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CONSISTENT CONJECTURES IN SYMMETRIC EQUILIBRIA

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Abstract

This note articulates an important limitation in a popularly-invoked framework in industrial organization: the conjectural-variations model of oligopoly. Using a well-defined notion of "consistency," it is shown that under the conditions implicit in the majority of empirical applications of the model, pure cartel behavior is the only form of conduct that is consistent with the observed equilibrium. This general result is derived independently from specific assumptions about the forms of the relevant cost and demand functions. Some implications are derived for future empirical work.

Consistent Conjectures in Symmetric Equilibria

Introduction

This note articulates an important limitation in a popularly-invoked framework in industrial organization: the conjectural-variations model of oligopoly. The conjectural-variations model has provided impetus for a large number of empirical investigations of noncompetitive behavior.[1] Although it has been the subject of criticism (e.g., Dixit),[2] it continues to provide an attractive framework for depicting departures from price-taking behavior. In this context the model has undoubtedly been of pedagogic use, which is due primarily to its ability to contrast various conduct scenarios through particular values assigned to a single parameter. From a conceptual standpoint this feature is desirable, both in analytical work and in empirical investigations in which a point estimate of this parameter is usually the main objective of the exercise.

A topic of importance—one of shared significance to both conceptual and empirical practitioners—is the set of conditions under which the *actual* behaviors of economic agents in question are consistent with those behaviors *predicted* by the model. In empirical investigations this question is usually resolved through the application of various statistical criteria; in conceptual analyses these criteria are generally less clear.

One notion of rationality that has gained general acceptance in the context of conjectural variations models is the concept of *consistent conjectures*. A firm's conjecture is defined to be consistent when its *ex post* behavior—as determined by the comparative-static properties of the initial equilibria—is consistent with its *ex ante* beliefs—as determined by the value of its conjectural elasticity. This well-defined concept has led to an extensive literature (Laitner; Bresnahan, 1981; Boyer and Moreaux; Kamien and Schwartz; Daughety; Makowski) that has examined the conditions under which particular conjectures are deemed consistent. Unfortunately, this literature has overlooked a particularly restrictive implication of the model when it is applied within the framework in which most empirical investigations are conducted. This is a framework in which firms' quantity responses are *symmetric*; under these conditions the monopolistic conjecture is the only consistent conjecture. In view of the large number of empirical applications in which this restriction is implicit (e.g., Appelbaum, 1979; Sumner; Appelbaum, 1982; Lopez; Schroeter; Just and Chern; Sullivan; Hall) and at least two conceptual papers that overlook this restriction (Perry, Quirmbach) a detailed investigation of this issue appears warranted.

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[1]See, for example, the works cited in Bresnahan (1989).

[2]As Dixit (p. 107) notes, these criticisms pertain to the model's static environment, within which the inherently dynamic concepts of conjectures and reactions are nebulous.

In the remainder of this note I consider a standard example of the conjectural-variations model and, from it, derive a formal definition of a consistent conjecture. I then consider a symmetric equilibrium and derive its restrictive implications in terms of the range of values implied in order for the conjectures to be consistent. Some implications for empirical applications of the model are discussed.

Conjectural Variations: A General Model

Consider an industry producing aggregate output Q and facing price $p = D(Q,d)$, where $D(\bullet)$ denotes a quantity-dependent schedule of prices and d represents an exogenous variable that shifts demand. Let q_i, $i \in \{1,2...N\}$, denote firm-specific output levels and define the corresponding cost functions by $C_i(q_i,s_i)$, $i \in \{1,2...N\}$, where s_i, $i \in \{1,2...N\}$, denote N firm-specific exogenous effects. Additionally, assume that firms form conjectures, $\theta_i \equiv (\partial Q/\partial q_i)(q_i/Q)$, $i \in \{1,2...N\}$, about how aggregate, industry output responds to adjustments in own output levels. The parameters θ_i, $i \in \{1,2...N\}$, denote the *conjectural elasticities* of each of the firms. They are defined over the unit interval, with particular reference points, $\theta_i = 0$, corresponding to competitive bahavior; $\theta_i = 1$, corresponding to monopolistc bahavior; and $\theta_i = q_i/Q$, corresponding to Cournot behavior.

An oligopolistic equilibrium for the industry is defined by the N+2 equation system:

$$p \;=\; D(Q,d)\,, \tag{1}$$

$$Q \;=\; \sum_{i=1}^{N} q_i\,, \tag{2}$$

$$p(1 + \theta_i/\eta) \;-\; \partial C_i(q_i,s_i)/\partial q_i \;\equiv\; g_i(q_i,d,s_i) \;=\; 0, \qquad i = 1,2...N; \tag{3}$$

where η denotes the demand elasticity, $\partial C_i(q_i,s_i)/\partial q_i$, $i \in \{1,2...N\}$, define firm's marginal costs, and $g_i(\bullet)$, $i \in \{1,2...N\}$, denote implicit supply schedules. Allowing for displacements in each of the exogenous variables, we can express these, and the equilibrating adjustments in each of the endogenous variables, p, Q, q_i, $i \in \{1,2...N\}$, in proportional change terms (i.e., $\tilde{x} \equiv \Delta x/x$) as follows:

$$\tilde{p} \;=\; \eta^{-1}\,\tilde{Q} \;+\; \xi_d\,\tilde{d}\,, \tag{4}$$

$$\tilde{Q} \;=\; \sum_{i=1}^{N} (q_i/Q)\,\tilde{q}_i\,, \tag{5}$$

$$\xi_{di}\,\tilde{d} \;+\; \xi_{qi}\,\tilde{q}_i \;+\; \xi_{si}\,\tilde{s}_i \;=\; 0, \qquad i = 1,2...N; \tag{6}$$

where $\xi_d \equiv (\partial D(\bullet)/\partial d)(d/D(\bullet))$ denotes the elasticity of the price schedule with respect to the demand shifter; $\xi_{si} \equiv (\partial g_i(\bullet)/\partial s_i)(s_i/g_i(\bullet))$ denote elasticities with respect to firm-specific supply shifters; and $\xi_{qi} \equiv (\partial g_i(\bullet)/\partial q_i)(q_i/g_i(\bullet))$ denote elasticities with respect to firm-specific quantities. We note, in passing, that each of the latter elasticities will be functions of the conjectural-variations parameters.

The 2+N equations in (4)-(6) can be solved in order to determine the equilibrating movements in each of the endogenous variables. In general, each of these solutions will be a function of each of the exogenous variables in the system, and will be dependent on each of the parameters that appear in equations (4)-(6). Given the specific structures of the equations implicit in (6), the system can be solved recursively with solutions:

$$\tilde{q}_i = \tilde{q}_i(\tilde{d}, \tilde{s}_i \mid \theta_i), \qquad i = 1,2...N; \tag{7}$$

$$\tilde{Q} = \tilde{Q}(\tilde{d}, \tilde{s}_1, \tilde{s}_2 \ldots \tilde{s}_N \mid \theta_1, \theta_2 \ldots \theta_N), \tag{8}$$

$$\tilde{p} = \tilde{p}(\tilde{d}, \tilde{s}_1, \tilde{s}_2 \ldots \tilde{s}_N \mid \theta_1, \theta_2 \ldots \theta_N), \tag{9}$$

where the conditional dependence on each of the conjectural elasticities is acknowledged explicitly. Using this framework, we are now in a position to formally define a consistent conjecture.[3]

Definition: *A set of consistent-conjectures is a sequence of values* $\{\theta_1^*, \theta_2^* \ldots \theta_N^*\}$*, such that, in a neighbourhood of the initial equilibrium,*

$$\theta_i^* = \frac{\tilde{Q}(\tilde{d}, \tilde{s}_1, \tilde{s}_2 \ldots \tilde{s}_N \mid \theta_1^*, \theta_2^* \ldots \theta_N^*)}{\tilde{q}_i(\tilde{d}, \tilde{s}_i \mid \theta_i^*)}, \qquad \forall\, i = 1,2...N.$$

The values implicit in the left-hand side of this equality are the responses *predicted* by the firms; the values implicit on the right-hand side are the *actual* responses. Since the latter is represented by a ratio of comparative-static expressions it is determined by the values of particular variables in the initial equilibrium. Hence, *consistency* simply requires local equality between the predicted and actual responses of the individual firms in question.

[3]One should compare this definition with the variants proposed by the authors of the works cited in the previous section.

Symmetric Equilibria

Several authors have examined oligopoly models in which firms have equal market share; that is: $q_i = q_j$, $\forall$ i,j = 1,2...N. For most industries casual empiricism refutes the hypothesis of a symmetric equilibrium. Nevertheless, this assumption remains in use due, no doubt, to its convenient mathematical properties. When an industry is depicted in this manner, one must make explicit account of the potentially significant roles of the entry and exit decisions of firms. Neglect of this issue implies the following restriction on the equilibrium being considered:

> ***Proposition:*** *In the symmetric equilibrium in the absence of entry and exit, the consistent conjecture is the monopolistic conjecture:* $\theta^* = 1$.
>
> ***Proof:*** *Let* $q \equiv q_i = q_j$, $\forall\, i,j = 1,2...N$, *denote the output level of a representative firm. Replacing equation (2) with* $Q = N q$, *we derive:* $\tilde{Q} = \tilde{N} + \tilde{q}$. *Setting* $\tilde{N} = 0$, *it follows that* $\tilde{Q} = \tilde{q}$. *Substituting for* $\tilde{Q}$ *in the numerator of the expression defining consistent conjectures, it follows that* $\theta^* = 1$. *Q.E.D.*

Surprisingly, this simple result appears to have gone unnoticed in at least two key analyses of oligopoly in symmetric equilibria (Perry, Quirmbach). Fortunately, in wholly conceptual analyses the problem is easily circumvented by explicitly modelling N as an endogenous variable.[4] In empirical applications, remedies are less obvious, but it is clear that some account must be made of changes in firm numbers in order to derive consistent estimates of the conjectural elasticity. We consider this issue in more detail, below.

Endogenizing Entry and Exit

To consider the problem explicitly within the framework of most empirical studies, we invoke the specific case of constant marginal costs. Recent applications (e.g., Lopez; Schroeter) have popularized this assumption by postulating cost functions of the so-called *Gorman-Polar form* (Blackorby *et al.*). These allow for inter-firm differences in fixed costs, but restrict marginal costs to be constant and equal across firms. Appelbaum (1982, p.

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[4]We note—in Perry's defense—attempts to model these decisions in the specific case of free entry (Perry, pp. 203-05). Not surprisingly, this leads to some rather obvious conclusions (see Proposition 3, p. 204).

291) has justified this form by appealing to its frequency of use in most aggregate studies of production behavior.[5] To endogenize N assume that firms enter the industry until profits are exactly zero.[6] Assume also that upon entering the industry firms incur a fixed cost, κ. These assumptions combine to yield a symmetric version of equations (1)-(3) in which entry and exit are endogenous:

$$p \;=\; D(Q,d), \tag{10}$$

$$Q \;=\; Nq, \tag{11}$$

$$p(1+\theta/\eta) \;-\; C(s) \;\equiv\; g(q,d,s) \;=\; 0, \tag{12}$$

$$pq \;-\; C(s)q \;-\; \kappa \;=\; 0, \tag{13}$$

where C(•) denotes marginal costs.

These above equations implicitly define equilibrium values for the 4 endogenous variables in the system; namely Q, p, q, and N. As before, comparative statics can be performed and restrictions on the feasible range of values for the consistent conjectures can be determined given additional information about the specific forms of particular functions in the system. In general, the domain of the consistent conjecture is unrestricted over the entire unit interval. This can be seen from substituting $\tilde{N} + \tilde{q}$ for $\tilde{Q}$ in the numerator of the right-hand side of the expression defining the consistent conjecture. We note that this does not imply that *each* of the values along the unit interval are consistent; only those values that satisfy the explicit relation implied between $\tilde{N}$, $\tilde{q}$, $\tilde{Q}$, can be construed *consistent*.

By defining $\omega \equiv \kappa/pq$ as the share of fixed costs in the revenues of a representative firm, we note from equations (12) and (13), that the feasible range of conjectures can be restricted to a particular point; namely: $\theta = -\eta\omega$. This relationship between the conduct variable, θ, and the market-structure variables η and ω, indicates the conditions under which θ takes on its limiting values, zero and one. Specifically, the case of pure competition, $\theta = 0$, is achieved whenever demand is perfectly inelastic or the "entry fee" is zero. As the share of fixed costs increases the industry becomes less competitive until the limiting case of monopolistic behavior is attained. Thus, while the relationship $\theta = -\eta\omega$ serves to restrict the feasible value of the conjectural elasticity, it also derives an explicit linkage between the conjectural-variations model and the precursor to modern industrial-organization theory: the

[5]As noted by Lopez (p. 222), the Gorman-Polar form satisfies the sufficient condition for the existence of an aggregate cost function, namely, that the individual cost functions are quasihomothetic.
[6]Here, of course, we ignore the integer problem.

structure-conduct-performance paradigm. In particular, market structure—as depicted by particular values for the demand parameter η and the technology parameter ω—and firm conduct—as depicted by values of θ—are seen to be determined simultaneously. Once obtained, particular values of θ can then be used to derive explicit relationships between this measure and common performance measures such as the *Lerner* index or the *Harberger* measure of deadweight loss.

Implications for Future Empirical Work

The derivation of consistent estimates of firm conduct remains one of the major challenges facing applied economists. The analytical results presented above contain several implications that are relevant to this objective.

First, there is a fairly clear indication that it is necessary to make some account of the numbers of firms in the industry when applying the Appelbaum approach to making inferences about conduct—especially when applying the Gorman-Polar cost function. It seems intuitive that firm numbers should be accounted for even when consistency considerations are not a major focus of the research effort. In cases where theoretical consistency is an overriding concern, it is imperative that firm numbers be modelled explicitly.

A second implication is that the latter comments apply equally well in situations where there may be possibilities for inter-firm differences in market shares: When constant marginal costs are assumed, the conjectural elasticity is the same for all firms. Since the numerator in the definition of the conjectural elasticity is common, the denominator must be also. Hence, assuming constant marginal costs—which is designed to dissipate complications arising from unequal divisions of output among the firms—leads to a form of "symmetry" in the output responses of individual firms. In this case, the definition of consistency suggests a nonparametric approach to estimating conduct from the application of time series data on proportional changes in firm numbers and movements in aggregate output.

A third implication of the results above is that consistent estimates of the conduct parameter may be obtained from an econometric model in which the structural equation depicting firm numbers is included in the estimation. However, since this method relies on the validity of data on levels of fixed costs, it remains to be seen whether the procedure can be easily implemented.

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