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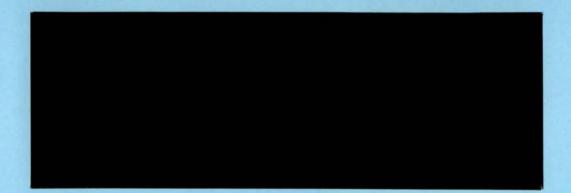
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Money



### **WORKING PAPER SERIES**

MONEY IN A SEQUENCE ECONOMY: A CORRECTION

by

Ross M. Starr

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Department of Economics University of California Davis, California Money in a Sequence Economy: A Correction\*

by

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In Starrett's elegant and important study [1], there is an oversight in the statement of Lemma 1 (p. 446). The sense of the inequality in equation (23) and the definition of the interest factor  $S_w^n$  are incorrect as printed. An alternative correct formulation of the lemma is:

Lemma 1': The problem (19) is equivalent to  

$$(x_h^w, y_h^w)$$
 maximizes U<sup>h</sup> ...(22)

subject to nonnegativity of  $c_h$ ,  $x_h^w$ ,  $y_h^w$ , and subject to  $\sum_{w} \sigma_w^{n(h)} (q^w y_h^w - p^w x_h^w) \le 0$ 

or, equivalently,

$$\overline{S}_{w}^{n}(q^{w}y_{h}^{w} - p^{w}x_{h}^{w}) \leq 0, \qquad \dots (23B)$$

)

..(23A)

where  $\sigma_{w}^{n(h)} \equiv \Pi_{t=w}^{n(h)-1} S^{t}$ ,  $\sigma_{n(h)}^{n(h)} \equiv 1$ ,

and  $\overline{S}_{w}^{n} \equiv \prod_{t=n}^{w-1} \frac{1}{c^{t}}$ , for n an arbitrary base date  $(n \leq b(h)-1)$ .

The original statement in [1] differs in the sense of the inequality and by using  $S_w^n$  in place of  $\overline{S}_w^n(S_w^n \equiv \pi_{t=n}^w S^t)$ .  $\phi_w^{n(h)}$  does not appear in [1]. Recall that  $S^t$  is an interest factor on the household's indebtedness at date t.

As a plausibility argument that a revision of Lemma 1 is required, recall that (23) derives from the requirement (18) that the household's net debt in its terminal period  $d_h^{n(h)}$  be nonpositive ( $\leq 0$ ). The LHS of (23) is the weighted sum of debts accumulated at dates w, so the revised sense of the

<sup>\*</sup>Notation and the numbering of equations are taken from [1]. I am indebted to Rhonda Price for assistance in research and to David Starrett for a helpful conversation. Errors are my responsibility.

inequality in (23') is appropriate. Debt incurred at early dates w should carry a heavy weight of accumulated interest at n(h) (the household's death date) compared with debt of w close to n(h). Hence the revised definition of  $\overline{S}_{w}^{n}$  appears the more appropriate. The original formluation of  $S_{w}^{n}$  in [1] is inadequate since it leads to a LHS of

> $\Sigma(\pi s^t)(q^w y_h^w - p^w x_h^w)$  bearing no particular relationship to  $d_h^{n(h)}$ . w t=n

To prove lemma 1', follow the prescription for proving lemma 1 in [1]. That is, substitute recursively for  $d_h^w$  in

$$d_{h}^{w+1} = d_{h}^{w}S^{w} + q^{w+1}y_{h}^{w+1} - p^{w+1}x_{h}^{w+1} \qquad \dots (17)$$

using

$$d_h^{b(h)-1} = 0; \ d_h^{n(h)} \leq 0.$$
 (18)

Recall that b(h), n(h) are respectively the first and last dates of the household's life. We have

$$d_{h}^{n(h)} = d_{h}^{n(h)-1} s^{n(h)-1} + q^{n(h)} y_{h}^{n(h)} - p^{n(h)} x_{h}^{n(h)}$$

$$= d_{h}^{n(h)-2} s^{n(h)-2} s^{n(h)-1} + (q^{n(h)-1} y_{h}^{n(h)-1} - p^{n(h)-1} x_{h}^{n(h)-1}) s^{n(h)-1}$$

$$+ q^{n(h)} y_{h}^{n(h)} - p^{n(h)} x_{h}^{n(h)}$$

$$= \dots \leq 0$$

That is,  $d_h^{n(h)} = \sum_{\substack{w \\ w}} (\pi_{t=w}^{n(h)-1} s^t) (q^w y_h^w - p^w x_h^w) \leq 0$ . This is precisely (23A).

Birth and death dates, b(h), n(h), depend on the household, but it is convenient to have a version of (23A) independent of these dates. Choose the base date n smaller than b(h)-l for all h. Note that

$$\overline{S}_{w}^{n} \equiv \left( \prod_{t=n}^{n(h)-1} \frac{1}{s^{t}} \right) \sigma_{w}^{n(h)}$$

Then (23B) comes from (23A) by multiplying through by  $\begin{pmatrix} n(h)-1 \\ I \\ t=n \\ S^t \end{pmatrix}$ . This completes the proof of lemma 1'.

The definition of  $\overline{S}^n_w$  is formally very similar to  $S^n_w$  in [1] and may

represent what that term was intended to convey.  $S_w^n = \frac{s^w}{S_w^n}$  Theorems 2 and 3 follow as before. In the proofs we substitute  $\overline{S}_w^n$  for  $S_w^n$ . Note that in Theorem 2, where lifetimes differ, this substitution is obligatory to make use of the base date n which is common to the households though their lifetimes differ. In Theorem 3, where all lifetimes coincide,  $S_w^n$  can be replaced in the proof by  $\overline{S}_w^n$  or by  $\sigma_w^{n(h)}$ .

#### REFERENCE

 Starrett, David, "Inefficiency and the Demand for 'Money' in a Sequence Economy," <u>Review of Economic Studies</u>, v. XL(4), no. 124, October 1973, pp. 437-448.

