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WORKING PAPER SERIES

CONTINUING CONTROVERSY ON THE FALLING RATE OF PROFIT: FIXED CAPITAL AND OTHER ISSUES

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1. **The Need for Microfoundations: Methodology**

For the most part, discussion of the Marxian falling rate of profit (FRP) theory is marked by lack of attention to microeconomic detail. Precisely: how do the anarchic actions of atomistic capitals give rise to a falling rate of profit? Marx's discussion of this issue in Capital, Volume III was formulated in a microeconomic way. Briefly, the profit-maximizing urge of capitalists directs them to replace workers with machinery, which raises the organic composition of capital, which lowers (or produces a tendency to lower) the profit rate. Whether or not this argument is correct, it must be admitted it is microeconomic in this sense: it claims to deduce a macroeconomic phenomenon, itself quite beyond the ability of any individual capitalist to realize, to the anarchic (competitive) behavior of atomized economic units. This type of economic reasoning, of deducing aggregate economic effects from the behavior of individual economic units, was employed by economists of all ideological bents in the nineteenth century. It is, indeed, one of the hallmarks of why Marxism is scientific socialism. The outcome of socialism (and of capitalist crisis) was argued, by Marx and Engels, not to be a utopian solution (and crisis fortuitious), but the predictable outcome of social forces which eventually were reducible to the actions of individuals and classes of individuals. That Marx determined individual behavior as a consequence of the social context and imperatives, while the neoclassical school postulated a hegemonic, ahistorical position for the individual, in no way weakens the claim that Marx's theory possesses a microeconomic foundation.

Weaknesses have been pointed out in Marx's FRP theory by various authors, presented originally in a formal way in a paper of Okishio (1961). The argument, which shall be developed in more detail later, was briefly this: if capitalists introduce technical innovation when and only when it is cost-reducing, then the equilibrium rate of profit will rise, in a situation when prices are determined by competition, assuming the real wage remains fixed. Although no one believes the real wage does remain fixed, the problem has been to understand whether a FRP can be construed to be due to technical innovation itself, independent of changes in the real wage.

Responses to this claim, of Okishio and others, have been of three types. These are, first, what Fine and Harris (1976) call fundamentalist positions on FRP. These consist, it appears to me, of postulating FRP as part of the definition of capital. Somehow, FRP is inherent in capital, hence the proposition is not a proposition, is not falsifiable. While this position may have been adopted as an invincible counter to critiques of the theory, it renders the theory completely uninteresting and powerless. Second are empirical discussions of whether or not the organic composition of capital is indeeed rising. While this sort of investigation may be useful, it does not bear upon the <u>theoretical</u> issue of whether or not the rate of profit falls due to technical change. That is, either such investigation will be consistent with the Okishio conclusion, or it will not be: in the latter case, it would show the need for a different microeconomic argument of capitalist technical innovation; it would not, however, show Okishio's <u>argument</u> to be wrong. The empirical investigations, then, are certainly necessary, but they cannot provide refutation of a theory. To some extent, they appear to be carried out without sufficient consciousness of the microeconomic arguments which exist. That is: if one believes Okishio's model, then there is no increase possible in the organic composition of capital so great as to reduce the rate of profit. What, then, is the point of tracking the organic composition, unless one first consciously questions the postulates of the Okishio model? Third, are arguments which argue for FRP, against the Okishio model, but on the same analytical level: that is, by postulating microeconomic behavior of capitalist technical innovation which will (may) lend to a falling rate of profit. Papers which contain elements of this position are Persky and Alberro (1978), Shaikh (1978a, 1978b) and Fine and Harris (1976). One common claim of these arguments is that if one takes fixed capital into account, as Okishio did not, then the rate of profit can be shown to fall (always, ceteris paribus, independent of wage changes).

It is the intent of this paper to examine more carefully the microeconomic foundations of the rising-rate-of-profit position. In a word, the conclusion is this: capitalist technical innovation, even in the presence of fixed capital, will produce a rising rate of profit, ceteris paribus. (As in all discussions of this type the ceteris paribus assumption includes real wages and realization of surplus value.) It is imperative to emphasize from the outset what this conclusion does not claim. First, it does not say the rate of profit does not fall. If one includes as part of a theory of technical change how the real wage responds to innovation, the rate of profit can be shown systematically to fall, under various realistic hypotheses. (See, for instance, Roemer (1978).) Second, it does not say there cannot exist a microeconomic theory of FRP: it says, more narrowly, that the usual competitive assumptions do not produce such a theory. (Indeed, Persky and Alberro (1978) put forth a pre-theory of FRP, with microeconomic foundations.)

Since the technique of exploring the "micro foundations" of economic behavior may seem to many Marxists to be a neoclassical (and hence forbidden) methodology, it should be emphasized that this is not the case. Indeed, as I have indicated, this approach is one of the attributes of Marxist analysis which render it scientific and not utopian. To put this point a little differently: an avoidance of microeconomic analysis can lead to functionalism. If one does not investigate the mechanism by which decisions are made and actions carried out, one can too easily fall into the error of claiming that what is good or necessary for the preservation of the economic order comes to prevail. Or, somewhat perversely, whatever is necessary for the demise of the system--such as a falling rate of profit--must come to prevail. (This latter sort of functionalism serves the end of justifying the demise of the capitalist mode of production, which is viewed as historically necessary.) Indeed, this paper may be taken to illustrate the methodology which I think should and can be fruitfully applied to other problems in Marxist political economy--the methodology of investigating the micro foundations of phenomena, as an insurance against functionalist and tautological theorizing. Examples of areas where such an approach is needed are: the theory of the state, theory of discrimination and internal labor markets, theory of the labor process, theory of imperialism. If this approach is not taken by Marxists, the field is surrendered to neo-classical argumentation by default, an argumentation which, in general, demonstrates that rational microeconomic behavior leads to outcomes which are in some sense desirable or pleasant.

2. Miscellaneous Arguments Against Okishio's Theorem

In this section, various arguments which have been advanced in reply to

Okishio's argument against a FRP are discussed. For this purpose, a pure circulating capital model is sufficient. In the next section, problems peculiar to models with fixed capital are discussed.

A. Okishio's Theorem

A brief review of Okishio's theorem is appropriate. The original reference is Okishio (1961); a more general and recent treatment is found in Roemer (1977). The argument has been reproduced in various forms by other authors as well, such as Himmelweit (1974) and Samuelson (1974).

We posit a pure circulating "capital" model where:

A is the n x n input matrix

L is the n-row vector of direct labor coefficients in worker days

b is the n-column vector which is the worker's daily subsistence bundle

π is the equilibrium rate of profit

p is the n-row vector of prices of production

The equations which specify equilibrium in the model are:

$$p = (1+\pi)(pA+L) \tag{2.1}$$

$$1 = pb \tag{2.2}$$

where the daily wage is taken as unity. These can be written:

$$\frac{1}{1+\pi}\, p = pM \tag{2.3}$$

where $M = A + bL$

M is the input-output matrix gotten by viewing all commodities as being produced by commodities, the augmented input coefficient matrix. If M is a productive matrix, then a positive solution π and non-negative solution p exist to (2.3); if M is in addition indecomposable then (π,p) is uniquely determined. (This is a consequence of the Frobenius-Perron theorems.)

Suppose, now, we are in such an equilibrium, and a technical innovation appears, which can be characterized as a new column A^{i*} for the matrix A, and a new coefficient L^{i*}. We say the innovation (A^{i*},L^{i*}) is viable if capitalists can cut costs, at current equilibrium prices, by using it; that is, if and only if:

$$pA^{i*} + L^{i*} < pA^{i} + L^{i} \tag{2.4}$$

If and only if the innovation is viable, it will be introduced. After its introduction there will, for the time being, be a higher rate of profit earned by the innovators in Sector i; eventually, through entry and price cutting a new equilibrium will be arrived at which we call (π^*,p^*):

$$p^* = (1+\pi^*)(p^*A^*+L^*) \tag{2.5}$$

$$p^*b = 1$$

where A^* is the old matrix A with column A^i replaced by the innovation A^{i*}, and similarly for L^*. Notice the real wage b is assumed to remain constant. Question: Can we say anything about the relative sizes of π^* and π? Answer: π^* will be greater than π, if M is indecomposable. (If M is decomposable, π^* may equal π.) This, in brief, is the crucial argument: viable technical changes at constant real wages raise the equilibrium profit rate.

B. The Maximum Rate of Profit

One of the arguments which has been advanced in an attempt to mollify the impact of the above argument is that although the actual rate of profit rises, the maximal rate of profit falls. This position has been put forth by Fine and Harris (1976) and Shaikh (1978a),(1978b) as significant--the claim being that if the maximal rate of profit falls over time, the system becomes more and more hemmed in, so to speak, and crisis-prone. To quote Shaikh:

> The proposition that mechanization lowers the maximum rate of profit would appear to imply that sooner or later the actual rate of profit must necessarily fall. And indeed this is exactly how it has been interpreted by many Marxists. The basic logic of Marx's argument therefore, seems to emerge unscathed. (1978b, p. 20)

I will argue the conclusions here do not follow from the premises.

By the maximal rate of profit is meant the rate of profit that would prevail under a given technology if the wage were reduced to zero: *i.e.*, what return capitalists would get if they had no direct labor costs. From our Equ. (2.1), this is seen to be that number $\overline{\pi}$, such that a price vector $\overline{p}$ exists such that:

$$\overline{p} = (1+\overline{\pi})\overline{p}A \tag{2.7}$$

That is, $\frac{1}{1+\overline{\pi}}$ is the eigenvalue of the matrix A. Let us demonstrate very simply how the maximum rate of profit can fall with viable technical innovation.

Suppose an innovation (A^{i*}, L^{i*}) is capital-using and labor-saving (CU-LS) by which is meant:

$$A^{i*} \geq A^i, \quad L^{i*} < L^i.$$

All material input coefficients increase or stay the same, and direct labor input decreases. This is the kind of technical change we think of as being common. There certainly exist viable, CU-LS technical changes. (For a complete discussion of this, see Roemer (1977).) Clearly with such innovations, $A^* \geq A$. Since the maximal rates of profit before and after the innovation are $\overline{\pi}$ and $\overline{\pi}^*$, defined by:

$\frac{1}{1+\overline{\pi}}$ is the eigenvalue of A

$\frac{1}{1+\overline{\pi}^*}$ is the eigenvalue of A* it follows immediately that $\overline{\pi}^* < \overline{\pi}^*$, since it is well-known (Frobenius-Perron) that if $A^* \geq A$ then the eigenvalue of A^* is greater than the eigenvalue of A. However, by Okishio's thereom, the actual rate of profit rises, as long as the change is viable: $\pi^* > \pi$.

Suppose, now, we have an infinite sequence of such viable CU-LS technical changes, one after the other. In each case the actual rate of profit must rise (always holding constant real wages b) and the maximal rate of profit must fall. We have:

$$\pi^1 < \pi^2 < \pi^3 < \ldots < \pi^t < \ldots < \overline{\pi}^t < \overline{\pi}^{t-1} < \ldots < \overline{\pi}^3 < \overline{\pi}^2 < \overline{\pi}^1$$

Certainly the $\{\overline{\pi}^i\}$ decrease: but they never cause the actual rate of profit to fall. In particular, any actual rate of profit π^* is a lower bound for all maximal rates of profit $\{\overline{\pi}^i\}_{i=1,\infty}$. In particular, the sequence of maximal rates of profit $\overline{\pi}^i$ does not converge to zero, but rather to some "large" positive number: large in the sense that it is larger than any actual rate of profit the system every achieved in the hypothetical history.

Another writer who discusses the "falling maximal rate of profit" phenomenon is Schefold (1976). It must be pointed out that Schefold is careful not to draw any false inferences from his demonstration; it is worth discussing here, however, for his mathematical model is sufficiently complex that readers might think the falling maximal rate of profit in that model does imply something about what happens to the actual rate. Schefold's model includes fixed capital. He demonstrates that "mechanization" leads to a falling maximal rate of profit. By "mechanization," Schefold means a technical innovation that uses at least the same amount of circulating capital (what he calls raw materials) as the old technique, an increased amout of fixed capital, and less direct labor. Since, in computing the maximal rate of profit, the direct labor has no impact since the wage is assumed to be zero, it is obvious that if we increase fixed capital and do not decrease circulating capital, the maximal rate of profit will fall. The economic intuition remains the same as the one given above for the pure circulating capital case, although the model with fixed capital is mathematically more complex. Hence, the fall in the maximal rate of profit is of no consequence for what happens to the actual rate.

Clearly, what does happen in the infinite history that has been proposed is that the actual and maximal rates of profit get closer to each other. As long as the real wage remains non-zero at b, however, the two sequences cannot converge to the same limit. If, in addition, we wish to allow the real wage to vary with technical changes, then the actual rate of profit will not increase so fast, or may even decrease. That is, suppose b^t is the real wage bundle associated with the technology $(A^*,L^*)^t$ which is extant at time t. Let us say workers succeed in raising the real wage with each viable innovation: $b^t \geq b^{t-1}$. Then we will certainly have that the sequence $\{\pi^*(b^t)\}$ of actual rates of profit (viewed as a function of the contemporary real wage) will increase less fast than the sequence $\{\pi^*(b)\}$ would have; and we may even have $\{\pi^*(b^t)\}$ to be a decreasing sequence in periods when the real wage rises sufficiently rapidly. In this case, of variable real wage, the maximal rates of profit (which remain the same as before) become even less "constraining" than they were before.

Another version of the falling maximal rate of profit theory is put forth by Okishio (1977). Using the Marxian categories S, C, V, and L he observes:

$$\text{value rate of profit} = \frac{S}{C+V} < \frac{S+V}{C+V} = \frac{L}{C+V} < \frac{L}{C} \qquad (2.8)$$

Hence, if $\frac{L}{C} \to 0$ with technical change, then π must approach zero. In particular, L/C is an upper bound in the rate of profit.

It may be intuitively pleasing to think of L/C approaching zero under advanced mechanization. Recall, however, that the value of constant capital, C, will get small if total direct labor L in the system gets small. So mechanization does not necessarily mean $L/C \to 0$. But more to the point, there are no micro foundations for the argument that $L/C \to 0$. In particular, we argue the following: if L/C approached zero then the value rates of profit in all sectors would necessarily approach zero. But it is well-known that the price rate of profit, π, is an average of the value rates of profit. Hence the price rate of profit π would necessarily approach zero. But we have shown this is impossible under the assumption of a fixed real wage, as π increases. Hence, under the usual competitive assumptions, there is not only no reason to suppose that L/C might approach zero, but there is proof that L/C cannot approach zero. It should be pointed out that Okishio says there is no evidence that $L/C \to 0$ in his paper.

Hence, the inferences that are frequently drawn concerning the actual rate of profit from the decrease in the maximal rate of profit are without foundation.

C. Rising Organic Composition of Capital Arguments

These arguments go back to Marx. Briefly they are based on this:

$$\rho = \frac{S}{C+V} = \frac{S/V}{C/V+1} = \frac{e}{C/V+1}$$

If the organic composition of capital (OCC) C/V rises over time as a consequence of technical change, then, "all other things being equal," ρ, the value of rate profit, falls. The argument is fallacious, under the assumption that the real wage remains fixed, because in that case e will always rise sufficiently to more than offset the rise in C/V, under the competitive scenario which we are assuming. (Actually, the value rate of profit may fall as a consequence of viable technical changes although the price rate of profit will never fall. Futhermore, since the price rate of profit is always an average of the sectoral value rates of profit, it is clear the value rates of profit cannot fall too much; precision on this point is not important here.)

From a logical point of view, this is all there is to say on the question of a rising OCC. Nevertheless, many discussions of FRP are still found around the question of how fast the OCC rises. For example, Mandel's (1974) discussion of FRP is based on claims that the OCC rises fast. Moreover the rebuttals to FRP arguments frequently take the form of arguing the OCC, empirically, has not risen very fast, or has stayed constant in the twentieth century. This is the tack taken by Hodgson (1974) and Rowthorn (1976). Now it may very well be true that the OCC has not risen fast: the point is, however, that invoking such an argument concedes too much. It implies that it is possible for the OCC to rise sufficiently fast for the rate of profit to fall. Under the assumption of a constant real wage, this is impossible; and if the real wage increases, looking at C/V is not sufficient anyway, one must also look at what happens to e.

It should probably be reiterated that I am not opposed to measuring the OCC over time--precisely because whether or not the rate of profit has fallen is an empirical question, as it rests entirely on the relation between technological change and the rate of change of the wage. We can, of course, put forth theories of the relation between technical change and changing real wages and examine the consequences for the rate of profit. (See, for instance, Roemer (1978).) Verification of such theories must be empirical. But the argument here is addressed to those who conclude that technical change <u>itself</u> (that is, in the absence of real wage change) can bring about a falling rate of profit in consequence of a rising OCC. There is no reason to examine the OCC either to demonstrate such an argument or to rebut it, unless one questions the competitive, cost-cutting theory of technical innovation. To my knowledge, these authors have not explicitly questioned such a theory. Indeed, I take these assumptions to be the basis of the Marxian theory of competition.

Although these remarks are sufficient to make the point--that the OCC is not the variable to examine--it may be useful to point out what does happen to the OCC under <u>viable</u> technical changes. We have:

<u>Proposition</u>. Let (A^{i*}, L^{i*}) be a viable, CU-LS technical change in sector i. The real wage b is fixed. Then:

(a) the OCC will rise in sector i

(b) the OCC may rise or fall in the other sectors

(c) the aggregate OCC may rise or fall.

(For the proof, see Appendix.)

Hence, we may in fact observe rising OCC with CU-LS innovations. But that has no bearing on theories of the FRP.

3. <u>The rising rate of profit with fixed capital: A Special Case</u>

Shaikh (1978b) has claimed that in a model with fixed capital, the rate of profit may fall due to rational, competitive capitalist innovation. Since the version of Okishio's theorem which is appropriate to the case of fixed capital does not seem to appear in the literature, it is appropriate to present such a theorem here. In this section a special case of a fixed capital model is presented: it is assumed that there are no joint products, and that all fixed capital lasts forever. (These assumptions are related: since fixed capital does not wear out, it does not have to be considered a joint product of a process in which it is used. The only process which "produces" an item of fixed capital is the one which manufactures it originally.) In the next section, we treat the <u>general</u> problem of fixed capital by examining the von Neumann model. However, it is worthwhile to treat the special case of non-depreciating fixed capital first, for several reasons: (1) it appears as a straightforward generalization of the pure circulating capital model, and the economic ideas embedded in the equations are therefore quite transparent; and (2) it is the polar opposite of the pure circulating capital case. That is: if the rate of profit can be shown to rise as a consequence of technical innovation in a model when fixed capital lasts forever, <u>a fortiori</u> it should rise when fixed capital wears out, this latter case being in some sense an average between the two polar cases.

Production, as before, consists of n processes producing n commodities. We define:

A is the n x n input matrix of circulating capital coefficients

L is the n-row vector of direct labor coefficients

b is the n-column vector of worker's subsistence

Φ is the n x n input matrix of fixed capital coefficients

In this case, the capital inputs into process i consist of a column A^i of inputs which are consumed in the process, and a column Φ^i which are used but not consumed. Notice no distinction has been made in labelling "fixed capital" goods and "circulating capital" goods and "consumption" goods. In general, we would therefore expect many components of A^i and Φ^i to be zero. (This formulation of the fixed capital model is due, first, to Schwartz (1961), I believe.)

What is the equilibrium price vector p and profit rate r in this model? It is that pair (p,r) which makes the present discounted value (PDV) of the revenue stream, from operating each process at unit level, equal to zero. That is:

$$-(p\Phi+pA+L) + \sum_{1}^{\infty} \frac{(p-(pA+L))}{(1+r)^i} = 0 \qquad (3.1)$$

Consider one component of this matrix equation (3.1), gotten by examining process i. The first term, $-(p\Phi^i+pA^i+L^i)$, is the cost incurred in the first period of operation, when fixed capital to operate the process must be purchased ($p\Phi^i$) as well as circulating capital (pA^i+L^i). There is no revenue in the first period, since output only appears in the second period. In the second period, gross revenue is p^i, from selling the output made last period, and gross costs are the circulating capital laid out for next period, (pA^i+L^i). There is no fixed capital cost, as the fixed capital set up in the first period works forever. For all subsequent periods, net revenue is $(p^i-(pA^i+L^i))$; hence equation (3.1).

Using the identity $\sum_{1}^{\infty}(\frac{1}{1+r})^i = \frac{1}{r}$, we can rewrite (3.1) as:

$$p = rp\Phi + (1+r)(pA+L) \qquad (3.2)$$

from which the economic interpretation is clear. In equilibrium, price consists of three components: costs of materials used (pA+L), the markup on materials used r(pA+L), and the markup on fixed capital used $rp\Phi$. There are no costs in consuming fixed capital, because depreciated over its infinite lifetime, zero fixed capital is used up each period. Equ. (3.2) is, therefore, the straightforward generalization of Equ. (2.1).

Now suppose an innovation appears in sector i--that is, a new technique $(\Phi^{i*}, A^{i*}, L^{i*})$. How does the capitalist decide whether to adopt it? If it is adopted, he must write off all his fixed capital currently in place: if and only if, despite this one time loss in the present period, his stream of discounted net revenues is positive under present prices and profit rate, then it is to his advantage to adopt the new technique. That is, the rational capitalist, treating prices as given, adopts the new technique if and only if:

$$-p\Phi^{i} - (p\Phi^{i*}+pA^{i*}+L^{i*}) + \sum_{1}^{\infty} \frac{(p^{i}-(pA^{i*}+L^{i*})}{(1+r)^{i}} > 0 \qquad (3.3)$$

This is equivalent to:

$$p^{i} > rp(\Phi^{i}+\Phi^{i*}) + (1+r)(pA^{i*}+L^{i*}) \qquad (3.4)$$

Again, the economics behind the innovation criterion (3.4) are clear. Since fixed capital lasts forever, if the capitalist adopts the new technique he must continue to treat the old fixed capital as a cost. Hence he must make the normal mark-up on his total fixed capital, $rp(\Phi^{i}+\Phi^{i*})$. His circulating costs become, of course, $(pA^{i*}+L^{i*})$, along with their mark-up, $r(pA^{i*}+L^{i*})$. If his price exceeds the sum of these costs and mark-ups, his transitional profit rate will have increased, and the technique is adopted.[1]

1. There is another way to understand the innovation criterion (3.4). The innovation is adopted if and only if the PDV of the net revenue stream from the innovation is greater than the PDV of the remaining net revenue stream from the old technique, discounted at current (p,r); that is:

$$-(p\Phi^{*}+pA^{*}+L^{*}) + \sum_{1}^{\infty} \frac{(p-(pA^{*}+L^{*}))}{(1+r)^{i}} > -(pA+L) + \sum_{1}^{\infty} \frac{(p-(pA+L))}{(1+r)^{i}}$$

But, by Equ. (3.1), the right hand side of this inequality is equal to $p\Phi$. Hence this inequality reduces to (3.3).

Another way to see this, perhaps more transparently, is to suppose the capitalist is borrowing money at interest rate r to pay for fixed and circulating capital. If the innovation occurs, he must pay the interest cost for setting up the new fixed capital, $rp\Phi^{i*}$, the interest costs on the old fixed capital which he cannot sell (if the innovation is to be adopted), $rp\Phi^{i}$, the interest cost on borrowing for the circulating capital $r(pA^{i*}+L^{i*})$, and the costs of circulating capital $(pA^{i*}+L^{i*})$. If all these costs add up to less than the price, which is condition (3.4), the capitalist innovates as he will make a windfall profit. (In equilibrium, under this scenario, the capitalist's profits are zero, all profits being imputed to interest.)

If (3.4) holds, and the innovation is adopted, a new equilibrium (p^*,r^*) must be reached under the new technique (Φ^*,A^*,L^*), which consists of the old technique (Φ,A,L) with column i replaced with the innovation $(\Phi^{i*},A^{i*},L^{i*})$. The generalization of Okishio's theorem becomes:

Theorem 3.1. Let (p,r) be the equilibrium for technique (Φ,A,L), satisfying:

$$p = rp\Phi + (1+r)(pA+L).$$

$$1 = pb$$

Let an innovation satisfy Inequality (3.4). Then, if (p^*,r^*) is the new equilibrium:

$$p^* = r^*p^*\hat{\Phi} + (1+r^*)(p^*A^*+L^*),$$

$$1 = p^*b.$$

It follows that $r^* > r$, where $\hat{\Phi}$ is the matrix Φ with column i replaced by $\Phi i + \Phi i^*$. (It is assumed that the matrix $A^* + bL^* + \Phi$ is indecomposable.)

Proof: See Appendix.

It is now necessary to remark on Shaikh's (1978b) observation that in the presence of fixed capital the equilibrium rate of profit may fall, a statement which cannot coexist with Theorem 3.1. Shaikh maintains that under the "Okishio criterion," capitalists simply evaluate whether the new technique permits lower circulating costs of production then the old one. If so, they innovate. Clearly this can lead to a fall in the rate of profit if there are large amounts of fixed capital. But this is only the Okishio criterion in the pure circulating capital case! Where there is fixed capital, any rational capitalist must take it into account. The way to take it into account is to ask, as has been done here, whether, when the old fixed capital is written off, switching to the new technique will still lower amortized unit costs (or raise the transitory profit rate, or produce windfall profits). This point deserves further elaboration as it is a source of confusion. Why will the capitalist not adopt the technique with lowest circulating costs, thereby enabling him to cut prices?Because price does not consist only of a circulating cost component, but of an interest charge or mark-up on fixed capital. To reject this is precisely equivalent to rejecting that Equ. (3.2) defines the equilibrium price and profit rate in this model.

Still, it may be objected, there must be some force which will move capitalists to adopt a more cost-efficient technique, even if it is not advantageous for them to scrap existing machinery--that is, even if (3.4) fails to hold. And in this case the rate of profit will fall. Let us examine this. Suppose the technique (Φ^*, A^*, L^*) is more cost-efficient at current prices (p, r). That is:

$$p^i > rp\Phi^{i*} + (1+r)(pA^{i*}+L^{i*}) \tag{3.5}$$

Then an entrepreneur who has not previously set up the old technique will enter, and set up the new technique. Old entrepreneurs, however, will not find it profitable to switch. We will have two techniques operating simultaneously. What happens? As long as only a few entrants operate the new technique, they earn a higher profit rate than the economy-wide rate; this profit rate differential can be viewed as a rent. When the entrants become numerous, however, prices shift and the rent falls, until eventually the new technique determines a new equilibirum price and profit rate (p^{**}, r^{**}). It can be shown, in a way similar to the proof of Theorem 3.2, that $r^{**} > r$. At some point in this shift from (p,r) to (p^{**}, r^{**}) it may become profitable for the capitalists operating the old technique (Φ^i, A^i, L^i) to innovate, according to Inequality (3.4). Capitalists will innovate at that point at which the new internal rate of return from operating the new technique, taking into account that old fixed capital must still be written off, exceeds the internal rate of return from operating the old technique. If fixed capital eventually wears out, then it will certainly become profitable for all capitalists to adopt the new technique. The general point, however, is this: the existence of an innovation which is an improvement over existing technology, but not such an improvement as to cause capitalists to voluntarily scrap old machines and adopt it, either eventually becomes profitable for all to use, or gives rise to differential rents in the economy. (In the case where fixed capital lasts forever, it always gives rise to permanent differential rents, since the internal rate of return of the capitalist who had to scrap equipment will never be as high as that of the capitalist who started fresh with the new technique.)

It does not, however, cause any sort of myopic action on the part of capitalists which would lead to a fall in the general profit rate.

A final technical note should be appended to this argument. Saying that capitalists seek to maximize the internal rate of return is not tautologically the same as saying the general rate of profit increases in the economy. A theorem is required to prove this (Theorem 3.1). Maximizing the internal rate of return is the relevant notion of cutting costs in the fixed capital model. Hence, we are not saying that the economy naturally gravitates to the furthest wage-profit curve "because it's there"; rather, that competitive cost-cutting pushes the economy to the frontier.

It is appropriate at this juncture to mention the interesting paper of Persky and Alberro (1978). I will take the liberty of phrasing their argument in terms of my model, and hope I am not thereby distorting it. Suppose an innovation appears, which should be introduced, according to Inequality (3.4). The new rate of return is r^*. Then, two years later, another innovation appears, which is again profitable to introduce, even considering that the two-year-old machines must be scrapped. (The new innovation yields an equilibrium profit rate $r^{**} > r^*$.) But then the actual rate of return for the two year period in which the first innovation operated was considerably less than r^*, for the infinite stream of positive net revenues never materialized. In this way, we see how a sequence of innovations can occur, each one of which will be adopted as it leads to a higher expected rate of return; but due to the truncated lifetimes of these innovations because of the unforeseen obsolescence, the actual rate of return falls. The most extreme case, and easiest one to see, is when the innovations occur every year, so the capitalist is every period incurring the large costs of new fixed capital, and receiving only the small revenue of last period's output. A rising expected profit rate is, therefore, consistent with a falling actual profit rate!

This is, indeed, a theory of the falling rate of profit with micro foundations. It depends, however, on an assumption which is of dubious validity: that there is a series of unforeseen technical innovations. Capitalists are, for some reason, consistently underestimating the speed of technical progress. It is certainly possible that this may happen for a short period, but after a while, capitalists will adjust their expectations, and assume innovations will occur at a reasonable rate. Mathematically, this takes the form of truncating the expected economic lifetime of a technique. They will, for instance, not sum $\sum_{1}^{\infty}$ in Ineq. (3.4), but $\sum_{1}^{5}$, say, which is to say, they will demand an innovation pay for itself at the going rate of return in five years to pass the test for adoption. Furthermore, when most innovations today come out of huge R & D labs, and are the result of planned and concerted development on a mass scale, it is reasonable to suppose that capitalists can forecast quite accurately the speed of innovation.

The Persky-Alberro proposal, then, does provide a story of a falling rate of profit. At best, it seems to work only for a short period; it cannot support a secular falling rate of profit story. It depends upon an unanticipated rate of technical change. It may be convenient to say the anarchy of capitalist production is captured in this unanticipated rate of innovation; it seems more realistic, however, to believe that capitalists are not caught unaware for long, especially given the institutional environment for successful technical development today.

4. The General Case of Fixed Capital: the von Neumann Model

In the previous section, a special model of fixed capital was considered where all fixed capital lasts forever, and there is no joint production. In this section, we treat the general von Neumann model, where fixed capital can wear out, joint production occurs, and production processes can have different periods of production. This is the most general linear model of production. It shall be our aim to examine what happens to the rate of profit following the introduction of a cost-reducing technical change, with the real wage, as always, fixed.

To review the von Neumann model: there are n commodities, and m processes, each of which uses some inputs and labor and produces some outputs. We represent the technology by a set $\{B,A,L,b\}$ where

B is n x m matrix of output coefficients

A is n x m matrix of input coefficients

L is m-row vector of direct labor inputs

b is n-column vector of subsistence wage.

The ith column B^i and A^i of the matrix B and A give the outputs and inputs of operating process i at unit level. We call $M = A + bL$ the augmented input coefficient matrix, and refer to the technology from now on as $\{B,M\}$. Recall that old machines are joint products themselves, which is why the joint product framework is so convenient for analyzing the general model with fixed capital. (For a more complete discussion of fixed capital as a joint product see, for instance, Morishima (1969, chapter 6).)

An equilibrium price vector and profit rate (p,π) must satisfy:

$$pB \leq (1+\pi)pM \qquad (4.1)$$

We shall call the von Neumann profit rate the minimum π for which there exists a price vector $p \geq 0$ satisfying (4.1). (This is, however, not a sufficient characterization of what constitutes an equilibrium, as will be discussed below.)

A. An Example: Fixed Capital which lasts forever.

It may be useful, in making the transition from the special case of Section 3 to the general formulation of fixed capital here, to show how the case of the previous section looks if we model it in the von Neumann way. To this end, let us assume we have a technology in which the only joint products are the fixed capital used in production. Otherwise, each process produces a single output. Furthermore, since fixed capital does not depreciate, there are no new processes for producing the commodities using old fixed capital (since old fixed capital does not exist). Consequently, the output matrix B, in this case, is a square matrix given by:

$$B = I + \Phi$$

where Φ, as in Section 3, is the matrix of fixed capital coefficients. The input matrix is $M + \Phi$. Inequality (4.1) becomes

$$p(I+\Phi) \leq (1+\pi)p(M+\Phi)$$

or
$$p \leq p(M+\pi(M+\Phi)) \tag{4.2}$$

Now by the Frobenius-Perron theorem, it is known that the minimum π for which there exists a non-negative vector p such that (4.2) holds is, in fact, the value π^* such that:

$$p^* = p^*(M+\pi^*(M+\Phi)). \tag{4.3}$$

If the matrix M is indecomposable, then the value π^* is unique, as is the price vector p^*, and in fact (p^*, π^*) is the equilibrium discussed above in Section 3.

Hence, the von Neumann formulation readily reduces to the characterization of equilibrium gotten by our present discounted value formulation of Section 3, in the special case there dealt with.

B. Marxian Equilibrium in the von Neumann Model

In the case of a simple Leontief model with no fixed capital, no joint production, and unit periods of production, we have the nice situation, assuming that the technology is indecomposable, that a unique price profit rate equilibrium exists. Indeed, it is unnecessary to consider at what levels the various outputs are produced: one need only look at the price equations of the economy. This is because at the equilibrium, all processes operate at the same (and therefore maximal) profit rates, and consequently capitalists can operate all processes, and produce any desired output combination. In the von Neumann case, this is not necessarily so: since some processes will not be operated at an equilibrium described in Equ. (4.1), as they do not produce the maximum attainable profit rate. It is necessary, then, to define an equilibrium in such a way that we are guaranteed extended reproduction is possible, while operating only those processes which produce a maximum profit rate. This might be done as follows:

Definition A. A price vector and profit rate (p,π) are an equilibrium for the von Neumann system (B,M) if:

(1) $pB \leqq (1+\pi)pM$

(2) $\exists x \geqq 0$ such that $pBx = (1+\pi)pMx$

(3) $Bx \geqq Mx$

(4) $pBx > 0$

<u>Interpretation</u>: Condition (3) says that if the economy operates the processes at levels x, then it is capable of supplying the inputs it needs--that is, reproduction takes place. Condition (2) says that at activity levels x, only those processes are being operated which produce the maximum profit rate (π) available at given price p. Capitalists will not, of course, operate any other processes. Condition (4) states that the value of output is positive, ruling out a trivial possibility. In particular, it guarantees that $x \neq 0 \neq p$ and that there are, by (2), some processes which support the profit rate π.

(Notice conditions (1)-(4) are weaker than the classical von Neumann conditions for a balanced growth equilibrium. In particular, we do not require that goods which are growing faster than the minimum growth rate be priced at zero.)

It is worth pointing out that conditions (1)-(4) are sufficient to guarantee that in the simple Leontief circulating capital model, the usual Frobenius root and eigenvector are the unique equilibrium:

<u>Proposition 4.1</u>. If $B = I$ and M is an indecomposable square matrix (pure circulating model) then the only equilibrium (p, π) satisfying Definition A is the Frobenius equilibrium.

<u>Proof</u>: See Appendix.

However, in the general case of a von Neumann technology (B,M), Conditions (1)-(4) are not sufficient to guarantee the existence of a unique equilibrium (p, π).

If the equilibrium (p, π) is not unique, then we are not even in a situation where the falling rate of profit can be discussed. For suppose the economy is operating at an equilibrium (p, π) but there also exists an equilibrium

(p',π'), with $\pi' < \pi$. Then, if the economy leaves the original equilibrium for some reason, there is no guarantee it will return there--it may, in fact, return to (p',π') and the rate of profit will have fallen. But notice this fall in the profit rate is not associated with technical change or with anything in particular. It has to do with problems of multiple equilibria. Thus, if there are multiple equilibria, the problem of traverse becomes an important one in analyzing changes in the rate of profit. This is a problem which is not discussed in the Marxian literature, and is not in the tradition of falling--profit-rate discussions.

We will, therefore, limit ourselves to cases where the profit rate which will be considered an equilibrium profit rate, is unique. There are several ways of accomplishing this: If we demand that an equilibrium (p, π) satisfy the von Neumann-Kemeny-Morgenstern-Thompson (1956) conditions:

Definition B. (p,π) is an equilibrium if there exist $x \geqq 0$ and a number g such that:

(i) $pB \leqq (1+\pi)pM$

(ii) $x \geq 0$, $pBx = (1+g)pMx$

(iii) $pBx \geqq (1+g)Mx$

(iv) $pBx > 0$

Then, if the technology (B,M) is irreducible (see Gale (1961, p. 314)) it follows that the equilibrium π is unique. (This definition corresponds to the Marxian equilibrium of extended reproduction (balanced growth), where goods which are produced at a rate faster than the prevailing growth rate become free.) If, on the other hand, we wish only to insist on the weaker definition of equilibrium given in Definition A, then we can define the equilibrium π as the minimum π satisfying the Conditions (1)-(4). (It is well-known that under

Definition B, $\pi = g$, for any equilibrium. Furthermore, if (B,M) is irreducible, then the unique π for which an equilibrium (p,π) exists is what I have called the von Neumann profit rate--the minimal π for which a non-negative, non-zero p exists satisfying (i). This fact can be deduced from Gale (1961).)

The purpose for engaging in the above discussion is to make clear that when one passes from the Leontief/Sraffa model to the von Neumann model one loses some convenient properties of the equilibrium price vectors: (1) in the latter model, one is not guaranteed that the price equilibrium is unique; (2) the price equilibrium cannot (necessarily) be defined independently of outputs, as it is in the Leontief system. For these reasons, the question of how one should define a Marxian equilibrium in a von Neumann model may have several answers; and, depending on the variant of answer chosen, the question of the falling rate of profit requires a different analysis. This discussion is beyond the scope of this paper. We shall, instead, restrict the discussion to a sufficiently general observation about what happens to the rate of profit in a von Neumann model, in which this ambiguity in what constitutes a good definition of Marxian equilibrium is avoided.
(For a complete discussion of general Marxian equilibria, see Roemer (1978b).)

C. The change in rate of profit in the von Neumann model in the presence of viable innovations.

Given the von Neumann technology (B,M) as specified above, we define an optimal price vector as any semi-positive price vector which minimizes the profit factor for the economy:

Definition. The minimal profit factor for (B,M) is that number $\rho^* = 1 + \pi^*$ which is minimal in the set

$\{\rho \geq 0 \,|\, (\exists p \geq 0)(pB \leq \rho pM)\}$

Any vector $p \geq 0$ satisfying

$$p^*B \leq \rho^* p^*M$$

is an optimal price vector.

Morishima (1974) has called π^* the guaranteed profit rate for the economy. Discussion above has shown that for some complete definitions of equilibrium, π^* is the only profit rate which makes sense. Finally, we might imagine that some competitive process drives the economy to minimal-profit-rate prices. Hence, optimal price vectors in the above sense are natural Marxian equilibrium prices.

An innovation in this economy is a new process which can be characterized as a new pair of columns to be added to the B and M matrices. Call the innovation (B^{m+1}, M^{m+1}) where B^{m+1} and M^{m+1} are the two new column vectors. Notice we do not replace columns of the production matrices with the innovations, as was the procedure in the simple Leontief model; rather, we append them to the old technology. In general, there are many alternative processes already in (B,M). Also, we may append many columns at once to (B,M).

We follow the same procedure for investigating changes in the rate of profit under innovation as is followed in the Leontief model in Section 2 above.

Definition. Let (p^*, π^*) be an optimal price vector at the minimal profit rate π^* for economy (B,M). An innovation (B^{m+1}, M^{m+1}) will be called viable with respect to p^* if and only if:

$$p^*B^{m+1} > (1+\pi^*)p^*M^{m+1} \tag{4.4}$$

If an innovation is viable, it upsets the old equilibrium, since it is more profitable than any process currently in operation at existing prices. (If a process is not viable, it does not upset the existing price equilibrium, and there is no question of a change in equilibrium.) Thus, we consider the <u>appended technology</u> $(\overline{B},\overline{M})$ where $\overline{B} = (B \vdots B^{m+1})$, $\overline{M} = (M \vdots M^{m+1})$, which includes the new viable innovation, and ask: what happens to the minimal profit factor of $(\overline{B},\overline{M})$? Does it necessarily rise or fall from π^*?

<u>Theorem 4.2</u>. Let (B,M) be a von Neumann technology with minimal profit rate π^* and optimal price vector p^*. Let (B^{m+1},M^{m+1}) be a viable innovation at prices p^*.

Then:

A. If the optimal price vector p^* is <u>unique</u> for (B,M), the minimal profit factor, $\overline{\pi}^*$, of the appended technology $(\overline{B},\overline{M})$ is greater than π^*.

B. If p^* is not unique, then there can always be constructed viable innovations (B^{m+1},M^{m+1}) which leave the minimal profit rate unchanged.

C. The minimal profit rate can never fall due to viable innovations.

This theorem is proved in the <u>Appendix</u>. It resolves the issue of what happens to the rate of profit in completely general fixed capital, joint production, alternative process models in consequence of the introduction of new techniques--subject to the caveats of what the definition of a Marxian equilibrium <u>is</u> in such general models. For our purposes here, the theorem makes this general point: no matter how one complicates the technology, the "competitive" profit rate can only rise a result of technical innovation, if the real wage remains unchanged.

A comment is worthwhile on the assumption of uniqueness of the optimal price vector p^*. If the optimal price vector p^* is not unique, then the theorem tells us the profit rate might stay the same after innovation. This is the generalization of what occurs in decomposable Leontief economies with no joint production: if an innovation occurs in a "luxury goods" process, the rate of profit will not be affected. If we are willing to assume that the proper specification of the economy has sufficient structure to guarantee unicity of equilibrium prices, then the profit rate must rise, unambiguously, with the innovation of a viable process--according to Theorem 4.2, Part A.

The question naturally arises: is there a condition on the von Neumann technology, such as indecomposability in the Leontief economy, which will guarantee unicity of the optimal price vector in the von Neumann model? The answer is yes, but the discussion of this question is beyond our scope here. The issue involved, anyway, is only whether the new profit rate rises or stays the same: it can in no case fall.

D. Some miscellaneous points on technical innovations

Two assumptions on the nature of technical change have been made in this paper, which are almost ubiquitous, it appears, in Marxian discussions of the falling rate of profit. One is that innovations take the form of inventing new processes, but not new commodities; the second is that innovations fall costlessly from the sky. A brief evaluation of what occurs upon relaxation of these assumptions follows.

I. It is not difficult to introduce the concept of the invention of new commodities into the von Neumann model. Recall each column of the matrix B (or M) is an n-vector of goods which appear as outputs (or inputs) from (or into) a particlar process. Suppose a process is invented which produces a new commodity--the $(n+1)^{st}$ commodity. The process is characterized by two $(n+1)$-column vectors B^{m+1}, M^{m+1}, where the $(n+1)^{st}$ components of the vectors are the outputs and inputs of the new commodity in the new process. We simply adjust the old technology (B,M) to the new commodity space by adding a zero component to each column of B and M--since the old processes involve the new commodity as neither input nor output. Hence the appended technology is

$$\left(\overline{B},\overline{M}\right) = \left(\begin{array}{c|c} B & B^{m+1} \\ 0 & \end{array}\right), \left(\begin{array}{c|c} M & M^{m+1} \\ 0 & \end{array}\right)$$

where the symbol 0, in both places, is a row vector of zeros m components long.

An optimal price vector for $\{\overline{B},\overline{M}\}$ is in $\mathbb{R}^{n+1}$. Let $\overline{p}$ be such an optimal price vector. Write $\overline{p} = (p,\overline{p}_{n+1})$ where p is the vector consisting of the first n components of $\overline{p}$ and $\overline{p}_{n+1}$ is the $(n+1)^{st}$ component. Now

$$\overline{p}\overline{B} \leq (1+\overline{\pi}^*)\overline{p}\overline{M} \tag{4.5}$$

which holds by definition of $\overline{p}$ where $\overline{\pi}^*$ is the minimal profit rate for $(\overline{B},\overline{M})$, implies

$$pB \leq (1+\overline{\pi}^*)pM \tag{4.6}$$

(This is easily seen from the definitions of $\overline{p},p,B,M,\overline{B},\overline{M}$.) Hence, by (4.6) as long as $p \neq 0$, $\overline{\pi}^*$ must be at least as large as π^*, the minimal profit rate for the old technology (B,M). But it is certainly reasonable to assume $p \neq 0$: for if $p = 0$, that is a statement that a price equilibrium exists in the new technology in which <u>all</u> old commodities become free goods. We can rule out such an occurrence as economically unrealistic. Hence an innovation which introduces a new commodity cannot but raise the minimal profit rate of the economy.

After the new good exists, it will gradually become used as input into many more processes: this phenomenon takes the form of adding many new columns to the technology $(\bar{B},\bar{M})$--the dynamics of which have already been discussed. Eventually all the original processes of (B,M) may become obsolete: but at each step the von Neumann profit rate rises or stays the same, according to Theorem 4.2.

II. It is certainly un-Marxian to assume that innovation falls costlessly from the sky. Innovations are socially determined. Indeed, as has been remarked above, in discussing the Persky-Alberro paper, technical change in modern capitalism is a costly and deliberate process, emerging out of huge R & D centers. How does this alter the FRP discussion?

We shall not engage in a thorough analysis of this question here: for even if such an analysis did give rise to a theory of falling rates of profit, that would be a genuinely new theory, and not a theory of the type this paper concentrates upon.

Several general comments can be made, however. Let us view the process of innovation as itself an activity: that is, it can be characterized by a set of inputs, and a set of outputs (new, improved techniques). Suppose there is a fair degree of certainty in what "outputs" will emerge from this process, for given input expenditure. Then capitalists can rationally allocate resources to the R & D process in such a way as to maximize total expected present discounted value--which is to say, they will never engage in R & D to such an extent as to lower the expected rate of profit. (Models verifying this intuition be easily worked out. The logic is the same as we have used above.)

Suppose, however, there is uncertainty in the operation of the R & D activity. This opens a Pandora's box, akin to the phenomenon pointed out by Persky-Alberro (1978). In this case, capitalists may commit large expenditures to R & D, which do not pan out, and actual profit rates fall (while expected rates rise, due to capitalists' misestimates). While this does generate a FRP "theory," one must ask: can such a theory explain secular or even cyclic falling rates of profit on an economy-wide level? Certainly not. Research and development has itself become such a large business, and so systematized and "rationalized" through division of labor, that the outputs are not random and subject to elements of individual genius, but quite predictable. One cannot appeal to uncertainty in the innovation process as a source for endemic falling rates of profit in capitalist economies.

One can approach the costly production of new technologies from another angle by asking what are the returns over time to R & D? Are there diminishing or increasing returns? It has been suggested to me that if there are diminishing returns to R & D then the rate of profit may fall. First, this is not the case. Under diminishing returns in the R & D industry, innovation would eventually stop (when the expected benefits from further R & D expenditure did not pay for the opportunity costs of shifting funds from direct profit-making production activities). But second, the assumption of diminishing returns in R & D is quite bizarre. If anything, casual empiricism suggests the opposite. The more one learns, the greater are the frontiers of knowledge, and the more beckoning the possibilities. (See the learning-by-doing literature.) Certainly a Marxist would not wish to base a theory of FRP on a finite supply of human inventive ingenuity. This would be a Malthusian or Ricardian theory of precisely the variety from which Marx endeavored to escape.

5. **Conclusion**.

For at least a generation, various writers have pointed out that there is no necessity for the rate of profit to fall as a consequence of technical change considered by itself (Robinson (1942); Sweezy (1942); Dobb (1945)). Okishio (1961) demonstrated in a simple and compelling model that the rate of profit would rise as a consequence of competitive innovation. In recent years, other writers have attempted to resurrect FRP theories, by poising more complicated, but still competitive, models. The argument of this paper is that there is no hope for producing a FRP theory in a competitive, laissez-faire environment.

It must be reiterated that this does not mean the rate of profit does not fall; it does not mean there cannot exist a theory of a falling rate of profit in capitalist economies. One must, however, relax some of the assumptions of the stark models discussed here to achieve such a FRP theory. Perhaps the most natural change to make concerns behavior of the real wage. If the real wage increases as a consequence of technical innovations, then a FRP may result. To produce a theory along these lines requires a rule which describes how the wage changes with innovation. An example of such a rule would be: the real wage changes to keep the share of wages in national income constant. It can be shown that with such a rule, viable technical innovations will give rise to a FRP (see Roemer (1978a)). The general point is this: if the rate of profit falls in such a changing real wage model it is a consequence of the class struggle which follows technical innovation, not because of the innovation <u>itself</u>. A second possibility for producing a FRP theory is to produce a theory of rising state expenditures, which eat into before-tax profits, thus rendering a fall in the after-tax profit rate. This, indeed, is the suggestion of much recent Marxist work on the state (see Wright (1975), O'Connor (1973)). A third possibility, suggested by Rowthorn (1976), is that the increase in bargaining power of the LDCs vis-à-vis the imperialist countries may have shifted the terms of trade against the latter, resulting in a lower rate of profit for imperial capital.

Clearly this list does not pretend to be exhaustive: the general point is that many FRP theories can exist if the pure competitive model is abandoned. The general attack by the "fundamentalists" on these attempts is this: the "new" theories of FRP do not deduce a falling rate of profit from the development of capital *itself*, but from various *ad hoc* phenomena, such as class struggle (rising real wages), increased role of state, *etc.* A favorite quote, from Marx, is "The real barrier of capitalist production is *capital itself*." (Marx (1967), p. 250) But if one wishes to construct an exegesis of FRP based on the implied theory of the development of capital from this quote, one can reply that the fundamentalists have taken far too narrow a definition of what "capital itself" is. Capital itself is a social relation, as we are so often reminded, and as such its development must include such phenomena as class struggle around the real wage, the increasing role of the state, uneven development of capitals (shifting terms of trade), and all the various influences which appear as *ad hoc* modifications of the rising organic composition model. These new theories are also called "profit-squeeze" theories. It is only the most narrow view of capital which classifies profit-squeeze theories as abandoning the view that capitalism develops according to its internal contradictions, and therfore that the fundamental barrier to capitalist production is capital itself.

Finally, I wish to comment on the political positions which are frequently said to necessarily coexist with the positions taken in the FRP debate. The fundamentalists often say or imply that the profit-squeeze theories lead to reformist politics, as the fall, if any, in the rate of profit becomes completely contingent on ad hoc and subjective elements. The rising OCC theories, on the other hand, imply capitalist crisis independent of the subjective wills of men/women and lead therefore to a more revolutionary politics. A profit-squeeze theorist, however, could reply the supposed deduction that a FRP and capitalist crisis occurs independently of the intervention of class struggle, as in the rising OCC theories, can give rise to an economist and mechanistic theory of politics, where the necessity for conscious organization is dissipated. I do not think either of these theories of political action follows logically from its respective premises: the connection between economic theory and political practice is considerably more circuitous and subtle. The point of the examples, then, is to call for a more dispassionate discussion of the issues. One should not have to fear advocating a particular position in the discussion, or in scientific work in general, because of consequential branding as a political heretic. Were the discussions conducted on this basis, whatever myths remain could be more easily cast aside, and progress would be made in developing a Marxian theory of crisis.

Appendix

Proposition 1. Let (A^{*i}, L^{*i}) be a viable, CU-LS technical change in sector i. The real wage b is fixed. Assume the augmented technology M is indecomposable. Then:

(a) the OCC will rise in sector i;

(b) the OCC may rise or fall in the other sectors;

(c) the aggregate OCC may rise or fall.

Proof:

Part (a):

Labor values of commodities are defined as the vector:

$\Lambda = L(I-A)^{-1}$.

The OCC in sector i is:

$$\omega_i = \frac{C_i}{V_i} = \frac{\Lambda A^i}{\frac{1}{1+e}L^i} = \frac{\Lambda A^i}{L^i(\Lambda b)} = \frac{L(I-A)^{-1}A^i}{L^i(\Lambda b)} = \frac{(L/L^i)(I-A)^{-1}A^i}{\Lambda b} \qquad (A.1)$$

If a CU-LS change occurs in sector i, then the vector L/L^i increases in all components except component i, where it remains equal to unity. The matrix $(I-A)^{-1}$ increases in all components, since the matrix A increases to the matrix A^*. The vector A^i increases. Hence, the numerator of the final expression in (A.1) increases.

Now it is known that viable, CU-LS innovations decrease all labor values in an indecomposable system. (This theorem is proved in Roemer (1977).) Hence the denominator, Λb, in (A.1) decreases.

Since the numerator in (A.1) increases and the denominator decreases, ω_i increases with viable, CU-LS technical change.

Part (b):

This is proved by constructing examples.

Suppose the technical coefficients in sector 1 are almost identical to the vector b. That is:

$$a_{j1} = b_j \qquad \forall_j \neq i$$
$$a_{i1} = b_i + \varepsilon_i, \qquad \varepsilon_i > 0$$

where i is the sector where technical change is occuring, and $i \neq 1$.

Then, from (A.1):

$$\omega_1 = \frac{\Lambda A^1}{L^1(\Lambda b)} = \frac{1}{L^1}\left(\frac{\Lambda b+\Lambda_i\varepsilon_i}{\Lambda b}\right) = \frac{1}{L^1}\left(1+\frac{\varepsilon_i}{\frac{\Lambda}{\Lambda_i}b}\right) \qquad (A.2)$$

With the technical change in sector i, notice L^1, ε_i, b all remain constant. $\frac{\Lambda}{\Lambda_i}$ changes. The vector $\frac{\Lambda}{\Lambda_i}$, in fact, <u>increases</u> in all components: for it is known that a technical change in sector i which decreases the value of commodity i will decrease the value of its own product relatively most. (See Morishima (1973), p. 33.) Hence, from (A.2), ω_i decreases.

Similarly, an example where ω_i increases is constructed by specifying the technology of sector 1 by:

$$a_{j1} = b_j \qquad \forall_j \neq i$$
$$a_{ji} = b_i - \varepsilon_i, \qquad \varepsilon_i > 0.$$

<u>Part (c)</u>:

Clearly, if the OCC falls in some sectors while it necessarily increases in sector i, the aggregate OCC may rise or fall depending on the composition of output (<u>i.e.</u>, the weights of aggregation). q.e.d.

<u>Theorem 3.1</u>. Let (p,r) be the equilibrium for the technique (Φ,A,L), satisfying:

$$p = rp\Phi + (1+r)(pA+L) \qquad (A.3)$$
$$1 = pb$$

Let an innovation satisfy:

$$p^i > rp(\Phi^i+\Phi^{i*}) + (1+r)(pA^{i*}+L^{i*}) \qquad (A.4)$$

Then, the new equilibrium (p^*, r^*) satisfies:

$$p^* = r^* p^* \hat{\Phi} + (1+r)(p^* A^* + L^*) \qquad (A.5)$$

$$1 = p^* b$$

and $r^* > r$, where $\hat{\Phi}$ is the matrix Φ with column i replaced by $\Phi^i + \Phi^{i*}$. (It is assumed that the matrix $(A^* + bL^* + \Phi)$ is indecomposable.)

Proof:

It is not proved here that a unique equilibrium exists. That is a consequence of the Frobenius theorem in the usual way.

Let $M \equiv A + bL$ be the augmented circulating input coefficient matrix. Then (A.3) can be re-written:

$$p = p[M + r(M + \Phi)] . \qquad (A.6)$$

Similarly, let $M^* \equiv A^* + bL^*$, where the matrix A^* (and vector L^*) are gotten, recall, by replacing the i^{th} column (component) of $A(L)$ with $A^{i*}(L^{i*})$.

According to (A.3) and (A.4) we have:

$$p_i > p(M^{i*} + r(M^{i*} + \hat{\Phi}^i))$$

$$p_j = p(M^{j*} + r(M^{j*} + \hat{\Phi}^j)) \quad \text{for } j \neq i \qquad (A.7)$$

Consider the matrix $(M^* + r(M^* + \hat{\Phi})) \equiv \Omega(r)$. According to (A.7), p is a non-negative vector which has the property:

$$p \geq p\Omega(r).$$

It is well-known that the existence of such a non-negative vector implies the Frobenius eigenvalue of $\Omega(r)$ is smaller than unity (Frobenius theorem). It is also well-known that increasing the components of $\Omega(r)$ will increase the matrix's Frobenius eigenvalue; this follows since $M^* + \Phi$ is indecomposable, and therefore so is $M^* + r(M^* + \hat{\Phi})$. Hence, increasing r, which increases the components of $\Omega(r)$, eventually produces a matrix $\Omega(r^*)$ with eigenvalue of unity, since an equilibrium exists. For r^*, there exists a positive eigenvalue for p^* such that:

$$p^* = p^*\Omega(r^*) \tag{A.8}$$

But (A.8) is equivalent to (A.5). It has been shown that $r^* > r$. q.e.d

<u>Proposition 4.1</u>. If $B = I$ and M is an indecomposable square matrix (pure circulating model) then the only equilibrium (p,π) satisfying Definition A is the Frobenius equilibrium.

<u>Proof of Proposition 4.1</u>:

Let (p,π) satisfy (1)-(4). There are two possibilities:

Case (i). $x = Mx$ (from (3)).

In this case, x is the Frobenius eigenvector of M, since $x \neq 0$ by Condition (4), and so $x > 0$, since M is indecomposable. It follows from Condition (2) that $p = (1+\pi)pM$ and hence p is the Frobenius row eigenvector of M. (Also, we have $\pi = 0$ in this case.)

<u>Case (ii)</u>. $x \geq Mx$ (from (3)).

Hence $(I-M)x \geq 0$. Since M is indecomposable, $(I-M)^{-1} > 0$ and so $x > 0$. As above, it follows that $p = (1+\pi)pM$, and (p,π) are unique. q.e.d.

<u>Theorem 4.2</u>. Let (B,M) be a von Neumann technology with minimal profit rate π^* and optimal price vector p^*. Let (B^{m+1},M^{m+1}) be a viable innovation at price p^*.

Then:

A. If the optimal price vector p^* is <u>unique</u>, the minimal profit factor, $\overline{\pi}^*$, of the appended technology $(\overline{B},\overline{M})$ is greater than π^*.

B. If p^* is not unique, then there can always be constructed viable innovations (B^{m+1},M^{m+1}) which leave the minimal profit rate unchanged.

C. The minimal profit rate can never fall due to viable innovations.

<u>Proof of Theorem 4.2.</u>

<u>Part C</u>. It is clear the minimal profit can never fall in the appended

technology. For let $(\overline{p}^*, \overline{\pi}^*)$ be an optimal price vector/optimal profit rate for $(\overline{B},\overline{M})$. Then $\overline{p}^*\overline{B} \leq (1+\overline{\pi}^*)\overline{p}^*M$. Thus $\overline{\pi}^* \geq \pi^*$ by definition of π^*.

Part A. Let p^* be the unique optimal price vector for (B,M). Suppose the minimal profit factor for $(\overline{B},\overline{M})$ were π^*--that is, it did not rise. Then:

$$\text{there exists } \overline{p}^* \geq 0 \text{ such that } \overline{p}^*\overline{B} \leq (1+\pi^*)\overline{p}^*\overline{M} \qquad (1)$$

In particular, $\overline{p}^*B \leq (1+\pi^*)\overline{p}^*M$. But by the unity of p^*, we must therefore have $\overline{p}^* = p^*$. This, however, is impossible: because by the assumption of viability, $p^*B^{m+1} > (1+\pi^*)p^*M^{m+1}$; but according to (1), $\overline{p}B^{m+1} \leq (1+\pi^*)p^*M^{m+1}$. Hence the minimal profit factor must rise in the appended technology.

Part B. Let p^* and p^{**} be two optimal intensity price vectors for (B,M). Since $p^* \neq p^{**}$, there can be chosen a vector C such that

$$p^* \cdot C < 0,\ p^{**} \cdot C \geq 0. \qquad (2)$$

(This is a result of the so-called separating hyperplane theorem.) We can always find non-negative vectors called B' and M' such that:

$$C = (1+\pi^*)M' - B'. \qquad (3)$$

Since B' and M' are non-negative, they qualify as a conceivable technical process, with inputs and outputs specified by M' and B' respectively.

By (2) and (3) it follows that the innovation (B',M') is viable with respect to prices p^*; however, from (2) we also have

$$p^{**}B' \leq (1+\pi^*)p^{**}M' \qquad (4)$$

which means that in the appended technology

$$\overline{B} = (B \mid B'),\ \overline{M} = (M \mid M') \text{ we have } p^{**}\overline{B} \leq (1+\pi^*)p^{**}\overline{M}.$$

Hence the minimal profit rate of $(\overline{B},\overline{M})$ remains unchanged at π^*. q.e.d.

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