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RECONSIDERATIONS ON RISK DEDUCTIONS IN PUBLIC PROJECT APPRAISAL*

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In an article published several years ago (Anderson 1983), the field of risk accounting in public project appraisal was explored and some methods suggested for dealing with risk in a practical and low-cost manner. The main empirical or, to be more precise, pseudo-empirical result was based on a small Monte Carlo study of hypothetical economies and projects of diverse size and riskiness. The criterion function chosen [equation (7) in Anderson 1983] was based on expressing the proportional risk reduction in a rather narrow way. The experiment on which the interpretation was based was also rather confined in its scope, because the riskiness of the national economy was held at a fixed level. This means that the equations applied only to economies of the same relative level of riskiness as the Australia-like standardised one used (coefficient of variation of 0·01). This level of riskiness significantly understates that experienced by many smaller economies, which are not as diversified as the Australian economy.

In this note, the restrictiveness of a fixed level of riskiness of the economy is avoided and the results are reworked using a measure of proportional risk reduction that is more appropriate.

Defining Proportional Risk Deduction

In the 1983 work, the proportional risk deduction was defined as

$$(1)\quad P=\{E[Y+X]-CE[Y+X]\}/E[X]$$

where Y is a measure of national income and X is project return.

Certainty-equivalent income $CE[Y+X]$ was found by inverting the utility function evaluated at sampled mean utility. This measure of proportional risk reduction is perhaps not too bad when the background risk in the national economy is held at a constant level. Its deficiencies become more apparent as one considers quite different levels of overall riskiness in the economy as is required, for instance, in any cross-country comparisons.

What seems to be needed is a more direct measure of the incremental risk deriving from a specific project relative to the overall risk in the economy. This risk is captured quite overtly in a measure derived by Wilson (1982).[1] Wilson's static incremental risk charge (*IRC*) for the case of (a) a project and an economy where the returns are distributed

*My reconsiderations were stimulated by some perceptive comments by Avinash K. Dixit, Princeton University, on an earlier version of the Anderson (1983) results.

[1] In the earlier piece by Anderson (1983), the work of Wilson was categorised as 1977 from his Stanford Working Paper but this was subsequently published in a 1982 Proceedings Volume by Resources for the Future.

according to the bivariate normal distribution, and (b) constant absolute risk aversion is given by his equation (50), namely:

(2a) $IRC = D(Y+X) - D(Y) = (0\cdot5/r)(V[X] + 2\text{cov}[Y, X])$

where D denotes risk charge (or the absolute risk deduction) for the economy with or without the project, r is Wilson's measure of risk tolerance [the inverse of the coefficient of absolute risk aversion which is $-U''(W)/U'(W)$ where U is utility, W wealth and the primes denote derivatives], and V and cov are the variance and covariance operators. The base risk charge in the economy is given as the difference between mean and respective certainty equivalent as $D[Y] = E[Y] - CE[Y]$ and the with-project case is $D[Y+X] = E[Y+X] - CE[Y+X]$. Thus, the incremental risk charge, in terms of the underlying certainty equivalents, is

(2b) $IRC = (E[Y+X] - \text{CE}[Y+X]) - (E[Y] - CE[Y])$

(2c) $\qquad = E[X] - (CE[Y+X] - CE[Y])$

Equation (2a) can also be expressed in a form analogous to those reported by Anderson (1983) by dividing IRC by mean project return $E[X]$, identifying this ratio as P, and by re-interpreting the risk tolerance parameter. This parameter is fixed and constant for equation (2a) to hold strictly. It can, however, be regarded as locally fixed for given national income but as a globally diminishing function in the manner speculated by Arrow (1965) and Pratt (1964). Let $1/r = A/E[Y]$, where A is the coefficient of relative risk aversion. With some simplification, the proportional version of equation (2a) can then be rewritten as

(3) $P = Ac_x\{c_xR/2 + \rho c_y\}$

where $R = E[X]/E[Y]$, c denotes respective coefficients of variation and ρ is the simple correlation between X and Y.

A Further Monte Carlo Study of Proportional Risk Deductions

In the manner of Anderson (1983, p. 234), a further experiment was conducted with five factors in complete factorial combinations with $3\times3\times3\times5\times4 = 540$ treatments: $A = (0\cdot5, 1, 2)$, $R = E[X]/E[Y] = (0\cdot01, 0\cdot04, 0\cdot16)$, $c_x = S[X]/E[X] = (0\cdot1, 0\cdot4, 0\cdot8)$, $\rho = (-0\cdot9, -0\cdot5, 0, 0\cdot5, 1)$, and $c_y = S[Y]/E[Y] = (0\cdot01, 0\cdot05, 0\cdot1, 0\cdot2)$. Again, social risk aversion was incorporated through a constant relative risk aversion utility function: $U(Y+X) = [1/(1-A)](Y+X)^{1-A}$ or, if $A = 1$, $U(Y+X) = \log(Y+X)$, where A is the coefficient of relative risk aversion. National income was again arbitrarily scaled at $E[Y] = 1000$ and generality was achieved by specifying the experimental variables in essentially unit-free measures. Pseudorandom samples of 2000 pairs of bivariate normal variates of X and Y were used and the certainty equivalents of equation (2c) were found by inverting the utility function at sampled mean utilities.

Given the different definition of P, the results differed from those of Anderson (1983) in several ways. Most obviously different were the negative, albeit usually very small (of the order of $-0\cdot001$), values of proportional risk deductions. These occurred at high negative levels of correlation. That this is to be expected is apparent from equations (2a) and (3). In the words of Wilson (1982, p. 221): 'The advantage of a project that is negatively correlated with existing sources of income is evident ... It should be clear as well that the incremental risk charge levied against any one project depends upon the correlation between its benefits and those of other projects adopted. Thus a project cannot be evaluated [properly] in isolation ...' and, in similar vein (p. 249), 'Thus, social insurance takes two forms, one being risk sharing, and the other being the selection of a balanced collection of public projects.'

It seems natural to seek to describe the Monte Carlo results with regression summary equations. The form of equation (3) is intrinsically non-linear in the variables. As a first step, equation (3) was applied to all the experimental treatments and these predictions were then compared with the Monte Carlo-generated results. The squared correlation coefficient was 0·95, indicating that there is little variation in these data that can be explained by either omitted variables or alternative mathematical specifications. Non-linear least squares estimates of power transformations of all the variables in equation (3) proved unstable. An ordinary least squares estimate using two composite explanatory variables from equation (3) yielded:

$$\text{(4)} \quad \hat{P} = \underset{(0\cdot001)}{0\cdot008} + \underset{(0\cdot03)}{0\cdot92}(Ac_x^2R/2) + \underset{(0\cdot01)}{1\cdot08}Ac_xc_y$$

$$R^2 = 0\cdot95$$

which does not differ significantly from equation (3) in both form and explanatory power.

It is relevant to consider how well the earlier reported analogous summary predictor [Anderson 1983, equation (8), p. 236] predicts these new data. Since that equation does not include c_y as a variable, and the experiment on which it was based set $c_y = 0\cdot01$, the one-quarter of the new data for which $c_y = 0\cdot01$ was used to compare corresponding predictions of P. The squared correlation coefficient in this case was 0·59 which, while considerably lower than the 0·95 value reported above, is surprisingly higher than the coefficient of determination (0·39) for its original (logarithmic) ordinary least squares estimation.

In summary, if national income and project return are approximately bivariate normal and if social risk aversion is approximately constant in either the absolute sense [Wilson 1982 and equation (2) above] or the relative sense [the above Monte Carlo data and equation (4)], then equation (3) provides a simple, reliable and superior means of computing proportional risk deductions. If correlation between project return and national income is small, unimportant or zero, equation (3) collapses to the special 'large project' case of Little and Mirrlees (1974).

The next question to consider is how robust is equation (3) with respect to departures from normality. National income might generally be approximately normal consisting as it usually does of the summation of many sources of largely independent random variation. The same type of Central Limit Theorem reasoning does not, however, so obviously apply to the return from a project. Accordingly, some further Monte Carlo results are sought to answer this question.

Choice of a bivariate distribution for this purpose is not very straightforward. Certainly, there is no other family of bivariate distributions that can match the bivariate normal in parametric parsimony and analytical convenience. In particular, complete capturing of stochastic dependence in just one parameter, the correlation coefficient, is only possible in a straightforward way with the bivariate normal. Rather cumbersome methods using linked generalised beta distributions have been used by Anderson (1975) in analogous circumstances but were avoided here in favour of a more simple albeit partial alternative.

The best choice seems to be a procedure suggested by Kleijnen (1974) and used in contexts somewhat analogous to the present by Anderson (1976). It is possible to deal with some special cases, such as bivariate normal and lognormal processes (Meija, Rodriguez-Iturbe and Cordova 1974). The pragmatic procedure of Kleijnen (1974) does, however, seem adequate for the present purpose.

The approach adopted is of taking an extremely positively skewed distribution, the lognormal, as the marginal distribution of project return x. The parameters of this distribution are determined by the specifications of the treatments in the experimental design. The two-parameter lognormal distribution (Johnson and Kotz 1970, pp. 112–7) is used wherein $x = \log(X)$ is normally distributed with mean and standard deviation. The parameters for the standard deviation and mean of the lognormal distribution, respectively, are found from the design parameters by

(5) $\sigma = [\log(c_x^2 + 1)]^{0 \cdot 5}$

and

(6) $\mu = \log(E[X]) - 0 \cdot 5\sigma^2$

To avoid problems arising from very large values of X, such as correspondingly very small and possibly negative values of Y when correlation is large and negative, the lognormal distribution is truncated at $X = \exp(\mu + 3\sigma)$.

With X pseudorandomly sampled, an appropriately correlated variate of Y is sampled using the Kleijnen (1974) method whereby the mean and variance of Y are preserved, along with its correlation with X, although its marginal distribution is indeterminate:

(7) $Y = E[Y] + (\rho S[Y]/S[X])(X - E[X]) + (1 - \rho^2)^{0 \cdot 5} S[Y] N$

where N is an independent standard normal variate.

This partial investigation of robustness proved to be positive in indicating that, in spite of the very different specification of probability distributions, the ability of equation (3) to provide reliable estimates of P is still very high. The squared correlation coefficient between predicted and generated data is 0·915 which is not too much less than that for the ideal bivariate normal case. It is concluded from this comparison that analysts could feel confident in using the suggested equation (3) even when they are aware that some of the variables under consideration are distinctly non-normal in their probability characteristics. Doubtless it would be possible to find more extreme types of probability distribution which would make the prediction equation less reliable but it will probably be the case that return distributions are unimodal and not too extremely skewed and that national income will be approximately normal in distribution. All this means that the simple equation (3) can serve as a safe standard approximation.

Conclusion

This note is intended to refine a previously proposed procedure. In doing so, the importance of allowing for risk in public project appraisals has been restated, although it can be seen that the magnitudes of deductions required are typically (still) quite small.

The importance of accounting for an additional factor, namely the extent of variation in national income itself, has been identified. This source of variation plays a potentially important role when projects are strongly (either positively or negatively) correlated with national income.

An observation through casual empiricism is that, in practice, few project analysts undertake any explicit accounting for risk in their project assessments.[2] Such a situation seems untenable and, even if the adjustments that should be made are typically small, endeavours to account appropriately for risk in public project appraisal should be routine.

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[2] This does not suggest that such accounting is a trivial exercise, but it need not be too demanding using pragmatic procedures summarised by Anderson (1983, pp. 237–8) and elaborated by Anderson (1989).