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MONEY IN A SEQUENCE ECONOMY: A CORRECTION

by

Ross M. Starr

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Department of Economics
University of California
Davis, California

Money in a Sequence Economy: A Correction*

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Ross M. Starr
University of California, Davis

In Starrett's elegant and important study [1], there is an oversight in the statement of Lemma 1 (p. 446). The sense of the inequality in equation (23) and the definition of the interest factor S^n_w are incorrect as printed. An alternative correct formulation of the lemma is:

Lemma 1': The problem (19) is equivalent to

$$(x^w_h, y^w_h) \text{ maximizes } U^h \qquad \ldots(22)$$

subject to nonnegativity of c_h, x^w_h, y^w_h, and

$$\text{subject to } \sum_w \sigma^{n(h)}_w (q^w y^w_h - p^w x^w_h) \leq 0 \qquad \ldots(23A)$$

or, equivalently,

$$\sum_w \bar{S}^n_w (q^w y^w_h - p^w x^w_h) \leq 0, \qquad \ldots(23B)$$

where $\sigma^{n(h)}_w \equiv \Pi^{n(h)-1}_{t=w} S^t$, $\sigma^{n(h)}_{n(h)} \equiv 1$,

and $\bar{S}^n_w \equiv \Pi^{w-1}_{t=n} \frac{1}{S^t}$, for n an arbitrary base date ($n \leq b(h)-1$).

The original statement in [1] differs in the sense of the inequality and by using S^n_w in place of $\bar{S}^n_w (S^n_w \equiv \Pi^w_{t=n} S^t)$. $\sigma^{n(h)}_w$ does not appear in [1]. Recall that S^t is an interest factor on the household's indebtedness at date t.

As a plausibility argument that a revision of Lemma 1 is required, recall that (23) derives from the requirement (18) that the household's net debt in its terminal period $d^{n(h)}_h$ be nonpositive (≤ 0). The LHS of (23) is the weighted sum of debts accumulated at dates w, so the revised sense of the

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*Notation and the numbering of equations are taken from [1]. I am indebted to Rhonda Price for assistance in research and to David Starrett for a helpful conversation. Errors are my responsibility.

inequality in (23') is appropriate. Debt incurred at early dates w should carry a heavy weight of accumulated interest at n(h) (the household's death date) compared with debt of w close to n(h). Hence the revised definition of $\bar{S}^n_w$ appears the more appropriate. The original formluation of S^n_w in [1] is inadequate since it leads to a LHS of

$$\sum_w \left(\prod_{t=n}^{w} S^t\right)(q^w y^w_h - p^w x^w_h) \text{ bearing no particular relationship to } d_h^{n(h)}.$$

To prove lemma 1', follow the prescription for proving lemma 1 in [1]. That is, substitute recursively for d^w_h in

$$d_h^{w+1} = d_h^w S^w + q^{w+1} y_h^{w+1} - p^{w+1} x_h^{w+1} \qquad \ldots(17)$$

using

$$d_h^{b(h)-1} = 0;\ d_h^{n(h)} \leq 0. \qquad \ldots(18).$$

Recall that b(h), n(h) are respectively the first and last dates of the household's life. We have

$$d_h^{n(h)} = d_h^{n(h)-1} S^{n(h)-1} + q^{n(h)} y_h^{n(h)} - p^{n(h)} x_h^{n(h)}$$

$$= d_h^{n(h)-2} S^{n(h)-2} S^{n(h)-1} + (q^{n(h)-1} y_h^{n(h)-1} - p^{n(h)-1} x_h^{n(h)-1}) S^{n(h)-1}$$

$$+ q^{n(h)} y_h^{n(h)} - p^{n(h)} x_h^{n(h)}$$

$$= \ldots \leq 0$$

That is, $d_h^{n(h)} = \sum_w (\prod_{t=w}^{n(h)-1} S^t)(q^w y^w_h - p^w x^w_h) \leq 0$. This is precisely (23A).

Birth and death dates, b(h), n(h), depend on the household, but it is convenient to have a version of (23A) independent of these dates. Choose the base date n smaller than b(h)-1 for all h. Note that

$$\bar{S}^n_w \equiv \left(\prod_{t=n}^{n(h)-1} \frac{1}{S^t}\right) \sigma_w^{n(h)}.$$

Then (23B) comes from (23A) by multiplying through by $\left(\prod_{t=n}^{n(h)-1} \frac{1}{S^t}\right)$. This completes the proof of lemma 1'.

The definition of $\bar{S}^n_w$ is formally very similar to S^n_w in [1] and may represent what that term was intended to convey. $\bar{S}^n_w = \frac{S^w}{S^n_w}$ Theorems 2 and 3 follow as before. In the proofs we substitute $\bar{S}^n_w$ for S^n_w. Note that in Theorem 2, where lifetimes differ, this substitution is obligatory to make use of the base date n which is common to the households though their lifetimes differ. In Theorem 3, where all lifetimes coincide, S^n_w can be replaced in the proof by $\bar{S}^n_w$ or by $\sigma^{n(h)}_w$.

REFERENCE

[1] Starrett, David, "Inefficiency and the Demand for 'Money' in a Sequence Economy," Review of Economic Studies, v. XL(4), no. 124, October 1973, pp. 437-448.