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CONJECTURAL VARIATIONS WITH FEWER APOLOGIES

by

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1. INTRODUCTION

The case of two producers, 'duopoly', may be illustrated by the following simple example:
Let the demand line be $p = c - k\,(x_1 + x_2)$, and the suppliers' lines $p_1 = l_1\, x_1$, $p_2 = l_2\, x_2$.
The first supplier varies x_1 to maximize $(c - k\,(x_1 + x_2) - l_1\, x_1)\, x_1$
so that he aims at x_1 given by $(c - 2\,(k + l_1)\, x_1 - kx_2 - kD_{x_1}(x_2) = 0.$
The second aims at x_2 given by $(c - 2\,(k + l_2)\, x_2 - kx_1 - kD_{x_2}(x_1) = 0.$
To solve these we should need to know x_2 as a function of x_1, and this depends on what each producer thinks the other is likely to do. There is then likely to be oscillation in the neighborhood of the price given by the equation marginal price for each = selling price,
unless they combine and arrange what each shall produce so as to maximize their combined profit (p. 38).

—A. L. Bowley, The Mathematical Groundwork of Economics, 1924.

This paper presents a complete and unified treatment of Bowley's (1924) conjectural variations. There are five objectives. The first is to provide a general solution to the consistent-conjectures problem, which received considerable attention throughout the last decade. The second is to explain the curious appearance of discrepant interpretations of the model. The third is to bring to attention hitherto unrecognized findings that link Bowley's contributions to the history of economic thought and the pedagogic development of the static theory of oligopoly. The fourth is to elucidate the essential similarities and dissimilarities between consistent-conjectures and Cournot-Nash. The fifth and final contribution is to provide a fairly complete menu of comparative-static predictions for consistent-conjectures oligopolies.

Interest in conjectural variations reached its peak in the early-to-mid 1980's following investigations of the so-called consistency criteria. Two of the earliest examples are Laitner (1980) and Bresnahan (1981). These focused on the duopoly situation. Later efforts sought generalizations, initially to identical-firm industries and subsequently to non-symmetric situations. There emerged two literatures in which the model was the center of attention. One, which is largely philosophical in purview, focuses on the rationality-cum-irrationality of the consistency criteria. Examples—too numerous to mention here—are listed in an appendix, which is available upon request. The other literature, stemming from Iwata (1974), is surveyed by Bresnahan (1989). It uses the model as a basis for empirical investigations of market power. These empirical studies fostered something of a revival for conjectural variations; the infusion of game theory into oligopoly had earlier spelled its demise. Whether game theory was the sole contributing factor remains upon to question. Irrespective of the reasons, indifference to Bowley's theory has left several issues unresolved. This paper addresses each of these in the context of a search for consistent conjectures.

The genesis of consistency actually dates to Leontief (1936). Interest in the topic, renewed or otherwise, did not emerge until the late '70's and early '80's. It is debatable, although logical, that the reason for this was the increased acceptance of rational expectations as an equilibrium concept in stochastic theory. The analogy, both intellectual and applicable, between rational expectations and consistent conjectures is apparent to anyone who has solved a rational-expectations model, retaining subjectivity in beliefs until the penultimate step in the analysis. No doubt, the ground for a similar approach in non-stochastic oligopoly was certainly more fertile during the '80's than it was in 1936. Clearly ahead of its time, Leontief's contribution was overlooked until quite recently (Pace and Gilley, 1990).

Leontief's work is noteworthy for another reason. It uncovers the formal link between the quantity-setting models of Cournot, Bowley, and Stackelberg. In his review of Stackelberg (1934), Leontief indicates, quite clearly, that Bowley's duopoly is a Stackelberg duopoly in which both firms behave as leaders; Cournot, on the other hand, is a Stackelberg duopoly in which both firms behave as followers. Written in German, Stackelberg's contribution remained mostly inaccessible until the appearance of Leontief's review, but the review appears also to have gone unnoticed. The interpretation of Bowley's firms as leaders in the Stackelberg duopoly has three important implications. First, it leads naturally to a correct formulation of the firm's problem. Second, it uncovers the logical flaw in conjectural variations and clarifies the essential difference between it and Cournot-Nash. Third, it leads ultimately to resolution of the consistent-conjectures problem.

I present some necessary technicalities in section 2 and, in section 3, review the significance of Stackelberg's contribution. Section 4 derives comparative statics from which the consistency criteria emerge and previews the strategy of the search. I begin the search in section 5 and examine convergence and existence, respectively, in sections 6 and 7. Conclusions are offered in section 8.

2. PRELIMINARIES

... j'appelle ces coefficients des coefficients conjecturaux ou élasticités conjecturales, par opposition aux élasticités de fait qui expriment ce qui se passe en réalité (p. 252).

—Ragnar Frisch, Monopole—Polypole—La Notion de Force dans l'Economie, in Supplement to Nationalokonomisk Tidsskrift, 1933.

There are N firms indexed { i i=1..N }. Each produces a positive, finite output { x_i i=1..N }. The outputs are perfect substitutes. Accordingly, firms face common demand

(1) $$p = D(x \mid \sigma_o),$$

where p denotes price, x denotes aggregate output, and σ_o represents a shift variable, the value of which is given exogenously. Industry and firms' outputs are related through the aggregation condition

(2) $$x = \sum_{i=1}^{N} x_i.$$

Let $\{ C_i(x_i \mid \sigma_i) \; i=1..N \}$ denote variable costs. When the shocks $\{ \sigma_o \{ \sigma_i \; i=1..N \}\}$ are perturbed firms adjust output in order to remain at their optima. They conjecture relations $\{ x_j = \chi_{ij}(x_i) \; j=1..N \}$ and solve, accordingly,

PROBLEM 1: $$\max_{x_i} \; \pi_i(x_i \mid \sigma_o, \sigma_i) \equiv \begin{cases} \max\limits_{x_i}: & p\, x_i - C_i(x_i \mid \sigma_i) \\ \text{subject to:} & \\ & p = D(x \mid \sigma_o) \\ & x = \sum_{j=1}^{N} x_j \\ & x_j = \chi_{ij}(x_i) \quad j=1..N \end{cases} \qquad i=1..N.$$

The third constraint in this problem makes explicit that firms have perceptions about each of their rivals. To each of these relations corresponds *a conjectural elasticity*, or simply, *a conjecture* depicting the firm's perception about rivals' behavior. For notational simplicity I include $\chi_{ii}(x_i) \equiv x_i$, depicting the firm's perception of itself. Let $\{ \hat{\theta}_{ij} \equiv (\partial\chi_{ij}(\cdot)/\partial x_i)(x_i/\chi_{ij}(\cdot)) \; i,j=1..N \}$ denote i's perception about j's response. If firms observe outputs and know how to count, equation (2) implies a set of correspondences, $\{ x = \chi_i(x_i) \equiv \sum_j \chi_{ij}(x_i) \; i=1..N \}$, relating own output to the industry aggregate; and another $\{ \hat{\theta}_i \equiv \sum_j (x_j/x) \, \hat{\theta}_{ij} \; i=1..N \}$, relating the inter-firm conjectures, $\{ \hat{\theta}_{ij} \; i,j=1..N \}$, to the set of aggregates, $\{ \hat{\theta}_i \equiv (\partial\chi_i(\cdot)/\partial x_i)(x_i/\chi_i(\cdot)) \; i=1..N \}$. Some substantial algebra is avoided by working with the aggregate conjectures alone, but these correspondences should be kept in mind as I proceed.

Defining $\varepsilon \equiv (\partial D(\cdot)/\partial x)(x/D(\cdot))$ as the demand flexibility and applying the usual manipulations, the first-order conditions corresponding to Problem 1 can be expressed

(3) $$p\,(1 + \varepsilon\, \hat{\theta}_i) - c_i(x_i \mid \sigma_i) \equiv \phi_i(\cdot) = 0 \qquad i=1..N.$$

The following regularity assumptions will be used throughout the paper.

ASSUMPTION 1 (Monotonicity): $\partial D(\cdot)/\partial x \leq 0$.

ASSUMPTION 2 (Concavity): $\{ \partial^2 \pi_i(\cdot)/\partial x_i^2 < 0 \; i=1..N \}$.

ASSUMPTION 3 (Stability): $\{ \dot{x}_i \equiv \partial x_i(t)/\partial t = \alpha_i \, \partial \pi_i(\cdot)/\partial x_i , \; \alpha_i > 0 \; i=1..N \}$.

ASSUMPTION 4 (Sustainability): $p \geq \max \{ c_i(x_i \mid \sigma_i) \; i=1..N \} > 0$.

Assumption 1 delineates the market. Assumption 2 is simply asserted. The precise conditions under which it holds will be established in section 7. Assumption 3 asserts that firms expand production if they perceive positive profits from doing so, where the parameters $\{ \alpha_i > 0 \; i=1..N \}$ denote speeds of adjustment. Finally, Assumption 4 rules out pricing below marginal cost. This assumption, although seemingly innocuous, is extremely important. It will prove useful in tying down the range of possible consistent conjectures during the initial stages of the search. Except for a minor digression in section 7 to consider entry, all discussion will be couched in terms of equations (1)-(3) and Assumptions 1-4.

3. THE LOGICAL FLAW

> "The Mathematical Groundwork of Economics" ... talks in the end of the paragraph on "several manufacturers, one commodity" (i.e., a "supply oligopoly") about the duopoly of supply. There, each duopolist looks at the quantity supplied by his competitor as being dependent on his own quantity supplied, i.e., as a function of his own quantity supplied. This function "depends on what each producer thinks the other is likely to do." It is obvious that in reality both functions cannot exist simultaneously. Here, each duopolist has the "position of independence." That is, the market situation described by Bowley is the form of duopoly that we have already analyzed as "Bowley's Duopoly." ... After all, priority for the idea of "quantity independence" has to be given to Bowley, and therefore we have named the market situation where oligopolists strive for "quantity independence" after Bowley (pp. 82-83).
>
> *—Heinrich von Stackelberg, Marktform und Gleichgewicht, 1934.*
> *(Translation by Barbara Hegenbart).*

Bowley's theory contains a logical flaw. It can be traced to a single, significant feature of the firm's problem and the corresponding first-order conditions: Unlike the Cournot model in which the outputs of rivals appear, the primal profit function of a conjectural-variations firm is defined solely with reference to own output.

To make this observation return to Section 2. In Problem 1 make a sequence of recursive substitutions. First, substitute the conjectures, $\{ x_j = \chi_{ij}(x_i) \; j=1..N \}$, into the aggregation condition, everywhere that the outputs of the rivals appear. Next, substitute for aggregate output in the demand function, $D(\cdot)$. Finally, substitute for price in the firm's objective function and observe that Problem 1 reduces to:

PROBLEM 2: $\max_{x_i} \pi_i(x_i \mid \sigma_o, \sigma_i) \equiv \max_{x_i} D(\chi_i(x_i) \mid \sigma_o) x_i - C_i(x_i \mid \sigma_i)$ i=1..N.

Inspection of Problem 2 uncovers a fundamental result:

LEMMA 1 (Independence): The first-order conditions $\{ p (1 + \varepsilon \hat{\theta}_i) - c_i(x_i \mid \sigma_i) \equiv \phi_i(x_i \mid \sigma_o, \sigma_i) = 0$ $i=1..N \}$ are independent of rivals' outputs.

A rich set of insights follow from the lemma. The first pertains to the rightful place of the so-called reaction functions. Observations about these are best illuminated with reference to Cournot. The essential difference between it and conjectural variations is the presence of the third constraint in Problem 1. This difference between the two models is extremely important. It formalizes a distinction in terms of a feature of Nash equilibrium, which is embedded in Cournot, but which is absent conjectural variations. This feature is that the strategies of the rivals must conform to the game. It is embedded in Cournot in the following way: *Given* that each of the other firms plays Cournot, the best strategy for the remaining firm is to play Cournot. It is the "given-ness" assumption with which the early theorists were grappling. Ex post Nash, we now embrace Cournot. However, in 1924 it was likely dissatisfaction with this assumption that led Bowley toward his ultimate conclusion—toward conjectural variations. Conjectures were his attempt to short-cut the adjustment process toward equilibrium. Rather than propagate a full-fledged dynamic model, he chose to characterize the process through a seemingly innocuous alternative—the inclusion of subjective information into the firm's objective function. Nowadays we embrace subjectivity as an important feature of most economies. We should ask, therefore, why Bowley's conjectures remain controversial. The controversy stems from the way in which subjectivity is depicted in Bowley's duopoly. In 1924 a formal explication of subjectivity was not far off (de Finetti, 1937). Inevitably, it would employ the probability calculus—that is, the *integral* calculus; Bowley employed the *differential* calculus. In so doing, comparative-static information was incorporated into the firm's decision problem. It is essentially this feature that renders the model controversial. There are two arguments. The first relates to the reaction functions, the second to an econometric dilemma.

The first argument is, once again, illuminated with reference to Cournot. Cournot firms generate responses, which we term reactions. The reaction of one firm to another is the response computed from applying the implicit-function theorem to the first-order condition of the reactive firm. However, from Lemma 1, this computation is no longer feasible. It follows that reaction functions have no place in Bowley's theory. There is thus an interesting conundrum within the static theory:

COROLLARY 1 (Duality): Cournot's theory postulates no reaction but generates one; Bowley's theory postulates reaction but generates none.

Although the motivation remains unclear, it was likely this observation that led Stackelberg (1934) to attempt a reconciliation between the two theories:

COROLLARY 2 (Unification): A Stackelberg duopoly is a game with a conjectural-variations leader and a Cournot follower. Hence, three theories are actually one.

This link between the static theories appears to have been overlooked by modern authors. It is an important oversight. The non-reactiveness of firms with conjectural variations arises as a consequence of substituting the relations $\{ x_j = \chi_{ij}(x_i)\ j=1..N \}$ *everywhere* that the outputs of rival firms appear. In Cournot these substitutions are omitted. The issue of whether to make these substitutions therefore represents a point of divergence between the two theories. There are two important implications. The first is an observation about differences in results obtained from otherwise identical models. These can be traced to *selective* substitution of the conjectures into the constraints and objective functions of conjectural-variations firms. The second implication is that the independence engendered by making complete these substitutions leads naturally to an alternative definition of consistency and, ultimately, to a general solution conforming to this definition. Most work during the 80's failed to make complete these substitutions. I depart from this precedent for several reasons: First, it is counter-intuitive to make selective substitutions of constraints in an optimization model. Second, if the substitutions are not made completely then a type of consistency is imposed prior to undertaking comparative statics. Third, if the substitutions are not made, we obtain Cournot; when made completely we model Stackelberg leaders. Therefore, when the substitutions are made selectively it can be argued that what is being modeled is some form of hybrid between these two extrema of a Stackelberg continuum—one in which all firms act as leaders and another in which all act as followers. In the remainder of the paper I perform the substitutions $\{ x_j = \chi_{ij}(x_i)\ j=1..N \}$ *everywhere* that the outputs of the rivals appear. This should be kept in mind when comparing my results to those of previous authors.

The second argument about the logical flaw also relates to the conjectural variations $\{ x_j = \chi_{ij}(x_i)\ j=1..N \}$. The arguments on the right-hand sides are choice variables of the firms. In transition between successive equilibria the firms observe correlation in output. However, from the definitions of the conjectures, this correlation interpretation must give way to one implying causality. In other words, the firms observe correlation but interpret this to be causal. In particular, they consider it to evolve from their own adjustment in output. It is here that we encounter a paradox familiar from econometrics of trying to infer causality from correlation. In short, conjectural-variations firms make an econometric mistake. There is, however, one situation in which causality and correlation coincide. It is unique. It is the consistent-conjectures equilibrium.

Before turning to the search, note three additional features of the equilibrium that follow from Lemma 1:

COROLLARY 3 (Recursion): Price is determined recursively: Equations (3) determine $\{ x_i\ i=1..N \}$ given $\{ \sigma_o, \{ \sigma_i\ i=1..N \}\}$; given $\{ x_i\ i=1..N \}$, equation (2) determines aggregate output, x; given x and σ_o, equation (1) determines price, p.

COROLLARY 4 (Uniqueness): Under Assumptions 1 and 2, there exists a locally unique $\{ p^*, x^*, \{ x_i^*\ i=1..N \}\}$ satisfying equations (1)-(3).

COROLLARY 5 (Stability): Under Assumptions 1, 2 and 3 the $\{ p^*, x^*, \{ x_i^*\ i=1..N \}\}$ that satisfy equations (1)-(3) are locally stable.

Recursion in price distinguishes an oligopoly from the price-taking model in which price and output are determined simultaneously. Uniqueness relies on local concavity of the objective functions. It is investigated in section 7. Stability is most easily observed from comparative statics, which are derived in the next section.

This discussion is important in directing the remainder of the paper. Several features are worth reaffirming. Dissatisfaction with the "given-ness" assumption implicit in Cournot led Bowley toward his theory, but the theory models subjective behavior via comparative statics. These comparative statics contain a logical flaw. It stems from two sources. The first is that the firms conjecture reaction but do not generate one. The second is that they make an econometric mistake. An attempt to circumvent the flaw led Stackelberg (1934) to his leader-follower model. Stability arguments in the leader-leader regime led Leontief to consistent conjectures ...

4. SEARCH STRATEGY

> ... Stackelberg distinguishes thus three possible situations: (1) each of the two competitors might assume that the other is going to behave like a leader and consequently act himself as follower, (2) each will expect the other to take the attitude of a follower, and both will actually behave as leaders; or finally, (3) one of them will act as a follower, expecting the other to be the leader, which this other will actually do on the (correct) assumption that the first is ready to follow. The first case leads to the solution of the duopoly problem which was envisaged by Cournot (also Amoroso and Schneider), while the second indicates another type of solution—that of Bowley ... Discussing the general scheme of possible duopolistic equilibriums, Stackelberg seemingly has overlooked one, not very probable, but nevertheless theoretically significant situation. His assertion that Bowley's setup, in which both competitors try to attain the leadership cannot possibly produce a stable equilibrium, is explained by the fact that neither of the two will actually behave according to the expectations of the other ... One can, however, easily think of a case in which each of the two sellers expects a definite adjustment policy on the part of the other, chooses the role of the leader, and finds that the behavior of his competitor actually corresponds to his expectations (pp. 555-56).
>
> —*Wassily Leontief, Stackelberg on Monopolistic Competition, Journal of Political Economy, 1936.*

I now begin the search. An issue arising as a consequence of Lemma 1 is the definition of the responses against which to infer consistency. The convention is to compute responses between pairs of firms in isolation from other competitors. This is achieved by allowing any pair to adjust outputs *ceteris paribus* and, subsequently, computing the responses associated with the corresponding first-order conditions. In view of the results above this practice is neither appropriate nor feasible. An alternative approach, which I now adopt, is more akin to traditional equilibrium analysis. It perturbs each of the exogenous variables in turn, and allows each endogenous variable to adjust *mutatis mutandis*. In other words, the method permits simultaneous movements in $\{ p, x, \{ x_i\ i=1..N \}\}$ in response to perturbations in each of the elements $\{ \sigma_o, \{ \sigma_i\ i=1..N \}\}$. It is essentially this difference—a difference stemming from interpretations about what is held constant—that represents a point of departure from previous work.

Displacements

A technical detail that will prove useful is to express derivatives in proportional-change terms. This approach allows not only the qualitative effects of certain adjustments to be ascertained, but also their relative magnitudes. Henceforth, a tilde above a variable will denote a change in its value relative to that in initial equilibrium. That is, for some variable, v, let $\tilde{v} \equiv \Delta v/v$ denote its proportional change. Results used repeatedly throughout the paper are those pertaining to multiples, namely, for $r = st$, $\tilde{r} = \tilde{s} + \tilde{t}$; quotients, namely, for $u = v/w$, $\tilde{u} = \tilde{v} - \tilde{w}$; and functions, namely for $x = g(y,z)$, $\tilde{x} = \varepsilon_{gy}\, \tilde{y} + \varepsilon_{gz}\, \tilde{z}$, where $\varepsilon_{gy} \equiv (\partial g(\cdot)/\partial y)(y/g(\cdot))$ and $\varepsilon_{gz} \equiv (\partial g(\cdot)/\partial z)(z/g(\cdot))$ denote partial elasticities. Note that when the partial derivatives $\partial g(\cdot)/\partial y$ and $\partial g(\cdot)/\partial z$ are unity—as they would be, for example, in equation (2)—the parameters ε_{gy} and ε_{gz} denote, respectively, the shares of y and z in x. Applying this calculus in equations (1)-(3), the adjustments $\{ \tilde{p}, \tilde{x}, \{ \tilde{x}_i \; i=1..N \}\}$ emanating from the shocks $\{ \tilde{\sigma}_o \{ \tilde{\sigma}_i \; i=1..N \}\}$ are as follows:

$$\text{(4)} \qquad \tilde{p} = \varepsilon \tilde{x} + \nu \tilde{\sigma}_o,$$

$$\text{(5)} \qquad \tilde{x} = \sum_{i=1}^{N} \frac{x_i}{x} \tilde{x}_i ,$$

$$\text{(6)} \qquad \upsilon_i \tilde{x}_i + \eta_i \tilde{\sigma}_o + \mu_i \tilde{\sigma}_i = 0 \qquad i=1..N,$$

where $\nu \equiv (\partial D(\cdot)/\partial\sigma_o)(\sigma_o/D(\cdot))$ denotes the partial effect on demand of the shock $\tilde{\sigma}_o$ and the remaining parameters $\upsilon_i \equiv x_i\partial\phi_i(\cdot)/\partial x_i \equiv \upsilon_i(\hat{\theta}_i)$, $\eta_i \equiv \sigma_o\partial\phi_i(\cdot)/\partial\sigma_o \equiv \eta_i(\hat{\theta}_i)$ and $\mu_i \equiv \sigma_i\partial\phi_i(\cdot)/\partial\sigma_i \equiv \mu_i(\hat{\theta}_i)$ depend implicitly on the conjectures. Deriving the exact forms of these expressions is problematic. Without specification of the conjectures it is not possible to differentiate explicitly in the first-order conditions $\{ \phi_i(x_i \mid \sigma_o, \sigma_i) = 0 \; i=1..N \}$. For example, are the conjectures functions of firms' outputs or are they parametric ? To answer, I need to establish the specific forms of the expressions $\{ \hat{\theta}_i \; i=1..N \}$ that are consistent with equilibrium. I do so in section 7. Results prior to that stage are independent of the forms of the conjectures.

Immediate interest lies in the adjustments in quantities. Hence, focus on the vector $\tilde{\mathbf{x}} \equiv (\tilde{x}, \tilde{x}_1 \,..\, \tilde{x}_N)^T$ and study its progression in response to the shocks $\tilde{\mathbf{z}} \equiv (\tilde{\sigma}_o , \tilde{\sigma}_1 \,..\, \tilde{\sigma}_N)^T$. Equations (5) and (6) yield

$$\text{(7)} \qquad \Phi \, \tilde{\mathbf{x}} = \Psi \, \tilde{\mathbf{z}} ,$$

where the coefficient matrix corresponding to the endogenous movements is

$$\text{(8)} \qquad \Phi \equiv \begin{pmatrix} 1 & -x_1/x & -x_2/x & .. & -x_N/x \\ & \upsilon_1 & & .. & \\ & & \upsilon_2 & .. & \\ \vdots & \vdots & \vdots & & \vdots \\ & & & .. & \upsilon_N \end{pmatrix},$$

and the other, corresponding to the shocks is

$$
(9) \qquad \Psi \equiv \begin{pmatrix} & & & \cdot\cdot & \\ -\eta_1 & -\mu_1 & & \cdot\cdot & \\ -\eta_2 & & -\mu_2 & \cdot\cdot & \\ \vdots & \vdots & \vdots & & \vdots \\ -\eta_N & & & \cdot\cdot & -\mu_N \end{pmatrix}.
$$

I assume, of course, that the pairs $\{ \eta_i, \mu_i \; i=1..N \}$ are non-zero and finite. If not the exercise would be quite uninteresting. Returning to stability, note that the square matrix of order N in the lower right of (8) is diagonal. Each of its elements, corresponding to a second-order condition, represents an eigenvalue. It follows that the system is locally stable whenever Assumptions 2 and 3 are maintained. In addition, Corollaries 3 and 4 imply:

LEMMA 2 (Existence): Under Assumptions 2 and 3 there exists a locally stable, unique solution $\tilde{\mathbf{x}} = \Phi^{-1} \Psi \tilde{\mathbf{z}}$ with finite adjustments $\{ \tilde{x} \{ \tilde{x}_i \neq 0 \; i=1..N \}\}$.

Note that the firm-level adjustments are assumed to be non-zero, while the aggregate adjustment is not. It would be a rare event that aggregate output remained stationary when the outputs of the firms adjust. This issue will play an important role in subsequent analysis. I return to it in section 5.

Consistency

Corresponding to Lemma 2 are ratios $\{ \tilde{x} / \tilde{x}_i \equiv \theta_i \; i=1..N \}$ and $\{ \tilde{x}_j / \tilde{x}_i \equiv \theta_{ij} \; i,j=1..N \}$ about which the firms form conjectures. For immediate purposes, however, only on the aggregate effects are of interest. In general these effects are functions of the conjectures themselves, namely $\{ \tilde{x}(\hat{\theta}_1 .. \hat{\theta}_N) / \tilde{x}_i(\hat{\theta}_i) \equiv \theta_i(\hat{\theta}_1 .. \hat{\theta}_N) \; i=1..N \}$. A set of conjectures are *consistent* if they solve the fixed-point problem that equates the subjective perceptions $\{ \hat{\theta}_i \; i=1..N \}$ to the set of true values $\{ \theta_i \; i=1..N \}$. Formally:

DEFINITION 1 (Consistent-Conjectures Equilibria): A consistent-conjectures equilibrium is a $\{ p^*, x^*, \{ x_i^* \; i=1..N \}\}$ that satisfy(1)-(3), a $\{ \tilde{p}, \tilde{x}, \{ \tilde{x}_i \; i=1..N \}\}$ that satisfy (4)-(6), and a set of solutions $\{ \hat{\theta}_i \; i=1..N \}$ to the fixed-point problem:

$$
\hat{\theta}_i = \frac{\tilde{x}(\hat{\theta}_1 .. \hat{\theta}_N)}{\tilde{x}_i(\hat{\theta}_i)} \qquad i=1..N.
$$

Search Criteria

Almost all previous investigations have sought *direct* solutions to the fixed-point problem in Definition 1. Existence of equilibrium is more often assumed rather than proven, and sets of restrictions on demand and costs are derived that ensure satisfaction of the fixed-point equations. Where general conclusions have been sought, success has been modest. Nevertheless, these attempts have made two significant contributions. First, they have focused attention toward Cournot as a consistent-cum-inconsistent conjecture. Second, they have motivated rethinking a strategy for deriving general results. I now pursue one of these.

A similarity shared between the current approach and previous ones is that I assume, at least initially, that an equilibrium exists. This convenience—Lemma 2—is defensible only in terms of its ample precedent. One should keep in mind that industries with consistent conjectures may not exist, or may be unlikely to exist under any liberal interpretation of existence. Before attempting to solve any fixed-point problem I first ask: Can one restrict *a priori* the range of the conjectures $\{ \hat{\theta}_i \; i=1..N \}$? The answer is yes. The strategy entails identifying a set of conditions that are necessary, but may not be sufficient for consistency. I term these conditions *admissibility criteria* and the conjectures that meet them *admissible conjectures*. There are three notions of admissibility. One, termed *firm-wise admissibility*, pertains to conjectures about aggregate responses. Another, termed *pair-wise admissibility*, pertains to conjectures between pairs of firms. The third, termed *group-wise admissibility*, pertains to the conjectures of all the firms viewed collectively. Each of these concepts is formalized as follows:

DEFINITION 2 (Firm-Wise Admissible Conjectures): A set of N conjectures $\{ \hat{\theta}_i \; i=1..N \}$ are firm-wise admissible if they that satisfy: $\hat{\theta}_i = \dfrac{\tilde{x}}{\tilde{x}_i} \quad i=1..N.$

DEFINITION 3 (Pair-Wise Admissible Conjectures): A set of N^2 pairs $\{ \hat{\theta}_i, \hat{\theta}_j \; i,j=1..N \}$ are pair-wise admissible if they satisfy: $\dfrac{\hat{\theta}_i}{\hat{\theta}_j} = \dfrac{\tilde{x}_j}{\tilde{x}_i} \quad i,j=1..N.$

DEFINITION 4 (Group-Wise Admissible Conjectures): A set of N conjectures $\{ \hat{\theta}_i \; i=1..N \}$ are group-wise admissible if they that satisfy simultaneously: $\tilde{x} = \sum_{i=1}^{N} \dfrac{x_i}{x} \tilde{x}_i$ and $\hat{\theta}_i = \dfrac{\tilde{x}}{\tilde{x}_i} \quad i=1..N$.

Any set of conjectures that satisfy Definition 1 must also satisfy Definitions 2, 3 and 4, but not the converse. Thus, admissibility is necessary but not sufficient for consistency. Note, also, that Definition 4 implies Definitions

2 and 3, but not the converse. Hence, firm- and pair-wise admissibility are necessary but not sufficient for group-wise admissibility. Definitions 2-4 narrow the search quite substantially. The following example illustrates.

An Example

Consider an industry with identical firms in which entry is restricted. From Definition 2, firm-wise admissibility requires $\{ \hat{\theta}_i = \tilde{x} / \tilde{x}_i \; i{=}1..N \}$. However, when the firms are identical these conditions revert to the single restriction $\tilde{x} - \tilde{x}_i \hat{\theta} = 0$. Under symmetry equation (4) becomes $\tilde{x} = \tilde{N} + \tilde{x}_i$. Hence, when entry is restricted the industry adjusts according to $\tilde{x} - \tilde{x}_i = 0$. Consequently, the group-wise admissibility criterion reduces to

$$\Sigma \, \tilde{\mathbf{x}} = \mathbf{0}, \tag{10}$$

where $\tilde{\mathbf{x}} \equiv (\tilde{x} , \tilde{x}_i)^T$, $\mathbf{0}$ denotes a null vector of length two, and the matrix Σ is defined

$$\Sigma \equiv \begin{pmatrix} 1 & -1 \\ 1 & -\hat{\theta} \end{pmatrix}. \tag{11}$$

For this system to yield nontrivial solutions in the vector $\tilde{\mathbf{x}}$ the determinant $| \Sigma | \equiv -\hat{\theta} + 1$, must equal zero. It follows that there exists a unique admissible conjecture. It is the monopolistic conjecture. Since admissibility is necessary for consistency, this conjecture, if it exists, must be the consistent conjecture. Entry is considered in section 7. This procedure works effectively, albeit less precisely, when the equilibrium is non-symmetric. A recurring theme throughout the search is the analogy between Cournot-Nash and consistent conjectural variations.

5. ADMISSIBLE CONJECTURES

> A realistic approach to oligopoly problems cannot be based on Cournot's theory. Yet now, after more than a century, it is still difficult to see what is involved in an oligopoly theory without showing how the theory is related to Cournot's basic construction ... The characteristic feature of the Cournot model is that if each duopolist continues to assume that the other will not change his rate of output, then ultimately they will prove to be correct although during the approach to equilibrium, for a limited period of time, they will be wrong ... This, of course, means that ultimately they prove to be "right" for the wrong reasons. Each *assumes* that his rival follows a policy of fixed output while *in reality* each follows a policy of adjusting his own output to the requirement of profit maximization, on the assumption that the other follows a policy of fixed output ... It is essential to realize that, as long as the firms make the Cournot assumptions concerning their rival's behavior, the analysis cannot be adjusted in such a way as to make the firms be right for the right reasons, instead of describing a situation in which they turn out to be right for the wrong reasons (pp. 57-63).
>
> *—William Fellner, Competition Among The Few, 1949.*

When the equilibrium is non-symmetric the problem is tackled sequentially. I begin by first applying the group-wise admissibility criterion. Subsequently, I consider admissibility among pairs. The pair-wise admissibility criterion will derive the most general form of the fixed-point equations that are consistent with equilibrium. The key result is to show that every pair-wise admissible equilibrium in which group-wise admissibility prevails is a consistent-conjectures equilibrium. There are two solutions. Each represents a Nash strategy in beliefs. One is the set of collusive conjectures. The other is a set of conjectures N times the Cournot conjectures, where N denotes the number of incumbents in restricted-entry equilibrium. Existence is considered subsequently. Some intermediate results will be necessary, but most of these can be traced to Definitions 2-4. Figure 1 summarizes the search.

(Insert Figure 1 about here.)

Group-Wise Admissibility

Applying the procedure outlined above to a non-symmetric equilibrium yields:

LEMMA 3 (Group-Wise Admissible Equilibria): Group-wise admissible configurations are combinations of market shares $\{ x_i/x \; i=1..N \}$ and conjectures $\{ \hat{\theta}_i \; i=1..N \}$ that satisfy $\prod_{i=1}^{N} - \hat{\theta}_i + \sum_{i=1}^{N} x_i/x \prod_{j \neq i}^{N} - \hat{\theta}_j = 0.$

PROOF: Non-zero movements in firms' outputs permit rearrangement of the N admissibility conditions as follows: $\{ \tilde{x} = \hat{\theta}_i \, \tilde{x}_i \; i=1..N \}$. Combining these with the aggregation condition in (5), yields $\Sigma \, \tilde{\mathbf{x}} = \mathbf{0}$, where $\tilde{\mathbf{x}} \equiv (\tilde{x}, \tilde{x}_1 .. \tilde{x}_N)^T$, $\mathbf{0}$ denotes the N+1 null vector, and Σ is defined

$$\Sigma \equiv \begin{pmatrix} 1 & -x_1/x & -x_2/x & .. & -x_N/x \\ 1 & -\hat{\theta}_1 & & .. & \\ 1 & & -\hat{\theta}_2 & .. & \\ \vdots & \vdots & \vdots & & \vdots \\ 1 & & & .. & -\hat{\theta}_N \end{pmatrix}.$$

For this system to yield nontrivial solutions in the vector $\tilde{\mathbf{x}}$, the determinant of this matrix must equal zero. This determinant is $\prod_{i=1}^{N} - \hat{\theta}_i + \sum_{i=1}^{N} x_i/x \prod_{j \neq i}^{N} - \hat{\theta}_j \equiv | \Sigma |$. Q.E.D.

The condition $|\Sigma| = 0$ specifies a relationship between the market shares { x_i/x i=1..N } and the conjectures { $\hat{\theta}_i$ i=1..N } that is necessary for consistency to prevail. As such, it circumscribes a relationship between structure and conduct in an admissible-conjectures setting. In so doing, admissibility links consistent conjectures to the structure-conduct-performance paradigm. The point that should be made is this: Under admissibility conduct is endogenous. In the absence of such a condition conduct is ad hoc subjective. Conduct is also endogenous in the Cournot setting, but in quite a different way. In both cases, however, it is neither the shares, nor the numbers of firms that creates this endogeneity. It is the primal concepts of costs and demand that determine firm numbers and the distribution of output among incumbents. Admissibility uses this dependence to rule out improbable conduct. This feature is also available from the Cournot model, but in a different way. Nevertheless, an intimate relationship exists between Cournot and a particular admissible conjecture. The analysis below will draw this out. This discussion should be kept in mind as I consider the feasibility of alternative configurations. I proceed by assigning, *in an ad hoc manner*, different values for the $\hat{\theta}_i$'s in the constraint in Lemma 3. These experiments assume that the market shares are given, but this implies, simply, that costs and demand are predetermined.

The accuracy of inter-firm predictions will later play an important role. However, for immediate purposes I can obtain results solely with reference to the aggregate conjectures. One advantage of working with the $\hat{\theta}_i$'s is that their domains are easily delimited. Combining equations (3) with Assumptions 1 and 4, the domains of the conjectures are restricted to the intervals { $\hat{\theta}_i \in [0,-\varepsilon^{-1})$ i=1..N }. Traditionally, conjectures are assumed to lie on the positive unit interval, with reference points { $\hat{\theta}_i = 0$ i=1..N } synthesizing competition, { $\hat{\theta}_i = 1$ i=1..N } depicting collusion, and { $\hat{\theta}_i = x_i/x$ i=1..N } representing Cournot conjectures. Note, however, that conjectures { $\hat{\theta}_i > 1$ i=1..N } are permissible, providing demand is sufficiently flexible. Specifically, neither equations (3), nor Assumptions 1-4 are sufficient to rule out conjectures that exceed one. Conversely, the conditions { $\hat{\theta}_i \geq 0$ i=1..N } are implied by combining equations (3) and Assumptions 1 and 4. These conditions will play a key role in restricting the ensuing search. I first consider whether competitive behavior could ever be admissible. Substituting into the constraint in Lemma 3 the conjectures { $\hat{\theta}_i = 0$ i=1..N } observe:

COROLLARY 6 (Competition): Competitive behavior { $\hat{\theta}_i = 0$ i=1..N } is group-wise admissible.

Competition therefore meets the necessary condition for consistency. This result should appeal to empiricists who focus attention toward this hypothesis. In conceptual work, oligopolies with a competitive fringe are frequently encountered. What can be said about separating equilibria in which a subset of the industry is non-competitive ?

COROLLARY 7 (Fringe Behavior): Non-competitive conduct by a subgroup { $\hat{\theta}_i \neq 0$ i=1..n } and a competitive fringe of two or more firms { $\hat{\theta}_j = 0$ j=n+1..N } is group-wise admissible.

Note that the smallest number of firms in the fringe is two. To see this, observe that the expression for the determinant of Σ contains summations of market-share terms weighted by product terms that exclude individual perceptions. If firm i is the sole competitive firm there exists a non-zero determinant $|\Sigma| = x_i/x \prod_{j \neq i} -\hat{\theta}_j$. Therefore, in a group-wise admissible oligopoly with one firm behaving competitively there must be at least one other firm that also behaves competitively. In other words, competitive behavior by more than one firm is necessary for admissibility. However, is competition ever likely ?

A belief that competition prevails is irrational. To illustrate, I make separate use of Definitions 2 and 4 and make a momentary excursion to consider pair-wise admissibility, Definition 3. There are two cases to consider, namely competitive beliefs by all firms—Corollary 6—and oligopoly with a fringe—Corollary 7. Equilibrium with a fringe implies, by Definition 4, that aggregate output must be non-stationary. That is, it requires $\tilde{x} \neq 0$. If not, then the entire industry is competitive. But then, for competition to be admissible, the output adjustments of firms in the fringe must be infinite. To see this recall, from Definition 2, the conditions { $\hat{\theta}_j = \tilde{x} / \tilde{x}_j = 0$ j=n+1..N }. Restricting attention to finite adjustments I can rule out oligopoly { $\hat{\theta}_i \neq 0$ i=1..n } with a fringe { $\hat{\theta}_j = 0$ j=n+1..N }. In the other scenario, now with finite adjustments, aggregate output must be stationary. That is, the condition $\tilde{x} = 0$ is required. In this case firms' adjustments must offset one another. But this is impossible unless one of two situations emerges. The first is that the output of each incumbent is stationary, which is ruled out. The second is that at least one pair of outputs moves in opposite directions. But then, by Definition 3, there must be at least one conjecture that is negative, contradicting Assumption 4. Hence:

COROLLARY 8 (Inconsistency): Competitive conjectures { $\hat{\theta}_i = 0$ i=1..N } are inconsistent.

Henceforth I restrict attention to the intervals { $\hat{\theta}_i \in (0,-\varepsilon^{-1})$ i=1..N }. Turning to the other polar case, I examine the admissibility of complete collusion among the firms.

COROLLARY 9 (Collusion): Collusion { $\hat{\theta}_i = 1$ i=1..N } is group-wise admissible.

Unlike its antithesis, collusion is explicable solely in terms of similarities among the firms. Lemma 3 depicts two ways in which firms may differ. One is through differences in beliefs and the other is through differences in market shares. The two, however, are intimately related.

COROLLARY 10 (Concordance): If conjectures are homogeneous { $\hat{\theta}_i = \hat{\theta}$ i=1..N }, or if the distribution of output is symmetric { $x_i/x = 1/N$ i=1..N }, then conjectures are collusive { $\hat{\theta}_i = 1$ i=1..N }.

The first result is obtained when { $\hat{\theta}_i = \hat{\theta}$ i=1..N } is substituted into the expression $|\Sigma| = 0$. The second is proved in the example in section 4. Three implications are noteworthy. The first is that likeness in beliefs and likeness in market share obligate the firms to a passive collusion. Note that this collusion is tacit since nothing is said of formal contracts among the firms. A second observation concerns symmetry. While it forces conjectures to be collusive, the converse is not the case. In other words, symmetry is sufficient but not necessary for collusion. This result is one variant of the structure-conduct-performance paradigm. It suggests that conduct is endogenous to market structure, not vice versa. Finally, one additional implication of Corollary 10 is important: If a consistent conjecture exists, it must synthesize collusion when there is symmetry among firms. Consequently, conjectures that meet this requirement must be functions of the market shares. What are the specific forms of these functions ? To answer, I now turn attention to the middle ground between competition and collusion and examine the feasibility of the Cournot conjectures { $\hat{\theta}_i = x_i/x$ i=1..N }.

A surprising amount of attention has focused on the conditions under which Cournot is consistent. This focus, is somewhat misguided. The reason is revealed in Corollary 1: Essentially, Cournot firms conjecture no reaction. It would therefore seem illegitimate to consider the circumstances in which Cournot is consistent. However, several insights evolve from examining this situation. Two key results follow from substituting the Cournot conjectures into the constraint in Lemma 3. For later reference, I present these independently. The first pertains to a general situation, the second to its exception.

COROLLARY 11 (Cournot): Cournot { $\hat{\theta}_i = x_i/x \quad i=1..N > 1$ } is inadmissible in oligopoly.

COROLLARY 12 (Monopoly): Cournot { $\hat{\theta}_N = x_N/x = N = 1$ } is admissible in monopoly.

These two results unfurl usefully the controversy surrounding Cournot. Only in monopoly does Cournot pass the necessary criteria to be a consistent conjecture. The reason is important. Only in monopoly do the *mutatis mutandis* effects of a change and the *ceteris paribus* effects coincide. In monopoly this distinction is moot, but in oligopoly it matters. Moreover, it is extremely important in the search for consistency. In particular, it provides an additional stepping-stone on the way to completing the search: If a consistent conjecture exists it must revert to the collusive conjecture whenever the number of firms degenerates.

What use are these results in further restricting the search ? I respond in three steps. The first is to transform the constraint in Lemma 3 into one that is more amenable to visual inspection. The second is to observe the value of this new condition under the assumption that firms conjecture Cournot. The third is to transform the Cournot conjectures by the value so observed.

Invoking { $\hat{\theta}_i \neq 0 \; i=1..N$ }, divide through in the constraint $|\Sigma| = 0$ with the product $\prod_{i=1}^{N} \hat{\theta}_i$ and observe:

COROLLARY 13 (The Gamma Criterion): Conjectures are admissible iff: $\sum_{i=1}^{N} \frac{x_i/x}{\hat{\theta}_i} \equiv \Gamma(\cdot) = 1.$

The function $\Gamma(\cdot)$ will reappear at several, subsequent junctures. Next, I rule out negative conjectures by assuming that demand slopes downward and that firms price above marginal cost. It follows from Corollary 13 that each firm's conjecture must exceed its market share. Consequently, I have derived a lower bound on the conjectures.

COROLLARY 14 (The Cournot Lower Bound): If they exist, the consistent conjectures are at least as collusive as the Cournot conjectures. That is { $\hat{\theta}_i > x_i/x \quad i=1..N$ }.

Hence, if conjectures are consistent they must lie on an open interval above Cournot. An obvious set of candidates emerges from examining the value of Gamma criterion when the firms have Cournot conjectures. In this case I obtain $\Gamma(\cdot) = N$. Thus, normalizing on N, I obtain:

COROLLARY 15 (Cournot Consistency): The conjectures { $\hat{\theta}_i = Nx_i/x \; i=1..N$ } are group-wise admissible.

I term these conjectures Bowley beliefs. They contain market-structure parameters and are therefore endogenous. They also fulfill each of the necessary conditions required for admissibility. In particular, they satisfy Corollary 10 (Concordance), Corollary 12 (Monopoly), Corollary 13 (The Gamma Criterion) and Corollary 14 (The Cournot Lower Bound). They are strong candidates for consistency, but are they consistent ? Moreover, if they are consistent, are they unique ? I will respond to both of these questions, in due course.

Summarizing, group-wise admissibility rules out Cournot conjectures, but through a simple transformation, returns to them as candidates for consistency. The parameter N enters the Bowley beliefs when the equilibrium is non-symmetric. It is present in the Cournot conjectures when symmetry prevails. As a description of market structure, which should one prefer ? Under symmetry Cournot contains an appealing convergence property, approaching the competitive limit as the size of the market relative to the firm grows without bound (Novshek, 1980). This property is lost when firms have conjectures of the Bowley kind; under symmetry the industry reverts to collusion. The Bowley beliefs suggest that, in this case, firm numbers are no longer relevant. Prima facie this is appropriate. If collusion can be maintained, albeit tacitly, then the number of member firms is irrelevant in determining equilibrium price, aggregate output and profits. This is not the case in non-symmetric settings. The number of incumbents among which there is unequal distribution of output should have an impact on price, output and the distribution of profits among the firms. Note, also, that a firm's perception about market structure should be quite different if it produces one half the industry output and there are two competitors than if it produced one half and faced one-hundred and two competitors. Bowley beliefs incorporate this crucial information into the firm's decision, Cournot conjectures do not. Another issue raised by the Bowley beliefs is the feasibility of conjectures that exceed the value one. Traditionally, this value is assumed to delimit an upper bound. This bound is attained in three situations, namely when there is collusion { $\hat{\theta}_i = 1$ i=1..N }, when there is concordance of a more general form { $\hat{\theta}_i = \hat{\theta}$ i=1..N }, or when symmetry prevails among the firms { $x_i/x = 1/N$ i=1..N }. This "bound," however is surpassed in non-symmetric situations. As yet there is no condition within equilibrium that can rule out conjectures exceeding one. Conventional wisdom that this value represents an upper bound has likely arisen from intuition under symmetry. This observation is pertinent to the feasibility of consistent conjectures under entry. I return to it in section 7. This discussion has drawn analogies between Cournot and consistent conjectures, but there exists another that is, arguably, more important.

Cournot-Nash and Nash Strategies in Conjectures

The concept of Nash equilibrium provides a logical foundation for the continued use of Cournot as *the* theoretically valid framework within the static theory. Given the Nash quantity choices of all but one oligopolist, the best strategy for that remaining firm is to play Cournot-Nash. This is well known. The key feature of this game is that it is static. In a static game comparative statics are irrelevant. One is only interested in a single equilibrium. The previous and subsequent equilibria are not pertinent to the firm's decision. Transition is irrelevant because a static game is, by definition, a static game.

In conjectural variations transition is everything. Conjectures need not be made about comparative-static phenomena, but when they are the game is no longer static. As noted earlier, the inclusion of comparative-static information in the firm's decision problem renders the model controversial. However, it also uncovers the redeeming feature of consistent conjectures vis-à-vis Cournot-Nash. The following example illustrates.

Consider a duopoly. The notation $\{ x_i^* \equiv \text{argmax} \{ \pi_i(x_i , x_j) \mid x_j = x_j^* \} \; i,j \; i \neq j \}$ formalizes its Cournot equilibrium, which is its Nash equilibrium in quantities. Each quantity choice is optimal given the optimizing choice of the rival. Now consider the situation when the firms play consistent conjectures. Its equilibrium is: $\{ x_i^* \equiv \text{argmax} \{ \pi_i(x_i , x_j) \mid x_j = \chi_{ij}(x_i^*) , \theta_{ij} = \hat{\theta}_{ij} \} \; i,j \; i \neq j \}$. Accordingly, each quantity choice is optimal given the choice of its rival *and* in transition between successive equilibria the local response of the rival is consistent with the conjecture. Therefore, unlike Cournot-Nash, there are two components to the consistent-conjectures game. The first is the static phase. The second is the subsequent comparative-static phase. Three observations are noteworthy. The first is that the consistent-conjectures equilibrium is necessarily more stringent than Cournot-Nash. In consistent conjectures, firms must be correct in the initial static stage, but also in the corresponding comparative-static stage. It follows that consistency places additional restrictions on the behavior of firms and, consequently, the generality of market structures that can support it. I investigate this further in a moment. A second point is that, although both phases are necessary, it is the subsequent comparative-static phase that overrides and dictates the equilibrium. Note the procedures employed in the current search. In particular, note that the search is first being narrowed by considering a set of necessary conditions in the comparative-static stage. Subsequently, the market structures that can support the equilibrium will be derived. A number of possibilities already are excluded. Finally, observe that the quantity choices in the initial stage of the game are Nash quantity choices. More precisely, they are Nash choices conditional on local correctness of the conjecture. It follows that the optimizing behavior of the rival is implicit in both stages of the game. Accordingly, concepts of Nash equilibrium are embedded in consistent conjectures. An explicit statement of this fact is available from the group-wise admissibility criterion. Focusing on the "best" strategy for the j^{th} firm, given the strategies of its rivals, rearrange the Gamma criterion as follows

$$(12) \qquad \hat{\theta}_j = \frac{x_j/x}{1 - \sum_{i \neq j}^{N} \frac{x_i/x}{\hat{\theta}_i}} .$$

There are two situations to consider. Suppose first that all firms other than the j^{th} conjecture collusively. That is, suppose $\{ \hat{\theta}_i = 1 \; i=1..N \; i \neq j \}$. Satisfaction of (12) then requires $\hat{\theta}_j = 1$. Recall that the Gamma criterion is necessary for consistency and interpret as follows: Given that the other firms conjecture collusively, firm j—wishing to form a consistent conjecture—must also conjecture collusively. Alternatively, suppose that the other firms each have Bowley beliefs. That is, suppose $\{ \hat{\theta}_i = Nx_i/x \;\; i=1..N \; i \neq j \}$. Satisfaction of (12) then requires $\hat{\theta}_j = Nx_j/x$. In this case, firm j must also play according to the Bowley rules. Note the important point. It is the desire for consistency that forces a type of Nash discipline on the remaining firm. Henceforth, I term conjectures that satisfy the Gamma criterion *Nash conjectures* and note, accordingly:

COROLLARY 15 (Nash Strategies in Conjectures): The collusive conjectures $\{ \hat{\theta}_i = 1 \; i=1..N \}$ and the Bowley beliefs $\{ \hat{\theta}_i = Nx_i/x \;\; i=1..N \}$ are Nash conjectures.

What can be said of competition ? I have argued previously that the conjectures $\{ \hat{\theta}_i = 0 \; i=1..N \}$ are inconsistent. However, they also fail to meet the Nash criterion. To observe this revert to the constraint in Lemma 3—recall that the Gamma criterion prohibits division by zero. From Corollary 7 (Fringe Behavior) observe that the constraint in Lemma 3 is satisfied when any pair of firms possess competitive conjectures. Note, in addition, that this holds irrespective of the conduct of the remaining firms. In this sense competition lacks the Nash discipline that Bowley beliefs and collusive conjectures impose on the remaining firm.

All of the observations within this section are obtained from a condition that places restrictions across imaginary variables. The condition is the Gamma criterion; the variables are the beliefs of the firms. I now consider constraints across real variables, namely firms' technologies.

Pair-Wise Admissibility

Until this point I have said little of firms' predictions about each other. Virtually all of the discussion has been framed within the context of predictions about aggregate output. If firms are correct in these aggregate predictions, but are incorrect about those of their rivals they are "right for the wrong reasons." Such behavior is inconsistent. I now focus on Definition 3 and examine admissibility in the context of inter-firm predictions. The question I wish to consider is the following one: How general are equilibria in which firms conjecture accurately among themselves ? Repeated application of Definition 3 yields:

LEMMA 4 (Pair-Wise Admissible Equilibria): Pair-wise admissible configurations are supply responses that satisfy $\{ \tilde{x}_i = \hat{\theta}_i^{-1} \ \phi \ \ i=1..N \}$, where ϕ is a common, non-zero scalar.

PROOF: Definition 3 requires $\{ \hat{\theta}_i / \hat{\theta}_j = \tilde{x}_j / \tilde{x}_i \ \ i,j=1..N \ i \neq j \}$. Since this must hold for all (i,j) pairs, this condition requires constancy of the multiples $\hat{\theta}_1 \tilde{x}_1 = ... = \hat{\theta}_N \tilde{x}_N \equiv \phi$. Q.E.D.

The key point here is that, in order for them to forecast accurately among themselves, the firms must be somewhat similar. The severity of this restriction has not been recognized previously. When firms conjecture accurately their supply responses must be proportional to their conjectures. Parameter ϕ, representing the proportionality factor, is indeterminate at present. In section 7 I will consider the conditions under which a common ϕ can exist. More precisely, I will ascertain the existence of consistent conjectures by considering the feasibility of cost and demand structures that generate a common ϕ. In the remainder of this section I examine the conditions prevailing in pair-wise admissible economies.

I first derive the reduced-forms. Summing $\{ \tilde{x}_i = \hat{\theta}_i^{-1} \phi \ i=1..N \}$ according to (5) and normalizing on each of the respective responses I obtain

$$\theta_i = \hat{\theta}_i \sum_{i=1}^{N} \frac{x_i/x}{\hat{\theta}_i} \qquad i=1..N. \tag{13}$$

These equations express the explicit dependence of the true effects $\{ \theta_i \ i=1..N \}$ on the conjectures $\{ \hat{\theta}_i \ i=1..N \}$. They reveal three important features of the equilibrium. First, they specify the structure of the fixed-point problems that must be solved in order to attain consistency. Second, they link the search to the literature on rational expectations in which agents must "forecast the forecasts of others" (Townsend, 1983). Third, they allow me to assess the impact on the true effects of changes in the beliefs of the firms. These comparative statics are important for understanding the way in which admissible economies evolve over time. Firms should wish to know the impacts on the true effects of three phenomena, namely a change in one's own belief, a change in the belief of a rival firm, and a change in the beliefs of all the firms occurring simultaneously. For this purpose, note that equations (13) can be written $\{ \theta_i = \hat{\theta}_i \Gamma(\cdot)\ i=1..N \}$ and note, also, that the Gamma function satisfies $\Gamma(\cdot) \equiv \sum_i \Gamma_i(\cdot) \equiv \Gamma_{-i}(\cdot) + \Gamma_i(\cdot)$, and $\Gamma_i(\cdot) \in (0,\Gamma(\cdot))$. Using these results, the following comparative statics are obtained.

COROLLARY 16 (Inflexion): The relative effect of a change in one's own belief is positive and inelastic. That is, $\tilde{\theta}_i \,/\, \tilde{\hat{\theta}}_i = \Gamma_{-i}(\cdot)/\Gamma(\cdot) \in (0,1) \quad i=1..N$.

Hence, a firm cannot influence its own outcome by an amount greater than its contribution to the Gamma function. In contrast, the effect of change in a rival's perception is quite different:

COROLLARY 17 (Coercion): The relative effect of a change in a rival's belief is negative an inelastic. That is, $\tilde{\theta}_i \,/\, \tilde{\hat{\theta}}_j = -\Gamma_j(\cdot)/\Gamma(\cdot) \in (-1,0)\ i,j=1..N\ \ i \neq j$.

What of simultaneous changes in all the beliefs ? Equations (13) are homogeneous of degree zero in the beliefs. Consequently:

COROLLARY 18 (Innovation): Equi-proportional adjustments in all beliefs are inconsequential.

Three features of these comparative statics are noteworthy. First, to the extent that they both affect and effect action, the mental capacities of the firms have a significant impact on equilibrium. This issue is discussed by Guesnerie (1993), as a justification for rational expectations, under the nomenclature "eductive reasoning." Eduction refers explicitly to the abilities of agents to forecast the forecasts of others. Its alternative, "evolutive reasoning," refers to agents' abilities to acquire knowledge from repeated observations on the equilibrium. This theme is pursued further in section 6. A second observation concerns strategic interaction. The reduced forms make clear that when firms adjust their own perceptions they also alter the adjustments of their rivals. A third observation concerns stability.

The economy is unaffected by innovation. That is, any received shock that scales all firms perceptions by the same amount leaves the equilibrium unchanged. Hence, the equilibrium is stable under such innovations.

The results above are partial in nature; each experiment assumes that something is held constant. Lemma 3, however, shows that the conjectures depend on the distribution of output among the firms. Under Bowley beliefs the relationship between the market shares and the conjectures is direct, but there is also an indirect effect. It is manifested by the Gamma function, which enters the reduced forms. Hence, for completeness, note:

COROLLARY 19 (Changing Stance): The relative effect of a change in market share is positive and inelastic. That is, $\tilde{\theta}_i / (x_j/\tilde{x}) = \Gamma_j(\cdot)/\Gamma(\cdot) \in (0,1)$ i,j=1..N.

Finally, equations (13) permit me to state and prove a key result in the search for consistency, namely:

LEMMA 5 (Consistent-Conjectures Equilibria): Every pair-wise admissible equilibrium that is group-wise admissible is a consistent-conjectures equilibrium.

PROOF: Pair-wise admissibility is necessary for consistency. Hence, $\{ \theta_i = \hat{\theta}_i \Gamma(\cdot)$ i=1..N $\}$ are the consistent-conjectures reduced forms. From Corollary 13, however, $\Gamma(\cdot) = 1$ is necessary and sufficient for group-wise admissibility; but, $\Gamma(\cdot) = 1$ solves the fixed-point problems. Q.E.D.

This lemma permits two concluding observations to be made. The first pertains to an intriguing aspect of the equilibrium, which has received attention elsewhere. Many types of Nash equilibria are independent of the distribution of parameters across agents. For example, Bergstrom and Varian (1985)—who appear to be the first to recognize this—demonstrate that Cournot equilibria are independent of the distribution of marginal costs among the firms; only the sum of the marginal costs matters for the determination of output and price. The pair-wise admissible equilibrium is a Nash equilibrium when the conjectures are consistent. It also possesses this independence property. Combining the responses in Lemma 4 with equation (5), and observing the structure of the fixed-point equations I obtain:

COROLLARY 20 (Independence): In pair-wise admissible equilibria, movments in aggregate output, $\tilde{x} = \Gamma(\cdot)\,\phi$, and the reduced forms, $\{ \theta_i = \hat{\theta}_i \Gamma(\cdot)$ i=1..N $\}$, are independent of the distribution of share-weighted beliefs.

The second observation is that the search for consistency is now restricted to two sets of candidate conjectures:

COROLLARY 21 (Candidacy): Pair-wise admissible collusive conjectures $\{ \hat{\theta}_i = 1 \; i=1..N \}$ and pair-wise admissible Bowley beliefs $\{ \hat{\theta}_i = Nx_i/x \; i=1..N \}$ are consistent conjectures.

6. REPEATED PLAY, LEARNING AND CONVERGENCE

> ... We can think of the following story. At time t-ε, ε > 0, both duopolists announce their output decisions for the period to begin at time t, each firm attempting to maximize its (one period) profits in light of its conjectural-variation function. After the first announcement both firms reevaluate their decisions and issue a (simultaneous) announcement of corrections. The time is t-δ, ε > δ > 0. The process continues until the corrected output bundles satisfy both firms (pp. 641-42).
>
> *—John Laitner, 'Rational' Duopoly Equilibria, Quarterly Journal of Economics, 1980.*

Two interesting questions now arise. The first concerns convergence to equilibrium, the second pertains to its existence. In this section I focus on convergence; existence is considered subsequently. Suppose I give agents a chance of being consistent. That is, suppose I restrict attention to pair-wise admissible configurations. Does the economy converge to either set of candidate consistent conjectures ? To shed light on this issue, I implement Laitner's sequence of adjustments, using $\{ t \; t=0...\infty \}$ to denote stopping points in the iterations: At the beginning of period t firms conjecture $\{ \hat{\theta}_i(t) \; i=1..N \}$ about the true effects $\{ \theta_i(t) \; i=1..N \}$, which are observed at the end of this period. They receive a shock $\tilde{\sigma}(t)$ and make their pair-wise admissible responses $\{ \tilde{x}_i(t) = \hat{\theta}_i(t)^{-1} \; \tilde{\sigma}(t) \quad i=1..N \}$. At the end of period t they observe each other's response and they compute the adjustment in aggregate output. They compare their conjectured responses to the ones that actually occur and they update their conjectures accordingly. At the beginning of period $t+1$ the firms receive another shock $\tilde{\sigma}(t+1)$ and the game is repeated until convergence, if ever, is achieved. This process yields the $2N+1$ sequences

$$\text{(14)} \qquad \tilde{x}_i(t) = \hat{\theta}_i(t)^{-1} \; \tilde{\sigma}(t), \qquad i=1..N,$$

$$\text{(15)} \qquad \tilde{x}(t) = \sum_{i=1}^{N} \frac{x_i(t)}{x(t)} \; \tilde{x}_i(t),$$

$$\text{(16)} \qquad \theta_i(t) = \tilde{x}(t) \; / \; \tilde{x}_i(t) \qquad i=1..N.$$

Interest lies in the evolution of the endogenous variables,

(17) $$\{ \tilde{x}_i(t) \ \ i=1..N \}_{t=o}^{\infty} ,$$

(18) $$\{ \tilde{x}(t) \}_{t=o}^{\infty} ,$$

(19) $$\{ \theta_i(t) \ i=1..N \}_{t=o}^{\infty} ,$$

in response to the shocks

(20) $$\{ \tilde{\sigma}(t) \}_{t=o}^{\infty} ,$$

which are exogenous. Convergence is achieved if the sequence

(21) $$\{ \hat{\theta}_i(t) \ i=1..N \}_{t=o}^{\infty} ,$$

converges to the sequence in (19). At each iteration the market shares are parametric. This is consistent with "one-shot," traditional comparative-statics, in which conjectural variations are usually applied. However, the sequence

(22) $$\{ x_i(t)/x(t) \ i=1..N \}_{t=o}^{\infty} ,$$

is actually endogenous—a fact that follows from (14) and (15).

The shares plays a key role in dictating the type of equilibrium that the economy attains. Their influence is two-fold. There is a direct effect and an indirect effect. The direct effect occurs through the impact of the shares on the conjectures. The indirect effect evolves from their impact on the sequence of Gamma effects, namely

(23) $$\{ \Gamma(t) \equiv \sum_{i=1}^{N} \frac{x_i(t)/x(t)}{\hat{\theta}_i(t)} \}_{t=o}^{\infty} .$$

This sequence, in turn, has an impact on the reduced forms for the economy, which are

(24) $$\theta_i(t) = \hat{\theta}_i(t)\,\Gamma(t) \qquad\qquad i=1..N,$$

and are, thus, time-dependent versions of equations (13).

There are two types of equilibria I wish to examine. I term *stationary* an equilibrium in which the conjectures and the market shares are constant through time and I term *non-stationary* one in which they vary. I define these with reference to some convergent period, "τ," as follows.

DEFINITION 5 (Repeated-Play, Stationary Equilibria): A repeated-play, stationary equilibrium is a set of time-invariant sequences $\{\ \hat{\theta}_i(t) = \theta_i(t) = \theta_i \ i=1..N\ \}_{t=\tau}^{\infty}$ and $\{\ x_i(t)/x(t) = x_i/x \ i=1..N\ \}_{t=\tau}^{\infty}$.

DEFINITION 6 (Repeated-Play, Non-Stationary Equilibria): A repeated-play, non-stationary equilibrium is a time-varying sequence: $\{\ \hat{\theta}_i(t) = \theta_i(t) \ i=1..N\ \}_{t=\tau}^{\infty}$.

My objective in the remainder of this section is to characterize equilibria satisfying each of these definitions. Before examining convergence, I consider adjustments assuming that the conjectures are consistent. The contemporaneous decision rules that generate consistency are $\{\ \hat{\theta}_i(t) = 1 \ i=1..N\ \}$ and $\{\ \hat{\theta}_i(t) = Nx_i(t)/x(t) \ i=1..N\ \}$. In the first case the firms resolve to collude; in the second they acquire knowledge from their market shares. Under both of these schemes the firms find themselves on an equilibrium path—in each and every period their expectations are fulfilled. What do the dynamics of adjustment look like under each of these schemes ?

In depicting adjustments there are several measures of interest. Given a sequence of shocks $\tilde{\sigma}(t)$ $t=0..T$ I wish to observe the corresponding adjustments in the conjectures $\{\ \hat{\theta}_i(t) \ i=1..N\ \}$ $t=0..T$, in the shares $\{\ x_i(t)/x(t) \ i=1..N\ \}$ $t=0..T$, and in the sequence of Gamma effects $\Gamma(t)$ $t=0..T$. It would also be useful to know whether the industry has, in some sense, become more or less "concentrated." Several indices exist for this purpose, but the index

$$f(t) \equiv \frac{\prod_{i=1}^{N} x_i(t)/x(t)}{N^{-N}} \tag{25}$$

is desirable for two reasons. First, it is conveniently defined over the unit interval. It has a well defined maximum at the value one, which occurs when the distribution is symmetric. It has a minimum of zero, which is approached asymptotically as the market share of any firm becomes negligible. Second, this index is stable whenever the market shares of each of the firms are stable, but it adjusts whenever the share of any single firm adjusts. It follows that the sequence $f(t)$ $t=0..T$, should prove convenient as a dual indicator of concentration and stability of the equilibrium. Another index, which proves useful in relation to the former, is the index:

$$g(t) \equiv \prod_{i=1}^{N} \hat{\theta}_i(t) . \tag{26}$$

The two indices converge when the conjectures of each of the firms converge to the Bowley beliefs. In the other Nash regime—the collusive equilibrium—the index $g(t)$ is stable at the value one.

Turning to the adjustments, I first consider collusion. This case is rather artificial, but insightful because it illustrates precisely how outcomes are effected simply by "willing" them to occur. When all firms resolve to collude, collusion actually prevails. To observe this formally, note that the sequence $\{ \hat{\theta}_i(t) = 1 \; i=1..N \}$, $t=0..T$ is consistent. That is, it implies the sequence $\Gamma(t) = 1$, $t=0..T$, and convergence is achieved in every period. The remaining question is whether the equilibrium is stationary. It is. To observe so, consider proportional changes in the shares. These can be written $\{ \widetilde{x_i(t)/x(t)} \equiv \tilde{x}_i(t) - \tilde{x}(t) \; i=1..N \}$. It follows, combining (14) and (15) with (23) and (24), that the proportional changes in the shares evolve according to

$$\text{(27)} \qquad \widetilde{x_i(t)/x(t)} = \frac{1 - \theta_i(t)}{\hat{\theta}_i(t)} \tilde{\sigma}(t) \qquad i=1..N.$$

Under collusion, the sequence $\{ \hat{\theta}_i(t) = 1 = \theta_i(t) \; i=1..N \}$, $t=0..T$, is attained. It follows that the sequence of collusive conjectures constitutes a stationary equilibrium.

Turning attention to the other Nash regime, consider the situation when firms have Bowley beliefs. As in the previous case, convergence is achieved in every period. The question of interest is whether the equilibrium is stationary. In terms of (27), an interesting phenomenon emerges. Bowley behavior implies the conditions $\{ \hat{\theta}_i(t) = Nx_i(t)/x(t) = \theta_i(t) \; i=1..N \}$, $t=0..T$. Hence, for a firm with a market share at time t that lies above the average, which is $1/N$, the coefficient of the shock on the right side of (27) must be negative. In contrast, for a firm with a share below the average the coefficient must be positive. Thus, the direction of movement in the shares depends on two components: its size prior to the shock, and the sign of the shock. When the shock is positive, the shares converge to the average; when negative, they diverge away from it. In the former case the industry becomes more symmetric; in the latter it becomes more concentrated. Decision rules of this type link Bowley beliefs to the empirical literature on relationships between concentration and the "booms" and "busts" of the business cycle. I interpret a "favorable" shock as one in which incumbents expand output. From (14), a favorable shock is a positive $\tilde{\sigma}(t)$, conversely for an unfavorable shock. It follows that the model predicts heightened diffusion of business activity during "upswings" and increases in concentration during "downswings." In other words, the model predicts negative correlation between concentration and the signs of the shocks. Figure 2 presents an example for a duopoly. The firms have Bowley beliefs, and the indices $f(t)$ and $g(t)$ therefore coincide. Shocks are drawn randomly around a trend, which is declining. The two indices rise when the sequence of shocks is positive and decline when it becomes negative. The patterns of adjustment are much the same for industries of five and ten firms. Examples are presented in the appendix.

(Insert Figure 2 about here.)

In the two previous scenarios the economy is void of learning. I now consider a simple rule through which learning may occur. I restrict attention to the class of rules that are compatible with agents who conjecture. These agents are necessarily myopic: They conjecture only one step ahead. They predict only into the comparative-static phase of the current static game, but there are additional restrictions. Recall that the firms are not afforded any opportunity to apply the integral calculus. The only calculus available to them is the differential kind. Accordingly, their subjective perceptions about the one-step-ahead forecast must be based on previous perturbations of the economy. They cannot set-up and solve a dynamic optimization with forecasts defined over an infinite horizon, which is now commonplace in dynamic models. The firms' predictions must be derived from the data they observe in the sequence of transitions between consecutive equilibria. Accordingly, the rules adopted must be ones involving "hindsight." Arguably, the simplest of such rules is the following (Ezekiel, 1938):

$$(28) \qquad \hat{\theta}_i(t) = \theta_i(t\text{-}1) \qquad i=1..N.$$

To analyze adjustments under this scheme, I make use of the reduced forms in (24). Rewriting these in accordance with (28), I obtain

$$(29) \qquad \theta_i(t) = \theta_i(t\text{-}1)\ \Gamma(t,t\text{-}1) \qquad i=1..N,$$

where, in general, $\Gamma(t,t\text{-}k) \equiv \sum_{i=1}^{N} \frac{x_i(t)/x(t)}{\theta_i(t\text{-}k)}$, k=1..t .

Convergence under hindsight requires { $\hat{\theta}_i(t) = \theta_i(t\text{-}1) = \theta_i(t)$ i=1..N } in some period $t\text{-}1 \geq \tau$. In other words, in the convergent period, the conjectures and the true effects must be stationary. Note that stationarity can be verified from the identity $\Gamma(t,t) \equiv 1$ and the corresponding equality $\Gamma(t,t\text{-}1) = \Gamma(t,t)$ which must exist in the convergent period. Both the collusive conjectures { $\hat{\theta}_i(t) = 1$ i=1..N } and the Bowley beliefs { $\hat{\theta}_i(t) = Nx_i(t)/x(t)$ i=1..N } are candidates for this criteria. To which, if either, of these two sets will the economy converge ? To answer, begin iterating equations (29) and obtain

$$\theta_i(1) = \theta_i(0)\ \Gamma(1,0) \qquad i=1..N. \tag{30}$$

$$\theta_i(2) = \theta_i(1)\ \Gamma(2,1)$$

$$= \theta_i(0)\ \Gamma(1,0)\ \Gamma(2,1) \qquad i=1..N,$$

and so on. It follows that equations (29) can be rewritten

$$\theta_i(t) = \theta_i(0) \prod_{s=1}^{t} \Gamma(s,s-1) \qquad i=1..N. \tag{31}$$

Regressing one period and substituting for $\theta_i(t-1)$ into the definition of $\Gamma(t,t-1)$ yields

$$\Gamma(t,t-1) = \sum_{i=1}^{N} \frac{x_i(t)/x(t)}{\theta_i(0) \prod_{s=1}^{t-1} \Gamma(s,s-1)} . \tag{32}$$

Factoring out the product term in the denominator, implies $\prod_{s=1}^{t} \Gamma(s,s-1) = \Gamma(t,0)$. Consequently, (31) becomes

$$\theta_i(t) = \theta_i(0)\ \Gamma(t,0) \qquad i=1..N. \tag{33}$$

These expressions depend on two sets of parameters. The first is the set of initial beliefs $\{ \theta_i(0)\ i=1..N \}$. The second is the set of contemporaneous shares $\{ x_i(t)/x(t)\ i=1..N \}$. In the convergent period $\{ \theta_i(t+1) = \theta_i(t)\ i=1..N \}$ is implied, but then, from (33), $\Gamma(t+1,0) = \Gamma(t,0)$ is also implied. Accordingly, in any $t \geq \tau$

$$\sum_{i=1}^{N} \frac{x_i(t+1)/x(t+1) - x_i(t)/x(t)}{\theta_i(0)} = 0. \tag{34}$$

This condition is satisfied for any homogeneous set of initial beliefs. If $\{ \theta_i(0) = \alpha, \alpha > 0, i=1..N \}$ are employed the economy converges in a single period. In this case the convergent sequence is the set of collusive beliefs $\{ \hat{\theta}_i(t) = 1\ i=1..N \}\ t=0..\infty$. Thus, the arbitrary initial distribution $\{ x_i(0)/x(0)\ i=1..N \}$ is maintained at each iteration. What if the initial beliefs are not homogeneous ? The stationary equilibrium, $\{ \hat{\theta}_i(t) = 1\ i=1..N \}\ t=\tau..T$, is a possibility, but it is not the only one. In general a non-stationary equilibrium exists. Figure 3 presents an example for a duopoly in which the firms initially play Cournot. That is, in the initial period, they conjecture $\{ \hat{\theta}_i(0) = x_i(0)/x(0)\ i=1..N \}$, where the shares are drawn randomly. Learning occurs quite rapidly: The economy converges to Bowley behavior—Figure 3b—after only a few iterations, at which point the indices $f(t)$ and $g(t)$ then move in unison. Similar patterns emerge within five- and ten-firm oligopolies.

(Insert Figure 3 about here.)

For comparison, I consider two variants of the hindsight rule in (28). The first is the mean of past histories:

$$\hat{\theta}_i(t) = \frac{1}{t} \sum_{s=0}^{t-1} \theta_i(s) \qquad i=1..N. \tag{35}$$

Figure 4 presents an example for a duopoly. As above, the firms play Cournot in the initial period in which their market shares are drawn randomly. Like the cobweb rule in (28), convergence is achieved after only a few iterations, but then the equilibrium destabilizes before convergence is again achieved. The indices $f(t)$ and $g(t)$ diverge whenever $\Gamma(t)$ diverges from its equilibrium value, which is one. In other words, divergence occurs whenever the economy departs from the Nash equilibrium with Bowley behavior. Similar patterns emerge with five and ten firms.

(Insert Figure 4 about here.)

The second variant of the rule in (28) is adaptive expectations (Nerlove, 1958):

$$\hat{\theta}_i(t) = \hat{\theta}_i(t-1) + \alpha\Big(\theta_i(t-1) - \hat{\theta}_i(t-1) \Big), 0 < \alpha < 1, \qquad i=1..N. \tag{36}$$

Figures 5 and 6 present examples for duopolies with decision rules α=0.1 and α=0.5, respectively, in which the firms play Cournot in the initial period. In the first case, the initial distribution is almost symmetric; convergence to the stationary, symmetric equilibrium is smooth and direct. In the second case, the initial distribution is rather more concentrated; convergence toward the stationary, symmetric equilibrium is more sluggish.

(Insert Figures 5 and 6 about here.)

Some additional experiments suggest three factors upon which convergence depends. One is the starting values assumed in the initial period; the second is the sign and magnitude of the shocks; the third is the number of firms. In most situations the economy approaches an equilibrium with Nash strategies in conjectures. Accordingly, I conclude:

LEMMA 6 (Equilibrium in Repeated Play): Pair-wise-admissible economies that begin from Cournot and learn according to the rules of hindsight, $\{\ \hat{\theta}_i(t) = \theta_i(t\text{-}1)\ i{=}1..N\ \}$; the mean of past histories, $\{\ \hat{\theta}_i(t) = 1/t \sum_s \theta_i(s)\ i{=}1..N\ \}$, or adaptive expectations, $\{\ \hat{\theta}_i(t) = \hat{\theta}_i(t\text{-}1) + \alpha \left(\theta_i(t\text{-}1) - \hat{\theta}_i(t\text{-}1) \right),\ 0 < \alpha < 1\ i{=}1..N\ \}$; approach the Nash equilibrium with Bowley beliefs $\{\ \hat{\theta}_i(t) = Nx_i(t)/x(t)\ i{=}1..N\ \}$. With sufficient positive shocks, the economy approaches the stationary, symmetric equilibrium with collusive conjectures.

7. CONSISTENT CONJECTURES

> The comparative statics of equilibrium give firms enough information to recover one another' behavior. Suppose that some variable exogenous to the oligopoly (say, costs, the location of the demand curve) is changed. Equilibrium prices and quantities will change whatever the nature of the equilibrium concept. Suppose that firms learn nothing about one another from the dynamic process from which the new equilibrium is obtained. They can still learn one anothers' reactions from the location of the new equilibrium ... The natural experiment, the movement of exogenous variables, can reveal to firms that their conjectures are inconsistent. This argument does not depend in any critical way on the belief that the equilibrium comparative statics "actually happen" ... If firms have inconsistent conjectures and it is possible for them to learn how their industry reacts to exogenous shocks, they will learn that their conjectures are wrong. If they have consistent conjectures, nothing in the comparative statics of equilibrium will reveal those conjectures to be wrong. By what dynamic process the conjectures will come to be consistent is an unsolved problem, as is the possibility of an informationally consistent, stable dynamic for oligopoly prices and quantities (pp. 942-43).
>
> —*Timothy Bresnahan, Duopoly Models With Consistent Conjectures, American Economic Review, 1981.*

The results just derived employ a key assumption, that supply responses are pair-wise admissible. This condition, recall, is necessary for consistency—it is a prerequisite for the accuracy of inter-firm conjectures. In this section I examine the validity of this assumption and, ultimately, the existence of consistent conjectures.

One reason for studying oligopoly is its possibility for generating "perverse effects." These effects are comparative-static predictions that are counter-intuitive. Unfortunately, most conventional wisdom is derived from the purely competitive model, which is neither representative nor appropriate for a large number of circumstances. Nevertheless, a precedent exists and the model's predictions are well-known: When demand shifts out, price rises and firm and industry output expand in the face of entry. When costs increase, price rises, but output contracts and attrition occurs. In both situations adjustment proceeds until the industry is returned to zero profits. This need not be the case in oligopoly, but apart from this innocuous conclusion, more specific predictions are scant. Authors have discovered adjustments that move prices and quantities in counter-intuitive directions. For example, it is occasionally observed that positive shocks to demand and negative shocks to costs may cause output to contract. In the context of the search an interesting question arises: Can the consistency criteria rule out these so-called perverse effects ? This question is answered in two situations. The first, considered below, assesses the effects of shocks under the assumption that N is fixed. In other words, under the assumption that entry is restricted. The second, considered subsequently, allows N to vary. It should be noted, at the outset, that each of these endeavors involves some rather tedious derivations, but the investigation would be incomplete without this excursion. Readers interested in technicalities are referred to the appendix.

Existence

From Lemma 4, recall that the responses $\{ \tilde{x}_i = \hat{\theta}_i^{-1} \phi \ i=1..N \}$ are pair-wise admissible. From equations (6), however, I obtain $\{ \tilde{x}_i = -\upsilon_i(\hat{\theta}_i)^{-1} \eta_i(\hat{\theta}_i) \tilde{\sigma}_0 \ i=1..N \}$ when demand shifts, and $\{ \tilde{x}_i = -\upsilon_i(\hat{\theta}_i)^{-1} \mu_i(\hat{\theta}_i) \tilde{\sigma}_i \ i=1..N \}$ when variable costs shift. It follows that an equilibrium is pair-wise admissible if and only if

$$(37) \qquad \begin{aligned} -\upsilon_i(\hat{\theta}_i)^{-1} \eta_i(\hat{\theta}_i) \tilde{\sigma}_0 &\equiv \hat{\theta}_i^{-1} \phi \\ -\upsilon_i(\hat{\theta}_i)^{-1} \mu_i(\hat{\theta}_i) \tilde{\sigma}_i &\equiv \hat{\theta}_i^{-1} \phi \end{aligned} \qquad \forall \ i=1..N.$$

These conditions are expressed as identities for an important reason: They must hold throughout the entire range of values for the conjectures. Determining the forms of the conjectures that satisfy these constraints requires three steps.

The first step consists of specifying a factorization implied by (37). Note that the shocks $\{ \tilde{\sigma}_0 \{ \tilde{\sigma}_i \ i=1..N \}\}$ are exogenous. Hence, they cannot possibly be functions of the conjectures. Consequently, in the left sides of (37) I seek factorizations of the form $\{ \upsilon_i(\hat{\theta}_i)^{-1} \eta_i(\hat{\theta}_i) \propto \hat{\theta}_i^{-1} \ i=1..N \}$ and $\{ \upsilon_i(\hat{\theta}_i)^{-1} \mu_i(\hat{\theta}_i) \propto \hat{\theta}_i^{-1} \ i=1..N \}$. These restrictions do not necessarily require firms to be identical; they merely constrain their degree of heterogeneity. When the conjectures are different each firm responds differently in a way the depends crucially on the value of its conjecture.

The second step rules out firm-specific shocks. In the first constraint in (37) there is no problem because the shock $\tilde{\sigma}_0$ is common among firms. In the second, however, the factorizations restrict attention to shocks of the form $\{ \tilde{\sigma}_i \propto \phi \ i=1..N \}$. Thus, the shocks must be common among firms. This requirement is not too restrictive. For example, it is consistent with oligopolies that face given factor prices. Moreover, in comparative statics the common-shocks assumption is invoked more frequently than it is not. Nevertheless, this condition does restrict the focus and, while I continue to employ the notation { $\tilde{\sigma}_i$ i=1..N }, the shocks are now assumed to be common.

The third and final step is to determine the conditions under which the factorizations implied by (37) exist. The derivatives that I seek depend crucially on the conjectures. However, neither the values nor the forms of these elasticities are known. Consequently, derivatives in (3) are assessed in accordance with (6), under the most general set of assumptions possible. In the appendix I take derivatives assuming that the conjectural variations have the forms { $\chi_i(\cdot) \equiv \chi_i(x_i, \sigma^i)$ i=1..N }. In other words, I assume that the conjectural variations are general functions of two variables. One is the output level of the firm that forms the conjecture. The other is some variable beyond its control, but which influences its beliefs about output. The existence of the second variable is important and is outlined later. In the derivations immediately below I suppress σ^i. None of the results depend on this convenience. Proceeding, note that the structural parameters in (6) are not unique. Any non-zero transformation of the left-hand side will satisfy the condition on the right. In deriving the structural forms it will prove convenient to normalize on the cost-shift parameter. Hence, taking derivatives in (3) and rearranging I obtain

$$
(38) \qquad
\begin{aligned}
\upsilon_i(\hat{\theta}_i) &\equiv \frac{-\varepsilon\hat{\theta}_i}{\rho_i\,(1+\varepsilon\hat{\theta}_i)}\left(2 + D''(\cdot)^{x}_{x}\,\hat{\theta}_i + \chi_i''(\cdot)^{x_i}_{x_i} \right) + \frac{\omega_i}{\rho_i} \\
\eta_i(\hat{\theta}_i) &\equiv \frac{-1}{\rho_i\,(1+\varepsilon\hat{\theta}_i)}\left(\nu + D''(\cdot)^{x}_{\sigma_o}\,\varepsilon\hat{\theta}_i \right) \qquad\qquad i=1..N, \\
\mu_i(\cdot) &\equiv 1
\end{aligned}
$$

where $\omega_i \equiv (\partial c_i(\cdot)/\partial x_i)(x_i/c_i(\cdot))$, $\rho_i \equiv (\partial c_i(\cdot)/\partial \sigma_i)(\sigma_i/c_i(\cdot))$, $D''(\cdot)^{x}_{x} \equiv (\partial(\partial D(\cdot)/\partial x)/\partial x)(x/(\partial D(\cdot)/\partial x))$, $D''(\cdot)^{x}_{\sigma_o} \equiv (\partial(\partial D(\cdot)/\partial x)/\partial \sigma_o)(\sigma_o/(\partial D(\cdot)/\partial x))$ and $\chi_i''(\cdot)^{x_i}_{x_i} \equiv (\partial(\partial \chi_i(\cdot)/\partial x_i)/\partial x_i)(x_i/(\partial \chi_i(\cdot)/\partial x_i))$.

Note in (38) two features of the normalization. The first, of course, is that the $\mu_i(\cdot)$'s are constant across firms. The second is that they no longer depend on the conjectures. It follows from the second line of conditions (37) that if a set of factorizations exist then they must be the ones { $\upsilon_i(\hat{\theta}_i) \propto \hat{\theta}_i$ i=1..N }. But, these conditions then imply, in the first line in (37), that the $\eta_i(\cdot)$'s also must be constant across firms. Consequently, a set of joint restrictions must hold in order for conjectures to be pair-wise admissible:

LEMMA 7 (Pair-Wise Admissible Structural Forms): Necessary and sufficient conditions for pair-wise admissibility are the existence of joint restrictions { $\upsilon_i(\hat{\theta}_i) \propto \hat{\theta}_i$ i=1..N } and { $\eta_i(\hat{\theta}_i) \propto \phi$ i=1..N }.

I consider each of these restrictions in turn. The issue at hand is whether these restrictions permit beliefs that are heterogeneous among the firms. In other words, whether conjectures of the form $\{ \hat{\theta}_i = Nx_i/x \ i=1..N \}$ are compatible with equilibrium.

From inspection of (38), the factorizations $\{ \upsilon_i(\hat{\theta}_i) \propto \hat{\theta}_i \ i=1..N \}$ require marginal costs to be constant. That is, they require the restrictions $\{ \omega_i = 0 \ i=1..N \}$ across firms. Focusing on the remaining terms in the top line of (38), two additional restrictions are implied. The first is constancy of the bracketed expression; the second is constancy of the multiplicative term $\rho_i (1+\varepsilon\hat{\theta}_i)$. In the first case, heterogeneity in beliefs is retained if and only if two conditions prevail. The first is that the demand function is linear; the second is that the conjectural variations are affine in the outputs of each of the firms. When these joint conditions hold, the bracketed expression reverts to the number two, which is clearly constant across firms. Turning to the multiplicative term $\rho_i (1+\varepsilon\hat{\theta}_i)$, note two facts. The first is that the mark-up term $(1+\varepsilon\hat{\theta}_i)$ represents marginal cost divided by price. The second follows from combining this observation with the definition of ρ_i, which appears below (38): The multiplicative term $\rho_i (1+\varepsilon\hat{\theta}_i)$ is proportional to $(\partial c_i(\cdot)/\partial\sigma_i)\sigma_i$. However, since the σ_i's are common, constancy of $\rho_i (1+\varepsilon\hat{\theta}_i)$ is equivalent to restricting the slopes of the marginal cost functions to be common across firms. In other words, constancy of $\rho_i (1+\varepsilon\hat{\theta}_i)$ requires the conditions $\{ \partial c_i(\cdot)/\partial\sigma_i = \alpha \ i=1..N \}$, for some $\alpha > 0$. Integrating over the σ_i's I obtain $\{ c_i(\cdot) \equiv \alpha\sigma_i+\varphi_i \ i=1..N \}$, where the φ_i's are arbitrary constants. Hence, marginal costs, although constant, are constant at different levels. If not, beliefs must be homogeneous—a fact that follows from equations (3). Now, using the marginal costs $\{ c_i(\cdot) \equiv \alpha\sigma_i+\varphi_i \ i=1..N \}$ in $\{ p (1+\varepsilon\hat{\theta}_i) = c_i(\cdot) \ i=1..N \}$, I can derive equalities between the beliefs $\{ \hat{\theta}_i \ i=1..N \}$ and the remaining terms in equations (3). In particular, I derive $\{ \hat{\theta}_i = - \varepsilon \lambda_i \ i=1..N \}$, where the λ_i's, denoting Lerner indices, are defined $\{ \lambda_i \equiv (p-c_i(\cdot))/p \ i=1..N \}$. Using $\{ \hat{\theta}_i \equiv (\partial\chi_i(\cdot)/\partial x_i)(x_i/x) \ i=1..N \}$, the Lerner equalities imply $\{ \partial\chi_i(\cdot)/\partial x_i = - \varepsilon \lambda_i x/x_i \ i=1..N \}$. Integrating over the x_i's I obtain $\{ \chi_i(\cdot) \equiv - \varepsilon \lambda_i x \log(x_i)+\zeta_i \ i=1..N \}$, where the ζ_i's are arbitrary. These expressions are the forms of the conjectural variations when firms have heterogeneous beliefs. They are clearly not affine in firm output, which contradicts an initial assumption. Consequently there exist no consistent conjectures when beliefs are heterogeneous.

The analysis draws out an important fact: When the firms are heterogeneous, there are too many restrictions that must be met in order for consistency to prevail. This leads me to consider the alternative, namely an equilibrium with homogeneous beliefs. Repeating the steps above, I seek factorizations $\{ \upsilon_i(\hat{\theta}_i) \propto \hat{\theta}_i \ i=1..N \}$ in the top line of (38). As before, this requires marginal costs to be constant. However, when beliefs are homogeneous { $\hat{\theta}_i = \hat{\theta}$ i=1..N }, constancy of the bracketed expression no longer restricts the curvature of demand. The restrictions implied on the conjectural variations are also much weaker: Although constant across firms, they need not be affine. Constancy of $\rho_i\,(1+\varepsilon\hat{\theta}_i)$ is still required, and since the $\hat{\theta}_i$'s are common, so too are marginal costs. Consequently, the ρ_i's are also common. In the second equality in (38), these conditions are sufficient to guarantee constancy of the $\eta_i(\cdot)$'s. It follows that the equilibrium can support homogeneous beliefs. In this case, however, Corollary 10 restricts attention to collusion, and I conclude, accordingly:

LEMMA 8 (Existence): A consistent-conjectures equilibrium exists in which identical incumbents produce with constant returns to scale and conjecture collusively { $\hat{\theta}_i = 1$ i=1..N }.

The conclusion that firms must be identical stems from the facts that both their technologies and their beliefs must be the same. It follows that the distribution of output must be symmetric. This feature of the equilibrium begs closer scrutiny—for several reasons. First, the identical-firms assumption is commonplace in the literature in oligopoly, as it is elsewhere in economics. However, unlike its employment in non-strategic models, it indicts oligopolies to a specific interaction: The agents must act alike. This "likeness" implicates the firms to a tacit collusion. These observations are pertinent to two literatures, one simulating conceptual experiments and the other testing for market power. In both situations the identical-firms assumption is the rule rather than the exception. In the former case, my results suggest that experiments should be conducted recognizing collusion as a distinct possibility. I turn to this in a moment. For the empirical literature my results suggest an additional, relevant hypothesis, which may be testable, and is engendered by the equilibrium assumptions. This is the possibility of tacit collusion. Finally, I should mention that collusion—tacit or otherwise—was recognized by Bowley (p. 38) as an inherent possibility.

Comparative Statics

I wish now to examine comparative statics generated by consistent-conjectures oligopolies. Two preliminary results will assist in this venture. The first pertains to the form of the conjectural variations; the second to the structure of demand that is consistent with equilibrium. Regarding the conjectural variations, note that the conditions { $\hat{\theta}_i \equiv$

$(\partial\chi_i(\cdot)/\partial x_i)(x_i/x) = 1$ i=1..N } imply, in turn, { $\partial\chi_i(\cdot)/\partial x_i = x/x_i$ i=1..N }, but since the equilibrium is symmetric, these conditions become { $\partial\chi_i(\cdot)/\partial x_i = N$ i=1..N }. Integrating over the x_i's I obtain { $\chi_i(\cdot) \equiv Nx_i+\psi_i$ i=1..N }, where the ψ_i's are, once again, arbitrary. To tie down these constants, I note that, since the equilibrium is symmetric, the conjectural variations will be inconsistent throughout all ranges of output unless each ψ_i is zero. Accordingly, the *consistent* conjectural variations are the functions

$$(39) \qquad \chi_i(x_i , \sigma^i) \equiv N x_i \qquad i=1..N .$$

These relations are important for several reasons. First, they illustrate explicitly the sense in which the consistent-conjectures equilibrium is a rational-expectations equilibrium: The consistent conjectural variations must coincide exactly with the structural equation about which predictions are formed. If the firms have consistent conjectural variations they will always be on a rational-expectations time path. In each and every period, their one-step-ahead forecasts will be accurate. Second, as discussed in the derivations leading to (38), I can now determine the exogenous variable affecting firms' perceptions. It is, of course, N, which is clearly beyond the control of the firm and certainly affects output. The third implication of (39) is that I can now tie down a parameter in the first line of (38): The second-order, cross-partial of the consistent conjectural variation is zero. Finally, the relations in (39) illustrate the unique situation in which causality and correlation coincide: In the symmetric equilibrium the causal connotations of the conjectures are legitimate.

Regarding restrictions on demand, note that Assumption 2 is satisfied whenever { $\upsilon_i(\hat{\theta}_i) > 0$ i=1..N }. There exist a wide class of demand configurations that are consistent with these conditions. In particular, they are satisfied as long as the second-order effect of demand is not less than minus two. In other words, a locally unique equilibrium exists if demand is sufficiently "non-concave."

Turning to the adjustments, an increase in costs { $\tilde{\sigma}_i > 0$ i=1..N } leads to the following responses.

$$(40) \qquad \begin{aligned} \tilde{x}_i &= - \upsilon_i(\hat{\theta}_i)^{-1} \tilde{\sigma}_i &< 0 \\ \tilde{x} &= - \upsilon_i(\hat{\theta}_i)^{-1} \tilde{\sigma}_i &< 0 \\ \tilde{p} &= - \varepsilon\, \upsilon_i(\hat{\theta}_i)^{-1} \tilde{\sigma}_i &> 0 \end{aligned} \qquad i=1..N.$$

Hence, firm and industry output contract and price rises. Consequently, a cost increase yields results that are analogous to those under perfect competition. However, in oligopoly a cost increase will, almost surely, lead to a change in profits. The impact on profits can be written

$$\text{(41)} \qquad \Delta\pi_i = p x_i \left(\tilde{p} + \tilde{x}_i \right) - c(\cdot)\, x_i \left(\tilde{x}_i + \tilde{c}(\cdot) \right) \qquad i=1..N,$$

where $\tilde{c}(\cdot) \equiv \Delta c(\cdot)/c(\cdot)$ denotes the proportional change in marginal costs. It can be derived that the first effect, the revenue effect, is strictly negative. However, the sign of the second effect, the cost effect, is ambiguous. The precise impact of the change depends on the level of profits and the flexibility of demand in the initial equilibrium. A perverse-profits effect—a cost increase that enhances profits—remains a distinct possibility.

In contrast to the effects of a cost shift, the effects of demand shifts $\tilde{\sigma}_0 > 0$ are generally ambiguous. In particular, I obtain

$$\text{(42)} \qquad \begin{aligned} \tilde{x}_i &= -\eta_i(\hat{\theta}_i)\, \upsilon_i(\hat{\theta}_i)^{-1}\, \tilde{\sigma}_0 && \gtrless 0 \\ \tilde{x} &= -\eta_i(\hat{\theta}_i)\, \upsilon_i(\hat{\theta}_i)^{-1}\, \tilde{\sigma}_0 && \gtrless 0 \qquad i=1..N. \\ \tilde{p} &= \left(-\varepsilon\, \eta_i(\hat{\theta}_i)\, \upsilon_i(\hat{\theta}_i)^{-1} + \nu \right) \tilde{\sigma}_0 && \gtrless 0 \end{aligned}$$

The precise effects depend crucially on the way in which the shock affects the slope of demand. A sufficient condition for an expansion in output is that the demand slope becomes more negative as a result of the shock. Yet, even in this "normal" case, price may still decline. However, a perverse output effect—output contracting when demand expands—can never occur when demand is linear. This is not the case with respect to profits: A demand increase that lowers profits is an intriguing possibility.

Entry

Results thus far have assumed that firm numbers are fixed. One final issue remains unresolved. Can consistent conjectures prevail in the face of entry ? The identical-firms assumption will prove useful in this context. I interpret increases in N to imply entry, conversely, exit. The assumption also permits the usual comparative statics, treating N as real-, rather than integer-valued. This technique, although frequently employed, requires one caveat. This is that the effects of any perturbation do not alter in sign between successive integer realizations of N (Seade, p.

482). In other words, it requires monotonic adjustments in price and quantity within the range of adjustments in N. One is often interested in two issues. The first is the effect on price and quantity of autonomous changes in N. The second is the effect of a change that causes N to adjust endogenously. A related issue is the possible presence of a "business-stealing effect." Business stealing prevails whenever entry causes incumbent production to contract. This effect—sometimes referred to as "accommodation"—is considered by some to reflect conventional wisdom. However, an equally plausible counter-argument exists: Entry may be stimulated by the very factors that cause expansions in output. In this case one expects simultaneous increases in firm numbers and incumbents' outputs, or "predation." Although unresolved, the issue of which case seems more likely appears to rest on one key assumption—whether entry is cause or effect. In other words, whether entry is endogenous or exogenous to the equilibrium.

In concluding the search for consistent conjectures this debate is salient: The consistent-conjectures equilibrium represents a dividing line between the presence or absence of business stealing. I shed light on this debate in the course of introducing a few details. The first is a definition of aggregate output in the symmetric equilibrium:

$$x = N x_i . \tag{43}$$

When firms are identical equation (43) is used instead of (2). Allowing for perturbations in the neighborhood of equilibrium, I derive

$$\tilde{x} = \tilde{N} + \tilde{x}_i . \tag{44}$$

Finally, normalizing on the representative incumbent's output, I obtain

$$\theta_i \equiv \frac{\tilde{x}}{\tilde{x}_i} = \frac{\tilde{N}}{\tilde{x}_i} + 1, \tag{45}$$

from which the following observations are apparent. The expression on the right-hand side represents the true effects about which the firms form conjectures. Since these effects are common across firms, admissibility reduces to the single equality $\hat{\theta} = \theta$. Business stealing is present when the ratio $\tilde{N} / \tilde{x}_i$ is strictly negative; it is absent when this ratio is strictly positive. It follows that for business stealing to prevail, the admissible conjecture must be contained on the open interval $\hat{\theta} \in (0,1)$; conversely, $\hat{\theta} \in (1,-\varepsilon^{-1})$ when it is absent. In the first case the industry becomes more competitive; in the latter it becomes more collusive. However, neither situation can prevail in a consistent-conjectures equilibrium. In order for consistency to prevail the condition $\hat{\theta} = 1$ is required. In other words, the consistent-conjectures equilibrium is incompatible with entry and exit.

There are three arguments. A heuristic one proceeds as follows: Suppose entry occurs and business stealing is absent. Then the condition $\theta > 1$ prevails; but when firms are identical, the condition $\hat{\theta} > 1$ is inconsistent with optimizing behavior in the static phase of the game. In other words, when the firms are identical, pricing above monopoly is sub optimal. For the same reasons, I can also rule out $\theta < 1$. In this case too, the condition, $\hat{\theta} < 1$, is inconsistent with the optimizing assumptions embedded in the initial phase of the game. A second argument stems from a distinction between partial and total differentials, which arises as a consequence of the definitions in (39). In particular, the partial derivatives $\{ \hat{\theta}_i \equiv (\partial\chi_i(\cdot)/\partial x_i)(x_i/x)\ i=1..N \}$ can never be consistent with the total differentials of $\{ \chi_i(x_i , \sigma^i) \equiv N\, x_i\ i=1..N \}$ when N and x_i adjust simultaneously. A third argument stems from redefining the conjectural variations in an attempt to reconcile this situation. Suppose I permit firms to form conjectures about N. Suppose, also, that these conjectures depend on own output. In this case, the conjectural variations revert to univariate expressions of the form $\{ \chi_i(x_i) \equiv N(x_i)\, x_i\ \ i=1..N \}$. If the firm conjectures business stealing it conjectures a derivative of $N(\cdot)$ with respect to own output that is negative; conversely if it conjectures to the contrary. In the appendix I show that neither of these two conjectures are compatible with equilibrium. Thus:

LEMMA 9 (Non-Existence): No consistent conjecture exists in equilibria with entry. The restricted-entry equilibria is unique.

8. CONCLUSION

> ... I have not intended to advance any new theorems in economics, nor do I claim any originality in mathematical results, for the few theorems which I have not consciously adapted from others may in fact already have been published. Perhaps, however, there is in my analysis a more definite attempt than has been usual to deal equally with the hypotheses of competition and monopoly, to find a place for incomplete monopoly and to indicate how perfect competition and perfect monopoly are mathematically the extreme cases of a more general conception.
>
> *—A. L. Bowley, Preface, The Mathematical Groundwork of Economics, 1924.*

A consistent conjecture exists in identical-firms, restricted-entry equilibria. It is the collusive conjecture. It is unique. It is a Nash strategy in a two-stage game with an initial static stage and a subsequent comparative-static phase. It is derived from a rational-expectations conjectural variation. These conclusions complete a search that was initiated by Leontief in 1936.

Despite their conceptual flaws, conjectural variations continue to be employed. They are viewed as convenient devices for synthesizing outcomes on the spectrum between pure competition and monopoly. Interestingly, in these applications the assumption that firms are identical appears more commonly than not. In this respect my findings are quite pertinent: Results should be synthesized recognizing collusion as a distinct possibility. In this way one may use Bowley's conjectural variations with fewer apologies than were necessary in the past. The general applicability of the theory, of course, is quite a different issue, and this paper has done little to belie the criticism that conjectures are conceptually misconstrued. At the least, however, my results are useful in three respects. I have identified the specific limitations of the theory; I have explained the appearance of dissimilar interpretations of the model; and I have linked its origins to the history of thought about the static theory of oligopoly. As to whether Professor Bowley would be happy with these conclusions, one may only conjecture.

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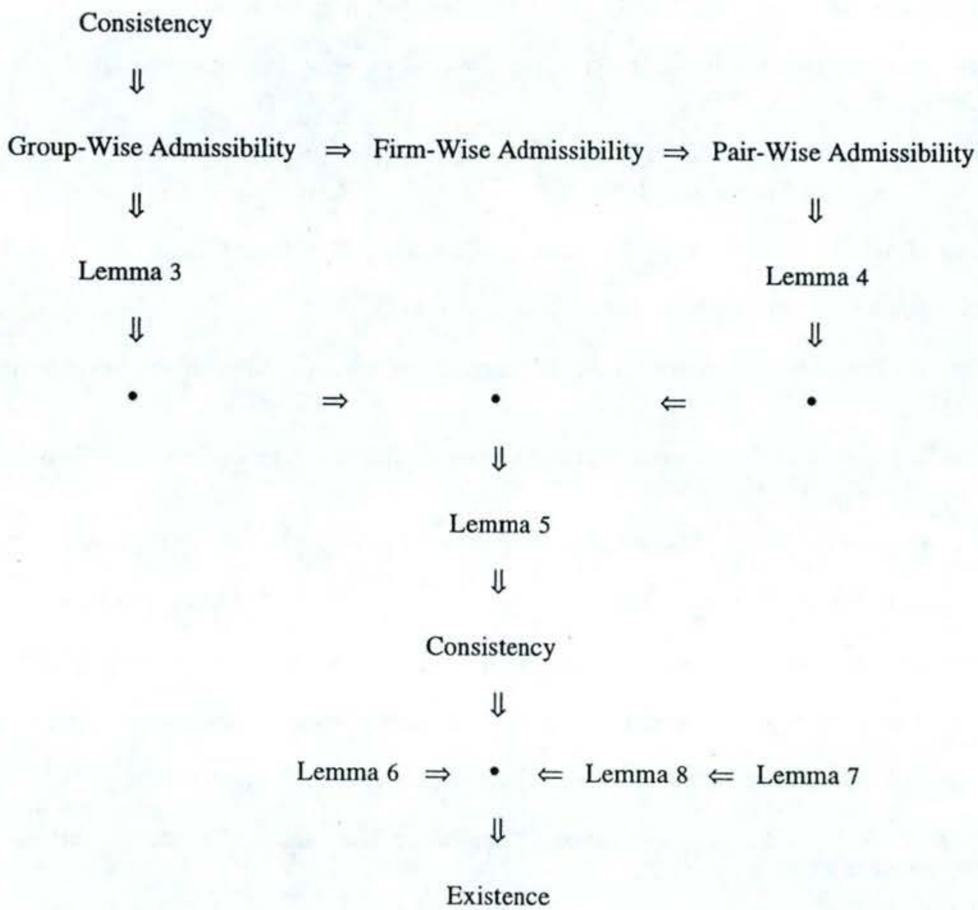

FIGURE 1. Search Strategy

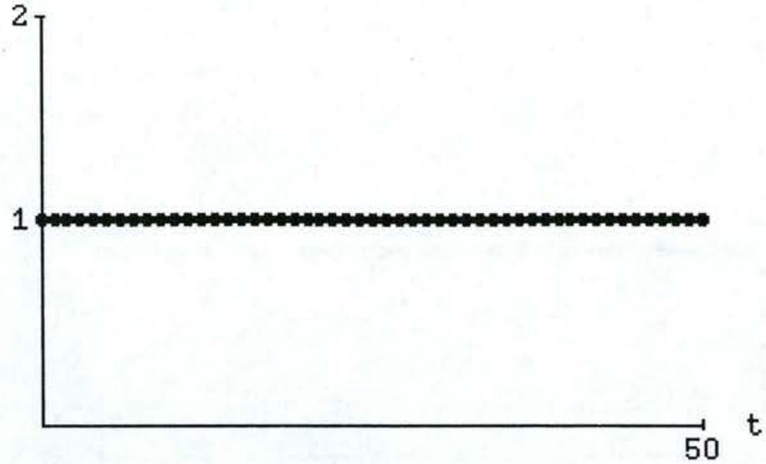

FIGURE 2a. — $\Gamma(t)$

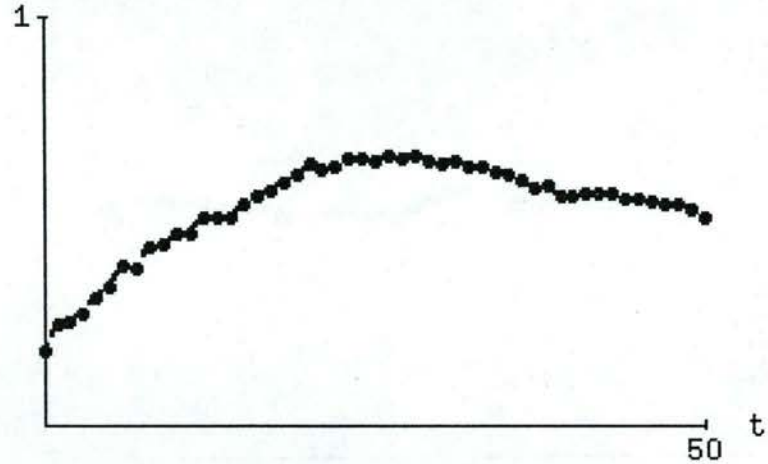

FIGURE 2b. — $f(t)$ and $g(t)$

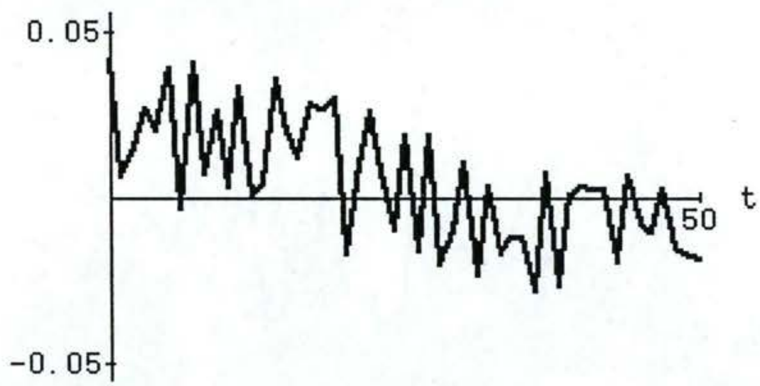

FIGURE 2c. — $\tilde{\sigma}(t)$

FIGURE 2. — Concentration Under Bowley Beliefs { $\hat{\theta}_i(t) = Nx_i(t)/x(t)$ i=1..N }
(N=2)

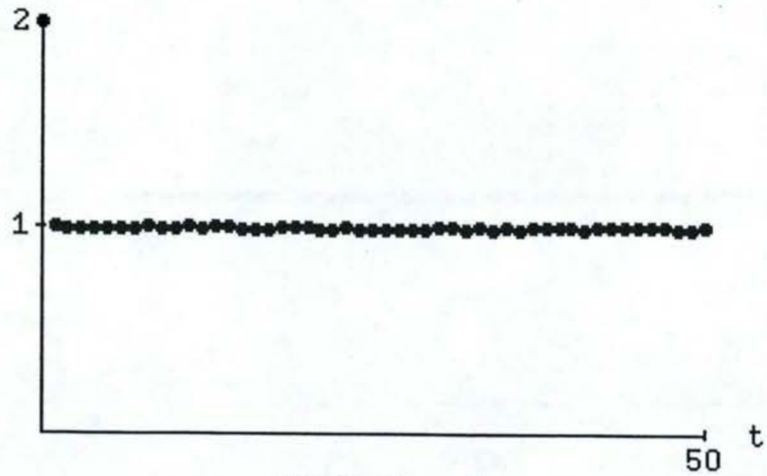

FIGURE 3a. — $\Gamma(t)$

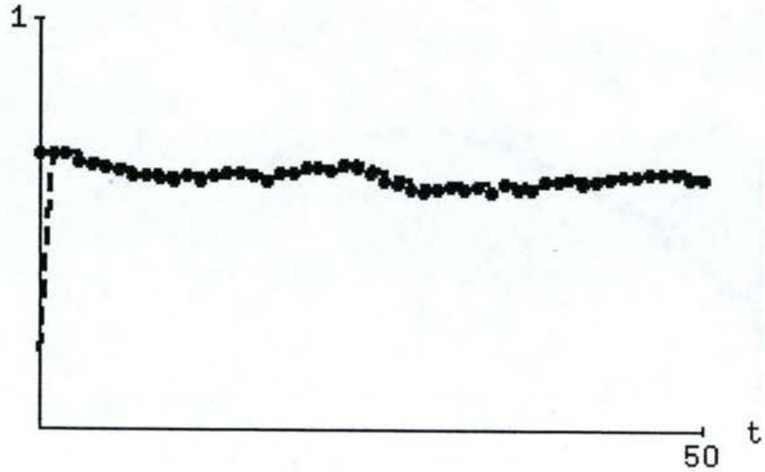

FIGURE 3b. — $f(t)$ • • and $g(t)$ – –

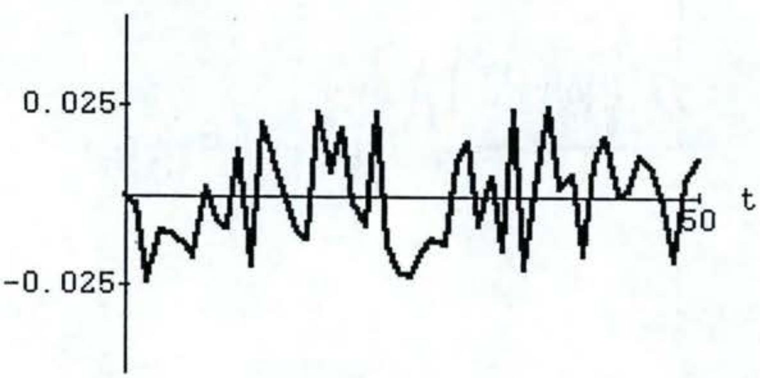

FIGURE 3c. — $\tilde{\sigma}(t)$

FIGURE 3. — Convergence Under Hindsight { $\hat{\theta}_i(t) = \theta_i(t-1)$ i=1..N }
(N=2)

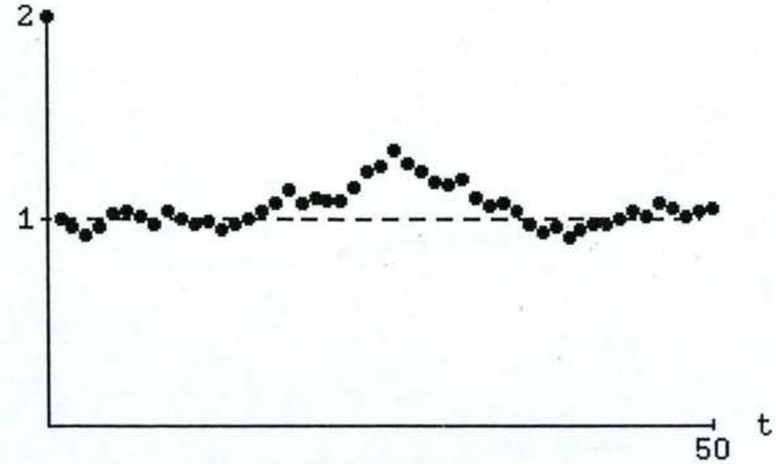

FIGURE 4a. — $\Gamma(t)$

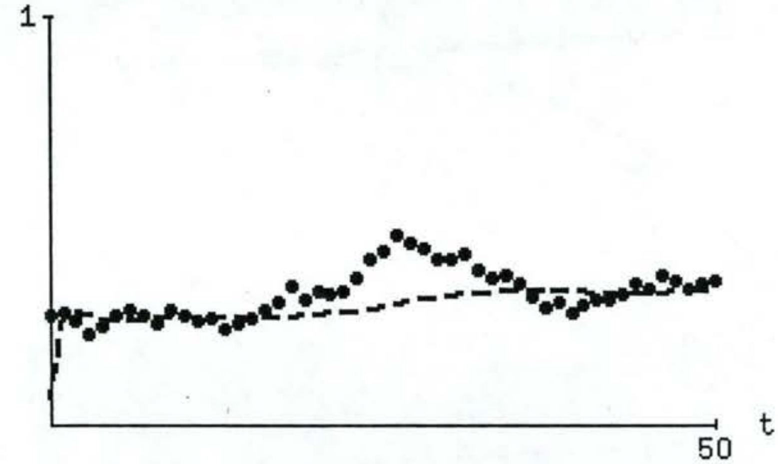

FIGURE 4b. — $f(t)$ • • and $g(t)$ – –

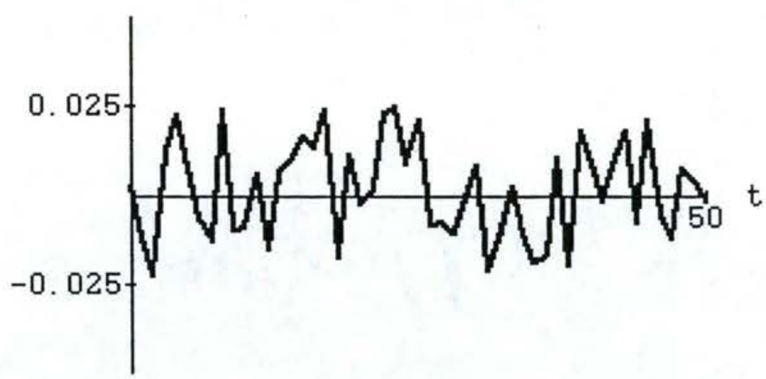

FIGURE 4c. — $\tilde{\sigma}(t)$

FIGURE 4. — Convergence Under Means of Past Histories $\{ \hat{\theta}_i(t) = \frac{1}{t} \sum_{s=0}^{t-1} \theta_i(s) \quad i=1..N \}$

(N=2)

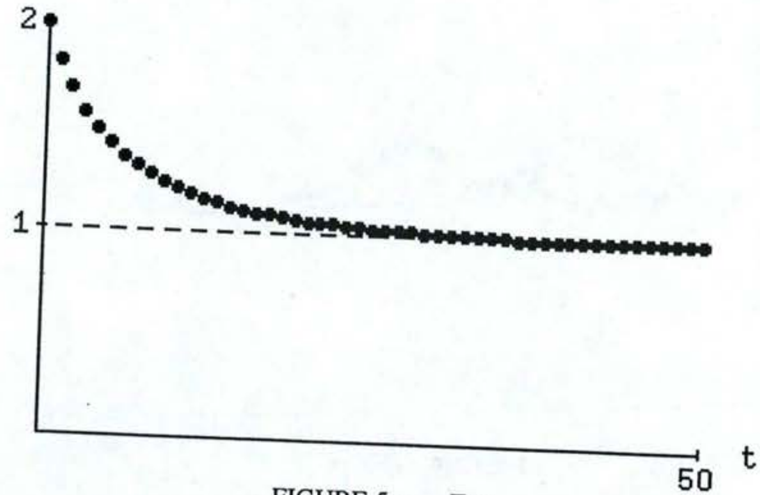

FIGURE 5a. — $\Gamma(t)$

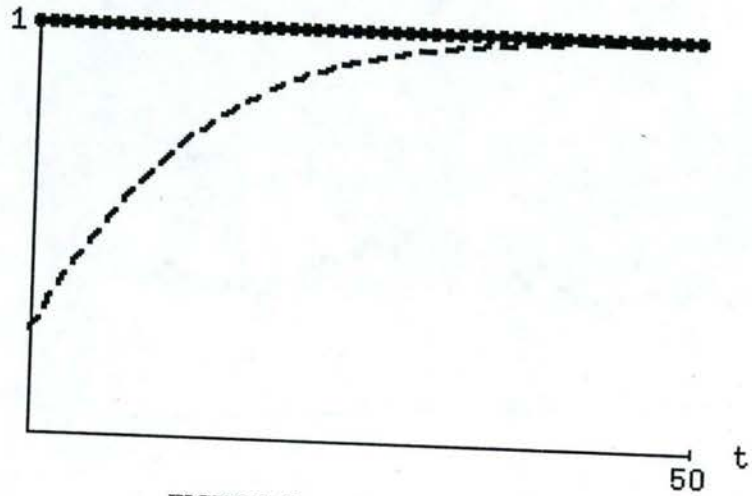

FIGURE 5b. — $f(t)$ • • and $g(t)$ – –

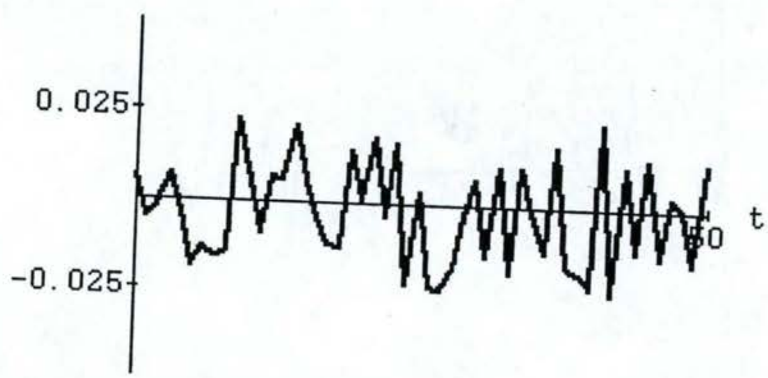

FIGURE 5c. — $\tilde{\sigma}(t)$

FIGURE 5. — Convergence Under Adaptive Behavior $\{ \hat{\theta}_i(t) = \hat{\theta}_i(t-1) + \alpha \left(\theta_i(t-1) - \hat{\theta}_i(t-1) \right) i=1..N \}$
(N=2, α=0.1)

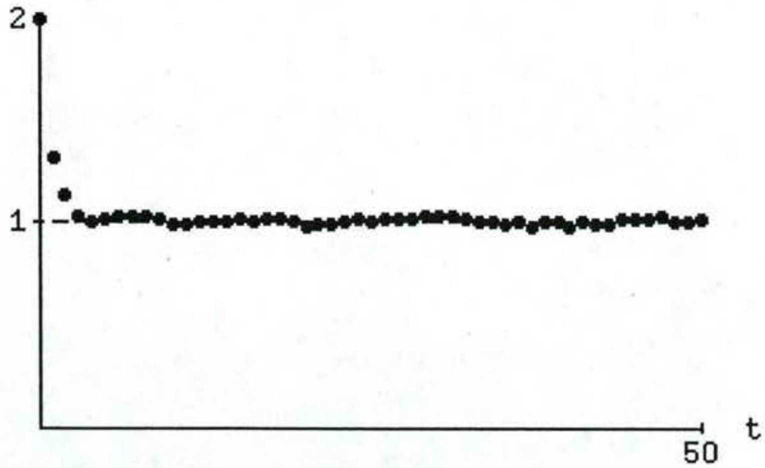

FIGURE 6a. — $\Gamma(t)$

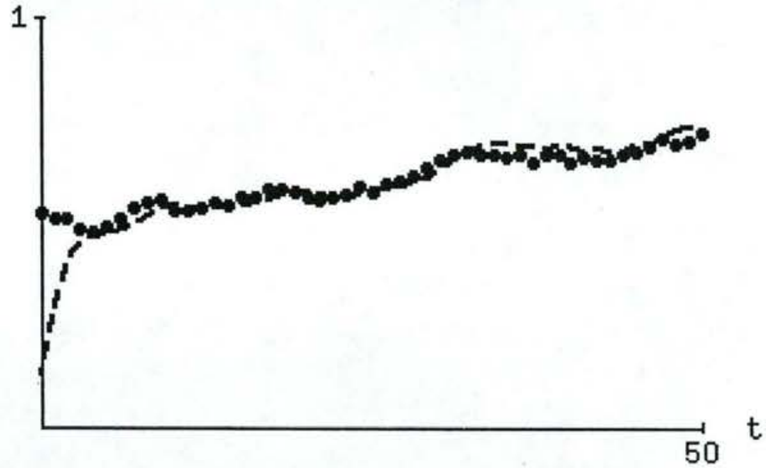

FIGURE 6b. — $f(t)$ • • and $g(t)$ – –

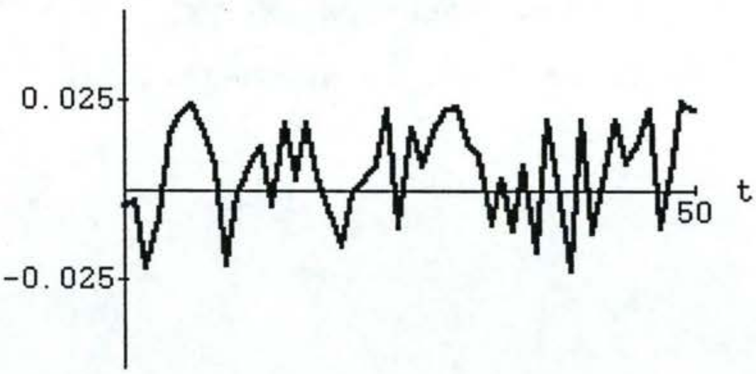

FIGURE 6c. — $\tilde{\sigma}(t)$

FIGURE 6. — Convergence Under Adaptive Behavior $\{ \hat{\theta}_i(t) = \hat{\theta}_i(t-1) + \alpha \left(\theta_i(t-1) - \hat{\theta}_i(t-1) \right) \; i=1..N \}$
$(N=2,\ \alpha=0.5)$

APPENDICES

(Not intended for publication)

Contents

APPENDIX ONE

LITERATURE REVIEWED

This appendix lists studies compiled, initially, from a search of the *Journal of Economic Literature* database (CD ROM, 1969 to 1994) using keywords "conjectural variations" and "conjectures."

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APPENDIX TWO

DERIVATION OF TEXT EQUATIONS (38)

Assuming $\{ \chi_i(\cdot) \equiv \chi_i(x_i, \sigma^i) \; i=1..N \}$, the first-order conditions appearing in (3) become

$$p(1 + \varepsilon \hat{\theta}_i) - c_i(x_i \mid \sigma_i) \equiv \phi_i(x_i \mid \sigma_0, \sigma_i, \sigma^i) = 0 \qquad i=1..N. \tag{a1}$$

The implicit forms of the derivatives corresponding to this equation are:

$$\upsilon_i(\hat{\theta}_i)\, \tilde{x}_i + \kappa_i(\hat{\theta}_i)\, \tilde{\sigma}^i + \eta_i(\hat{\theta}_i)\, \tilde{\sigma}_0 + \mu_i(\hat{\theta}_i)\, \tilde{\sigma}_i = 0 \qquad i=1..N, \tag{a2}$$

where $\upsilon_i(\hat{\theta}_i) \equiv x_i \partial\phi_i(\cdot)/\partial x_i$, $\eta_i(\hat{\theta}_i) \equiv \sigma_0 \partial\phi_i(\cdot)/\partial\sigma_0$, $\mu_i(\hat{\theta}_i) \equiv \sigma_i \partial\phi_i(\cdot)/\partial\sigma_i$ and $\kappa_i(\hat{\theta}_i) \equiv \sigma^i \partial\phi_i(\cdot)/\partial\sigma^i$. Note that equations (a2) differ from equations (6) due to the appearance of the exogenous $\{ \sigma^i \; i=1..N \}$, here assumed to enter the conjectural variations. To derive the explicit forms, take derivatives in Problem 2 and obtain

$$D(\cdot) + \frac{\partial D(\cdot)}{\partial \chi_i(\cdot)} \frac{\partial \chi_i(\cdot)}{\partial x_i} x_i - c_i(x_i \mid \sigma_i) \equiv \phi_i(x_i \mid \sigma_0, \sigma_i, \sigma^i) = 0 \qquad i=1..N, \tag{a3}$$

where we suppress, but acknowledge, the explicit dependence $D(\cdot) \equiv D(\chi_i(x_i \mid \sigma^i), \sigma_0)$.

To aid exposition, the ensuing steps are presented on consecutive pages.

Totally differentiating in (a3), repeated applications of the chain rule lead to

(a4)

$$\frac{\partial D(\cdot)}{\partial \chi_i(\cdot)} \frac{\partial \chi_i(\cdot)}{\partial x_i} \Delta x_i$$

$$+ \quad \frac{\partial D(\cdot)}{\partial \chi_i(\cdot)} \frac{\partial \chi_i(\cdot)}{\partial \sigma^i} \Delta \sigma^i$$

$$+ \quad \frac{\partial D(\cdot)}{\partial \sigma_o} \Delta \sigma_o$$

$$+ \quad \frac{\partial \chi_i(\cdot)}{\partial x_i} \; x_i \left\{ \frac{\partial\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)}{\partial \chi_i(\cdot)} \frac{\partial \chi_i(\cdot)}{\partial x_i} \Delta x_i + \frac{\partial\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)}{\partial \chi_i(\cdot)} \frac{\partial \chi_i(\cdot)}{\partial \sigma^i} \Delta \sigma^i + \frac{\partial\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)}{\partial \sigma_o} \Delta \sigma_o \right\}$$

$$+ \quad \frac{\partial D(\cdot)}{\partial \chi_i(\cdot)} \; x_i \left\{ \frac{\partial\,(\,\partial \chi_i(\cdot)/\partial x_i\,)}{\partial x_i} \Delta x_i + \frac{\partial\,(\,\partial \chi_i(\cdot)/\partial x_i\,)}{\partial \sigma^i} \Delta \sigma^i \right\}$$

$$+ \quad \frac{\partial D(\cdot)}{\partial \chi_i(\cdot)} \frac{\partial \chi_i(\cdot)}{\partial x_i} \Delta x_i$$

$$- \quad \frac{\partial c_i(\cdot)}{\partial x_i} \Delta x_i$$

$$- \quad \frac{\partial c_i(\cdot)}{\partial \sigma_i} \Delta \sigma_i \quad = \quad 0, \qquad i=1..N.$$

Converting to elasticities, one has:

(a5)

$$\frac{\partial D(\cdot)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,p}\,\frac{\partial \chi_i(\cdot)\,x_i}{\partial x_i\,\chi_i(\cdot)}\,p\,\frac{\Delta x_i}{x_i}$$

$$+\;\frac{\partial D(\cdot)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,p}\,\frac{\partial \chi_i(\cdot)\,\sigma^i}{\partial \sigma^i\,\chi_i(\cdot)}\,p\,\frac{\Delta \sigma^i}{\sigma^i}$$

$$+\;\frac{\partial D(\cdot)\,\sigma_o}{\partial \sigma_o\,p}\,p\,\frac{\Delta \sigma_o}{\sigma_o}$$

$$+\;\frac{\partial \chi_i(\cdot)\,x_i}{\partial x_i\,\chi_i(\cdot)}\left\{\frac{\partial\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)}\,\frac{\partial D(\cdot)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,p}\,\frac{\partial \chi_i(\cdot)\,x_i}{\partial x_i\,\chi_i(\cdot)}\,p\,\frac{\Delta x_i}{x_i}\right.$$

$$+\;\frac{\partial\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)}\,\frac{\partial D(\cdot)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,p}\,\frac{\partial \chi_i(\cdot)\,\sigma^i}{\partial \sigma^i\,\chi_i(\cdot)}\,p\,\frac{\Delta \sigma^i}{\sigma^i}$$

$$\left.+\;\frac{\partial\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)\,\sigma_o}{\partial \sigma_o\,(\,\partial D(\cdot)/\partial \chi_i(\cdot)\,)}\,\frac{\partial D(\cdot)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,p}\,p\,\frac{\Delta \sigma_o}{\sigma_o}\right\}$$

$$+\;\frac{\partial D(\cdot)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,p}\left\{\frac{\partial\,(\,\partial \chi_i(\cdot)/\partial x_i\,)\,x_i}{\partial x_i\,(\,\partial \chi_i(\cdot)/\partial x_i\,)}\,\frac{\partial \chi_i(\cdot)\,x_i}{\partial x_i\,\chi_i(\cdot)}\,p\,\frac{\Delta x_i}{x_i}\right.$$

$$\left.+\;\frac{\partial\,(\,\partial \chi_i(\cdot)/\partial x_i\,)\,\sigma^i}{\partial \sigma^i\,(\,\partial \chi_i(\cdot)/\partial x_i\,)}\,\frac{\partial \chi_i(\cdot)\,x_i}{\partial x_i\,\chi_i(\cdot)}\,p\,\frac{\Delta \sigma^i}{\sigma^i}\right\}$$

$$+\;\frac{\partial D(\cdot)\,\chi_i(\cdot)}{\partial \chi_i(\cdot)\,p}\,\frac{\partial \chi_i(\cdot)\,x_i}{\partial x_i\,\chi_i(\cdot)}\,p\,\frac{\Delta x_i}{x_i}$$

$$-\;\frac{\partial c_i(\cdot)\,x_i}{\partial x_i\,c_i(\cdot)}\,c_i(\cdot)\,\frac{\Delta x_i}{x_i}$$

$$-\;\frac{\partial c_i(\cdot)\,\sigma_i}{\partial \sigma_i\,c_i(\cdot)}\,c_i(\cdot)\,\frac{\Delta \sigma_i}{\sigma_i}\;=\;0, \qquad i=1..N.$$

Redefining terms:

(a6)

$$
\begin{aligned}
& \varepsilon\, \hat{\theta}_i\, p\, \frac{\Delta x_i}{x_i} \\
+ \quad & \varepsilon\, \eta^i\, p\, \frac{\Delta \sigma^i}{\sigma^i} \\
+ \quad & \nu\, p\, \frac{\Delta \sigma_o}{\sigma_o} \\
+ \quad & \hat{\theta}_i \quad \Big\{ \quad D''(\cdot)^x_x\, \varepsilon\, \hat{\theta}_i\, p\, \frac{\Delta x_i}{x_i} \\
& \qquad + \quad D''(\cdot)^x_x\, \varepsilon\, \eta^i\, p\, \frac{\Delta \sigma^i}{\sigma^i} \\
& \qquad + \quad D''(\cdot)^x_{\sigma_o}\, \varepsilon\, \hat{\theta}_i\, p\, \frac{\Delta \sigma_o}{\sigma_o} \quad \Big\} \\
+ \quad & \varepsilon \quad \Big\{ \quad \chi_i''(\cdot)^{x_i}_{x_i}\, \hat{\theta}_i\, p\, \frac{\Delta x_i}{x_i} \\
& \qquad + \quad \chi_i''(\cdot)^{x_i}_{\sigma^i}\, \hat{\theta}_i\, p\, \frac{\Delta \sigma^i}{\sigma^i} \quad \Big\} \\
+ \quad & \varepsilon\, \hat{\theta}_i\, p\, \frac{\Delta x_i}{x_i} \\
- \quad & \omega_i\, c_i(\cdot)\, \frac{\Delta x_i}{x_i} \\
- \quad & \rho_i\, c_i(\cdot)\, \frac{\Delta \sigma_i}{\sigma_i} \quad = \quad 0, \qquad i=1..N.
\end{aligned}
$$

- Grouping terms:

$$
\begin{aligned}
\text{(a7)} \qquad & \Big\{ \ \varepsilon\, \hat{\theta}_i\, p \\
& \quad + D''(\cdot)^{X}_{X}\, \varepsilon\, \hat{\theta}_i\, \hat{\theta}_i\, p \\
& \quad + \chi_i''(\cdot)^{x_i}_{x_i}\, \varepsilon\, \hat{\theta}_i\, p \\
& \quad + \varepsilon\, \hat{\theta}_i\, p \\
& \quad - \omega_i\, c_i(\cdot) \ \Big\} \ \frac{\Delta x_i}{x_i} \\
& + \Big\{ \ \varepsilon\, \eta^i\, p \\
& \quad + D''(\cdot)^{X}_{X}\, \varepsilon\, \eta^i\, \hat{\theta}_i\, p \\
& \quad + \chi_i''(\cdot)^{x_i}_{\sigma^i}\, \varepsilon\, \hat{\theta}_i\, p \ \Big\} \ \frac{\Delta\sigma^i}{\sigma^i} \\
& + \Big\{ \ \nu\, p \\
& \quad + D''(\cdot)^{X}_{\sigma_o}\, \varepsilon\, \hat{\theta}_i\, \hat{\theta}_i\, p \ \Big\} \ \frac{\Delta\sigma_o}{\sigma_o} \\
& + \Big\{ \ -\rho_i\, c_i(\cdot) \ \Big\} \ \frac{\Delta\sigma_i}{\sigma_i} \ = \ 0, \qquad\qquad i=1..N.
\end{aligned}
$$

Now, using the first-order equalities $\{ p(1+\varepsilon\hat{\theta}_i) - c_i(\cdot) = 0 \ i=1..N \}$, normalize on the last set of structural parameters, namely $\{ -\rho_i c_i(\cdot) = -\rho_i p(1+\varepsilon\hat{\theta}_i) \ i=1..N \}$. This yields:

(a8)

$$
\begin{aligned}
& \Big\{ \varepsilon\hat{\theta}_i \, p \\
& \quad + D''(\cdot)^{X}_{X} \, \varepsilon\hat{\theta}_i \, \hat{\theta}_i \, p \\
& \quad + \chi_i''(\cdot)^{x_i}_{x_i} \, \varepsilon\hat{\theta}_i \, p \\
& \quad + \varepsilon\hat{\theta}_i \, p \\
& \quad - \omega_i c_i(\cdot) \Big\} \Big\{ -\rho_i p (1+\varepsilon\hat{\theta}_i) \Big\}^{-1} \frac{\Delta x_i}{x_i} \\
+ & \Big\{ \varepsilon\eta^i p \\
& \quad + D''(\cdot)^{X}_{X} \, \varepsilon\,\eta^i \, \hat{\theta}_i \, p \\
& \quad + \chi_i''(\cdot)^{x_i}_{\sigma^i} \, \varepsilon\hat{\theta}_i \, p \Big\} \Big\{ -\rho_i p (1+\varepsilon\hat{\theta}_i) \Big\}^{-1} \frac{\Delta\sigma^i}{\sigma^i} \\
+ & \Big\{ \nu p \\
& \quad + D''(\cdot)^{X}_{\sigma_o} \, \varepsilon\hat{\theta}_i \, \hat{\theta}_i \, p \Big\} \Big\{ -\rho_i p (1+\varepsilon\hat{\theta}_i) \Big\}^{-1} \frac{\Delta\sigma_o}{\sigma_o} \\
+ & \Big\{ -\rho_i c_i(\cdot) \Big\} \Big\{ -\rho_i p (1+\varepsilon\hat{\theta}_i) \Big\}^{-1} \frac{\Delta\sigma_i}{\sigma_i} = 0, \qquad i=1..N.
\end{aligned}
$$

Finally, factoring $\{ \varepsilon \hat{\theta}_i \ i=1..N \}$ and rearranging the remaining terms yields the definitions of the parameters appearing in (a2), namely:

$$\text{(a9)} \qquad \begin{aligned} \upsilon_i(\hat{\theta}_i) &\equiv \frac{-\varepsilon\hat{\theta}_i}{\rho_i\,(1+\varepsilon\hat{\theta}_i)}\left(2 + D''(\cdot)^x_x\,\hat{\theta}_i + \chi_i''(\cdot)^{x_i}_{x_i} \right) + \frac{\omega_i}{\rho_i} \\ \kappa_i(\hat{\theta}_i) &\equiv \frac{-1}{\rho_i\,(1+\varepsilon\hat{\theta}_i)}\left(\varepsilon\,\eta^i + D''(\cdot)^x_x\,\varepsilon\,\eta^i\,\hat{\theta}_i + \chi_i''(\cdot)^{x_i}_{\sigma^i}\,\varepsilon\,\hat{\theta}_i \right) \\ \eta_i(\hat{\theta}_i) &\equiv \frac{-1}{\rho_i\,(1+\varepsilon\hat{\theta}_i)}\left(\nu + D''(\cdot)^x_{\sigma_o}\,\varepsilon\hat{\theta}_i \right) \\ \mu_i(\cdot) &\equiv 1 \end{aligned} \qquad i=1..N.$$

Subsequent to deriving the consistent conjectural variations I obtain $\{ \sigma^i \equiv N \ i=1..N \}$. Text equations (38) are given under the assumption that entry is restricted. That is $\{ \Delta\,\sigma^i \equiv \Delta\,N = 0 \ i=1..N \}$.

APPENDIX THREE

PROOF OF NON-EXISTENCE UNDER ENTRY

To investigate existence under entry, recall the symmetric equilibrium with N identical firms. Each contributes output x_i to the industry aggregate, x. Entry costs κ and production incurs variable cost $C(\cdot)$. Firms face inverse demand $D(\cdot)$, product price p and price flexibility $\varepsilon \equiv (\partial D(\cdot)/\partial x)(x/D(\cdot))$. Firms form conjectures $x = \chi(x_i)$ and solve: max $\{\ \pi(x_i) \equiv px_i - C(x_i) \mid p = D(x),\ x = \chi(x_i)\ \}$, where $\hat{\theta} \equiv (\partial\chi(\cdot)/\partial x_i)(x_i/\chi(\cdot))$ denotes the conjectural elasticity of each firm. It is assumed that

(a10) $\quad p = D(x \mid \sigma_o)$

(a11) $\quad x = Nx_i$

(a12) $\quad p\,(1 + \varepsilon\,\hat{\theta}\,) = c(x_i \mid \sigma_i\,)$

(a13) $\quad px_i - C(x_i \mid \sigma_i\,) - \kappa = 0,$

defines an equilibrium in p, x, x_i and N, with σ_o and σ_i given exogenously.

To investigate existence, substitute (a11) into (a10) and (a10) into (a13). This yields the two-equation system

(a14) $\quad \Phi(\,x_i \mid \sigma_o, \sigma_i\,) \equiv D(\,\chi(x_i) \mid \sigma_o\,)\Big[\,1 + \varepsilon(\,\chi(x_i) \mid \sigma_o\,)\,\hat{\theta}\,(\,\chi(x_i), x_i\,)\,\Big] - c(\,x_i \mid \sigma_o\,) = 0,$

(a15) $\quad \Psi(\,x_i, N \mid \sigma_o, \sigma_i\,) \equiv D(\,Nx_i \mid \sigma_o\,)\,x_i - C(\,x_i \mid \sigma_i\,) - \kappa = 0.$

Taking derivatives, one obtains:

(a16) $$\begin{pmatrix} \phi_x & 0 \\ \psi_x & \psi_N \end{pmatrix} \begin{pmatrix} \tilde{x}_i \\ \tilde{N} \end{pmatrix} = \begin{pmatrix} \phi_o & \phi_i \\ \psi_o & \psi_i \end{pmatrix} \begin{pmatrix} \tilde{\sigma}_o \\ \tilde{\sigma}_i \end{pmatrix},$$

where $\phi_x \equiv (\partial\Phi(\cdot)/\partial x_i)x_i$, $\phi_o \equiv -(\partial\Phi(\cdot)/\partial\sigma_o)\sigma_o$, $\phi_i \equiv -(\partial\Phi(\cdot)/\partial\sigma_i)\sigma_i$, $\psi_x \equiv (\partial\Psi(\cdot)/\partial x_i)x_i$, $\psi_N \equiv (\partial\Psi(\cdot)/\partial N)N$, $\psi_o \equiv -(\partial\Psi(\cdot)/\partial\sigma_o)\sigma_o$, and $\psi_i \equiv -(\partial\Psi(\cdot)/\partial\sigma_i)\sigma_i$. Except for two of these definitions expansions are unnecessary. The two in question are related through the condition $\psi_x \equiv (1-\hat{\theta})\, p\varepsilon x_i \equiv (1-\hat{\theta})\, \psi_N$, which I obtain from evaluating ψ_x and ψ_N in the initial equilibrium. I now state and prove:

LEMMA: In symmetric, free-entry equilibria with conjectural variation $\chi(x_i)$ no consistent conjecture exists.

PROOF: I first prove that no admissible conjecture exists. Since admissibility is necessary for consistency there exists no consistent conjecture. The aggregation condition under free entry is $\tilde{x} = \tilde{N} + \tilde{x}_i$. Substituting for $\tilde{x}$ admissibility requires: $\hat{\theta} = \tilde{x} / \tilde{x}_i = 1 + \tilde{N} / \tilde{x}_i$. Solving (a16) for $\tilde{x}_i$ and $\tilde{N}$, invoking $\psi_x = (1-\hat{\theta})\, \psi_N$, first setting $\tilde{\sigma}_o = 0$ and, subsequently, $\tilde{\sigma}_i = 0$, compute $\tilde{N} / \tilde{x}_i = \hat{\theta} - 1 + (\phi_x/\psi_N)\times(\psi_o/\phi_o)$ when demand shifts and $\tilde{N} / \tilde{x}_i = \hat{\theta} - 1 + (\phi_x/\psi_N)\times(\psi_i/\phi_i)$ when costs shift. Substituting for $\tilde{N} / \tilde{x}_i$ above, admissibility is incompatible with the computed ratios unless both $(\phi_x/\psi_N)\times(\psi_o/\phi_o)$ and $(\phi_x/\psi_N)\times(\psi_i/\phi_i)$ are zero. The condition $\phi_x = 0$ is incompatible with local uniqueness of the first-order condition. The condition $\psi_N \equiv p\varepsilon x_i = -\infty$ is ruled out by assuming that price is endogenous and firms produce finite output levels. This implies that ϕ_x/ψ_N is strictly positive. The conditions $\psi_o = 0$ and $\psi_i = 0$ are ruled out by assuming that σ_o shifts demand and σ_i shifts costs. The conditions $\phi_o = \infty$ and $\phi_i = \infty$ are inadmissible in comparative statics. It follows that ψ_o/ϕ_o and ψ_i/ϕ_i are also strictly positive. Consequently there exists no admissible conjecture. Accordingly, there exists no consistent conjecture. Q.E.D

APPENDIX FOUR

EXPERIMENTS WITH FIVE- AND TEN-FIRM OLIGOPOLIES

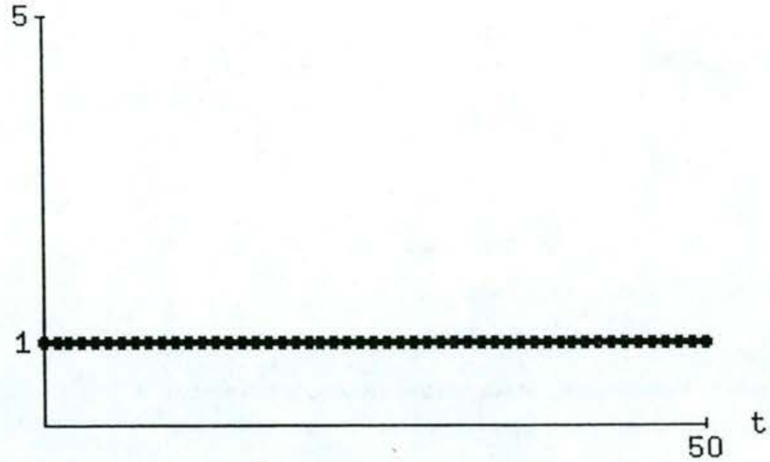

FIGURE A1a. — $\Gamma(t)$

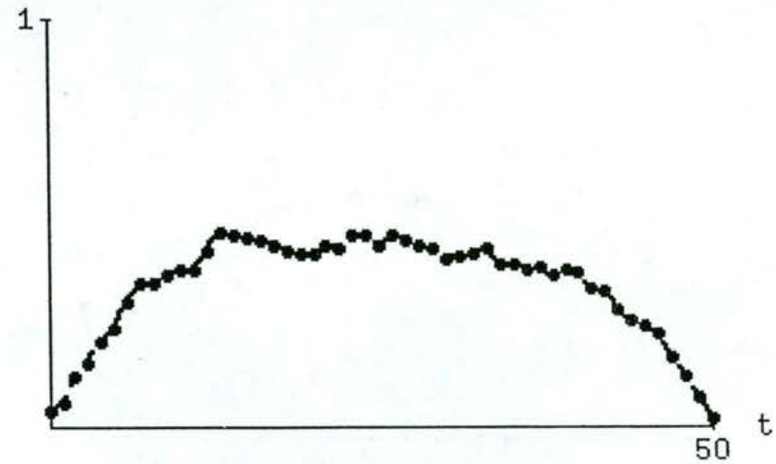

FIGURE A1b. — $f(t)$ and $g(t)$

FIGURE A1c. — $\tilde{\sigma}(t)$

FIGURE A1. — Concentration Under Bowley Beliefs { $\hat{\theta}_i(t) \doteq Nx_i(t)/x(t)$ i=1..N }
(N=5)

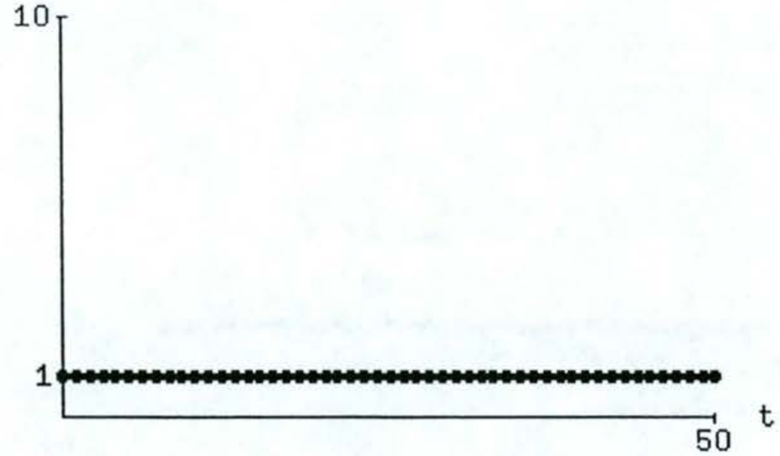

FIGURE A2a. — $\Gamma(t)$

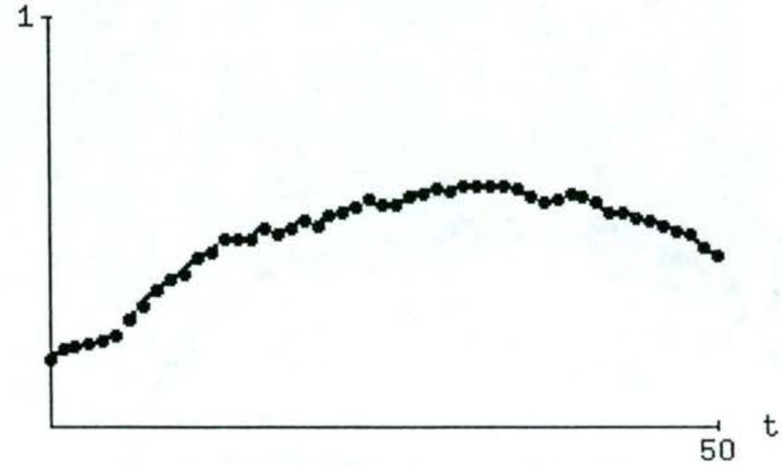

FIGURE A2b. — $f(t)$ and $g(t)$

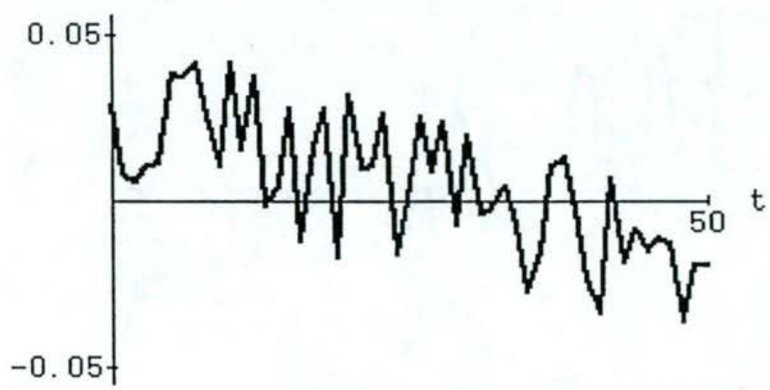

FIGURE A2c. — $\tilde{\sigma}(t)$

FIGURE A2. — Concentration Under Bowley Beliefs { $\hat{\theta}_i(t) = Nx_i(t)/x(t)$ i=1..N }
(N=10)

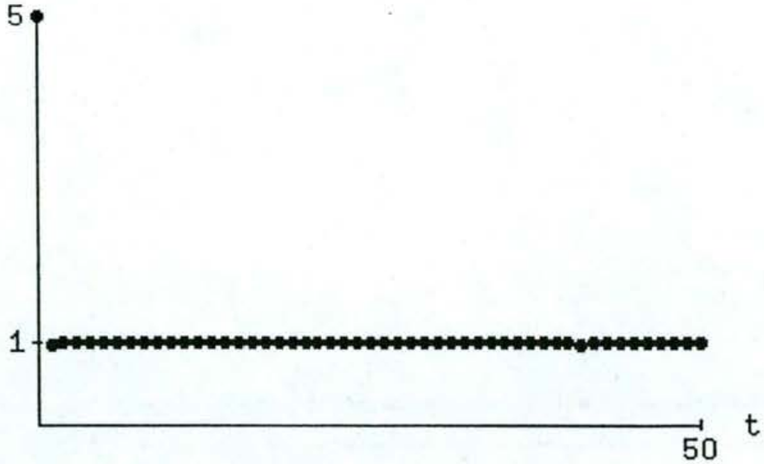

FIGURE A3a. — $\Gamma(t)$

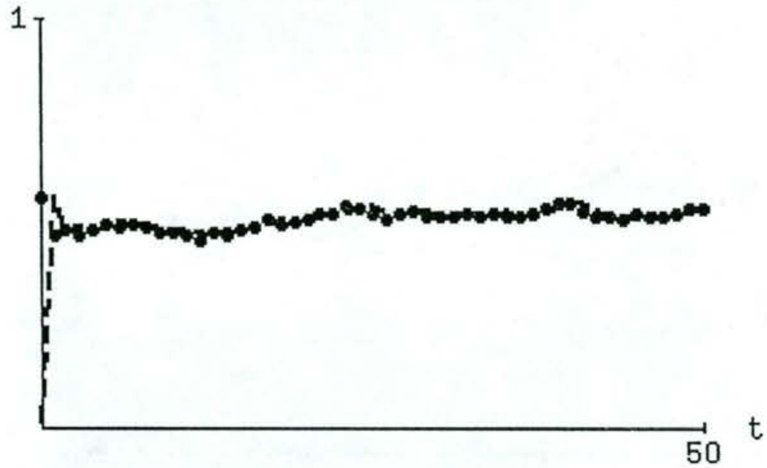

FIGURE A3b. — $f(t)$ • • and $g(t)$ – –

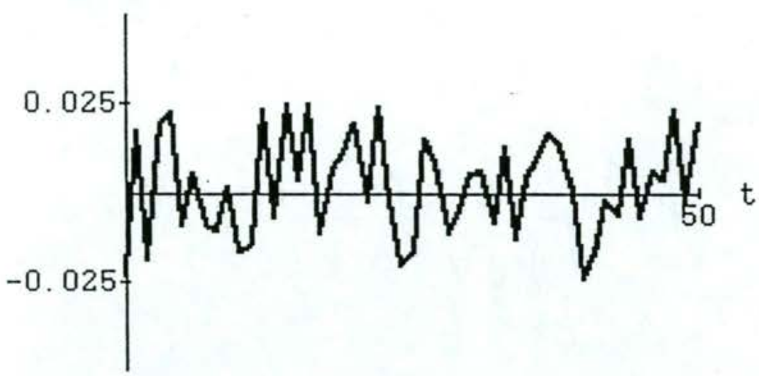

FIGURE A3c. — $\tilde{\sigma}(t)$

FIGURE A3. — Convergence Under Hindsight { $\hat{\theta}_i(t) = \theta_i(t\text{-}1)$ i=1..N }
(N=5)

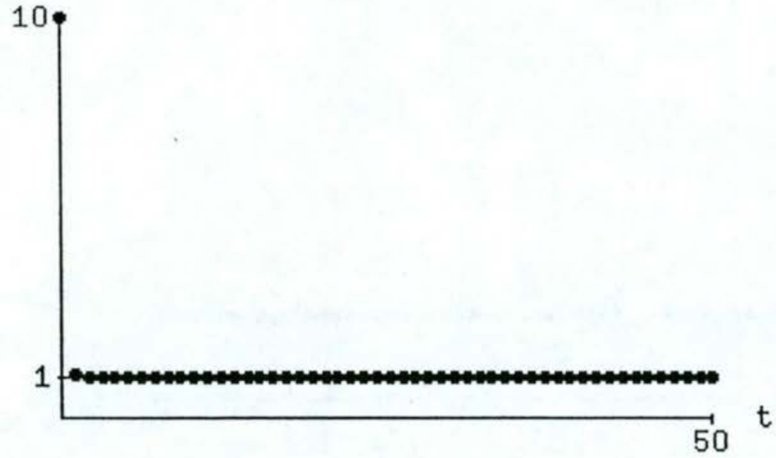

FIGURE A4a. — $\Gamma(t)$

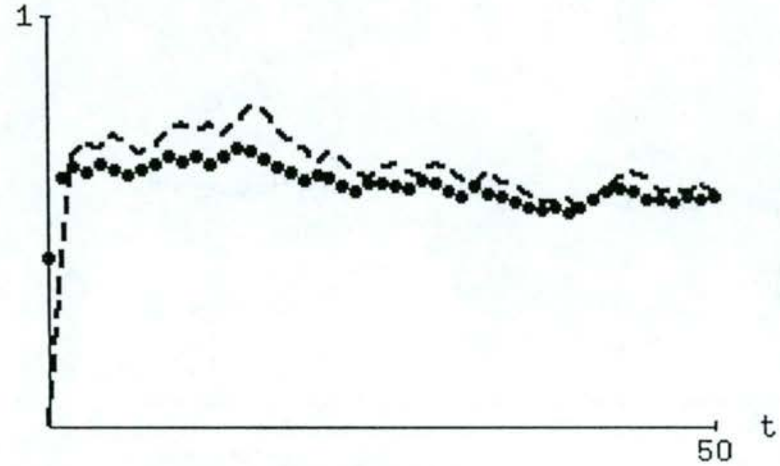

FIGURE A4b. — $f(t)$ • • and $g(t)$ – –

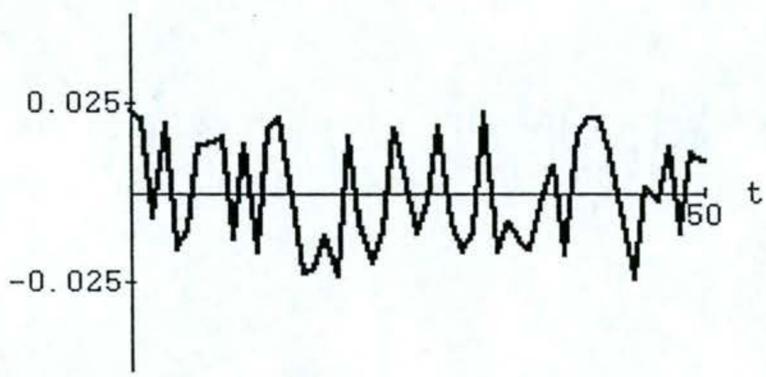

FIGURE A4c. — $\tilde{\sigma}(t)$

FIGURE A4. — Convergence Under Hindsight { $\hat{\theta}_i(t) = \theta_i(t\text{-}1)$ i=1..N }
(N=10)

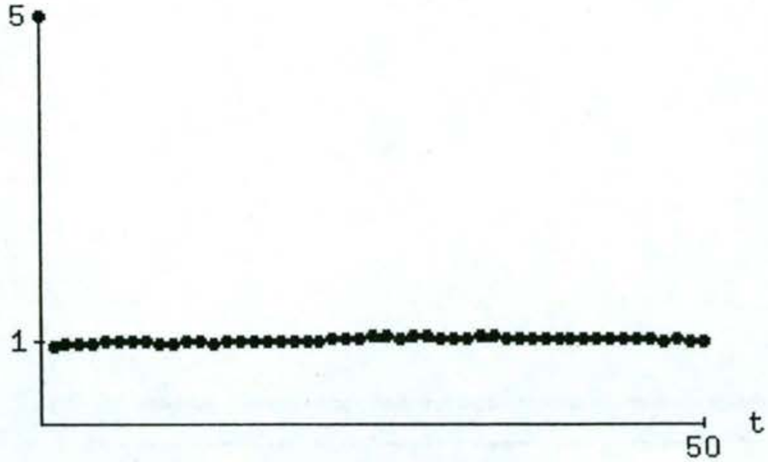

FIGURE A5a. — $\Gamma(t)$

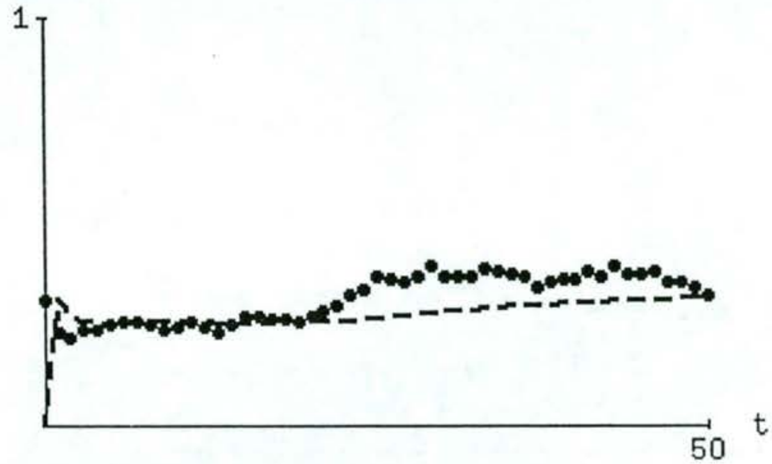

FIGURE A5b. — $f(t)$ • • and $g(t)$ – –

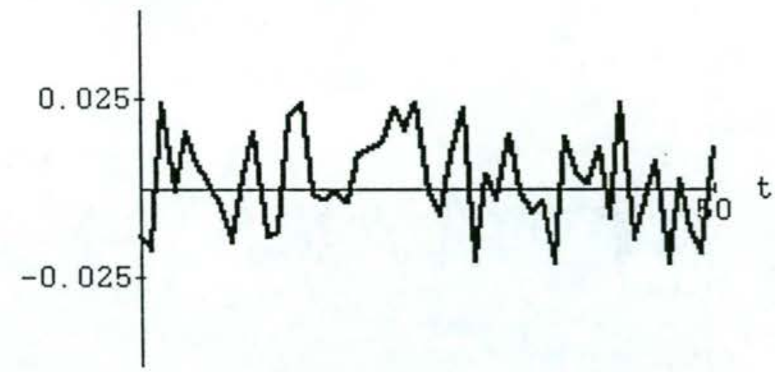

FIGURE A5c. — $\tilde{\sigma}(t)$

FIGURE A5. — Convergence Under Means of Past Histories { $\hat{\theta}_i(t) = \frac{1}{t} \sum_{s=0}^{t-1} \theta_i(s) \quad i=1..N$ }

(N=5)

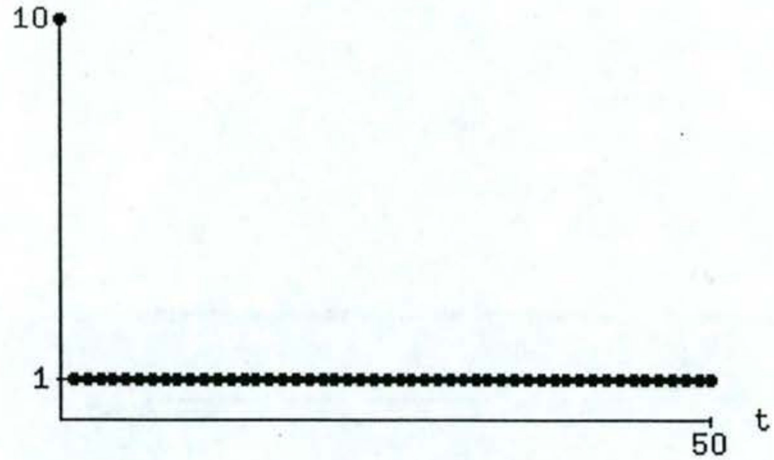

FIGURE A6a. — $\Gamma(t)$

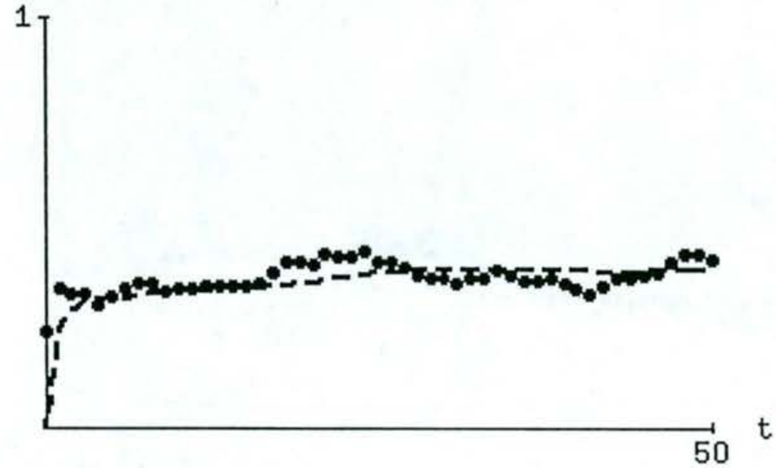

FIGURE A6b. — $f(t)$ • • and $g(t)$ – –

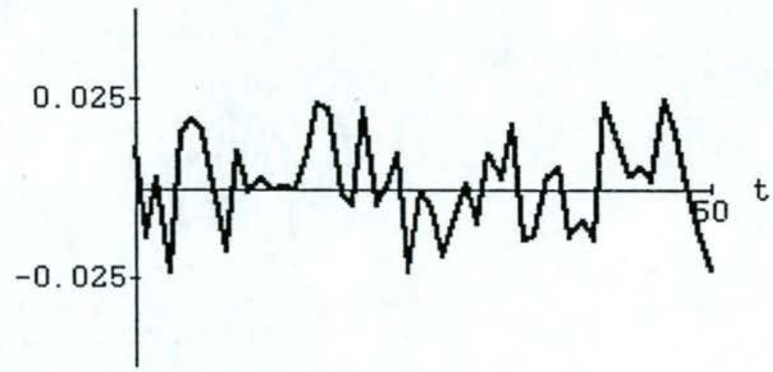

FIGURE A6c. — $\tilde{\sigma}(t)$

FIGURE A6. — Convergence Under Means of Past Histories { $\hat{\theta}_i(t) = \frac{1}{t} \sum_{s=0}^{t-1} \theta_i(s)$ i=1..N }

(N=10)

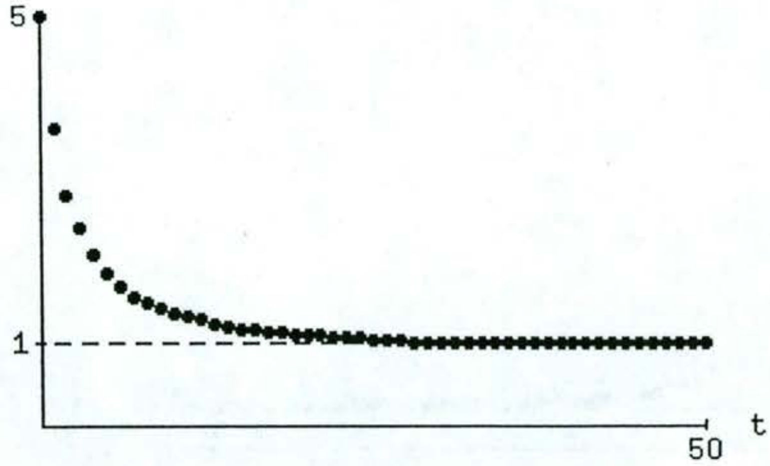

FIGURE A7a. — $\Gamma(t)$

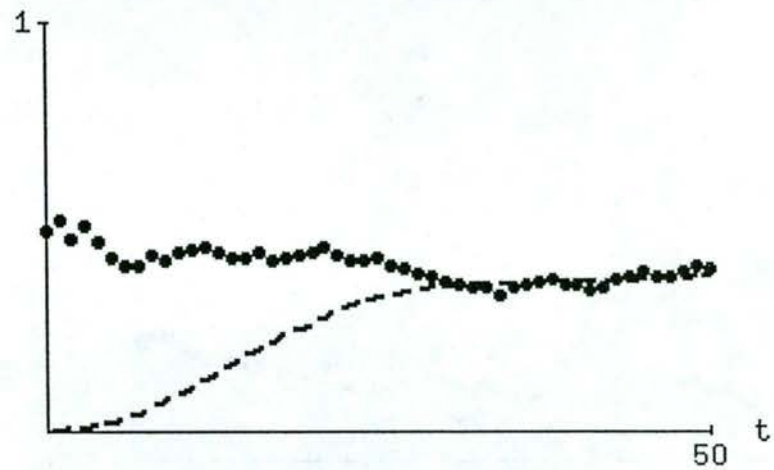

FIGURE A7b. — $f(t)$ • • and $g(t)$ – –

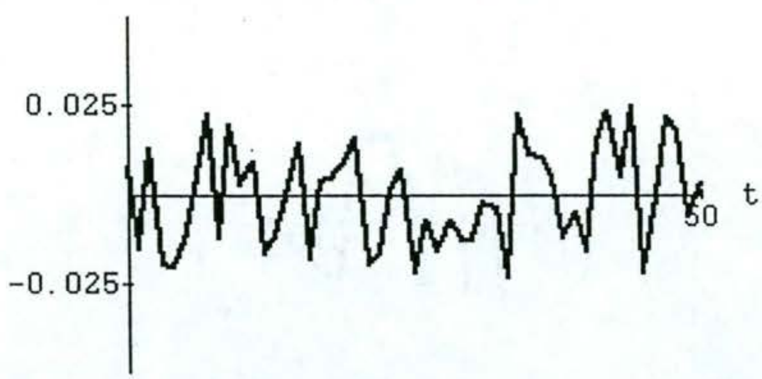

FIGURE A7c. — $\tilde{\sigma}(t)$

FIGURE A7. — Convergence Under Adaptive Behavior $\{\ \hat{\theta}_i(t) = \hat{\theta}_i(t\text{-}1) + \alpha \left(\theta_i(t\text{-}1) - \hat{\theta}_i(t\text{-}1) \right) \ i=1..N\ \}$
$(N=5,\ \alpha=0.1)$

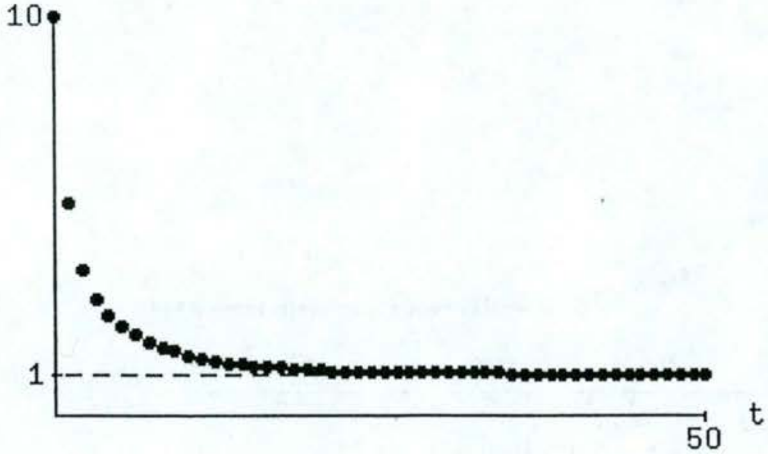

FIGURE A8a. — $\Gamma(t)$

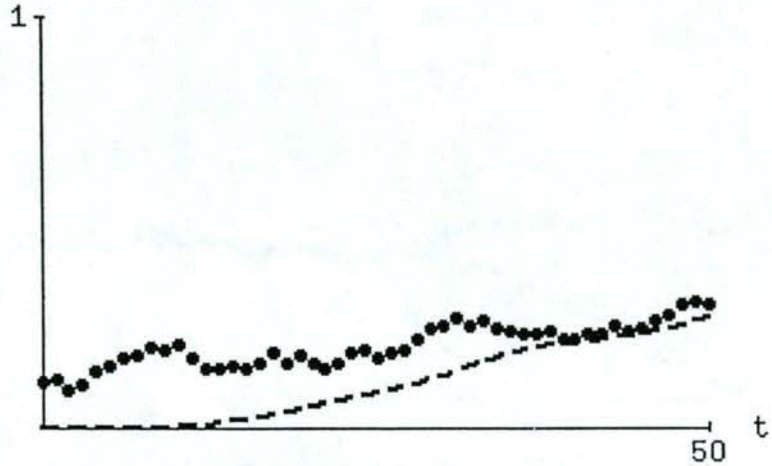

FIGURE A8b. — $f(t)$ • • and $g(t)$ – –

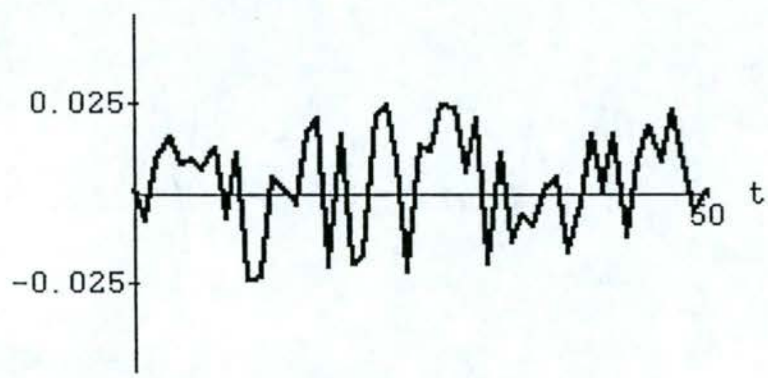

FIGURE A8c. — $\tilde{\sigma}(t)$

FIGURE A8. — Convergence Under Adaptive Behavior { $\hat{\theta}_i(t) = \hat{\theta}_i(t-1) + \alpha\left(\theta_i(t-1) - \hat{\theta}_i(t-1)\right)$ i=1..N }
(N=10, α=0.1)

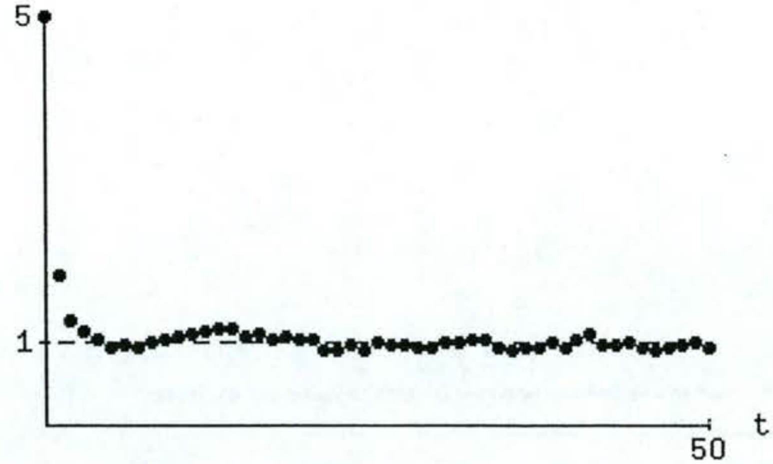

FIGURE A9a. — $\Gamma(t)$

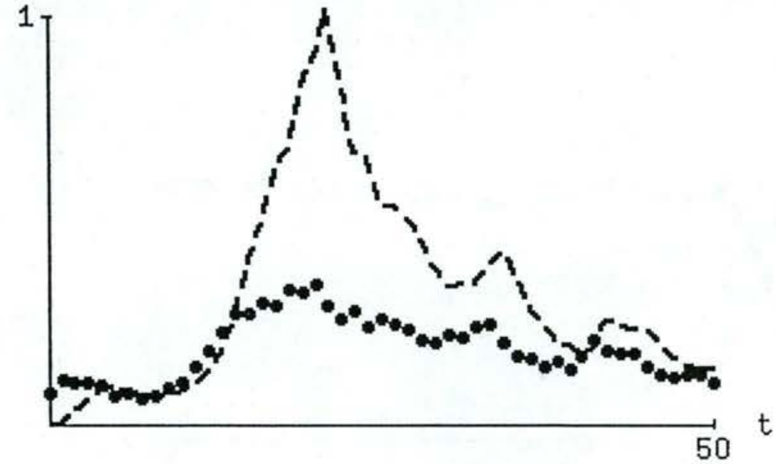

FIGURE A9b. — $f(t)$ • • and $g(t)$ – –

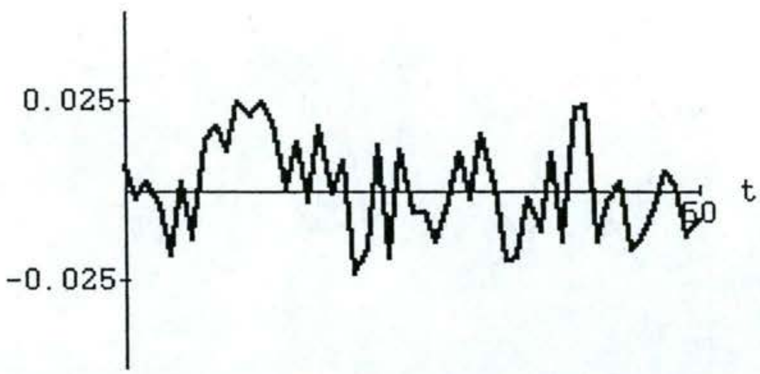

FIGURE A9c. — $\tilde{\sigma}(t)$

FIGURE A9. — Convergence Under Adaptive Behavior $\{ \hat{\theta}_i(t) = \hat{\theta}_i(t-1) + \alpha \left(\theta_i(t-1) - \hat{\theta}_i(t-1) \right) \; i=1..N \}$
$(N=5, \alpha=0.5)$

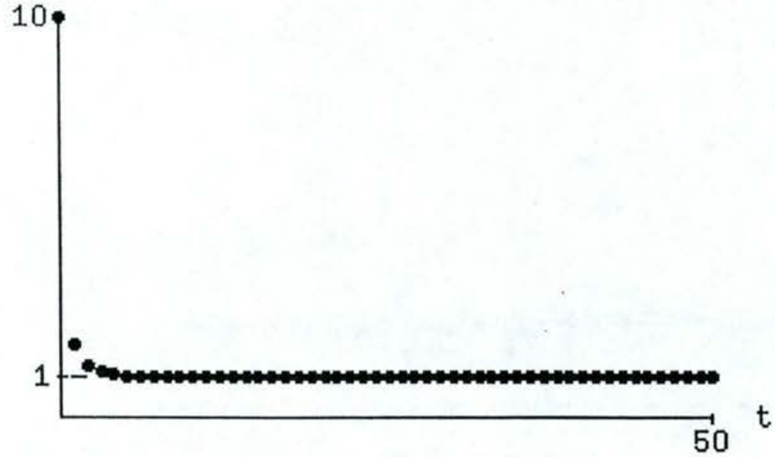

FIGURE A10a. — $\Gamma(t)$

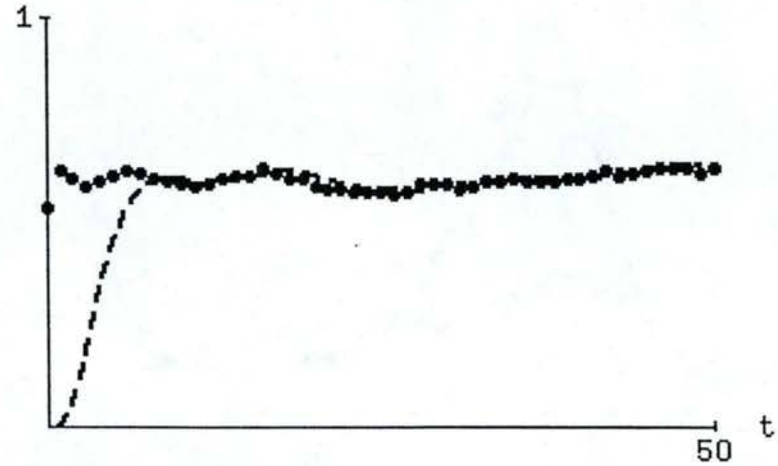

FIGURE A10b. — $f(t)$ • • and $g(t)$ – –

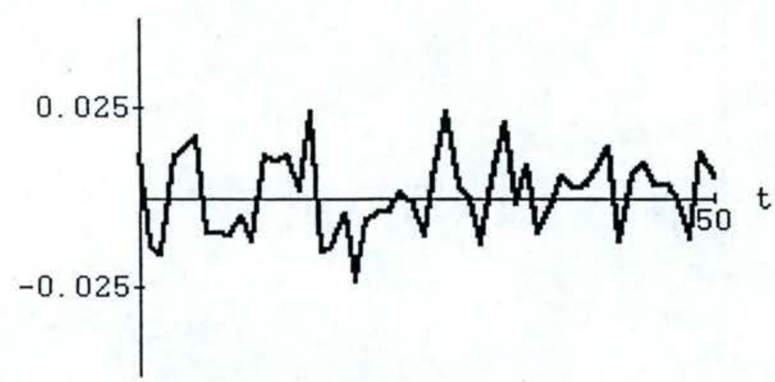

FIGURE A10c. — $\tilde{\sigma}(t)$

FIGURE A10. — Convergence Under Adaptive Behavior { $\hat{\theta}_i(t) = \hat{\theta}_i(t-1) + \alpha\left(\theta_i(t-1) - \hat{\theta}_i(t-1)\right)$ i=1..N }
(N=10, α=0.5)

76 p.

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Dept. of Agricultural Economics, U of W-Madison, December 2, 1994.
Dept. of Ag. and Res. Economics, NC State U., October 24, 1994.
Dept. of Ag. Economics, U. of New England, Armidale, February 17, 1994.

CONJECTURAL VARIATIONS WITH FEWER APOLOGIES

By Garth John Holloway