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MONEY IN A SEQUENCE ECONOMY: A CORRECTION

by
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Money in a Sequence Economy: A Correction*

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In Starrett's elegant and important study [1], there is an oversight in the statement of Lemma 1 (p. 446). The sense of the inequality in equation (23) and the definition of the interest factor S_w^n are incorrect as printed. An alternative correct formulation of the lemma is:

Lemma 1': The problem (19) is equivalent to

$$(x_h^w, y_h^w) \text{ maximizes } U^h \quad \dots(22)$$

subject to nonnegativity of c_h , x_h^w , y_h^w , and

$$\text{subject to } \sum_w \sigma_w^{n(h)} (q^w y_h^w - p^w x_h^w) \leq 0 \quad \dots(23A)$$

or, equivalently,

$$\sum_w \bar{S}_w^n (q^w y_h^w - p^w x_h^w) \leq 0, \quad \dots(23B)$$

where $\sigma_w^{n(h)} \equiv \prod_{t=w}^{n(h)-1} S^t$, $\sigma_{n(h)}^{n(h)} \equiv 1$,

and $\bar{S}_w^n \equiv \prod_{t=n}^{w-1} \frac{1}{S^t}$, for n an arbitrary base date ($n \leq b(h)-1$).

The original statement in [1] differs in the sense of the inequality and by using S_w^n in place of \bar{S}_w^n ($S_w^n \equiv \prod_{t=n}^w S^t$). $\sigma_w^{n(h)}$ does not appear in [1].

Recall that S^t is an interest factor on the household's indebtedness at date t .

As a plausibility argument that a revision of Lemma 1 is required, recall that (23) derives from the requirement (18) that the household's net debt in its terminal period $d_h^{n(h)}$ be nonpositive (≤ 0). The LHS of (23) is the weighted sum of debts accumulated at dates w , so the revised sense of the

*Notation and the numbering of equations are taken from [1]. I am indebted to Rhonda Price for assistance in research and to David Starrett for a helpful conversation. Errors are my responsibility.

inequality in (23') is appropriate. Debt incurred at early dates w should carry a heavy weight of accumulated interest at $n(h)$ (the household's death date) compared with debt of w close to $n(h)$. Hence the revised definition of \bar{S}_w^n appears the more appropriate. The original formulation of S_w^n in [1] is inadequate since it leads to a LHS of

$$\sum_{t=n}^w (S^t) (q^w y_h^w - p^w x_h^w) \text{ bearing no particular relationship to } d_h^{n(h)}.$$

To prove lemma 1', follow the prescription for proving lemma 1 in [1].

That is, substitute recursively for d_h^w in

$$d_h^{w+1} = d_h^w S^w + q^{w+1} y_h^{w+1} - p^{w+1} x_h^{w+1} \quad \dots(17)$$

using

$$d_h^{b(h)-1} = 0; d_h^{n(h)} \leq 0. \quad \dots(18).$$

Recall that $b(h)$, $n(h)$ are respectively the first and last dates of the household's life. We have

$$\begin{aligned} d_h^{n(h)} &= d_h^{n(h)-1} S^{n(h)-1} + q^{n(h)} y_h^{n(h)} - p^{n(h)} x_h^{n(h)} \\ &= d_h^{n(h)-2} S^{n(h)-2} S^{n(h)-1} + (q^{n(h)-1} y_h^{n(h)-1} - p^{n(h)-1} x_h^{n(h)-1}) S^{n(h)-1} \\ &\quad + q^{n(h)} y_h^{n(h)} - p^{n(h)} x_h^{n(h)} \\ &= \dots \leq 0 \end{aligned}$$

That is, $d_h^{n(h)} = \sum_w^{\Pi_{t=w}^{n(h)-1} S^t} (q^w y_h^w - p^w x_h^w) \leq 0$. This is precisely (23A).

Birth and death dates, $b(h)$, $n(h)$, depend on the household, but it is convenient to have a version of (23A) independent of these dates. Choose the base date n smaller than $b(h)-1$ for all h . Note that

$$\bar{S}_w^n \equiv \left(\prod_{t=n}^{n(h)-1} \frac{1}{S^t} \right) \sigma_w^{n(h)}.$$

Then (23B) comes from (23A) by multiplying through by $\left(\prod_{t=n}^{n(h)-1} \frac{1}{S^t} \right)$. This completes the proof of lemma 1'.

The definition of \bar{S}_w^n is formally very similar to S_w^n in [1] and may

represent what that term was intended to convey. $S_w^n = \frac{\sum_{i=1}^w S_w^n}{S_w^n}$ Theorems 2 and 3 follow as before. In the proofs we substitute \bar{S}_w^n for S_w^n . Note that in Theorem 2, where lifetimes differ, this substitution is obligatory to make use of the base date n which is common to the households though their lifetimes differ. In Theorem 3, where all lifetimes coincide, S_w^n can be replaced in the proof by \bar{S}_w^n or by $\sigma_w^{n(h)}$.

REFERENCE

- [1] Starrett, David, "Inefficiency and the Demand for 'Money' in a Sequence Economy," Review of Economic Studies, v. XL(4), no. 124, October 1973, pp. 437-448.

