MONEY IN A SEQUENCE ECONOMY: A CORRECTION

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Working Paper Series
No. 72

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November 1976

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In Starrett's elegant and important study [1], there is an oversight in the statement of Lemma 1 (p. 446). The sense of the inequality in equation (23) and the definition of the interest factor $S^n_w$ are incorrect as printed. An alternative correct formulation of the lemma is:

**Lemma 1':** The problem (19) is equivalent to

$$(w, y_h^w) \text{ maximizes } U^h$$

subject to nonnegativity of $c_h^w$, $x_h^w$, $y_h^w$, and

subject to $\sum_{w} (q_w^w y_h^w - p_w^w x_h^w) \leq 0$ ... (23A)

or, equivalently,

$$\sum_{w} (q_w^w y_h^w - p_w^w x_h^w) \leq 0,$$ ... (23B)

where $\sigma_n(h) = \prod_{t=n}^{b(h)-1} S_t^w$, $\sigma_n(h) = 1$,

and $S^n_w = \prod_{t=n}^{b(h)-1} s_t^w$, for $n$ an arbitrary base date ($n \leq b(h)-1$).

The original statement in [1] differs in the sense of the inequality and by using $S^n_w$ in place of $S^n_w$ in the revised statement. The LHS of (23) does not appear in [1].

As a plausibility argument that a revision of Lemma 1 is required, recall that (23) derives from the requirement (18) that the household's net debt in its terminal period $d_n^t(h)$ be nonpositive ($\leq 0$). The LHS of (23) is the weighted sum of debts accumulated at dates $w$, so the revised sense of the

*Notation and the numbering of equations are taken from [1]. I am indebted to Rhonda Price for assistance in research and to David Starrett for a helpful conversation. Errors are my responsibility.
inequality in (23') is appropriate. Debt incurred at early dates \( w \) should carry a heavy weight of accumulated interest at \( n(h) \) (the household's death date) compared with debt of \( w \) close to \( n(h) \). Hence the revised definition of \( S^n_w \) appears the more appropriate. The original formulation of \( S^n_w \) in [1] is inadequate since it leads to a LHS of

\[
\sum_{t=n}^{w} \Gamma(t) (Q_{y}^t - p_{x}^t) \text{ bearing no particular relationship to } d^n_h.
\]

To prove lemma 1', follow the prescription for proving lemma 1 in [1]. That is, substitute lemma recursively for \( d^n_w \) in

\[
d^{n+1}_h = d^n_h s^n + q^{n+1} w + p^{n+1} w \quad \text{(17)}
\]

using

\[
d_{b(h)-1} = 0; \quad d^n_h \leq 0. \quad \text{(18)}
\]

Recall that \( b(h), n(h) \) are respectively the first and last dates of the household's life. We have

\[
d^n_h = d^n_h - s^n_h - 1 + q^n y^n_h - p^n x^n_h
\]

\[
= d^n_h - 2 s^n_h - 2 q^n y^n_h - 1 - p^n x^n_h - 1 s^n_h
\]

\[
+ q^n y^n_h - p^n x^n_h
\]

\[
= ... \leq 0
\]

That is, \( d^n_h = \sum_{t=n}^{w} \Gamma(t) (Q_{y}^t - p_{x}^t) \leq 0. \) This is precisely (23A).

Birth and death dates, \( b(h), n(h) \), depend on the household, but it is convenient to have a version of (23A) independent of these dates. Choose the base date \( n \) smaller than \( b(h)-1 \) for all \( h \). Note that

\[
\bar{S}^n_w \equiv (\sum_{t=n}^{w} 1 \cdot \Gamma(t) s^n_w).
\]

Then (23B) comes from (23A) by multiplying through by \((\sum_{t=n}^{w} 1 \cdot \Gamma(t))\). This completes the proof of lemma 1'.

The definition of \( \bar{S}^n_w \) is formally very similar to \( S^n_w \) in [1] and may
represent what that term was intended to convey. \( S^w_n = \frac{s^w_n}{s^w} \) Theorems 2 and 3 follow as before. In the proofs we substitute \( \overline{s}^n_w \) for \( s^w_n \). Note that in Theorem 2, where lifetimes differ, this substitution is obligatory to make use of the base date \( n \) which is common to the households though their lifetimes differ. In Theorem 3, where all lifetimes coincide, \( s^w_n \) can be replaced in the proof by \( \overline{s}^n_w \) or by \( a^w_n(h) \).
REFERENCE
