of columns (b',a') which may be appended to the matrices (B,A). The innovation will be called viable at prices p if and only if:

$$pb' > \rho_{min} pa'$$

A viable innovation will immediately be adopted by capitalists who treat prices as given, as they will make super-profits from its operation. If a viable innovation appears, then it is reasonable to append it to the old technology, creating a new technology (B',A') where $B' = (B|b')$, $A' = (A|a')$, and ask: what happens to the minimal profit rate in passing from (B,A) to (B',A')? It is easy to see the minimal profit rate cannot fall; it may, however, not rise either.

The central task of the paper is to provide conditions which guarantee that the minimal profit does rise. This turns out to be akin to defining a kind of indecomposability for von Neumann economies. It is known that the same phenomenon occurs in Leontief models: there, a viable innovation in an indecomposable Leontief economy produces an increase in the rate of profit (which in that model is related to the eigenvalue of a matrix), whereas the rate of profit may remain constant in a decomposable economy.

At the mathematical level, then, this study investigates indecompsability in von Neumann models. It turns out that the previous definition of indecomposibility in the von Neumann literature, Gale's irreducibility (Gale, p. 314), is not a sufficiently strong condition to provide what is needed here. It shall be seen, also, that the question is equivalent to asking for a condition which guarantees that a unique price ray exists at ρ_{min}. Interest in the unicity of the von Neumann price ray has appeared elsewhere (Balinski & Young, 1974). From the economic point of view, this study shows that the rising-profit-rate story which has been told for the Leontief model generalizes suitably to the general activity analysis of von Neumann. In particular, the existence of fixed capital does not change the effect of technological change on the profit rate from the simpler circulating capital story. (For the view of the von Neumann model as a model of fixed capital, see Morishima (1969).)

The argument employed is geometric. In the next section, the geometric point of view is developed. In the third section, the questions of indecomposability and rising profit rate are studied.

2. Geometry of the von Neumann model

Definition 2.1. A von Neumann equilibrium for the model (B,A) is a triplet (p,x,ρ) where $\rho \in \mathbb{R}$, $\rho > 0$, p an n-row vector, $p \geq 0$; x an m-column vector, $x \geq 0$, such that:

(a) $pB \leqq \rho pA$

(b) $Bx = \rho pAx$

(c) $Bx \geqq \rho Ax$.

(Note that (b) is redundant given (a) and (c).)

We say p is a price vector for ρ, and x is an intensity vector for ρ. In addition, if:

(d) $pBx > 0$

then (p,x,ρ) is an economic von Neumann equilibrium. (For the study of economic von Neumann equilibria, see Kemeny, Morgenstern and Thompson (K-M-T) (1956).)

Definition 2.2. For any $\rho > 0$, define

$$P(\rho) = \{p \geqq 0 \mid pB \leqq \rho pA\}$$

$$\Gamma(\rho) = \{B^i - \rho A^i \mid p(B^i - \rho A^i) = 0 \quad \forall p \in P(\rho)\}$$

$$\cup \{-e^i \mid p\, e^i = 0 \quad \forall p \in P(\rho)\}$$

$$\cup \{0\}.$$

(e^i is the i^{th} unit vector in $\mathbb{R}^n$.)

$P(\rho)$ is the set of all price vectors for ρ; $\Gamma(\rho)$ is derived from the processes which are profitable, or binding, at profit factor ρ, for all price equilibria. (We may view $-e^i$ as equivalient to a disposal activity $0 - \rho e^i$, this last being written in the form $B^i - \rho A^i$. The outputs of the i^{th} disposal activity, $i = 1, n$, are the zero vector; the inputs are given by the unit vector.) The vector 0 is appended to the set $\Gamma(\rho)$ in case the set is otherwise empty, for notational convenience.

The geometry of the von Neumann model relevant for our inquiry is summarized in this theorem:

Theorem 2.1. $\forall \rho > 0$, $P^\perp(\rho) = \text{Cone}\ \Gamma(\rho)$.

($P^\perp(\rho) = \{v \in \mathbb{R}^n \mid p \cdot v = 0 \quad \forall p \in P(\rho)\}$; Cone $\Gamma(\rho)$ is the convex cone generated by the set $\Gamma(\rho)$.)

Proof:

It is clear that Cone $\Gamma(\rho) \subseteq P^\perp(\rho)$.

To show the converse, let

$$\overline{\Gamma}(\rho) = \{B^i - \rho A^i \mid \forall\, i = 1,m\} \cup \{-e^i \mid \forall\, i = 1,n\}.$$

It is claimed that

$P^{\perp}(\rho) \subseteq$ Cone $\overline{\Gamma}(\rho)$ implies $P^{\perp}(\rho) \subseteq$ Cone $\Gamma(\rho)$. For suppose not. Then $\exists v \in P^{\perp}(\rho)$, $v \in$ Cone $\overline{\Gamma}(\rho)$ - Cone $\Gamma(\rho)$. Then $v = \sum_1^m \alpha^i(B^i-\rho A^i) + \sum_1^n \beta^i(-e^i)$; $\alpha^i, \beta^i \geq 0$. Since p annihilates v, p annihilates every term $(B^i-\rho A^i)$ or $(-e^i)$ which appears with positive coefficient α^i or β^i in the sum -- since every term in the sum $p \cdot v$ is non-positive. Consequently,

$$(\forall\ i : \alpha^i>0)(\forall\ p \in P(\rho))(p(B^i-\rho A^i) = 0)$$

$$(\forall\ i\ \beta^i>0)(\forall\ p \in P(\rho))(pe^i=0)$$

which means precisely that $v \in$ Cone $\Gamma(\rho)$.

It is enough to show, therefore, that $P^{\perp}(\rho) \subseteq$ Cone $\overline{\Gamma}(\rho)$. Suppose not: $\exists v \in P^{\perp}(\rho)$, $v \notin$ Cone $\overline{\Gamma}(\rho)$. There exists a hyperplane separating $\{v\}$ from Cone $\overline{\Gamma}(\rho)$. That is: $\exists q \in \mathbb{R}^n$. $q \cdot v > 0$, $q\overline{\Gamma}(\rho) \leqq 0$. The latter condition means $q \geqq 0$ and $qB \leqq \rho qA$; hence q is a price vector, that is, $q \in P(\rho)$. But then $q \cdot v = 0$ since $v \in P^{\perp}(\rho)$. This contradicts the choice of q. q.e.d.

As has been remarked, under reasonable conditions on (B,A), there exists a minimal positive ρ for which semi-positive price vectors exist. For $\rho < \rho_{min}$, the cone generated by the binding constraints, which for these values ρ is all constraints (since $P(\rho) = \{0\}$) is $\mathbb{R}^n$. At $\rho = \rho_{min}$ this cone shrinks to become a proper subspace of R^n. Since the set $P(\rho)$ increases as ρ increases, the subspace $P(\rho)$ can only decrease. It is interesting to ask what special properties are enjoyed by values ρ where the dimensionality of $P(\rho)$ changes.

To do this, we define g_{max} in an analogous way to ρ_{min}:

<u>Definition 2.3</u>. $g_{max} = \max\{\rho \mid (\exists x \geq 0)(Bx \geqq \rho Ax)\}$.

Under Gale's assumptions, mentioned above, g_{max} is finite, and $g_{max} \geq \rho_{min}$. (See Gale, p. 314.)

We define the dual concepts to $P(\rho)$ and Cone $\Gamma(\rho)$:

Definition 2.4. For $\rho > 0$, let:

$$X(\rho) = \{x \geqq 0 | Bx \geqq \rho Ax\}$$

$$\Omega(\rho) = \{B_j - \rho A_j | (B_j - \rho A_j)x = 0 \quad \forall\, x \in X(\rho)\}$$
$$\cup \{-e_j | e_j x = 0 \quad \forall\, x \in X(\rho)\}$$
$$\cup \{0\}.$$

where $\{e_j\}$ are row unit vectors in $\mathbb{R}^m$.

Theorem 2.2. For $\rho > 0$, $X^{\perp}(\rho)$ = Cone $\Omega(\rho)$.

Proof: Same as for Theorem 2.1.

Notice the cone $X(\rho)$ decreases as ρ increases, becoming trivial for $\rho \geq g_{max}$. Hence the dimension of the subspace $X^{\perp}(\rho)$ increase as ρ increases. Therefore, as ρ increases (starting from a small positive number), the dimension of $P^{\perp}(\rho)$ changes, at most n times, and dim $X^{\perp}(\rho)$ changes, at most m times. The values of ρ where the dimensionality of the cones generated by $\Gamma(\rho)$ and $\Omega(\rho)$ change are related to a classical result of the K-M-T paper:

Theorem (K-M-T). There are a finite number, r, of values ρ for which economic von Neumann equilibria exist. Furthermore, $r \leq \min(m,n)$.

We shall show the values ρ where economic solutions exist must be, simultaneously, "jump values" of dim Cone $\Gamma(\rho)$ and dim Cone $\Omega(\rho)$.

Definition 2.5. Let

$$R_k = \{\rho | \text{dim Cone } \Gamma(\rho) = k\}$$
$$\text{and } \underline{\rho}_k = \inf R_k$$
$$S_k = \{\rho | \text{dim Cone } \Omega(\rho) = k\}$$
$$\text{and } \overline{\rho}_k = \sup S_k.$$

(Note: $\underline{\rho}_n = -\infty$ and $\overline{\rho}_m = +\infty$.)

Theorem 2.6. The values ρ where economic solutions exist must be common values of $\underline{\rho}_i$ and $\overline{\rho}_j$. Hence there can be at most min(m,n) of them.

Note: To prove this theorem, we need not assume the full strength of the K-M-T theorem but only that there do not exist an interval of values ρ for which economic solutions exist.

Proof: Notice:

$$\exists \text{ economic solutions at } \rho \Longleftrightarrow P(\rho)\cdot B\cdot X(\rho) = 0$$
$$\Longleftrightarrow B\cdot X(\rho) \subseteq P^{\perp}(\rho)$$
$$\Longleftrightarrow B\cdot[X(\rho)] \subseteq P^{\perp}(\rho),$$

where $[X]$ means the subspace generated by X.

As ρ increases, $P(\rho)$ increases as a set so $P^{\perp}(\rho)$ decreases as a set. $P^{\perp}(\rho)$ decreases only at the values $\underline{\rho}_i$. Consequently $[P(\rho)]$ increases as a set only at values $\underline{\rho}_i$. Similarly, $[X(\rho)]$ decreases as a set only at values $\overline{\rho}_j$.

Suppose there were an economic solution at value ρ, and $\rho \neq \underline{\rho}_i$ for any i. Then by definition there is a neighborhood $(\rho-\varepsilon,\rho)$ to the left of ρ where dim $P^{\perp}(\rho)$ does not change. To the left of ρ, $[X(\rho)]$ can only get larger. At ρ, $B\cdot[X(\rho)] \nsubseteq P^{\perp}(\rho)$ since there is an economic solution at ρ: but since $P^{\perp}(\rho)$ stays constant in $(\rho-\varepsilon,\rho)$ and $[X(\rho)]$ can only get larger in that interval this set inequality continues to hold $\forall \rho \in (\rho-\varepsilon,\rho)$. Hence there exist economic solutions in an interval of ρ's, an impossibility.

A similar argument demonstrates $\rho = \overline{\rho}_j$, some j. q.e.d.

3. Indecomposable von Neumann systems and the profit rate

Definition 3.1. Let $p \geq 0$ be a price vector associated with factor ρ. An innovation (b',a') is viable at prices $p \Longleftrightarrow pb' > \rho pa'$.

Assumption: We assume throughout that $B_j \geq 0 \ \forall j = 1,n$ and $A^i \geq 0$ $\forall i = 1,m$.

By Gale (p. 314), this is sufficient to guarantee that $\rho_{min} \leq g_{max}$ and von Neumann solutions exist for all $\rho \epsilon [\rho_{min}, g_{max}]$. In particular, for such ρ, (p,x) is a von Neumann solution if and only if $p \epsilon P(\rho)$ and $x \epsilon X(\rho)$.

Theorem 3.1. Let p be a price vector for the minimal profit factor, ρ_{min}, of system (B,A). Let (b',a') be a viable innovation at prices p, and let ρ'_{min} be the minimal profit factor for the appended technology (B',A'). then:

(a) $\rho'_{min} \geq \rho_{min}$

(b) $\rho'_{min} > \rho_{min}$ for every viable innovation at p if and only if p is the unique price ray at ρ_{min}.

Proof:

Part (a) is immediate, since $p'B' \leqq \rho p'A' \Rightarrow p'B \leqq \rho p'A$.

Part (b): $\Longleftarrow$

Let p' be a price vector for (B',A') at ρ'_{min}:

$$p'B' \leqq \rho'_{min} p'A'. \tag{3.1}$$

In particular:

$$p'B \leqq \rho'_{min} p'A.$$

If $\rho'_{min} = \rho_{min}$ then, by hypothesis, p' is a multiple of p, the unique price vector for (B,A) at ρ_{min}. But $pb' > \rho_{min} pa'$ by viability, which contradicts (3.1).

$\Longrightarrow$. Let q and p be two (independent) price vectors for (B,A) at ρ_{min}. Let v be a vector separating p from q:

$$p \cdot v < 0, \quad q \cdot v \geq 0.$$

Write $v^i = \rho A^i - B^i \quad i = 1,m.$ Then

$$p \cdot v^i \geq 0, \quad q\, v^i \geq 0.$$

Choose semi-positive vectors a' and b' in such a way that

$$v = \rho_{min} a' - b'. \tag{3.2}$$

Since $p \cdot v < 0$, the constructed innovation (b',a') is viable for (B,A) at (p, ρ_{min}). However, the minimal profit factor does not increase in passing from (B,A) to (B',A'), since $q \cdot v^i \geq 0 \;\; \forall\, i = 1,n$ and $q \cdot v \geq 0$. That is, q is a price vector for (B',A') at ρ_{min}. q.e.d.

To guarantee that ρ_{min} rises, then, under viable innovation, we must guarantee that the price ray at ρ_{min} is unique. By Theorem 2.1, this is equivalent to the condition that dim Cone $\Gamma(\rho_{min}) = n - 1$.

We recall Gale's concept of an irreducible von Neumann economy:

Definition 3.2. (Gale, p. 314) A set of goods is independent if it is possible to produce each good in the set without consuming goods outside the set. The model (B,A) is irreducible iff it has no proper independent subset. Formally: the set of goods $S \subseteq \{1,\ldots,n\}$ is independent if there is a subset of processes indexed by $T \subseteq \{1,\ldots,m\}$ such that: $(\forall i \in S)(\exists j \in T)\,(b_{ij} > 0)$ and $(\forall j \in T)(\forall i \notin S)(a_{ij} = 0)$.

We introduce a stronger notion:

Definition 3.3. A model (B,A) is indecomposable iff all intensity vectors x at ρ_{min} use at least n processes. That is: $x \in X(\rho_{min})$, $x \neq 0$, implies $x_j > 0$ for at least n components j.

Remark: (B,A) is indecomposable if and only if any semi-positive intensity vector, for any factor $\rho \geq \rho_{min}$, requires n positive intensity levels. For $\rho \geq \rho_{min} \Longrightarrow X(\rho) \subseteq X(\rho_{min})$. Since von Neumann equilibria exist only for $\rho \in [\rho_{min}, g_{max}]$, indecomposability means the system (B,A) cannot reproduce itself at any equilibrium unless it operates at least as many processes as there are goods.

Theorem 3.2. If (B,A) is indecomposable then dim Cone $\Gamma(\rho_{min}) = n - 1$.

Corollary 3.3. If (B,A) is indecomposable, then the minimal profit factor rises with the appending of any viable innovation.

Proof of Corollary: By Theorem 2.1 and Theorem 3.1, from Theorem 3.2.

Proof of Theorem 3.2:

By Theorem 2.1, Cone $\Gamma(\rho_{min})$ is a proper subspace of $\mathbb{R}^n$. Suppose dim Cone $\Gamma(\rho_{min}) = r < n - 1$. By Caratheodory's theorem,[2] any point in the convex hull of Γ can be expressed as a convex combination of at most $r + 1$ points of Γ. In particular, since Cone Γ is a subspace, $0 \in \text{Hull}\,\Gamma$ and 0 may be so expressed:

$$0 = \Sigma\alpha^i(B^i - \rho_{min}A^i) - \Sigma\beta^i e^i$$

where at most $r + 1$ terms occur in the sums together. It follows that

$$\Sigma\alpha^i B^i \geqq \rho_{min}\ \Sigma\alpha^i A^i$$

where there are fewer than n terms in the sums, since $r + 1 < n$ by hypothesis. But the vector α comprises a von Neumann intensity vector at ρ_{min} with fewer than n positive intensity levels, which contradicts indecomposability. q.e.d.

We next investigate the relationship between irreducibility and indecomposability.

Definition 3.4. Let H be the class of indecomposable models, and G the class of irreducible models.

Remark: Leontief models (I,A) which are indecomposable in the classical sense are members of $G \cap H$.

Theorem 3.4. $H \subset G$ but $G \not\subset H$.

Lemma 3.5. Every economy (B,A) can reproduce itself using not more than n processes at ρ_{min}. (That is: $\exists x \in X(\rho_{min})$ with at most n positive components.)

Proof of Lemma:

By the von Neumann-Gale existence theorem, there is a semi-positive intensity vector x at ρ_{min}:

$x \geq 0,\ v = (B-\rho_{min}A)x \geqq 0.$

Let $\Delta = \{B^i - \rho A^i \mid i = 1,m\}$. The point v in the convex hull of Δ lies in the non-negative orthant of R^n. However, no point of Hull Δ lies in the positive orthant: for if w were such a point then

$w = (B-\rho A)y > 0,\ y \geq 0$

which, by complementary slackness (Definition 2.1, part (b)) implies that $P(\rho) = \{0\}$, which is false. Hence, v lies in the edge, not the interior, of Hull Δ. Therefore v can be expressed as a convex combination of at most n elements of Δ. q.e.d.

Proof of Theorem 3.4: $G \not\subset H$:

Let (B,A) be a model which consists of two independent processes (b',a') and (b",a"), and several other processes which are positive convex combinations of these two. Let there be three goods in the model. This can easily be constructed to have no proper independent subsets of goods. Then $(B,A) \epsilon G$. Clearly, however, if x is an intensity vector which reproduces the system ($Bx \geqq \rho Ax$), the same results can be achieved by operating only the first two processes, and so $(B,A) \notin H$.

$H \subseteq G$:

Let $(B,A) \notin G$. Let $S = \{1,\ldots,s\}$ index a proper independent subset of goods. Let the processes T which are used to produce the goods in S, by using goods in S only, be indexed as the first t processes: $T = \{1,..,t\}$. Consider B, and delete those columns j, $m \geq j > t$. Delete also the rows of B (corresponding to goods) i, $n \geq i > s$. Call the reduced matrix $\hat{B}$. In like manner, define $\hat{A}$. Notice $\hat{A}$ has the property that every one of its processes (i.e., columns) uses some good in S: since every process of A used some good, and the processes left in $\hat{A}$ used no goods outside of S, by hypothesis. Similarly, $\hat{B}$ has the property that each of its rows contains a positive element: because by hypothesis all goods 1,...,s are produced by the processes in T.

Therefore, a von Neumann equilibrium $(\hat{p},\hat{x},\hat{\rho})$ exists for $(\hat{B},\hat{A})$, since $\hat{B}_j \geq 0$ for j =1,s and $\hat{A}^i \geq 0$ for i = 1,t.

Let p be the n-vector which is gotten by appending to $\hat{p}$ a string of zeros which correspond to prices for goods not in S; let x be the m-vector which is gotten by appending to $\hat{x}$ a string of zeros which are intensity levels for processes not in T. It immediately follows that $(p,x,\hat{\rho})$ is a von Neumann equilibrium for (B,A).

By Lemma 3.5, $(\hat{B},\hat{A})$ possesses an intensity vector $\hat{x}$ which uses not more than s processes. By the present construction, the existence of an intensity vector x for (B,A) has been shown using not more than s processes. Since s < n, $(B,A) \notin H$. q.e.d.

We next ask a related question. Suppose the economy is operating at a von Neumann equilibrium and a viable innovation appears and is appended to the technology. At the new equilibrium, will the new process in fact be used? A degenerate situation certainly exists if it is not used.

Theorem 3.6. Let ρ_{min}, g_{max} be the min profit factor and max growth factor for (B,A). Let ρ'_{min} be the min profit factor for the economy (B',A') after appending an innovation (b',a') viable at ρ_{min} for (B,A). If $\rho'_{min} > g_{max}$ then all von Neumann equilibria (p',x') for (B',A') use the innovation with positive intensity. Conversely, if all von Neumann intensity vectors for (B',A') use the innovation, then $\rho'_{min} > g_{max}$.

Proof: $\Longrightarrow$.

Let x' be an intensity vector for (B',A'), at any factor $\rho \geq \rho'_{min}$:

$B'x' \geqq \rho A'x'$.

If x' were zero in its last component, giving an intensity of zero for the new process, then the m-vector x consisting of the first m components of x' would be a semi-positive intensity vector for (B,A) at ρ which is impossible, since $\rho > g_{max}$.

$\Longleftarrow$. Conversely, if $g_{max} \geq \rho'_{min}$ then there is a semi-positive intensity vector for (B,A) at ρ'_{min}. Appending a zero component to this vector produces a semi-positive intensity vector for (B',A') at ρ'_{min}. q.e.d.

Corollary 3.7. If (B,A) is indecomposable, then all viable innovations will be used with positive intensity at all von Neumann equilibria in the appended technology.

Proof:

Gale (p. 315) has shown that $(B,A)\epsilon G$ implies $g_{max} = \rho_{min}$. Since $(B,A)\epsilon H \subset G$ (Theorem 3.4), we have $g_{max} = \rho_{min}$. Since $(B,A)\epsilon H$, it also follows that $\rho'_{min} > \rho_{min} = g_{max}$, and by Theorem 3.6. q.e.d.

Some final comments are warranted on the uniqueness of von Neumann equilibria. If we demand that the von Neumann equilibria be economic, then K-M-T have shown as quoted above that there exist a finite number of values ρ at which solutions exist. If $\rho_{min} = g_{max}$ then there exists at most one such solution. (In particular, if $(B,A)\epsilon G$ we need not speak of a minimal profit factor or guaranteed profit rate, since there is only profit factor capable of sustaining a full equilibrium, in the von Neumann sense.) An examination of Gale's duality theorem (Gale, p. 315) shows that if $(B,A)\epsilon G$ then, in fact, $Bx > 0$ for any intensity vector at ρ_{min}, and a fortiori, it follows that all von

Neumann equilibria are economic. Hence, if $(B,A)\epsilon H$, all von Neumann equilibria are economic, the equilibrium profit factor is unique, and the equilibrium price ray vector is unique.

It is, however, not clear that indecomposability or irreducibility are good economic assumptions in modelling fixed capital. The principal joint products with which we are concerned in modelling fixed capital are old capital goods which exit from the production process in a depreciated but still potentially useful state. In the von Neumann model, every capital good of every vintage counts as a separate commodity. A natural kind of indecomposability to assume for such an economy would be that the economy can reproduce itself using not more processes than than there are new goods. (That is, if there were some process for producing each new good reasonably efficiently using only new goods--a reasonable assumption--such indecomposability would exist.) If such were the case, the economy would not be in the class H, nor even in the class G (as the set of new goods would comprise a proper independent subset).

Hence, the results of this paper cannot in all likelihood be taken to apply to real fixed capital economics. Indeed, one might be tempted to expect that, due to the highly decomposable and reducible nature of real fixed capital economies, in the technical senses of this paper, the positive conclusions of this paper concerning the rising minimal rate of profit and uniqueness of equilibrium will not hold.

Footnotes

* Credit for the discovery that indecomposability, in the sense defined here, is a sufficient condition for Theorem 3.2 to hold, goes to the mathematician Roger E. Howe. Any mistakes in this rendition are, however, mine.

1. Convention on vector inequalities: $A \geqq B$ mean $A_i \geqq B_i$ $\forall$ components i; $A \geq B$ means $A \geqq B$ and $A \neq B$.

2. Caratheodory's Theorem (Mangasarian, p. 43) Let $\Gamma \subseteq R^n$. If x is in the convex hull of Γ then x is a convex combination of $n + 1$ or fewer points of Γ.

References

1. M. L. Balinski and H. P. Young, "Interpreting von Neumann Model Prices as Marginal Values," Journal of Economic Theory 9 (1974), pp. 449-463.

2. David Gale, The Theory of Linear Economic Models, New York: McGraw Hill, 1960.

3. J. G. Kemeny, O. Morgenstern, and G. L. Thompson, "A Generalization of the von Neumann Model of an Expanding Economy," Econometrica 24 (1956), pp. 115-135.

4. Karl Marx, Capital, Volume III, Moscow: Progress Publishers, 1966.

5. M. Morishima, Theory of Economic Growth, Oxford: Clarendon Press, 1969.

6. _________, "Marx in the Light of Modern Economic Theory," Econometrica 42, (1974), pp. 611-632.

7. N. Okishio, "Technical Change and the Rate of Profit," Kobe University Economic Review 7 (1961), pp. 86-99.

8. J. E. Roemer, "Technical Change and the 'Tendency of the Rate of Profit to Fall,'" Journal of Economic Theory 16 (1977), pp. 403-424.

9. _________, "Continuing Controversy on the Falling Rate of Profit: Fixed Capital and Other Issues," Department of Economics Working Paper No. 105, University of California, Davis (1978).