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COMPARING HYPOTHESES ABOUT COMPETITION

by
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Abstract

Consider two possible scenarios for a particular industry: perfect competition in factor and product markets and oligopoly with competitive factor markets. Under perfect competition, factor and product prices are determined simultaneously; although less well-known, it is nonetheless intuitive that under the oligopoly scenario product prices are determined recursively. In this latter case, firms take the prices of factors as predetermined when making their decisions about quantities to supply to the markets in which they perceive an ability to influence price. Hence, the price-determination process exhibits a causal relationship that flows from the factor to the product markets. This paper formalizes these notions through a generalization of the variable profit function in order to derive a hitherto unrecognized, but most appealing premise for discerning whether a market is "competitive." The approach that we propose is conceptually appealing and empirically attractive due to its modest data requirements and its ability to circumvent problems that are typically encountered in empirical analyses of noncompetitive conduct. To illustrate the procedure we apply the model to investigate a contentious and controversial issue in the US food system -- the nature of competition in the food industries.

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I. Introduction

In contrast to the normative implications of competitive markets, a number of potential policy issues arise as the result of departures from perfect competition. It is therefore not surprising that so much applied economic research has focused on formulating models and devising procedures to better discern whether a market is competitive. Dubbed by Bresnahan "the new empirical industrial organization," the most recent contributions in this area are typified by several characteristics that distinguish these studies from those of their precedents.[1] These characteristics include precise statements of the optimizing behavior of the relevant firms in the industry and a clear and explicit articulation of the null and alternative hypotheses being considered. As Bresnahan (1989, p. 1012) notes:

> "Firm and industry conduct are viewed as unknown parameters to be estimated. The behavioral equations by which firms set price and quantity will be estimated, and parameters of those equations can be directly linked to analytical notions of firm and industry conduct ... As a result, the nature of inference of market power is made clear, since the set of alternative hypotheses is made explicit. The alternative hypothesis of no strategic interaction, typically a perfectly competitive hypothesis, is clearly articulated and is one of the alternatives among which the data can choose."

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1 Here we refer, generally, to inter-industry studies of the structure-conduct-performance paradigm which, as Peltzman points out (p. 208), " ... is the longest running show in empirical industrial organization." A comprehensive review of these studies is made by Schmalensee.

In addition to these considerations, Bresnahan alludes to an overriding concern of most of the studies within this category; namely, the applicability of the theoretical model to the available data and, hence, an acceptable set of maintained hypotheses:

> "Firms' price-cost margins are not taken to be observables; economic marginal cost (MC) cannot be directly or straightforwardly observed. The analyst infers MC from firm behavior, uses differences between closely related markets to trace the effects of changes in MC, or comes to a quantification of market power without measuring cost at all."

This paper contributes to these themes by presenting an alternative procedure for making inferences about competition in an industry. In particular, we consider a number of clearly defined states of the world, each of which is derived from the optimizing behavior of firms in the industry; we evaluate the likelihood that each one of these may have generated the data; and we implement the statistical procedure with due regard for the robustness of the empirical model. Apart from these consistencies, however, the method we propose differs from previous approaches in several, rather significant ways.

First, we ask several questions that are fundamental in nature, but which appear to have been overlooked thus far in the relevant literature. The first of these is: What are the features that characterize the indirect objective functions of firms in perfect competition and, therefore, distinguish these from those derived under alternative modes of conduct? Second, what are the observable distinctions available from the equilibrium models derived from these alternative specifications and what are the empirically refutable propositions that follow? Third, under what conditions can we examine these in practice? Successive responses to these questions leads to our procedure, with the following distinguishing features.

First, the hypothesis of perfect competition is postulated as one that is distinctly different from the alternatives. This results in a comparison between nonnested hypotheses and therefore differs from the more usual type of evaluation in which the null hypothesis of competition is nested among a variety of alternatives. Second, probabilistic indices of competition are derived in a direct and conceptually appealing manner as the product of the hypotheses comparisons. Moreover, our approach gives accountability for the potential losses incurred in accepting a false hypothesis. This contrasts with the usual, indirect procedure whereby an index of the degree of competition is first derived and the validity of a particular hypothesis is then evaluated from an econometric procedure in which this parameter is restricted to a specific value. Third, no parametric assumptions about the forms of the relevant demand or cost functions are necessary and the technique makes no use of any possibly unreliable cost data. Hence, minimal restrictions are imposed across the preferences of consumers and upon the technologies of the firms in question. Fourth, the data requirements are extremely modest, requiring only observations on movements in prices and exogenous variables that shift the demand and supply functions relevant to the industry in question. Since observations on industry output are not used, the procedure circumvents the usual problems encountered when applying aggregate, industry-level data to a firm-level model.

The approach makes use of a fundamental premise about the firm in perfect competition: it takes the prices of each of the commodities that it trades as given. Since this in general is not the case in an oligopolistic setting, this simple observation provides an appealing, yet hitherto unexploited basis for discerning between the purely competitive behavior of firms in an industry and its alternatives. The distinction between the different behavioral modes is formalized through a generalization of the variable profit function to the case where the firm perceives an ability to influence the prices of some of the commodities that it trades. In the markets in which firms behave competitively the price determination process exhibits a well-known simultaneity. Conversely, prices in the markets in which firms behave noncompetitively are predetermined by the former prices, and this fact is exploited in deriving an appropriate procedure for evaluating the validity of the alternative hypotheses.

We outline the test procedure following the development of the key ideas in the approach. We show how these concepts are derived from previously unrelated and distinct contributions in the literature, and we subsequently formalize them through the development of a simple model depicting equilibrium in the industry. An example of the use of the method is illustrated in an application to several of the major U.S. food industries, for which the hypothesis of noncompetitive behavior has long been a contentious issue. The paper concludes with a summary of the main points and by considering the model's potential for applications in other areas.

II. Deriving an Appropriate Inference Strategy

Previous empirical work in the context of Bresnahan's nomenclature has, arguably, received its greatest impetus from the now-familiar model of homogeneous-product, quantity-setting firms in a conjectural-variations oligopoly. While the conjectural-variations model has been the subject of some appropriate criticism (Dixit),[2] it is defensible on a number of grounds. These stem from the model's conceptual appeal and, in particular, from its ability to characterize a broad range of firm conduct through values ascribed to a single parameter. This is an attractive feature for comparative-static investigations in which a wide range of equilibrium outcomes are being considered (e.g., Quirmbach). It is also attractive in empirical applications in which the derivation of a point estimate of the

2 As Dixit notes (p. 107), these criticisms pertain to the model's static environment, within which the inherently dynamic concepts of conjectures and reactions are nebulous.

conjectural-variations parameter is usually the main objective of the exercise.[3] These applications are typically made within a static-oligopoly framework and use one of two distinct procedures for making inferences about competition.[4] We classify these procedures on the basis of the technique that is used as either *static* or *comparative-static*.

Static Procedures

Early papers that have used static procedures within the conjectural-variations framework have applied their models to firm-level data (e.g., Iwata; Gollop and Roberts; Roberts). In principle, this permits estimation of the "wedges" between price and the marginal costs of each of the firms in question, under rather general specifications of their technologies. However, these examples represent the exception rather than the rule, since data is typically available only at an aggregate, industry level. In this case, a problem arises in empirical applications of the theoretical model since the latter is typically derived from the first-order conditions of a particular firm, and the behavior of the latter will differ in general from that of the industry whenever firms are heterogeneous. Applications of the model under these more usual circumstances have led to a variety of responses to this problem, including disregarding it (Appelbaum, 1979), rationalizing that the potential bias induced through its neglect is negligible (Sumner), and imposing restrictions upon individual firms' cost functions that permit the model to be reinterpreted at an aggregate level (Appelbaum, 1982). The study by Sumner is especially noteworthy since it appears to be the first attempt to deal explicitly with the aggregation problem. This is achieved through a formal explication of the conditions under which one may comfortably ignore any potential

3 Some authors prefer to avoid "conjectural-variations" language and simply refer to the outcome in the industry as the result of some potentially more general, but unknown game. Since we require some precision in the ensuing discussion, we retain the former terminology throughout this section.

4 Although we restrict attention in this section to firms' output markets, the procedures being considered are equally applicable to firms' factor markets.

aggregation bias. As Sumner shows (p. 1012), this is possible whenever the variation in the elasticities of demand facing the individual firms is small, there is little variation in their marginal costs at their equilibrium output levels, or the covariance between the demand elasticities and marginal costs across the firms is negligible.

In more recent applications of the conjectural-variations model (e.g., Lopez, 1984; Schroeter) the equivalence in firms' marginal costs is imposed directly, rather than assumed. This is achieved by postulating cost functions of the so-called Gorman-Polar form (Blackorby *et al.*) and justifying it by appealing to its frequency of use in most aggregate studies of production behavior (Appelbaum, 1982, p. 291).[5] Albeit restrictive, this approach has the advantage of deriving a single parameter -- such as θ in figure 1 -- that provides an entire summary of the relevant information about conduct in the industry. An added advantage is that θ can then be related to indices of industry performance.[6] Despite this conceptual appeal, an aggregate, industry measure of θ is an artifact of its own making. That is, it arises as the result of restrictions implied in the empirical model from which its estimate is derived. Hence, it seems rather contentious to simply measure differences between price and marginal cost and attribute this difference solely to the noncompetitive behavior of firms in an industry.

(Insert figure 1 about here.)

Comparative-Static Procedures

In contrast to the static approach, the comparative-static approach used in more recent applications is potentially more robust. The fundamental notion that is exploited in

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5 As noted by Lopez (p. 222), the Gorman-Polar form satisifies the sufficient condition for the existence of an aggregate cost function; namely, that the individual cost functions are quasihomothetic.

6 These include the Lerner index, the Harberger measure of deadweight loss, or -- with further restrictions on the modes of conduct -- the Herfindahl index of market power.

these studies is that industries characterized by firms with different modes of conduct will respond differently to certain types of exogenous shocks. This point is illustrated in figures 2a and 2b. In these diagrams it is assumed that there are two exogenous changes that result in equilibrating adjustments in prices and quantities. These are, respectively, a reduction in marginal costs and a shift outwards in the demand schedule. In both cases, there is an inverse and monotonic relationship between the absolute magnitudes of change in prices and quantities and the value of the conjectural-variations parameter.[7] This suggests that inferences about "competition" may be made from observations on movements in prices, quantities, and exogenous variables that shift the input-supply and product-demand functions facing the industry.

(Insert figure 2 about here.)

An early example of the use of the comparative-static technique is provided by Just and Chern in an investigation of monopsonistic behavior in the tomato processing industry. This preceded independent investigations by Bresnahan (1982) and Lau (1982) of the conditions that are necessary and sufficient for identifying the oligopoly solution concept using industry price and output data. Two of the several intuitive observations that are made by Bresnahan -- subsequently formalized by Lau -- are especially important in the context of this study. First, in order to identify the equilibrium concept, the exogenous variables in the demand function must enter in such a way as to shift both the intercept and the slope of this function.[8] Second, when the latter is the case, the hypotheses of competition and monopoly are distinctively different. As Bresnahan (p. 92) notes:[9]

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7 A first-order approximation of this result holds independently of the forms assumed for the demand and marginal-cost functions.

8 Figure 2b provides an example of such a shift.

9 This intuition is also presented graphically in Bresnahan's figures 1 and 2 (pp. 90-91).

"Rotations of the demand curve around the equilibrium point will reveal the degree of market power. ... In general, such rotations will have no effect on the equilibrium if pricing is competitive, but will have an effect if there is market power."

The first of these comments has obvious implications for deriving point estimates of θ using comparative-static techniques. However, the second point is potentially more significant. An implication is that the hypothesis of competition should be considered one that is distinctively different -- possibly structurally -- from the broad spectrum of oligopoly outcomes.[10] This observation sheds further doubt on the general procedure of evaluating hypotheses about competition from the results of restricted estimations in which θ is constrained to equal zero. However, it suggests possible alternative strategies for the hypotheses comparisons when the perfectly competitive hypothesis is formulated as one that is structurally different from the alternatives.

An example of the methods circumscribed by the latter category is provided by Hall. His technique is particularly significant because it appears to be the first to approach the problem without making parametric assumptions about the relevant cost and demand functions. However, the paper does not address the issue of firm-level agggregation, which is an issue that is considered explicitly in an earlier study by Sullivan. This study is noteworthy because it represents one of the few approaches in which the empirical model and estimable equations are derived entirely from first principles. More importantly, however, it relaxes the assumption of equally-sized firms. In order to fully appreciate the significance of this feature, a brief review of some previous theoretical work is necessary.

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10 More formally, one possible inference that can be derived from this discussion is that the domain of θ should be restricted to an open interval; namely, $\theta \in (0,1]$.

Symmetric versus Nonsymmetric Equilibria

There are two notions pertaining to the *symmetry* of firms in an industry: symmetry in quantities and symmetry in beliefs. The first concept corresponds to firms whose output levels are of the same size and the second corresponds to firms for which the values of their conjectures are the same when evaluated at their equilibrium output levels. In general, neither is necessary nor sufficient for the other. However, they are intimately related.

The significance for the equilibrium outcome of heterogeneous beliefs among firms appears to have gone largely unnoticed. This is not surprising given the inherent difficulty of obtaining insightful results from nonsymmetric models (e.g., Dixit) and a consequent preoccupation with models of firms of equal size (e.g., Perry, Quirmbach). The importance of heterogeneity in beliefs is identified in an early paper by Kamien and Schwartz in which they show that the degree of departure from marginal cost pricing in an industry depends on the degree of heterogeneity in firms' conjectures. In particular, it is shown (equations (1)-(6), pp. 198-97) that industry output and price are, respectively, increasing and decreasing in the degree to which firms' conjectures differ. Another important finding made by Kamien and Schwartz is that the static equilibrium outcome in the industry tends toward the competitive one as the conjecture of *any single firm* approaches its smallest limit. In this case, the observable outcome in the industry is the same as if *each* firm behaved competitively with price set equal to marginal cost.

In considering the empirical relevance of these results, note must be made of an extensive literature (Laitner; Bresnahan, 1981; Boyer and Moreaux; Makowski; Perry; Kamien and Schwartz; Daughety) that examines the conditions under which firms' conjectures are "rational." In this context, conjectures are defined to be *rational* if the *ex ante* beliefs of the firms are consistent *ex post* with the comparative-static properties of the resultant equilibrium.[11] Applying this definition to the symmetric equilibrium in quantities, one can derive a fundamental result that appears to have gone largely unnoticed in the theoretical literature. This result is that the monopolisitic conjecture is the only consistent conjecture in the symmetric equilibrium (Holloway, 1991). This is quite intuitive because there is no reason to believe that the responses of individual firms would differ if they were otherwise "identical." Since the only basis upon which firms may differ -- namely, their respective sizes -- is ruled out by assumption they are, for all intents and purposes, identical. When there exists an opportunity to influence price and all firms recognize that they are identical, the only rational outcome is for the industry to operate as a perfect cartel. Combining this finding with the aforementioned results of Kamien and Schwartz suggests that it is imperative to allow for some form of heterogeneity among the firms in question. This, however, is typically not possible during static applications using aggregate data. In particular, heterogeneity is usually precluded by the necessity of equating constant marginal costs to perceived marginal revenues.

Deriving Inferences from Exogenous Changes

Since the static approach seems inadequate to model industries with heterogeneous firms, we pursue the *comparative*-static approach in the remainder of the paper. Reviewing figure 2 it may appear, at first, that this approach has little additional information to offer. For example, in response to an exogenous increase in demand one observes output expansions and nonnegative price movements in each of the three respective cases of monopoly, oligopoly, and competition. Although these adjustments are of different magnitudes, they follow the same general pattern. However, implicit in these responses is the distinction that competitive firms respond *indirectly* to the shift in the demand schedule

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11 Here, of course, we encounter the fundamental criticism about the implicitly static nature of the model.

by responding to movements in prices; since firms in the other two regimes perceive an ability to influence price, they respond *directly* to movements in the demand schedule.[12] The former case is easily verified from the supply functions of these firms -- they are defined over prices. Since these functions are implicitly derived from corresponding dual profit functions, this posits the question about the appropriate forms for these functions under the noncompetitive scenarios. In the monopolistic regime we observe a single firm adjusting output so as to maintain equivalence between its marginal revenues and costs. Since the firm has complete control over price -- albeit indirectly, through the allocation of output -- it responds to the change in marginal revenue that arises *directly* from the shift in the demand schedule. In the oligopoly scenario a similar pattern of adjustment follows, with the only modification being that the firm adjusts to changes in *perceived* marginal revenues. It follows that in the latter two cases the "supply schedules" of such firms should reflect the fact that their outputs are directly dependent on the exogenous variables that shift the demand function; hence, so too are their corresponding profit functions. In the next section we examine this distinction more formally. The objective is to derive empirically refutable propositions that permit comparisons to be made between the competing hypotheses of pure competition and its alternatives.

In summary, early attempts to identify market power using formal models of firm behavior have suffered from a limitation in the availability of firm-level data. The application of these models to industry price and output data has led to a variety of responses to the aggregation problem inherent in the approach. By far the most popular strategy has been to impose restrictions on the heterogeneity of firms in the industry in question. We have, however, argued that this appears to be a dubious practice since the equilibrium outcome can be shown to depend on the degree to which firms are heterogeneous. In reexamining the comparative-static approach, an intuitive distinction is

12 These results would, of course, be the same had the demand schedule been perfectly elastic.

drawn between the processes of adjustment under the competitive and the noncompetitive regimes. The intuitive conclusions derived from this distinction are pursued in a more rigorous manner in the section that follows.

III. Theoretical Model

The model presented in this section has its genesis in some observations made by Diewert (pp. 584-86), although the initial ideas are attributed to Lau (1974, pp. 193-94; 1978) and some earlier remarks by Hotelling (p. 609). Diewert, however, appears to be the first to acknowledge explicitly that the profit function of a monopolistic firm is defined over the prices that the firm takes as given and exogenous variables that enter the demand function corresponding to the commodity for which the firm has market power. Under standard assumptions -- convexity of the production technology and differentiability of the profit function -- Diewert then proceeds to obtain results that are analogous to those derived for the competitive firm. In particular, he derives input-demand functions and, what he terms the "deflated sales function," which corresponds implicitly to the firm's supply behavior in the market within which it behaves monopolistically.[13]

To extend Diewert's approach to the case of oligopoly or oligopsony, we begin from comments made about the generality of dual techniques for analyzing noncompetitive behavior (p. 588):

> "Of course, the above techniques can also be used in situations where the firm is not behaving monopolistically or monopsonistically in an exploitative sense, but merely faces prices for its outputs or inputs that

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13 In addition, with modest restrictions on the deflated sales function, its form can be completely recovered from its profit function. As Diewert notes (p. 585), if the deflated sales function is concave then the normalized profit function will be its *conjugate* function. If the deflated sales function is not concave, but a maximum exists over the relevant range of the arguments of the profit function, then the latter can be used to represent the relevant part of the former.

depend on the quantity sold or purchased for any number of reasons, including transactions costs or quantity discounts."

One such reason, we propose, is the presence of a conjecture that the firm uses to relate movements in its own quantities to those of the industry as a whole and, therefore, to prices, which are common to all firms. In what follows, we use this feature to distinguish between the dichotomous modes of firm behavior, and thereby classify markets as being either *competitive* or *noncompetitive*. Hence, we formalize the notion of a *competitive industry* as one in which the firms *take prices as given*. Such firms do not form conjectures in their markets; whether it would be advantageous to do so is an entirely different question that we do not consider here. We simply take as *datum* the absence of conjectures in these markets. We do assume, however, that when conjectures are formed this is done so in a rational manner in which the adjustments in quantities that are derived from each of the firms -- hence, in aggregate output -- are entirely *consistent* with firms' perceptions. However, since our objective is to derive inferences based on industry data, we need to consider the dichotomous nature of firm behavior within an industry setting.

Consider for the moment an oligopoly in which firms take the prices of factors as given when making their output decisions. It follows that the derived supplies of all of the firms should reflect the predeterminedness of factor prices. In an equilibrium setting in which we equate the aggregate of these supplies to demand, it follows that the prices of the products supplied are predetermined by factor prices. Hence, the price-determination process for such an industry -- if observable -- should reflect a unidirectional causality that flows from the factor markets to the product markets. It necessarily follows that the converse should hold if firms perceive product prices to be given and they form conjectures in their factor markets. That is, one should observe unidirectional causality flowing from the product markets to the factor markets. Extending this logic, it follows that this concept may be applied to any particular subset of the relevant markets for which the hypothesis of competition is in contention. The objective in the remainder of this section is to establish these propositions more formally.

Consider an industry that produces a vector of outputs from a vector of variable inputs and is comprised of firms that perceive an ability to influence price in a subset of their factor and product markets.[14] Partition the vector of aggregate quantities, $\mathbf{Q}$, into subvectors of commodities traded in the competitive markets and the noncompetitive markets, respectively, such that $\mathbf{Q}\equiv(\mathbf{Q_c},\mathbf{Q_n})$ denotes these quantities and $\mathbf{p}\equiv(\mathbf{p_c},\mathbf{p_n})$ denotes the corresponding prices.[15] In these expressions the subscript $c\in\{1,2,...C\}$ indexes the commodities traded in the competitive markets and the subscript $n\in\{1,2,...N\}$ indexes those in the noncompetitive markets. There are, thus, $C+N=M$ markets in total that are relevant to the industry. If we also index the firms in this industry by the subscript $i\in\{1,2,...I\}$, we can denote the quantity vectors of each of the firms by $\mathbf{q_i}\equiv(\mathbf{q_{ci}},\mathbf{q_{ni}})$, $i\in\{1,2,...I\}$. Let $Q_n=K_{ni}(q_{ni})$, $n\in\{1,2,...N\}$, denote the N equations that depict firm i's conjectures about how the aggregate commodities in the noncompetitive markets are affected by changes in the firm's own quantity levels. Hence, we assume that the right-hand-sides of these N equations correspond to univariate functions, each defined over a particular element of the vector of quantities that corresponds to the markets in which firms perceive an ability to influence price. For later purposes it will be convenient to refer to these as a group and, hence, we denote these N equations collectively in the "vector": $\mathbf{Q_n}=\mathbf{K_{ni}}(\mathbf{q_{ni}})$.

In these noncompetitive markets firms face N inverse supply and demand functions of the form: $\mathbf{p_n}=\mathbf{w_n}\mathbf{I_n}(\mathbf{Q_n})$. The elements in this subvector comprise the last N components of the vector of all the relevant functions $\mathbf{p}=\mathbf{w}\mathbf{I}(\mathbf{Q})$, in which the subvector $\mathbf{p_c}=\mathbf{w_c}\mathbf{I_c}(\mathbf{Q_c})$ contains the first C components. In these expressions $\mathbf{w}\equiv(\mathbf{w_c},\mathbf{w_n})$

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14 We assume, for simplicity, that the firm's technology is not restricted on either outputs or inputs, although the model could be extended to encompass either of these particular cases.

15 We follow the familiar convention of using bold letters to denote vectors and matrices.

denotes an M-dimensional vector of variables that represent the effects of "other factors" in the relevant demand and supply functions. These effects are taken to be exogenous to the industry in question and are, therefore, beyond the control of its member firms. As Diewert suggests (p. 585), when the relevant function pertains to the market for a consumer good, a typical element of **w** could represent disposable income; it could also represent the price of a close substitute, or a linearly homogeneous function of income and the prices of other goods consumed. In the case where the function is the derived demand or supply function of another industry, an element of **w** could represent a linearly homogeneous function of all of the other relevant prices facing that industry. The forms implicit in $\mathbf{p}=\mathbf{w}\mathbf{I}(\mathbf{Q})$ are the result of the linear homogeneity restriction on the indirect objective functions of the agents from which these expressions are derived.[16] When the inverse demand and supply functions follow more general specifications the following analysis is complicated considerably. We choose to employ the above formulation for the sake of clarity, our key results being independent of this assumption.

Within the above setting, the maximal profits attainable by firm i are given by:[17]

$$\pi_i(\mathbf{p}_c,\mathbf{w}_n) \equiv \max_{\mathbf{q}_i} \{ \mathbf{p} \bullet \mathbf{q}_i \mid \mathbf{p}_n=\mathbf{w}_n\mathbf{I}_n(\mathbf{Q}_n),\ \mathbf{Q}_n=\mathbf{K}_{ni}(\mathbf{q}_{ni}),\ \mathbf{q}_i\in \tau_i \}$$

$$\equiv \max_{(\mathbf{q}_{ci},\mathbf{q}_{ni})} \{ \mathbf{p}_c \bullet \mathbf{q}_{ci} + \mathbf{p}_n \bullet \mathbf{q}_{ni} \mid \mathbf{p}_n=\mathbf{w}_n\mathbf{I}_n(\mathbf{Q}_n),\ \mathbf{Q}_n=\mathbf{K}_{ni}(\mathbf{q}_{ni}),\ (\mathbf{q}_{ci},\mathbf{q}_{ni})\in \tau_i \},$$

$$\equiv \max_{(\mathbf{q}_{ci},\mathbf{q}_{ni})} \{ \mathbf{p}_c \bullet \mathbf{q}_{ci} + \mathbf{w}_n \bullet \mathbf{S}_{ni}(\mathbf{q}_{ni}) \mid (\mathbf{q}_{ci},\mathbf{q}_{ni})\in \tau_i \},$$

16 Implicit is the assumption that an appropriate aggregator function exists for the relevant prices.

17 In the following, we denote by negative real numbers the elements of $\mathbf{q}_i$ that correspond to factors of production.

where the symbol "•" is used to denote the inner-product of the adjacent vectors; $\mathbf{S}_{ni}(\bullet) \equiv \mathbf{I}_n(\mathbf{K}_{ni}(\mathbf{q}_{ni})\mathbf{q}_{ni}$ denotes the N-dimensional vector of functions derived from substituting $\mathbf{K}_{ni}(\bullet)$ and $\mathbf{w}_n\mathbf{I}_n(\bullet)$ for $\mathbf{Q}_n$ and $\mathbf{p}_n$, respectively; and τ_i denotes the technological possibilities available to firm i.

Before progressing, it is worth discussing some of the special features of this formulation of the firm's problem. First, no additional restrictions are being placed on the firm's technology other than those necessary to guarantee that a well defined solution to the maximization problem exists. Second, although firms are permitted to be heterogeneous, their individual profit functions are defined completely over variables that are common to all of them.[18] This allows us to aggregate the supplies and demands of the firms in question in a consistent and straight-forward manner. When it is possible to make inferences about noncompetitive behavior solely from these aggregate functions, an empirical model can be formulated in a theoretically robust manner. Finally, observe that the above problem reveals an inherent similarity to the profit maximization problem of the perfectly competitive firm (e.g., Diewert, pp. 133-41). This should not be surprising since the latter is, of course, a special case of the above formulation. Indeed, if a well-defined solution exists it can be shown that $\pi_i(\bullet)$ shares most of the properties that one usually associates with the profit function of the purely competitive firm, but in this case in the modified "price" vector $(\mathbf{p}_c,\mathbf{w}_n)$. We formalize two of these properties through the following proposition:

PROPOSITION: *If a solution to the above maximization problem exists then* $\pi_i(\bullet)$ *is linearly homogeneous and convex in* $(\mathbf{p}_c,\mathbf{w}_n)$.

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18 At this point a natural question to ask is why firms' technologies may differ when their profit functions are defined over these same variables. However, the objective of the study is to provide a method for circumventing a problem associated with heterogeneity among the firms, rather than deriving an explanantion for its existence. We therefore leave this question unanswered. That an equilibrium may exist with heterogeneous firms, without requiring there to be differences in firm-specific endowments of fixed factors, is a notion that is implicitly acknowledged in several previous empirical studies (Appelbaum, 1982; Sumner; Lopez).

PROOF: *See* ***Appendix***.

While these two properties will be useful in empirical applications of the model, current interest lies principally in the derivative properties of $\pi_i(\bullet)$.[19] Specifically, our interest lies in establishing a generalization of Hotelling's lemma.

PROPOSITION: *If a solution to the profit maximization problem exists and, in addition,* $\pi_i(\bullet)$ *is differentiable in* $(\boldsymbol{p}_c, \boldsymbol{w}_n)$ *then a modified version of Hotelling's lemma holds:*

$$\begin{aligned} \boldsymbol{q}_{ci} &= \nabla_{\boldsymbol{p}_c} \pi_i(\boldsymbol{p}_c, \boldsymbol{w}_n), \\ \boldsymbol{S}_{ni}(\boldsymbol{q}_{ni}) &= \nabla_{\boldsymbol{w}_n} \pi_i(\boldsymbol{p}_c, \boldsymbol{w}_n), \end{aligned}$$

where $\boldsymbol{S}_{ni}(\bullet) \equiv \boldsymbol{I}_n(\boldsymbol{K}_{ni}(\boldsymbol{q}_{ni}))\boldsymbol{q}_{ni}$ *denotes the vector of the firm's deflated revenue and expenditure functions in the noncompetitive product and factor markets.*

PROOF: *See* ***Appendix***.

The vector of functions $\mathbf{S}_{ni}(\bullet)$ extends Diewert's deflated sales function to the cases of oligopoly and oligopsony. An obvious question is how this extension is useful, empirically, in identifying whether a particular market is competitive. Since the vector of functions $\mathbf{S}_{ni}(\bullet)$ implicitly defines the firm's derived demand and supply behavior in the noncompetitive markets, it depicts quantity responses to exogenous changes that enable us to distinguish this behavior from that of perfect competition. Moreover, the vector $\mathbf{S}_{ni}(\bullet)$ is a construct that allows us to aggregate the individual responses to an industry level, thus

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19 Under certain conditions, it can also be derived that $\pi_i(\bullet)$ is monotonic and continuous in $(\mathbf{p}_c, \mathbf{w}_n)$.

facilitating these observations to be made from aggregate data. To illustrate this, we now examine the equilibrium in the industry.

Industry Equilibrium

For the sake of clarity, it will be useful to explicitly acknowledge the indices over which certain aggregations are performed. Hence, we momentarily revert to scalar notation and consider the partial equilibrium defined by three sets of equations. First, consider the $M=C+N$ inverse demand and supply functions in the C competitive and the N noncompetitive markets, respectively:

$$p_c = w_c I_c(Q_c), \qquad c=1,2...C;$$

$$p_n = w_n I_n(Q_n), \qquad n=1,2...N. \qquad (1)$$

With these we combine the M aggregation conditions that sum the supplies and demands over each of the firms:

$$Q_c = \sum_{i=1}^{I} q_{ci}\ , \qquad c=1,2...C;$$

$$Q_n = \sum_{i=1}^{I} q_{ni}\ , \qquad n=1,2...N; \qquad (2)$$

and close the model, by deriving the MI derived demands and supplies over which these aggregations occur:[20]

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20 We now denote all quantity variables by positive real numbers. Thus, negative signs are implicit in the functions on the right-hand-sides of equations (3) whenever the commodity in question is an input.

$$q_{ci} = \partial\pi_i(\mathbf{p_c},\mathbf{w_n})/\partial p_c, \qquad c=1,2...C;$$

$$I_n(K_{ni}(q_{ni}))q_{ni} = \partial\pi_i(\mathbf{p_c},\mathbf{w_n})/\partial w_n, \qquad n=1,2...N;$$

$$i=1,2...I. \qquad (3)$$

These (2+I)M equations implicitly define a partial equilibrium for the (2+I)M endogenous variables in the system; namely, the 2M components of the elements of **p** and **Q**, and the IM quantities corresponding to the derived demands and supplies of each of the respective firms. We will denote the latter, collectively, by the IM-dimensional vector **q**.

Comparative Statics

Since we wish to investigate the price determination process in the above equilibrium, it is insightful to allow for displacements in each of the exogenous variables. Expressing these and the equilibrating adjustments in each of the endogenous variables in proportional change terms (i.e., $\tilde{x} \equiv \Delta x/x$), we derive:

$$\tilde{p}_c = \tilde{w}_c + \xi_c \tilde{Q}_c, \qquad c=1,2...C;$$

$$\tilde{p}_n = \tilde{w}_n + \xi_n \tilde{Q}_n, \qquad n=1,2...N; \qquad (4)$$

$$\tilde{Q}_c = \sum_{i=1}^{I} \alpha_{ci}\, \tilde{q}_{ci}, \qquad c=1,2...C;$$

$$\tilde{Q}_n = \sum_{i=1}^{I} \alpha_{ni}\, \tilde{q}_{ni}, \qquad n=1,2...N; \qquad (5)$$

$$\tilde{q}_{ci} = \sum_{j=1}^{C} \eta_{cji}\, \tilde{p}_j + \sum_{j=1}^{N} \nu_{cji}\, \tilde{w}_j, \qquad c=1,2...C;$$

$$(1 + \xi_n \theta_{ni})\, \tilde{q}_{ni} = \sum_{j=1}^{C} \eta_{nji}\, \tilde{p}_j + \sum_{j=1}^{N} \nu_{nji}\, \tilde{w}_j, \qquad n=1,2...N;$$

$$i=1,2...I. \qquad (6)$$

In these equations ξ_c and ξ_n refer to price flexibilities; α_{ci} and α_{ni} denote firms' market shares; and the terms η_{cji}, η_{nji} and ν_{cji}, ν_{nji} represent elasticities that depict firms' supply and demand behavior with respect to prices in the competitive and the noncompetitive markets, respectively. These elasticities are analogous to the usual elasticities derived from competitive firms' variable profit functions, except for two distinctions. First, these relationships are defined over the prices of the products traded in the competitive markets and the vector of exogenous variables that shift the supplies and demands in the noncompetitive markets. This, of course, follows as a consequence of the particular specification of the profit functions, which is defined over these same variables. Second, the derived demand and supply relationships in the noncompetitive markets are conspicuous by the presence of the bracketed expression on the left-hand-sides of the second group of equations in (6). In these expressions, the term θ_{ni} denotes the conjectural-variation elasticity of firm i in the market for commodity n. We note that particular values ascribed to this parameter can be used to synthesize various noncompetitive scenarios for the firm in question, including the cases of pure monopoly, $\theta_{ni} = 1$, and Cournot behavior, $\theta_{ni} = \alpha_{ni}$. It can also be used to *approximate* competitive behavior by observing the outcome as θ_{ni} approaches zero from above. However, it cannot be used to reflect *price-taking* behavior since the latter is synonymous with the absence of a conjecture. This distinction is summarized by defining the domain of this parameter as: $\theta_{ni} \in (0,1]$.

Since the presence of a conjecture in a particular market leads to the substitution of the relevant shift variable in place of its corresponding price, this conduct necessarily admits supply behavior that differs from that which occurs when firms take prices as given. An exception occurs, however, when the industry faces a perfectly elastic price schedule for all of the commodities in question. In this latter case -- indeed, only in this case -- the presence or absence of a conjecture is a moot point in deriving observational distinctions about the behavior of firms. To see this, consider equations (4) in the case where each of the parameters ξ_c, $n \in \{1,2..C\}$, and ξ_n, $n \in \{1,2..N\}$, is set equal to zero. In this case, there is direct equality between the movements in the prices and the movements in their coresponding shift variables, the bracketed terms on the left-hand sides of (6) collapse to one, and we can replace the shift variables on the right-hand sides of these expressions with their corresponding prices. The derived demands and supplies of purely competitive firms are what remains. Despite the long tradition of research that assumes the existence of these circumstances,[21] perfectly elastic demands and supplies in all of the markets is somewhat unrealistic. Moreover, this case is all but ruled out by the very definition of an industry in which at least one specific good is produced and, hence, firms face a demand function for at least one commodity that is downward sloping. In this case, the presence or absence of a conjecture matters, and we therefore continue with the derivation of the method under this more usual set of circumstances.

21 We draw to the reader's attention the large number of applied duality studies of production behavior using aggregate data for which a maintained hypothesis in the econometric procedure is that prices are exogenous.

Alternative Structural Models

It will be instructive to express the equilibrating adjustments in all of the relevant prices solely in terms of the exogenous variables in the system. That is, in the general form:

$$\Phi \tilde{\mathbf{p}} = \Psi \tilde{\mathbf{w}}, \tag{7}$$

where $\tilde{\mathbf{p}}$ and $\tilde{\mathbf{w}}$ denote the M-dimensional vectors of movements in the price variables, and Φ and Ψ denote square, order-M matrices of coefficient terms. To reduce the system in (4)-(6) in this form, first note that the latter equations can be written in the following manner:

$$\begin{pmatrix} \tilde{\mathbf{p}}_c \\ \tilde{\mathbf{p}}_n \end{pmatrix} = \begin{pmatrix} \tilde{\mathbf{w}}_c \\ \tilde{\mathbf{w}}_n \end{pmatrix} + \begin{pmatrix} \xi_c & 0 \\ 0 & \xi_n \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{Q}}_c \\ \tilde{\mathbf{Q}}_n \end{pmatrix}, \tag{8}$$

$$\begin{pmatrix} \tilde{\mathbf{Q}}_c \\ \tilde{\mathbf{Q}}_n \end{pmatrix} = \begin{pmatrix} \alpha_c & 0 \\ 0 & \alpha_n \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{q}}_c \\ \tilde{\mathbf{q}}_n \end{pmatrix}, \tag{9}$$

$$\begin{pmatrix} \tilde{\mathbf{q}}_c \\ \tilde{\mathbf{q}}_n \end{pmatrix} = \begin{pmatrix} \eta_c & 0 \\ \eta_n & 0 \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{p}}_c \\ \tilde{\mathbf{p}}_n \end{pmatrix} + \begin{pmatrix} 0 & \nu_c \\ 0 & \nu_n \end{pmatrix} \begin{pmatrix} \tilde{\mathbf{w}}_c \\ \tilde{\mathbf{w}}_n \end{pmatrix}, \tag{10}$$

where $\tilde{\mathbf{q}}_c$ is a subvector of length CI, which corresponds to firm-level quantity adjustments in the competitive markets and $\tilde{\mathbf{q}}_n$ denotes the NI-dimensional vector of responses in the noncompetitive markets. The structure of the coefficient matrices can be ascertained from (4)-(6). In particular, ξ_c and ξ_n are diagonal matrices of flexibilities of orders C and N, respectively; α_c and α_n are matrices comprised of market share terms and are, respectively, of orders CxCI and NxNI; η_c and η_n are matrices of firm-level elasticities of orders CxC and CxN, respectively; and ν_c and ν_n are matrices of dimensions CxN and NxN, respectively. In addition, we use the term **0** to denote a null matrix of conformable dimensions.

To reduce the above system to the form given by (7), substitute equations (10) into equations (9) and, subsequently, the resulting expressions into equations (8). This yields:

$$\begin{pmatrix} \phi_c & \mathbf{0} \\ \phi_n & \mathbf{I}_n \end{pmatrix} \begin{pmatrix} \tilde{p}_c \\ \tilde{p}_n \end{pmatrix} = \begin{pmatrix} \mathbf{I}_c & \psi_c \\ \mathbf{0} & \psi_n \end{pmatrix} \begin{pmatrix} \tilde{w}_c \\ \tilde{w}_n \end{pmatrix}, \tag{11}$$

where ϕ_c, ϕ_n, ψ_c, and ψ_n are coefficient submatrices of orders CxC, NxC, CxN, and NxN, respectively;[22] $\mathbf{I}_c$ and $\mathbf{I}_n$ are identity matrices of orders C and N; and the **0** terms are again used to denote null matrices of appropriate dimension.

The above system represents the structural model depicting the price-determination process for the industry. The placement of the null matrices suggests an intriguing feature about the system: *It is block-recursive in the prices of the commodities traded in the markets in which firms behave noncompetitively.* This feature represents the fundamental distinction between competitive and noncompetitive markets: *prices in competitive markets determine those in noncompetitive markets.* Given our previous discussion, this finding should not be surprising; it is merely a restatement of the earlier intuition we offered about the observational distinctions between firms in perfect competition and the alternative. Indeed, it is the very simplicity and intuitively straight-forward nature of this result which

22 Their precise definitions are: $\phi_c \equiv \mathbf{I}_c - \xi_c \alpha_c \eta_c$, $\phi_n \equiv -\xi_n \alpha_n \eta_n$, $\psi_c \equiv \xi_c \alpha_c \nu_c$, and $\phi_n \equiv \mathbf{I}_n + \xi_n \alpha_n \nu_n$.

makes it so appealing. Moreover, it is based on a very modest set of assumptions about the structure of the industry in question and on the technological possibilities available to the firms.

Equations (11) lend themselves readily to empirical analysis. Furthermore, no knowledge is required about the specific forms of the relevant cost or revenue functions, nor about the specificities of the price schedules facing the firms in the industry. The only requisite knowledge concerns the appropriate dimension of the null and identity matrices that appear on either sides of the above system of equations. Since this cannot always be ascertained *a priori*, this necessarily forms a basis for making inferences about competition. In particular, the statistical evaluation of the appropriate dimension of these submatrices provides a basis for comparing the alternative hypotheses.

By way of example, reconsider the case where firms operate in an oligopolistic environment but take the prices of their factors as given. The vector $\tilde{\mathbf{p}}_c$ would then correspond to movements in these latter prices. Hence, the vector of adjustments in the product prices, $\tilde{\mathbf{p}}_n$, would then be predetermined by the former and thus *factor-price movements cause product-price movements*. Conversely, in an oligopsonistic setting whereby firms are price takers in their product markets but form conjectures in their factor markets, the vector $\tilde{\mathbf{p}}_n$ would then correspond to movements in the factor prices and $\tilde{\mathbf{p}}_c$ would then represent the corresponding price movements in the product markets. In this case *product-price movements cause factor-price movements*.

Next, consider the polar case of pure price-taking behavior. By extending the logic used above, the vector $\tilde{\mathbf{p}}_c$ would then assume the length M. Consequently, the submatrices $\boldsymbol{\phi}_c$ and $\mathbf{I}_c$ expand to completely dominate the coefficient matrices on either sides of equations (11).[23] In terms of the formulation in (7) we therefore have:

23 This fact can be deduced from a reformulation of the model presented in equations (1)-(3).

$$\phi_m \tilde{\mathbf{p}} = \mathbf{I}_m \tilde{\mathbf{w}}, \tag{12}$$

where ϕ_m is an MxM matrix of coefficient terms and $\mathbf{I}_m$ is the order-M identity matrix. Hence, we obtain the familiar result that under perfect competition all prices are determined simultaneously. At the other extreme, consider the case in which firms form conjectures in all of their markets. Under this scenario the vector $\tilde{\mathbf{p}}_n$ is of length M, and the submatrices $\mathbf{I}_n$ and ψ_n completely dominate the system. In terms of equations (7), we obtain:

$$\mathbf{I}_m \tilde{\mathbf{p}} = \Psi_m \tilde{\mathbf{w}}, \tag{13}$$

where Ψ_m is an MxM matrix of coefficient terms and $\mathbf{I}_m$ is the corresponding identity matrix. In this case, all prices are predetermined by the movements in the demand- and supply-shift variables; there is no simultaneity and, hence, *both the structural and reduced form models are equivalent.*

These cases presented above are but two of many possible scenarios for the industry.[24] Despite their pedagogic significance, they are likely of little interest empirically. In most applications of the model, interest lies in comparing hypotheses about competition in some subset of the relevant markets. However, the polar cases given in (12) and (13) serve to reveal an important feature about the hypotheses to be compared: these hypotheses are nonnested. That is, none of the competing models can be obtained from another through the imposition of parametric restrictions.

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24 Specifically, when there are M markets relevant to the industry in question, the total number of possible scenarios is 2^M.

IV. Statistical Procedure

Judge *et al.* note one unfortunate outcome of the classical approach to evaluating nonnested hypotheses: the possible rejection of all alternatives. Despite this limitation, several authors (Pesaran and Deaton; Davidson and MacKinnon) have made progress in extending the original approach due to Cox (1961, 1962) to consider competing systems of nonlinear equations. A survey of some of the more recent developments in this area is provided by White.[25]

We consider the possibility of rejecting *all* alternatives to be a rather significant limitation of the classical approach. It may also become a computationally frustrating procedure when there are a large number of alternative hypotheses to evaluate and each hypothesis may require the estimation of a possibly nonlinear system of equations. In general, when there are H hypotheses to compare there are $H(H-1)/2$ comparisons which must be made, with no guarantee that the exercise will yield any additional information. In the context of the current application, this problem can clearly become quite serious. For example, when there are M markets for which the hypothesis of competition is in contention the total number of comparisons that must be made is $2^{M-1}(2^{M}-1)$. Hence, in the case of two markets, there are six comparisons that must be made; but when there are three markets, the number of comparisons totals twenty-eight.

25 Two approaches to testing causality relationships within the context of classical methods are the so-called Granger causality test (Granger and Newbold; Granger) and the Wu-Hausman test for endogeneity (Hausman; Wu, 1973, 1983). While the Granger approach suffers from a number of conceptual problems the Wu-Hausman procedure is not directly applicable to evaluating hypotheses in the nonnested environment.

Bayesian Methods

As an alternative to the classical approach, Bayesian methods have been advocated independently by Zellner and Leamer. In the Bayesian context one has available data **X**, a set of unknown parameters Θ, and a set of competing hypotheses that one wishes to compare H_j, $j \in \{1,2..J\}$. The task is to combine the sample and prior information in order to compute numerical values representing the likelihood that a particular hypothesis is the one that generated the data. These computed values are, of course, posterior probabilities which are defined over the unit interval. In the context of this paper, they may be interpreted as probabilistic indices of "competitiveness." That is, these values denote "degrees of belief" that firms with a particular mode of conduct generated the data. Hence, they may be viewed as Bayesian analogues to the usual point estimates of the conjectural variations elasticities, which are also defined over the unit interval and typically estimated within a classical framework. We find this analogy to be an appealing one which, coupled with the conceptual difficulties arising in the classical approach, suggest that the Bayesian procedure may be a preferable one to employ. We therefore pursue it in the remainder of the paper.

The evaluation of nonnested hypotheses provides no conceptual difficulties in the Bayesian environment (Leamer, pp. 90-91). Unfortunately however, there may be other problems that arise during implementation of this method. In particular, the evaluation of the posterior probabilities requires integrations to be performed over the entire domain of the parameter space. Depending on the specificity of each particular problem, this may be a difficult task and, in certain situations it may be intractable, at least analytically. This appears to be the case for the evaluation of hypotheses in the context of simultaneous-equations models, of which equations (11) are an example. Fortunately, procedures for applying Bayesian methods in the simultaneous-equations framework have been developed (Drèze and and Morales; Drèze and Richard; Bauwens) and we draw heavily on these works in implementing the statistical procedure. In the reminder of this section we document the key concepts of the method, referring the interested reader to additional detail offerred in the appendix and in the aforementioned papers.

The statistical implementation of equations (11) begins with an arrangement of T observations on the M equations in the system, and an assumption that the equality between the right- and left-hand sides of these equations holds with error. Making a distributional assumption about the process generating these errors,[26] we assess the likelihood of observing the parameters, given the data and under the null hypothesis $L(\Theta \mid \mathbf{X}, H_j)$. We combine this multiplicatively with a prior distribution that is conditional on the null hypothesis $f(\Theta \mid H_j)$ in order to derive a posterior distribution $f(\Theta \mid \mathbf{X}, H_j)$ which is conditional on this hypothesis and the given data, and is proportional to the likelihood and the prior:

$$f(\Theta \mid \mathbf{X}, H_j) \quad \propto \quad L(\Theta \mid \mathbf{X}, H_j)\ f(\Theta \mid H_j)\,. \tag{14}$$

To make statistical inferences with respect to this posterior density, we must perform integrations over certain intervals in the domain of Θ. As Bauwens notes, in the usual case where the error matrix is multivariate normal and the prior density function is chosen from the class of natural conjugate priors, analytical integration over the domains of all of the elements contained in Θ is not possible. Hence, the requisite integrations can only be performed approximately using numerical procedures. The technique advocated in the context of the simultaneous-equations model (Kloek and Van Dijk; Drèze and Richard) and in other contexts (Van Dijk and Kloek; Geweke) is the Monte Carlo procedure known

26 The usual assumption made here (Bauwens; Dréze and Richard) is that the errors are generated by a multivariate normal distribution. Indeed, we have been unable to find applications of the model under alternative distributional assumptions. Although the multivariate normal is commonly assumed in other situations, it is important to note that this rules out any form of serial correlation in the disturbances.

as *importance sampling*. We present below the key concepts in this procedure from Bauwens' exposition.

Numerical Integration

Denote the known part of the posterior density by:

$$k(\Theta \mid \mathbf{X},H_j) \quad \equiv \quad L(\Theta \mid \mathbf{X},H_j)\ f(\Theta \mid H_j)\,, \tag{15}$$

the unknown value of the integrating constant being given by:

$$\kappa \quad \equiv \quad \int_{\Theta} k(\Theta \mid \mathbf{X},H_j)\ d\Theta\ . \tag{16}$$

Suppose we wish to consider the likely value of some characteristic $g(\Theta)$; we would then wish to evaluate its posterior expectation:

$$E\{g(\Theta) \mid \mathbf{X},H_j\} \quad = \quad \frac{\int_{\Theta} g(\Theta)\ k(\Theta \mid \mathbf{X},H_j)\ d\Theta}{\int_{\Theta} k(\Theta \mid \mathbf{X},H_j)\ d\Theta}\,, \tag{17}$$

which obviously requires two integrations -- one representing the numerator and one corresponding to the denominator. Since the latter is a special case of the former, we focus on the numerator and note that this can be rewritten as the identity:

$$\int_{\Theta} g(\Theta)\ k(\Theta \mid \mathbf{X},H_j)\ d\Theta \quad \equiv \quad \int_{\Theta} g(\Theta)\ \frac{k(\Theta \mid \mathbf{X},H_j)}{f(\Theta)}\ f(\Theta)\ d\Theta, \tag{18}$$

where $f(\Theta)$ denotes any arbitrary function with a range noninclusive of zero.

When $f(\Theta)$ represents the known form of a probability density function, we can approximate the integral on the right-hand side of (18) by generating random samples from this density and applying the so-called *method-of-moments* estimator. That is, we compute

$$\frac{\sum_{i=1}^{N} g(\Theta_i) \frac{k(\Theta_i \mid \mathbf{X}, H_j)}{f(\Theta_i)}}{N} \approx \int_{\Theta} g(\Theta) \frac{k(\Theta \mid \mathbf{X}, H_j)}{f(\Theta)} f(\Theta)\, d\Theta\,, \qquad (19)$$

by sampling N replicates Θ_i, $i \in \{1,2...N\}$ from the probability density function $f(\Theta)$. Applying the same principle to the denominator on the right-hand side of equation (17), we approximate the expectation on the left-hand side of this equation by deriving the ratio of the sample estimates:

$$E\{g(\Theta) \mid \mathbf{X}, H_j\} \approx \frac{\sum_{i=1}^{N} g(\Theta_i) \frac{k(\Theta_i \mid \mathbf{X}, H_j)}{f(\Theta_i)}}{\sum_{i=1}^{N} \frac{k(\Theta_i \mid \mathbf{X}, H_j)}{f(\Theta_i)}}. \qquad (20)$$

In order to implement these computations empirically, the function $f(\Theta)$ -- known as the *importance* function -- must meet certain criteria. First, since a large number of replicates is desirable, this function must be chosen from the available class of densities wherefrom random samples may be efficiently generated. It must meet other criteria too, since an injudicious choice of $f(\Theta)$ may bias the results of posterior analyses. In general, it must provide as close an approximation as possible to the posterior density $f(\Theta \mid \mathbf{X}, H_j)$. However, this will often be difficult to achieve, since in most cases the form of the latter density will be intractable. Bauwens and others have experimented with a number of alternative specifications for these importance functions, including the multivariate *t* density, the product of independent *t* densities, and a variety of specifications in the so-called class of *poly-t* densities. Each of these appears to have performed satisfactorily in repeated experiments.

Estimate Precision

To evaluate the precision of the estimates, Bauwen's appeals to the central-limit theorem in order to derive an "upper bound" on the potential error committed in the estimation. Under rather mild conditions the above estimate is known to be asymptotically normally distributed with mean μ and variance σ^2/N. Hence, with level of "confidence" $1-\alpha$, the error committed with the estimator is no more than an upper bound given by:

$$\bar{\varepsilon} = (2/\sqrt{N})\, z_{\alpha/2}\, (\sigma/\mu)\,, \tag{21}$$

where $z_{\alpha/2}$ denotes the $1-\alpha/2$ fractile of the standard normal distribution and σ/μ denotes the coefficient of variation in the population. This definition can be used to compute approximate "confidence intervals" on the estimates derived from the Monte Carlo exercise, in which case σ and μ are replaced by their sample estimates.

This definition can also be used to consider the manner in which more precise estimates may be derived. Since $z_{\alpha/2}$ denotes a given constant, greater precision may be achieved in two ways; namely, by increasing the sample size, or by selecting the function $f(\Theta)$ in such a way that it minimizes the relative variance given by σ/μ. Following Kloek and Van Dijk, Drèze and Richard we note that this relative variance factor may be small in situations where the expression being evaluated corresponds to a ratio of integrals. This is certainly the case where an estimate of a posterior expectation is sought. It is also the case where one seeks to revise prior degrees of belief about hypotheses $p(H_j)$ through the computation of posterior probabilities:

$$p(\Theta \mid \mathbf{X}, H_j) = \frac{p(H_j) \int_{\Theta} k(\Theta \mid \mathbf{X}, H_j)\, d\Theta}{\sum_{j=1}^{J} p(H_j) \int_{\Theta} k(\Theta \mid \mathbf{X}, H_j)\, d\Theta}. \tag{22}$$

For the implied variance factors to be small a positive covariance is required between the estimates of the numerator and the denominator in this expression. In general, however, this will hinge on the particular application in question.

V. An Application

We choose to apply the above model and procedures to a subset of the U.S. food industries. We use the term "food industries" in a rather broad sense to include all of those industries involved in assembling and processing farm commodities and distributing food products to consumers. This choice of focus is a significant one for several reasons.

First, the economic transactions performed by firms in these industries -- being the principal purchasers of the outputs of the farm sector and the major domestic suppliers of food items -- makes them an important link in the food-marketing system. Hence, the conduct of firms in these industries can have significant implications, not only for the profitability of food-manufacturing itself, but also for the profitability of farming operations and the welfare of the consumers of food products. A second reason for focusing on these industries is the long history of contention over their degree of competitiveness. In particular, the perceived inequities and the alleged malallocation of resources that are attributable to departures from perfect competition have become perennial concerns of agricultural economists (e.g., Marion; Luttrell; Connor *et al.*). Third, the food industries are of historical significance to the study market power. Early attempts to counter their "unfair advantage" in the procurement of certain farm commodities contributed to the foundations of modern antitrust enforcement as embodied in the Sherman Act of 1890 (Thorelli). More recent supposition of the potential for market-power abuses is evidenced by the exceptions to antitrust law which the farm sector is accorded in "orderly selling" and the formation of farm-marketing cartels.

An additional reason for applying the test procedure to the food industries is that it suggests a hitherto unrecognized premise for explaining a phenomenon that has been the subject of some considerable debate throughout the last decade. This is the observed causality-*cum*-simultaneity in farm- and retail-price movements. A rather significant amount of literature in agricultural marketing has focused on estimating econometric models of price determination and establishing the direction of the causal relationship between these price movements (Heien; Bessler and Brandt; Ward; Kinnucan and Forker). Since the theoretical model makes precise the one-to-one correspondence between hypotheses of competition in the food industries and the causal direction of price determination, the study forges a link between a controversial issue in the U.S. food-marketing system and a perplexing empirical phenomenon in the food system.

Commodity Groups

The predominance of concerns about market-power abuses in the food industries have focused on the potential for monopsonistic procurement in the farm-commodity markets. This orientation stems from a belief that farm-commodity suppliers are in some way disadvantaged due to their atomistic structure in relation to the firms with which they trade. This is thought to be especially the case in markets that are regionally isolated (Connor *et al.*), of which many of the markets for live cattle are a case in point. Indeed, it was concern about market-power abuses in the cattle industry that led to the inception of the Sherman Act. Moreover, there is recent evidence of concerns about market power in this sector (Ball and Chambers). Concerns have also been expressed about the potential for noncompetitive behavior in the markets for live hogs and pork products (Miller and Harris; Hayenga *et al.*). Since there is no reason to believe that the markets for other farm commodities are exempted from potential market-power abuses, we apply the model to five commodity groups for which data are freely available -- namely, beef and veal, pork, poultry, eggs, and dairy products.

Before discussing the empirical model, it is noteworthy to consider the extent to which the theoretical model is able to circumvent some of the inherent limitations of more recent analyses of the food industries. Two issues that arise are the lack of appropriate data on outputs of retail products and the convention of placing a Leontief restriction on the technologies of food processors. These two issues are inextricably related. In the first case, a problem arises in using USDA estimates of retail-food production since this is formed under the assumption that the technologies of food-industry firms are Leontief (Wohlgenant, 1989). At first glance, this may seem a rather modest restriction for certain food industries, including the ones considered in this paper. However, in another context (Alston and Scobie; Freebairn *et al.*) the assumption of Leontief technologies is shown to lead to some rather significant findings which are invalidated through even slight departures from the fixed-proportions model. Perhaps more importantly, however, evidence available for a number of commodity groups refutes this hypothesis empirically (Wohlgenant, 1989). One is therefore led to consider the degree of potential bias imposed through its use. The stringency of the assumption for the analysis of food-industry conduct can be ascertained quite clearly from the paper by Schroeter: By imposing the condition that inputs are used in fixed proportions the conjectures of firms are forced to be identical in their product and factor markets. This would appear to be a rather stringent assumption, especially given the potential for vastly different structures in these markets. Since our method does not require observations on retail quantities, and since we make minimal assumptions on firms' technologies, we are able to circumvent restrictions implicit in most previous analyses of the food industries.

VI. Empirical Procedure and Results

To implement equations (11) empirically, we use a model that is, by now, so familiar to agricultural economists it has assumed the role of a *paradigm* in the analysis of food and fiber marketing systems. The framework is a one-output, two-input model that was originally formulated by Muth and was first applied to the food industries by Gardner in 1975. Since that time, the Muth framework has, arguably, provided the most popular model for the analysis of linkages between the markets for retail-food products and those of the farm commodities from which the former are derived. However, the model suffers from the limitation of assuming that the food industries are perfectly competitive. Given the controversial nature of this hypothesis, this draws into question the results of previous studies that have used the model to analyze a variety of marketing-system issues. These have included quantifying the benefits derived from "downstream" research (Alston and Scobie; Freebairn *et al.*; Holloway, 1989), characterizing the component elements of marketing-industry efficiency (Kilmer), and incorporating marketing-group behavior in modeling the demand for farm outputs (Wohlgenant).

For several reasons, the last of the above studies has particular relevance in the context of the current investigation. First, Wohlgenant's analysis showed how most of the important information about the marketing channel for a particular food product could be retrieved from a modicum of data; namely, observations on farm and retail prices and on exogenous variables relevant to the industry in question. In particular, one of the attractive features of his approach is that it made no use of any possibly unreliable estimates of quantity data at the retail level. Second, Wohlgenant's investigation made use of comparative-static methods that are very similar to the ones that form the foundations for equations (11). Hence, his models and procedures are almost directly applicable to the type of analysis envisaged in our theoretical framework. Third, and in view of the latter two observations, we make use of a subset of Wohlgenant's data and hence follow closely his procedures for implementing the Muth-Gardner model empirically.

Empirical Model

In the Muth-Gardner framework, we consider a collection of food-marketing firms, $i \in \{1,2...I\}$, who combine quantities of a farm-commodity input, q_{fi}, with quantities of all other nonfarm inputs, q_{mi}, to produce a retail product, q_{ri}.[27] In the market for the retail product, firms face a downward sloping demand schedule, which is given in inverse form by:

$$p_r = w_r I_r(Q_r), \tag{23}$$

where p_r and Q_r denote the price and quantity of the retail product, and w_r denotes an exogenous variable that shifts the demand for the food product. Following Wohlgenant, we construct w_r as a function of the prices of all other food items, all non-food items, and per-capita disposable income.

It is usually considered the case that the aggregate, nonfarm input is nonspecific to the food industry in question. Hence, we follow this precedent in assuming that the price

27 Hence, it is implicitly assumed that sufficient conditions exist to enable us to aggregate all nonfarm inputs -- labor, capital, materials, energy -- into a single aggregate input.

of this input is exogenous.[28] Hence, any particular firm takes the price of this input as given and predetermined when making its production decisions. It therefore faces the inverse supply function given by:

$$p_m = w_m , \tag{24}$$

in which p_m denotes the price of the nonfarm input and w_m denotes other, exogenous factors that define its value. In applying this equation, we follow Wohlgenant and define w_m as a weighted index of the prices of all nonfarm inputs used in the food-manufacturing industries, where the weights are given by the expenditure shares of each of the component series.

In the farm sector corresponding to a particular commodity group, we postulate a simple technology. Specifically, farmers are assumed to make input decisions defined over the quantity of a variable feed input, but restricted on a given level of the breeding stock that corresponds to the animal product being produced. Profit maximizing behavior under this technological scenario generates the following inverse-supply function facing the food industry:

$$p_f = w_f I_f(Q_f; w_k) , \tag{25}$$

where p_f and Q_f denote the price and quantity of the farm commodity; w_f denotes the price of the variable feed input; and w_k denotes the quantity of the breeding stock, which we assume to be fixed over the period in question. It is worth noting that, despite the inclusion of the fixed factor in farm production, the above supply schedule retains the same

28 This assumption can be relaxed with only slight modifications in the analysis which follows.

basic structure of the ones we considered in the theoretical model. This arises from the linear-homogeneity property of the individual farm-commodity suppliers' profit functions.

Hypotheses

Within this framework, and for each commodity group, there are four mutually exclusive hypotheses about the conduct of marketing firms that we consider: H_1 -- perfect competition in both the market for the retail-food product and the market for the farm-commodity input; H_2 -- perfect competition in the retail-product market, but noncompetitive behavior in the farm-commodity market; H_3 -- perfect competition in the factor market, but noncompetitive behavior in the product market; and H_4 -- noncompetitive behavior in both markets.

In our model, these four hypotheses completely exhaust all of the possible scenarios about conduct in the food industry in question. To derive the structural model depicting the equilibrium configuration under each one of these hypotheses, we proceed as above. Specifically, we consider the profit maximization problem facing a particular firm in the context of (23)-(25) and derive its implicit supply and demand behavior under each hypothesis. From this we derive the comparative-static linearization of the system as per equations (4)-(6). We then reduce this system to two equations that express the equilibrating adjustments in farm and retail prices solely in terms of the movements in each of the exogenous variables. These equations imply that the implicit functions from which they are derived are zero-degree homogeneous in the appropriate components of the modified price vector $(p_r, p_f, w_r, w_f, w_m)$.[29] It is convenient to normalize these equations on one of these variables and thereby reduce the number of parameters over which the numerical integrations must be performed. The price of the nonfarm input w_m is chosen

[29] This follows from the forms of the relevant demand and supply functions and the fact that firms' profit functions are linearly homogeneous in component subvectors of $(p_r, p_f, w_r, w_f, w_m)$.

as this numeraire. Hence, we use $\tilde{p}_{jm} \equiv \tilde{p}_j - \tilde{w}_m$ and $\tilde{w}_{jm} \equiv \tilde{w}_j - \tilde{w}_m$, $j \in \{r,f\}$, to denote proportional changes in the normalized prices, and denote the proportional change in the level of the breeding stock by $\tilde{w}_k$. Using these definitions, the *general* formulation of equations (11) can be expressed as:[30]

$$\begin{pmatrix} \phi_{rr} & \phi_{rf} \\ \phi_{fr} & \phi_{ff} \end{pmatrix} \begin{pmatrix} \tilde{p}_{rm} \\ \tilde{p}_{fm} \end{pmatrix} = \begin{pmatrix} \psi_{rr} & \psi_{rf} & \psi_{rk} \\ \psi_{fr} & \psi_{ff} & \psi_{fk} \end{pmatrix} \begin{pmatrix} \tilde{w}_{rm} \\ \tilde{w}_{fm} \\ \tilde{w}_k \end{pmatrix}. \tag{26}$$

With reference to to this expression, we are now in a position to denote the four hypotheses explicitly in terms of the dual profit function of a particular firm -- firm i -- and the parametric restrictions implied across the coefficients of this equation system. These are, specifically:[31]

H_1 -- competition-competition:	$\pi_i(p_r, w_m, p_f)$,	$(\psi_{rf}, \psi_{rk}, \psi_{fr}) = \mathbf{0}$;
H_2 -- competition-oligopsony:	$\pi_i(p_r, w_m, w_f, w_k)$,	$(\phi_{rf}, \psi_{fr}) = \mathbf{0}$;
H_3 -- oligopoly-competition:	$\pi_i(w_r, w_m, p_f)$,	$(\phi_{fr}, \psi_{rf}, \psi_{rk}) = \mathbf{0}$;
H_4 -- oligopoly-oligopsony:	$\pi_i(w_r, w_m, w_f, w_k)$,	$(\phi_{rf}, \phi_{fr}) = \mathbf{0}$.

30 It is important to note that we present this *general* formulation only for the purposes of contrasting the alternative hypotheses parametrically. No similarity is implied between this model -- which is excluded as a possible scenario since the four hypotheses are mutually exclusive and exhaustive -- and the procedure followed in some classical approaches to the nonnested problem of nesting all models within a more general framework.

31 Only the zero restrictions on the particular coefficients are of interest here; those coefficients that are restricted to be equal to one -- in line with the identity matrices referred to in (11)-(13) -- are unrestricted upon normalizing each equation in terms of a single endogenous variable prior to estimation.

Specific definitions of the coefficients can be derived by applying the manipulations presented in equations (1) through (13). These are of most significance in the implementation of the data and the prior information about these parameters. Before considering these topics, we note an important observation from the parametric restrictions implied by the respective hypotheses; namely, the causal-*cum*-simultaneous relationship between farm- and retail-price movements. In particular, we can now formalize the one-to-one correspondences between the respective hypotheses about competition and the alternate processes of price determination in the food-marketing system. These are, specifically: H_1 -- simultaneity in farm- and retail-price movements; H_2 -- causality flowing from retail prices to farm prices; H_3 -- causality flowing from farm prices to retail prices; and H_4 -- independent price determination in each market.

Data

The data used in this application are annual observations on farm and retail prices, an index of the prices of nonfarm inputs used in food manufacturing, and series pertaining to the relevant demand and supply shift variables. For the beef-and-veal, pork, and dairy commodity groups the observations cover the period 1955-79; for the poultry and egg groups the coverage is from 1956-79. The factor constraining this coverage is the availability of appropriate series to construct the farm-commodity supply shift variables, which are constructed from USDA sources. For each of the respective commodity groups the feed-price and breeding-stock variables are, respectively: (a) the annual average price of feed-corn and total numbers of breeding cows and heifers on US ranches during the preceding year, (b) a weighted-average index of the prices of hog feed rations and total numbers of sows farrowing in the preceding six months, (c) a weighted-average index of the price of poultry feed-rations and total numbers of broiler-type chicks hatched in the preceding six months, (d) a weighted-average index of layer-hen rations and total numbers of egg-type chicks hatched in the preceding six months, and (e) a weighted-average index of the prices of dairy rations and total numbers of dairy cows and heifers of milking age on farms during the preceding year. The remaining data are documented in detail by Wohlgenant. Prior to estimation, instantaneous proportional changes in the variables are replaced by first differences in their logarithms: $\tilde{x}_j \equiv \ln(x_{jt}) - \ln(x_{jt-1})$, where "t" and "t-1" denote successive time periods.

Prior Information

The prior information used in this study comes in the form of independent reports of demand and supply elasticities for each of the commodities in question, restrictions implied by the theoretical model upon which the application is founded, and the authors' own knowledge about reasonable ranges of values for the parameters in question. To incorporate this information in a formal manner, we follow Bauwens' suggestion and choose a prior probability density function from the class of natural conjugate priors. The multivariate *student* density is the one selected. As noted by Bauwens, one potential problem with the use of this density is the built-in prior independence between the coefficient vectors appearing in any two equations, but this is especially convenient when the prior information comes naturally in this form. This is the case here, since our prior parameter estimates are based, principally, on a collection of independent reports of elasticities, which we subsequently invert to yield flexibilities. Moreover, since we believe our knowledge about the likely covariances between the coefficients is rather diffuse, we specialize the multivariate *student* density to the product of independent t densities. The symmetry of this density about its mean is appropriate since we consider divergences from this point to be equally likely, *a priori*. Using the symmetry property and restrictions implied by an expansion of the terms that comprise each coefficient, we make a point estimate of the value of each particular parameter. Using results based on the standard normal we derive an approximate confidence interval within which the parameter value is likely to lie. We use the length of this interval and the prior point estimate of its mean to derive specific values for the parameters of the prior probability density function. Further details are presented in the appendix.

One alternative to the above procedure is to assume simply that we have diffuse prior information about the values of all parameters and, hence, use a noninformative prior. This, however, implies that the prior "density" does not integrate to one; it does not therefore qualify as a true probability density function. While this in itself is not a major problem -- numerical integrations can still be performed assuming this prior -- this practice throws doubt on the "probability" interpretation of the posterior measures being derived (Jeffreys; Zellner). More significantly, however, the diffuse prior implies that *any* values for the parameters along the real line are equally likely. We consider the informative prior that we use to be more appropriate.

To implement the data and prior information numerically, we select an importance function from the several that have been experimented with by Bauwens. The multivariate t density is chosen. While this performed marginally less favorably in Bauwen's experiments than members of the family of *poly-t* densities, one can generate random numbers quite conveniently from the t density. To generate these iterates we require values for the parameters of the importance function under each of the four respective hypotheses. We derive these from the application of three-stage least squares to each of the structural models.

Results

The results are based on samples of size $N = 100{,}000$. All computations were written in FORTRAN and performed on an IBM 3083 machine.[32] The principal results of interest are the estimates of the posterior probabilities in favor of each hypothesis.[33] These are reported in table 1, with the implied 95% "confidence intervals" given in parentheses.

Table 1
Posterior Probabilities

Market Retail-Product Farm-Commodity	Hypothesis H_1 *competition* *competition*	 H_2 *competition* *oligopsony*	 H_3 *oligopoly* *competition*	 H_4 *oligopoly* *oligopsony*
Beef and Veal	0.04 (±0.02)	0.59 (±0.01)	0.00 (±0.00)	0.37 (±0.01)
Pork	0.01 (±0.00)	0.87 (±0.01)	0.00 (±0.00)	0.12 (±0.01)
Poultry	0.00 (±0.00)	0.61 (±0.00)	0.00 (±0.00)	0.38 (±0.00)
Eggs	0.01 (±0.00)	0.78 (±0.01)	0.00 (±0.00)	0.21 (±0.01)
Dairy	0.16 (±0.07)	0.47 (±0.04)	0.00 (±0.00)	0.37 (±0.03)

Note: Implied 95% confidence intervals are given in parentheses.

32 Copies of these programs are available from the authors upon request.

33 Estimates of the first and second moments of the posterior densities $f(\Theta \mid \mathbf{X}, H_j)$, $j \in \{1,2..J\}$, are also available upon request.

The columns of this table present numerical estimates of the probabilities in favor of each hypothesis. Since they represent approximations, these measures need not sum to one. They are, however, accurate to two decimal places. For each of the five commodity groups, which are represented by the rows in the table, the results depict a consistent message: *the absence of competition in the farm-commodity markets, but competitive supply behavior in the markets for retail-food products*. Given our earlier preamble, and the general consensus of at least some departure from competition in these markets, this finding should not be surprising.[34] However, the relative evidence in favor of the dominant hypothesis *is* somewhat surprising. For example, in the beef-and-veal group -- arguably the commodity group for which the hypothesis of competition has been most contentious -- the posterior odds ratios imply that the data favor H_2 over the next most likely alternative -- H_4 -- by a factor of 1.6. Hence, it is more than one and one-half times as likely that these data were generated by firms that behave noncompetitively in the factor market, but competitively in the product market than it is that they were generated by firms that behaved noncompetitively in both markets. More striking, however, is the report of a zero probability in favor of H_3. This implies that *all* alternatives are infinitely more likely than the hypothesis of noncompetitive conduct in the retail-product market, but competitive behavior in the market for the farm commodity. Even more striking is the consistency of this result among the remaining commodity groups.

Two further observations are noteworthy. The first is the consistency of the rankings of the hypotheses among each of the groups. In each case the data favor H_2 over H_4, H_4 over H_1, and H_1 over. H_3. In our two-market setting, it seems intuitive

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34 It is interesting, however, to consider these findings in the light of other studies which have investigated the issue of departures from competition in the US food industries. Holloway (1991), in a forthcoming study that also employs Wohlgenant's data, finds no evidence of departures from competition in the retail-food markets of each of the above commodity groups. Hence the above findings are consistent with the latter ones. In his investigation of the US beef-packing industry, Schroeter found small but statistically significant departures from competition in both the retail-product and farm-commodity markets. The finding in the retail market may, however, be attributable to the assumption of a Leontief technology in food marketing.

to consider *competition-oligopsony* and *oligopoly-competition* as opposites, which these rankings seem to suggest. A second observation is that the implied confidence intervals suggest a good deal of precision in the estimates. These estimates were also shown to be fairly robust to alternative assumptions about the prior probability density function. Since these experiments were based on arbitrary priors, however, we choose to report the estimates in table 1 because we consider these to be most representative of the prior information.

There are several caveats on the above findings that stem from limitations in the empirical model. Specifically, these pertain to the degree to which the Muth-Gardner framework is able to adequately represent the structures of each of the commodity groups. Whereas the beef-and-veal and pork groups are adequately represented, the remaining groups depart somewhat from the basic model. In particular, the poultry and egg sectors are highly integrated between farm-supply and processing operations, and it is therefore difficult to interpret the finding that the farm-commodity market is noncompetitive.[35] The dairy sector, on the other hand, is virtually free of vertical integration of this kind. Its main departure from the model stems from farm ownership of processing operations in the form of cooperatives. Hence, in this case, the results at both the farm and retail levels may be questionable.

VII. Concluding Comments

The above results lend support to the supposition of noncompetitive behavior in certain farm-comodity markets. They suggest a number of unanswered questions pertaining to the regulation of potential market-power abuses by the food industries. These include, but are not restricted to, the efficacy of certain antitrust regulations and the

[35] The USDA reports of farm-commodity prices in these sectors are derived from a combination of sample estimates and imputed values.

potential counterveiling of monopsony power through the formation of farm-marketing cartels. The answers to these questions, however, lie outside the scope of this paper.

Our main objective has been to present an alternative approach for making inferences about competition in an industry. We have shown that this can be achieved in a theoretically consistent and empirically robust manner within a framework that places minimal restrictions on firms' technologies and on consumer preferences. Our main contribution has been to derive these results from a rather fundamental and intuitive distinction that differentiates firms in perfect competition from other modes of behavior. We have shown that this distinction can be observed from an equilibrium model in which prices are expressed as functions of exogenous variables that are relevant to the industry in question. Hence, the model and procedures are applicable to any industry for which one has data on prices and the relevant exogenous variables. While we illustrated these concepts in an application to several US food industries, further work may suggest additional useful applications of the model. This work should perhaps focus on refining some of the conditions that are necessary for the application of the model in order to derive more precise inferences about hypotheses pertaining to competition.

Appendix

Proofs of Propositions

To prove that $\pi_i(\bullet)$ is linearly homogeneous in $(\mathbf{p_c},\mathbf{w_n})$, first make the following substitutions:

$$\pi_i(\mathbf{p_c},\mathbf{w_n}) \equiv \max_{\mathbf{q_i}} \{ \mathbf{p} \bullet \mathbf{q_i} \mid \mathbf{p_n}=\mathbf{w_n}\mathbf{I_n}(\mathbf{Q_n}),\ \mathbf{Q_n}=\mathbf{K_{ni}}(\mathbf{q_{ni}}),\ \mathbf{q_i}\in \tau_i \},$$

$$\equiv \max_{(\mathbf{q_{ci}},\mathbf{q_{ni}})} \{ \mathbf{p_c} \bullet \mathbf{q_{ci}} + \mathbf{p_n} \bullet \mathbf{q_{ni}} \mid \mathbf{p_n}=\mathbf{w_n}\mathbf{I_n}(\mathbf{Q_n}),\ \mathbf{Q_n}=\mathbf{K_{ni}}(\mathbf{q_{ni}}),\ (\mathbf{q_{ci}},\mathbf{q_{ni}})\in \tau_i \},$$

$$\equiv \max_{(\mathbf{q_{ci}},\mathbf{q_{ni}})} \{ \mathbf{p_c} \bullet \mathbf{q_{ci}} + \mathbf{w_n} \bullet \mathbf{S_{ni}}(\mathbf{q_{ni}}) \mid (\mathbf{q_{ci}},\mathbf{q_{ni}})\in \tau_i \},$$

where $\mathbf{S_{ni}}(\bullet) \equiv \mathbf{I_n}(\mathbf{K_{ni}}(\mathbf{q_{ni}})\mathbf{q_{ni}}$ denotes the N-dimensional vector of functions derived from substituting $\mathbf{K_{ni}}(\bullet)$ and $\mathbf{w_n}\mathbf{I_n}(\bullet)$ for $\mathbf{Q_n}$ and $\mathbf{p_n}$, respectively. Let λ be a strictly positive scalar. Then:

$$\pi_i(\lambda\mathbf{p_c},\lambda\mathbf{w_n}) \equiv \max_{(\mathbf{q_{ci}},\mathbf{q_{ni}})} \{ \lambda\mathbf{p_c} \bullet \mathbf{q_{ci}} + \lambda\mathbf{w_n} \bullet \mathbf{S_{ni}}(\mathbf{q_{ni}}) \mid (\mathbf{q_{ci}},\mathbf{q_{ni}})\in \tau_i \},$$

$$= \lambda \max_{(\mathbf{q_{ci}},\mathbf{q_{ni}})} \{ \mathbf{p_c} \bullet \mathbf{q_{ci}} + \mathbf{w_n} \bullet \mathbf{S_{ni}}(\mathbf{q_{ni}}) \mid (\mathbf{q_{ci}},\mathbf{q_{ni}})\in \tau_i \},$$

$$\equiv \lambda\, \pi_i(\mathbf{p_c},\mathbf{w_n}).$$

Hence $\pi_i(\bullet)$ is linearly homogeneous in $(\mathbf{p_c},\mathbf{w_n})$. Q.E.D.

To prove that $\pi_i(\bullet)$ is convex in $(\mathbf{p}_c,\mathbf{w}_n)$, let $(\mathbf{q}^o_{ci},\mathbf{q}^o_{ni})$ and $(\mathbf{q}^*_{ci},\mathbf{q}^*_{ni})$ denote two profit-maximizing quantity choices when "prices" are $(\mathbf{p}^o_c,\mathbf{w}^o_n)$ and $(\mathbf{p}^*_c,\mathbf{w}^*_n)$, respectively. That is, define these quantity vectors by:

$$\pi_i(\mathbf{p}^o_c,\mathbf{w}^o_n) \equiv \max_{(\mathbf{q}_{ci},\mathbf{q}_{ni})} \{ \mathbf{p}^o_c \bullet \mathbf{q}_{ci} + \mathbf{w}^o_n \bullet \mathbf{S}_n(\mathbf{q}_{ni}) \mid (\mathbf{q}_{ci},\mathbf{q}_{ni}) \in \tau_i \}$$

$$= \mathbf{p}^o_c \bullet \mathbf{q}^o_{ci} + \mathbf{w}^o_n \bullet \mathbf{S}_n(\mathbf{q}^o_{ni}) \ ,$$

and

$$\pi_i(\mathbf{p}^*_c,\mathbf{w}^*_n) \equiv \max_{(\mathbf{q}_{ci},\mathbf{q}_{ni})} \{ \mathbf{p}^*_c \bullet \mathbf{q}_{ci} + \mathbf{w}^*_n \bullet \mathbf{S}_n(\mathbf{q}_{ni}) \mid (\mathbf{q}_{ci},\mathbf{q}_{ni}) \in \tau_i \}$$

$$= \mathbf{p}^*_c \bullet \mathbf{q}^*_{ci} + \mathbf{w}^*_n \bullet \mathbf{S}_n(\mathbf{q}^*_{ni}) \ ,$$

respectively. Let λ denote a nonnegative scalar, such that: $0 \leq \lambda \leq 1$; and define $(\mathbf{q}^\lambda_{ci},\mathbf{q}^\lambda_{ni})$ as the profit-maximizing quantity vector when "prices" are: $(\mathbf{p}^\lambda_c,\mathbf{w}^\lambda_n) \equiv \lambda(\mathbf{p}^o_c,\mathbf{w}^o_n) + (1\text{-}\lambda)(\mathbf{p}^*_c,\mathbf{w}^*_n)$. From these definitions it follows that:

$$\pi_i(\mathbf{p}^\lambda_c,\mathbf{w}^\lambda_n) \equiv \pi_i(\lambda(\mathbf{p}^o_c,\mathbf{w}^o_n) + (1\text{-}\lambda)(\mathbf{p}^*_c,\mathbf{w}^*_n))$$

$$\equiv \max_{(\mathbf{q}_{ci},\mathbf{q}_{ni})} \{ (\lambda\mathbf{p}^o_c+(1\text{-}\lambda)\mathbf{p}^*_c) \bullet \mathbf{q}_{ci} + (\lambda\mathbf{w}^o_n+(1\text{-}\lambda)\mathbf{w}^*_n) \bullet \mathbf{S}_n(\mathbf{q}_{ni}) \mid (\mathbf{q}_{ci},\mathbf{q}_{ni}) \in \tau_i \}$$

$$= \quad (\lambda p_c^o + (1-\lambda)p_c^*) \bullet q_{ci}^\lambda + (\lambda w_n^o + (1-\lambda)w_n^*) \bullet S_n(q_{ni}^\lambda)$$

$$= \quad \lambda(p_c^o \bullet q_{ci}^\lambda + w_n^o \bullet S_n(q_{ni}^\lambda)) + (1-\lambda)(p_c^* \bullet q_{ci}^\lambda + w_n^* \bullet S_n(q_{ni}^\lambda))$$

$$\leq \quad \lambda(p_c^o \bullet q_{ci}^o + w_n^o \bullet S_n(q_{ni}^o)) + (1-\lambda)(p_c^* \bullet q_{ci}^* + w_n^* \bullet S_n(q_{ni}^*))$$

$$\equiv \quad \lambda\, \pi_i(p_c^o, w_n^o) + (1-\lambda)\, \pi_i(p_c^*, w_n^*),$$

where the inequality follows from the fact that $(q_{ci}^\lambda, q_{ni}^\lambda)$ is a feasible production plan, but does not necessarily maximize profits when prices are (p_c^o, w_n^o) or (p_c^*, w_n^*). Hence, $\pi_i(\bullet)$ is convex in (p_c, w_n). Q.E.D.

To prove the derivative properties of $\pi_i(\bullet)$ let $q_i^o \equiv (q_{ci}^o, q_{ni}^o)$ denote the solution to the firm's maximization problem when prices $p^o \equiv (p_c^o, p_n^o)$ and $w^o \equiv (w_c^o, w_n^o)$ prevail. That is, define q_i^o by:

$$\pi_i(p_c^o, w_n^o) \equiv \max_{q_i} \{\, p^o \bullet q_i \mid p_n^o = w_n^o I_n(Q_n),\ Q_n = K_{ni}(q_{ni}),\ q_i \in \tau_i \,\}$$

$$= p^o \bullet q_i^o\,,$$

$$\equiv p_c^o \bullet q_{ci}^o \quad + \quad p_n^o \bullet q_{ni}^o\,,$$

$$\equiv p_c^o \bullet q_{ci}^o \quad + \quad w_n^o \bullet S_{ni}(q_{ni}^o)\,.$$

Since $\mathbf{q}_i^o$ is a feasible production plan, but is not necessarily optimal for any other vector of prices we have the following weak inequality: $\pi_i(\mathbf{p}_c,\mathbf{w}_n) \geq \mathbf{p}_c \bullet \mathbf{q}_{ci}^o + \mathbf{w}_n \bullet \mathbf{S}_{ni}(\mathbf{q}_{ni}^o)$. Define the function: $G(\mathbf{p}_c,\mathbf{w}_n) \equiv \pi_i(\mathbf{p}_c,\mathbf{w}_n) - (\mathbf{p}_c \bullet \mathbf{q}_{ci}^o + \mathbf{w}_n \bullet \mathbf{S}_{ni}(\mathbf{q}_{ni}^o))$. Since $(\mathbf{q}_{ci}^o,\mathbf{q}_{ni}^o)$ is the profit-maximizing plan when prices are $(\mathbf{p}_c^o,\mathbf{w}_n^o)$, $G(\bullet)$ attains a global minimum at $G(\mathbf{p}_c^o,\mathbf{w}_n^o)$. It must, therefore, satisfy the first-order conditions that are necessary at this point; namely:

$$\nabla_{\mathbf{p}_c^o} G(\mathbf{p}_c^o,\mathbf{w}_n^o) = \nabla_{\mathbf{p}_c^o} \pi_i(\mathbf{p}_c^o,\mathbf{w}_n^o) - \mathbf{q}_{ci}^o = \mathbf{0},$$

$$\nabla_{\mathbf{w}_n^o} G(\mathbf{p}_c^o,\mathbf{w}_n^o) = \nabla_{\mathbf{w}_n^o} \pi_i(\mathbf{p}_c^o,\mathbf{w}_n^o) - \mathbf{S}_{ni}(\mathbf{q}_{ni}^o) = \mathbf{0}.$$

Since the above holds for all price vectors, the result in the proposition is proven. Q.E.D.

The following is an expansion of some of the main points that are discussed in the text. It summarizes most of the key steps in Bauwens' derivations.

Assuming equations (10) to hold with error, the system can be transposed, normalized, and rewritten as

$$\mathbf{Y}\,\mathrm{B} + \mathbf{Z}\,\Gamma = \mathbf{U},$$

where $\mathbf{Y}$ denotes a T×M matrix of the T observtions on the M endogenous variables; $\mathbf{Z}$ is a T×N matrix of observations on the N exogenous variables; B is an order-M, square, nonsingular matrix of coefficients on the endogenous variables in which the diagonal elements equal one; Γ is an N×M matrix of coefficients on the exogenous variables; and U is a T×M matrix of unobserved disturbance terms. Hence, in terms of equations (11)-(13), the T observations are made on the vectors $\mathbf{Y} \equiv \tilde{\mathbf{p}}^{T}$ and $\mathbf{Z} \equiv \tilde{\mathbf{w}}^{\mathbf{T}}$, where "T" denotes the transpose operation. Hence, in equations (12) $\Phi_m \equiv B^{\mathbf{T}}$ and in equations (13) $\Psi_{\mathbf{m}} \equiv \Gamma^{\mathbf{T}}$. It should also be clear that those equations consider the case where Γ is M×M. It is assumed that the elements of $\mathbf{U}$ have a multivariate normal distribution:

$$f(\mathbf{U}) = ((2\pi)^{TM}\,|\Sigma|^{T})^{-1/2}\exp(-1/2(\mathbf{trace}(\Sigma^{-1}\mathbf{U}^{T}\mathbf{U}))),$$

where Σ is a positive-definite symmetric matrix of order M, "exp" denotes the natural exponent, and "**trace**" denotes the matrix-algebra trace operation. The important feature in this formulation is the absence of any form of serial correlation in the errors.

To derive the likelihood function corresponding to the density of $\mathbf{U}$, a useful reformulation of the structural model consists of rewriting the i^{th} equation in the system as:

$$\mathbf{y}_i = \mathbf{x}_i\,\delta_i + \mathbf{u}_i,$$

where $\mathbf{y}_i$ denotes the i^{th} column of $\mathbf{Y}$; $\mathbf{x}_i$ is the $T \times k_i$ matrix of observations on the k_i explanatory variables appearing in the i^{th} equation; δ_i is the k_i vector of unrestricted coefficients in the i^{th} equation; and $\mathbf{u}_i$ is the i^{th} column of $\mathbf{U}$. Hence the matrices B and Γ together contain $K = \sum_{i=1}^{M} k_i$ unknown coefficients. Defining $\Theta \equiv (B, \Gamma, \Sigma)$ and $\mathbf{X} \equiv (\mathbf{Y}, \mathbf{Z})$, we can write the likelihood function conditional on a particular hypothesis H_j as:

$$L(\Theta|\mathbf{X}, H_j) \propto \|B\|^{T} |\Sigma|^{-T/2} \exp(-1/2(\mathbf{trace}(\Sigma^{-1}(\mathbf{S}+(\Delta-\hat{\Delta})^{\mathbf{T}}\mathbf{M}(\Delta-\hat{\Delta}))))) ,$$

where the definitions of the matrices are as follows: $\mathbf{S} \equiv (\mathbf{Y}-\Xi\Delta)^{\mathbf{T}}(\mathbf{Y}-\Xi\Delta)$; $\hat{\Delta} \equiv \mathbf{M}^{+}\Xi^{\mathbf{T}}\mathbf{Y}^{\mathbf{T}}$; $\mathbf{M}^{+}$ denotes the generalized inverse of $\mathbf{M}$; $\mathbf{M} \equiv \Xi^{\mathbf{T}}\Xi$; Ξ is defined by:

$$\Xi \equiv [\ \mathbf{x}_1\ \mathbf{x}_2\ \dots\ \mathbf{x}_M\ ] ,$$

and Δ is defined by:

$$\Delta \equiv \begin{pmatrix} \delta_1 & & \\ & \delta_2 & \\ & & \delta_M \end{pmatrix}.$$

To select a prior probability density function, one is chosen from the class of natural conjugate prior densities, which is given by:

$$f(\Theta|H_j) \propto \|B\|^{\tau_o} |\Sigma|^{-1/2(\nu_o+M+1)} \exp(-1/2(\mathbf{trace}(\Sigma^{-1} (\mathbf{S}_o+(\Delta-\hat{\Delta}_o)^{\mathbf{T}}\mathbf{M}_o(\Delta-\hat{\Delta}_o))))),$$

where $\mathbf{S_o}$ of order M, $\mathbf{M_o}$ of order K, and Δ_o of dimension KxM are matrices whose elements are to be assigned numerical values, *a priori*. The scalars τ_o and ν_o, also to be assigned, can be interpreted as "degrees of freedom" in a previous hypothetical sample.

In the natural-conjugate framework, the prior and the likelihood combine to yield a posterior measure that has the same functional form:

$$f(\Theta|\mathbf{X},H_j) \propto \|B\|^{\tau_*} |\Sigma|^{-1/2(\nu_*+M+1)} \exp(-1/2(\mathbf{trace}(\Sigma^{-1} (\mathbf{S}_*+(\Delta-\hat{\Delta}_*)^{\mathbf{T}}\mathbf{M}_*(\Delta-\hat{\Delta}_*))))),$$

where $\tau_* = \tau_o + T$, $\nu_* = \nu_o + T$, $\mathbf{M}_* = \mathbf{M}_o + \mathbf{M}$, $\Delta_* = \mathbf{M}_*^+(\mathbf{M}_o\Delta_o+\mathbf{M}\hat{\Delta}_o)$ and $\mathbf{S}_* = \mathbf{S}_o + \Delta_o^{\mathbf{T}}\mathbf{M}_o\Delta_o + \mathbf{S} + \hat{\Delta}^{\mathbf{T}}\mathbf{M}\hat{\Delta} - \Delta_*^{\mathbf{T}}\mathbf{M}_*\Delta_*$.

To conduct posterior analyses with respect to this density one needs to evaluate integrals of the domains of the parameters in $\Theta \equiv (\Delta,\Sigma)$. We may write the posterior density as the product of a conditional density over Σ and a marginal density upon Δ ,

$$f(\Theta|\mathbf{X},H_j) \propto f(\Sigma|\Delta,\mathbf{X},H_j)\, f(\Delta|\mathbf{X},H_j),$$

and proceed to integrate out the *nuisance* parameters contained in Σ. This is achieved by noting that the conditional density over Σ is proportional to the so-called Inverted-Wishart density:

$$\begin{aligned} f(\Sigma|\Delta,\mathbf{X},H_j) \propto\ & 2^{\nu_* M/2} \pi^{M(M-1)/4} \prod_{i=1}^{M} \Gamma((\tau_*+1-i)/2) \\ & \times\ |\mathbf{S}_*+(\Delta-\hat{\Delta}_*)^{\mathbf{T}}\mathbf{M}_*(\Delta-\hat{\Delta}_*)|^{\nu_*/2}\ |\Sigma|^{-(\nu_*+1-i)/2}) \\ & \times\ \exp(-1/2(\mathbf{trace}(\Sigma^{-1} (\mathbf{S}_*+(\Delta-\hat{\Delta}_*)^{\mathbf{T}}\mathbf{M}_*(\Delta-\hat{\Delta}_*))))), \end{aligned}$$

where $\Gamma(\bullet)$ denotes the gamma function, and all other symbols are previously defined. Since the function on the right-hand-side of this expression represents a true probability density function, the value of the integrand defined over the domain of Σ is one. Using this fact, we are left with:

$$f(\Delta|\mathbf{X},H_j) \propto ||B||^{\tau_*} \; |\mathbf{S}_* + (\Delta - \hat{\Delta}_*)^T \mathbf{M}_* (\Delta - \hat{\Delta}_*)|^{v_*/2} .$$

Since further analytical integration is not possible from this point on, posterior analysis with respect to the function on the right-hand side must be undertaken numerically.

Incorporating Prior Information

The purpose of this section is to give an example of how the intution about certain parameter values and otherwise diffuse information is implemented formally through particular values selected for the parameters of the prior probability density function. To this end, we expand some of the major points that were outlined in the text, beginning with the specific form of the prior density -- the product of multivariate *t* densities on each of the M equations:

$$f(\Theta|H_j) = \prod_{i=1}^{M} \pi^{-k_i/2}\,\Gamma(\nu_o/2)/\Gamma((\nu_o-k_i)/2)\; S_{oi}^{(\nu_o-k_i)/2}\,|\mathbf{M}_{oii}|^{1/2}$$

$$\times\ (\mathbf{S}_{oi}+(\delta_i-\delta_{oi})^{\mathbf{T}}\mathbf{M}_{oii}(\delta_i-\delta_{oi}))^{-\nu_o/2}\ .$$

It can be shown that this is a particular specification of the natural-conjugate prior density given above. It has prior means $E\{\bullet\}$ and covariances $V\{\bullet\}$ given, respectively, by:

$$E\{\delta_i\} = \delta_{oi}\,, \qquad i=1,2...M;$$

$$V\{\delta_i\} = (S_{oi}/(\nu_o-k_i-2))\,\mathbf{M}_{oii}^{-1}\,, \qquad i=1,2...M.$$

Hence, we select values for the parameters S_{oi}, ν_o and the elements of δ_{oi} and $\mathbf{M}_{oii}$ in accordance with the available prior information.

To illustrate the procedure, consider the structural model conditional on H_1 -- the hypothesis of price-taking behavior in both markets. Under this hypothesis we can normalize each of the equations in (26) on the respective endogenous variables -- thus, setting ϕ_{rr} and ϕ_{ff} equal to one -- and expand the term ψ_{rr} as:

$$\psi_{rr} \equiv (1 - \xi_r \sum_{i=1}^{I} \alpha_{ri}\,\eta_{rri})^{-1}\,,$$

where ξ_r denotes the price flexibility for the retail product; α_{ri} denotes firm i's market share in the supply of this product; and η_{rri} is the firm's direct-price elasticity of supply of the product. Hence ψ_{rr} is simply the inverse of one minus the multiple of the retail-demand flexibility and the sum of the share-weighted elasticities of supply. Given the convexity of the profit functions $\pi_i(p_r,w_m,p_f)$, $i \in \{1,2..I\}$, each of these elasticities must be nonnegative. Since the shares are also nonnegative and the price flexibility is nonpositive, it follows that the bracketed expression on the right-hand-side of the above identity is bounded between one and positive infinity. Hence its inverse, and thus the value of the parameter ψ_{rr}, lies in the unit interval. We also note that this must hold, almost surely. Thus, given the symmetric prior distribution for ψ_{rr}, we assume a prior point estimate:

$$E\{\psi_{rr}\} = 0.5 \equiv \delta_{orr},$$

and summarize our degree of belief about this estimate by constructing a confidence interval about this point. Before constructing this variance estimate, we use the prior point estimate to infer likely values for the remaining coefficients in the retail-price equation.

Rearranging the definition of ψ_{rr} above, and using its point estimate, a reasonable approximation is impled for the expected value of the sum of the share-weighted elasticities of supply; namely:

$$E\{\sum_{i=1}^{I} \alpha_{ri}\,\eta_{rri}\} \approx \frac{(E\{\psi_{rr}\})^{-1} - 1}{-\xi_r} = -\xi_r^{-1},$$

which is an approximation we use to derive a point estimate for the coefficient of the farm-price variable in this equation. This follows from taking expectations on both sides of the restriction implied by the linear homogeneity of $\pi_i(p_r, w_m, p_f)$, $i \in \{1,2..I\}$:

$$E\{\sum_{i=1}^{I} \alpha_{ri}\, \eta_{rfi}\} \;=\; -E\{\sum_{i=1}^{I} \alpha_{ri}\, \eta_{rri}\} - E\{\sum_{i=1}^{I} \alpha_{ri}\, \eta_{rmi}\},$$

where η_{rfi} and η_{rmi} denote elasticities of output supply with respect to the prices of the farm- and nonfarm-inputs, respectively. We have rather limited information about the magnitudes of each of these effects, but consider it likely that the former is considerably larger than the latter. Hence, we assume the following approximation:

$$E\{\sum_{i=1}^{I} \alpha_{ri}\, \eta_{rfi}\} \;\approx\; -E\{\sum_{i=1}^{I} \alpha_{ri}\, \eta_{rri}\},$$

and combine this logic with the results above it in order to derive:

$$E\{\sum_{i=1}^{I} \alpha_{ri}\, \eta_{rfi}\} \;\approx\; \xi_r^{-1}.$$

Using this result in the expanded definition of the parameter ϕ_{rf}, we use the following approximation:

$$E\{\phi_{rf}\} \;\approx\; \frac{\xi_r\, E\{\sum_{i=1}^{I} \alpha_{ri}\, \eta_{rfi}\}}{1 - \xi_r\, E\{\sum_{i=1}^{I} \alpha_{ri}\, \eta_{rri}\}},$$

in order to derive a point estimate for this parameter:

$$E\{\phi_{rf}\} = 0.5 \equiv \delta_{orf}.$$

Turning to the coefficients in the farm-price equation, we derive the definition of the coefficient of the feed-price variable as:

$$\psi_{ff} \equiv (1 - \xi_f \sum_{i=1}^{I} \alpha_{fi}\, \eta_{ffi})^{-1},$$

where ξ_f denotes the price flexibility of supply of the farm-commodity input; α_{fi} denotes firm i's share in the aggregate demand for this input; and η_{ffi} is its uncompensated elasticity of demand for the factor. Thus ψ_{ff} defines the input analogue of the parameter ψ_{rr}. Hence we use the same rationale as above in noting that the convexity of the profit function requires the demand elasticities to be nonpositive, and that the price flexibility is nonnegative. Therefore, ψ_{ff} almost surely lies in the unit interval and we use the symmetry of the prior density to obtain a prior point estimate of its mean; namely,

$$E\{\psi_{ff}\} = 0.5 \equiv \delta_{off}.$$

Similarly, we follow an analogous procedure to that used in deriving a point estimate for ϕ_{rf} to obtain:

$$E\{\phi_{fr}\} = 0.5 \equiv \delta_{ofr}.$$

The remaining coefficient to be considered under this hypothesis is the coefficient of the breeding-stock variable in the farm-price equation, ψ_{fk}. We derive its expanded definition as:

$$\psi_{fk} \equiv \frac{\xi_k}{1 - \xi_r \sum_{i=1}^{I} \alpha_{fi}\, \eta_{ffi}} ,$$

where ξ_k denotes the price flexibility of supply with respect to a change in the level of the breeding stock. Hence, this term refers to the proportional change in price which occurs from a movement *along* the supply schedule that results from a change in the level of this stock variable. Given the nature of the technologies we assume for the animal commodities in question, and the definitions of the breeding-stock data that we use, it seems reasonable to presume a unitary elasticity of response in supply emanating from changes in the breeding stock. Hence, expressing the farm-commodity supply function corresponding to (25) in proportional-change terms

$$\tilde{Q} = \xi_f^{-1} (\tilde{p}_f - \tilde{w}_f - \xi_k \tilde{w}_k),$$

it follows that

$$\xi_k \approx -\xi_f .$$

We use this approximation in the following expression

$$E\{\psi_{fk}\} \approx \frac{E\{\xi_k\}}{1 - \xi_f\, E\{\sum_{i=1}^{I} \alpha_{fi}\, \eta_{ffi}\}} .$$

Combining this with the point estimate of the sum of the share-weighted demand elasticities corresponding to $E\{\psi_{ff}\}$; namely,

$$E\{\sum_{i=1}^{I} \alpha_{fi}\,\eta_{ffi}\} \approx \frac{(E\{\psi_{ff}\})^{-1} - 1}{-\xi_f} = -\xi_f^{-1},$$

we obtain an approximate point estimate for the coefficient of the breeding stock variable as:

$$E\{\psi_{fk}\} = -\xi_f/2 \equiv \delta_{ofk}\,.$$

To implement this expectation numerically for each particular commodity group, we invert the numerical value of an external report of the farm-commodity supply elasticity.

To summarize thus far, under the null hypothesis of competition in factor and product markets -- H_1 -- we have obtained the following point estimates for the parameters of the retail-price equation,

$$E\{(\phi_{rf},\psi_{rr})\,|\,H_1\} = (0.5,0.5) \equiv \boldsymbol{\delta_{or}}\,,$$

and the farm-price equation

$$E\{(\phi_{fr},\psi_{ff},\psi_{fk})\,|\,H_1\} = (0.5,0.5,-\xi_f/2) \equiv \boldsymbol{\delta_{of}}\,.$$

In reviewing the logic and procedures for obtaining these estimates we note that we are able to restrict the domains of two of the parameters -- namely, ψ_{rr} and ψ_{ff} -- purely on theoretical grounds.[36] In contrast, we consider that the information used in the derivation of the remaining parameters is somewhat less precise. We wish, now, to derive

36 At the time of writing this appendix, we note another application of Bayesian methods to agricultural data, which is similar in its approach to constraining posterior estimates of parameters through the use of prior information (Chalfant *et al.*).

parameter values for the variance-covariance matrices that will reflect these disparities in belief. For simplicity, and in the absence of satisfactory information about the covariance between the parameters appearing in any single equation, we assume that the covariance terms are negligible.[37] Hence, we assume that the matrix $\mathbf{M}_{oii}$ is diagonal. This allows us to express the prior information about the coefficients through the product of independent t densities.

To construct an estimate of the variance for any particular parameter, it is convenient to use an approximation from the normal distribution, which the t distribution approximates in large samples. Let θ denote a parameter about which we wish to construct a confidence interval. Given the prior mean and variance of θ -- $E\{\bullet\}$ and $V\{\bullet\}$, respectively -- and the fact that θ is approximately distributed as a normal random variable the following measure:

$$z \equiv \frac{\theta - E\{\theta\}}{\sqrt{V\{\theta\}}}$$

is approximately distributed as a *standard* normal random variable. We choose upper and lower limits -- θ^- and θ^+, respectively -- symmetric about the known value $E\{\bullet\}$, and use these to define an interval such that approximately $1-\alpha$ percent of the values of θ fall within this range. It follows, therefore, that the event

$$-z_{\alpha/2} \leq \frac{\theta - E\{\theta\}}{\sqrt{V\{\theta\}}} \leq +z_{\alpha/2}$$

---

37 We acknowledge that this is not strictly correct, since we used estimates of the coefficients of the shift variables to derive estimates of the coefficients of the prices appearing in each equation. It does, however, seem to be an acceptable practice since the estimates of the posterior probabilities appeared robust to experiments using arbitrarily assigned covariance terms.

occurs with probability $1-\alpha$, where $z_{\alpha/2}$ denotes the $1-\alpha$ fractile of the standard-normal distribution. This has the same probability as the event:

$$\theta^- \le \theta \le \theta^+.$$

Hence we derive:

$$\theta^\pm = E\{\theta\} \pm z_{\alpha/2}\sqrt{V\{\theta\}}.$$

A convenient choice for α is 0.32. This yields $z_{\alpha/2} = 1.0$, and allows one to compute an approximate estimate for the variance from the limits of the confidence interval; namely:

$$\sqrt{V\{\theta\}} = E\{\theta\} - \theta^- = \theta^+ - E\{\theta\}.$$

Using this estimate, in each equation we select

$$\nu_o = \max\{k_i, i\in\{r,f\}\}+3,$$

and select S_{oi}, $i\in\{r,f\}$, such that $(S_{oi}/(\nu_o-k_i-2))$ equals one within each equation. This permits assignment of the diagonal elements of the matrix $\mathbf{M}_{oii}^{-1}$ solely in terms of the corresponding vector of variance estimates that are obtained from the 68% confidence interval. Specifically, we derive:

$$\mathbf{vec}(\mathbf{V}\{\delta_i\}) = \mathbf{vec}(\mathbf{M}_{oii}^{-1}).$$

We summarize the information obtained from the above procedures in table 2. The prior parameter estimates corresponding to the remaining hypotheses can be obtained in an entirely analogous manner. These are presented in tables 3-5. To implement numerically the parametric entries in these tables we use external reports of the relevant supply and demand elasticities and their corresponding flexibilities. These are presented in table 6.

Table 2

Parameters of $f(\Theta \mid H_j)$ Under

H_1 -- competition-competition: $\pi_i(p_r, w_m, p_f)$, $(\psi_{rf}, \psi_{rk}, \psi_{fr}) = \mathbf{0}$

θ	$E\{\theta\}$	68% Confidence Interval	$\sqrt{V\{\theta\}}$	δ_{o1}	$\mathbf{vec(M_{oll}^{-1})}$
ϕ_{rf}	+0.5	[-0.5,+1.5]	1.0	+0.5	1.00
ψ_{rr}	+0.5	[0.0,+1.0]	0.5	+0.5	0.25
ϕ_{fr}	+0.5	[-0.5,+1.5]	1.0	+0.5	1.00
ψ_{ff}	+0.5	[0.0,+1.0]	0.5	+0.5	0.25
ψ_{fk}	$-\xi_f/2$	$[-\xi_f, 0.0]$	$\xi_f/2$	$-\xi_f/2$	$(\xi_f/2)^2$

Table 3

Parameters of $f(\Theta | H_j)$ Under

H_2 -- competition-oligopsony: $\pi_i(p_r, w_m, w_f, w_k)$, $(\phi_{rf}, \psi_{fr}) = \mathbf{0}$;

θ	$E\{\theta\}$	68% Confidence Interval	$\sqrt{V\{\theta\}}$	δ_{ol}	$\mathbf{vec(M_{oli}^{-1})}$
ψ_{rr}	+0.5	[0.0,+1.0]	0.5	+0.5	0.25
ψ_{rf}	+0.5	[-0.5,+1.5]	1.0	+0.5	1.00
ψ_{rk}	$\xi_r/2$	$[\xi_r, 0.0]$	$-\xi_r/2$	$\xi_r/2$	$(-\xi_r/2)^2$
ϕ_{fr}	+0.5	[-0.5,+1.5]	1.0	+0.5	1.00
ψ_{ff}	+0.5	[0.0,+1.0]	0.5	+0.5	0.25
ψ_{fk}	$-\xi_f/2$	$[-\xi_f, 0.0]$	$\xi_f/2$	$-\xi_f/2$	$(\xi_f/2)^2$

Table 4

Parameters of $f(\Theta \mid H_j)$ Under

H_3 -- oligopoly-competition: $\pi_i(w_r,w_m,p_f)$, $(\phi_{fr},\psi_{rf},\psi_{rk}) = \mathbf{0}$;

θ	$E\{\theta\}$	68% Confidence Interval	$\sqrt{V\{\theta\}}$	δ_{ol}	$\mathbf{vec}(\mathbf{M}_{oli}^{-1})$
ϕ_{rf}	+0.5	[-0.5,+1.5]	1.0	+0.5	1.00
ψ_{rr}	+0.5	[0.0,+1.0]	0.5	+0.5	0.25
ψ_{fr}	+0.5	[-0.5,+1.5]	1.0	+0.5	1.00
ψ_{ff}	+0.5	[0.0,+1.0]	0.5	+0.5	0.25
ψ_{fk}	$-\xi_f/2$	$[-\xi_f,0.0]$	$\xi_f/2$	$-\xi_f/2$	$(\xi_f/2)^2$

Table 5

Parameters of $f(\Theta \mid H_j)$ Under

H_4 -- oligopoly-oligopsony: $\pi_i(w_r, w_m, w_f, w_k)$, $(\phi_{rf}, \phi_{fr}) = \mathbf{0}$;

θ	$E\{\theta\}$	68% Confidence Interval	$\sqrt{V\{\theta\}}$	δ_{oi}	$\mathbf{vec(M_{oii}^{-1})}$
ψ_{rr}	+0.5	[0.0,+1.0]	0.5	+0.5	0.25
ψ_{rf}	+0.5	[-0.5,+1.5]	1.0	+0.5	1.00
ψ_{rk}	$\xi_r/2$	$[\xi_r, 0.0]$	$-\xi_r/2$	$\xi_r/2$	$(-\xi_r/2)^2$
ψ_{fr}	+0.5	[-0.5,+1.5]	1.0	+0.5	1.00
ψ_{ff}	+0.5	[0.0,+1.0]	0.5	+0.5	0.25
ψ_{fk}	$-\xi_f/2$	$[-\xi_f, 0.0]$	$\xi_f/2$	$-\xi_f/2$	$(\xi_f/2)^2$

Table 6

Elasticities and Flexibilities

Commodity Group	Retail-Demand		Farm-Commodity Supply	
	elasticity ξ_r^{-1}	flexibility ξ_r	elasticity ξ_f^{-1}	flexibility ξ_f
Beef and Veal	-0.70	-1.43	0.65	1.54
Pork	-0.86	-1.16	1.00	1.00
Poultry	-0.56	-1.79	0.65	1.54
Eggs	-0.35	-2.86	0.55	1.82
Dairy	-0.65	-1.54	2.08	0.48

Source: Gardiner, W. H., V. O. Roningen, and K. Liu. *Elasticities in the Trade Liberalization Database*. United States Department of Agriculture, Economic Research Service, Washington DC, 1989.

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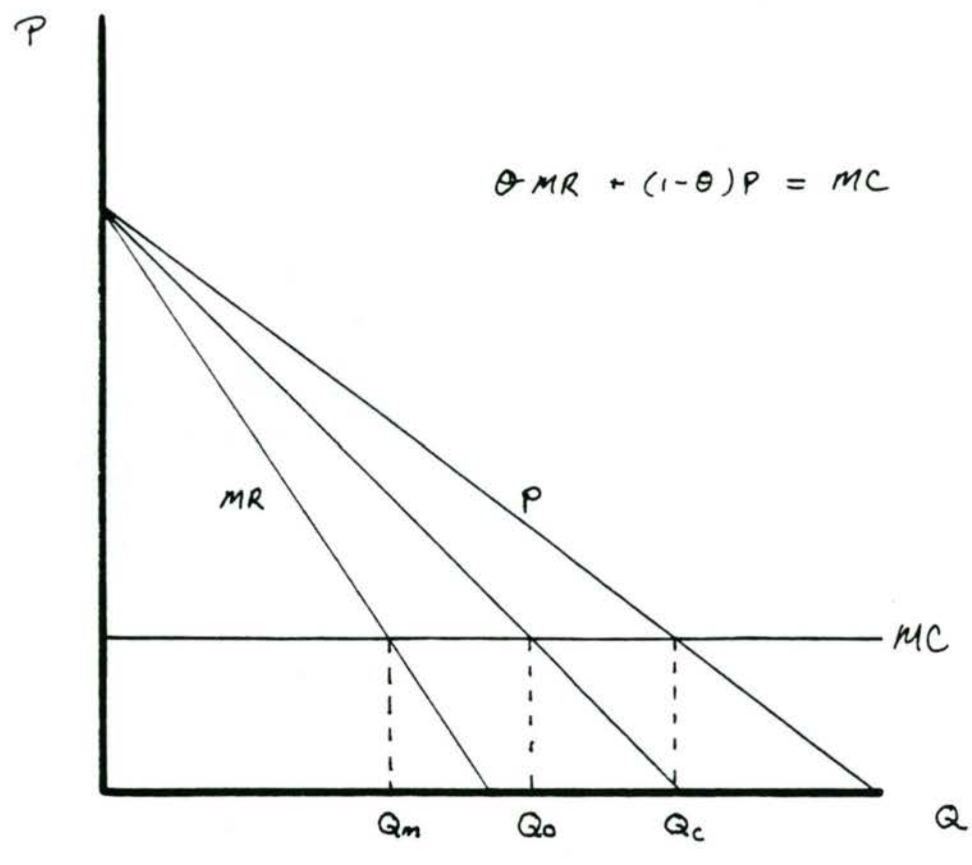
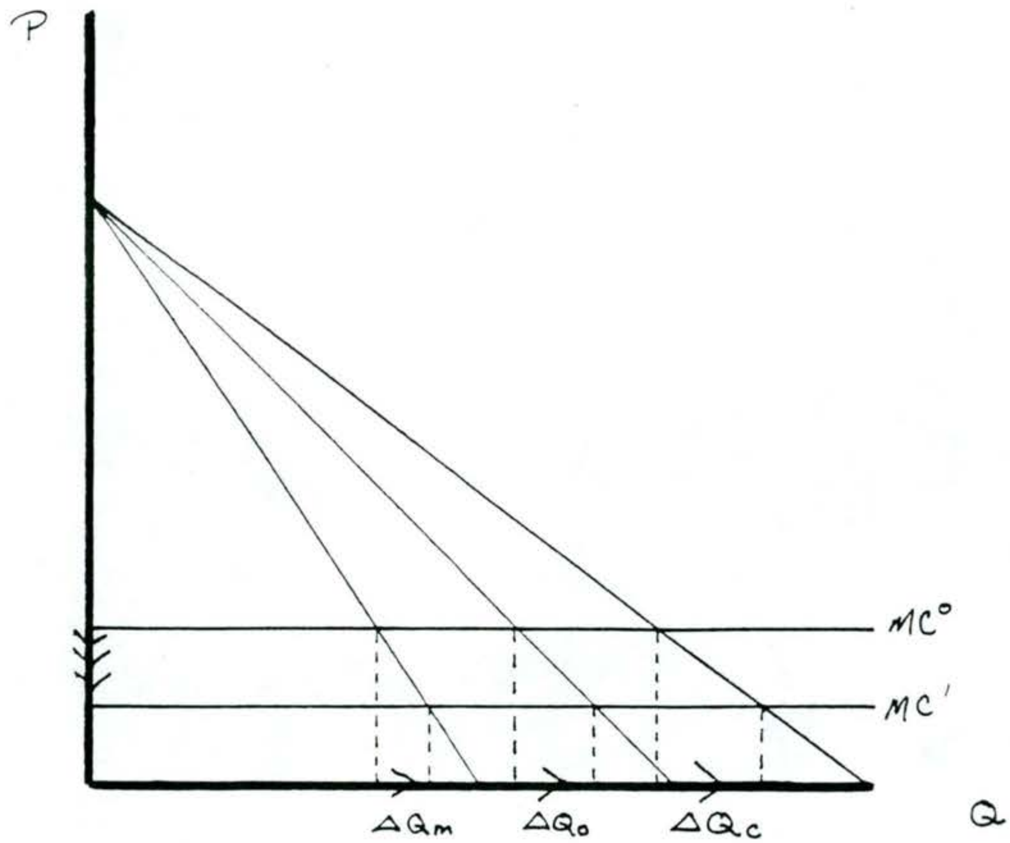
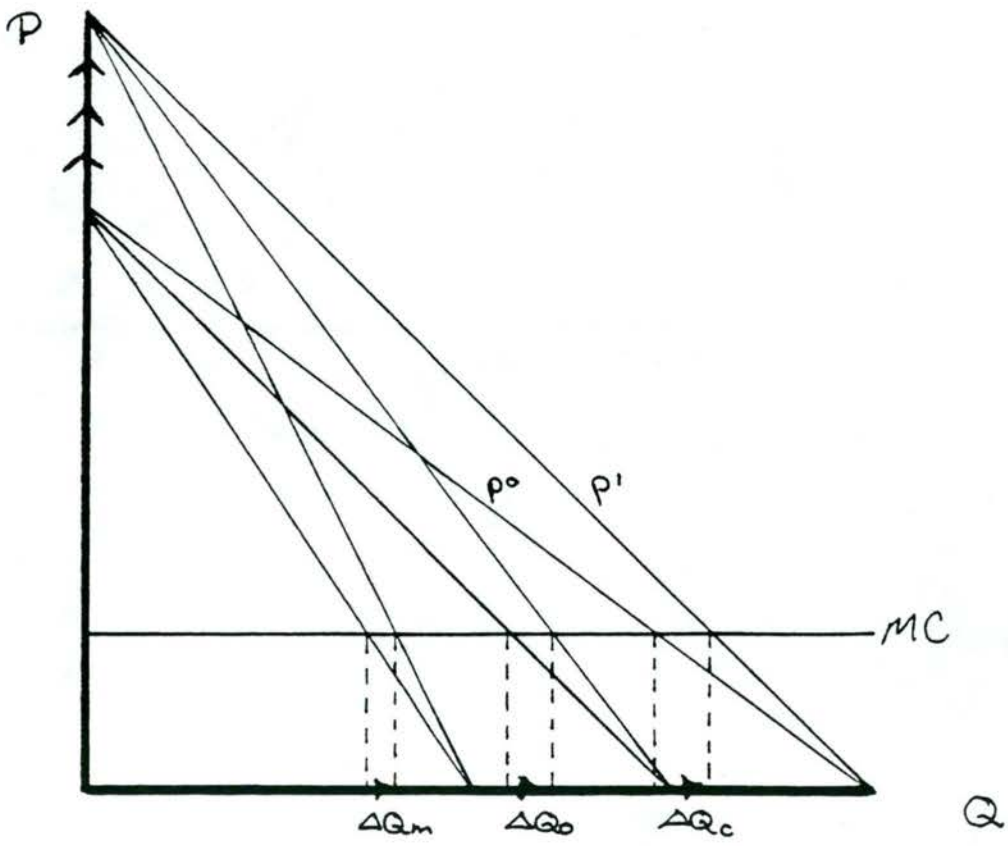


Figure 1. Static Equilibrium



2a. Supply Shift.



2b. Demand Shift

Figure 2. Comparative-Static Equilibria