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HEDGING OBJECTIVES, HEDGING MARKETS, AND THE RELEVANT RANGE OF HEDGE RATIOS

by

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ABSTRACT

The relationship between a hedger's objectives, choice of hedging market, and optimal hedge ratio is assessed. Propositions tested show that risk minimization is a subset of the utility maximization objective of all hedgers; optimal strategies can involve futures, options, or cash markets; and utility maximizing hedge ratios can be greater than one or less than zero.

HEDGING OBJECTIVES, HEDGING MARKETS, AND THE RELEVANT RANGE OF HEDGE RATIOS

The theory of hedging and empirical analyses of hedging activities both have not addressed three important issues: the objectives of hedging, the choice of which market to use when hedging (forward cash, futures, or options) and the relevant range of hedge ratios. Previous studies have assumed that the objective of hedging is either to minimize risk or to maximize utility (Cecchetti *et al.*, Chavas and Pope, Feder *et al.*), but have not directly considered how hedgers choose between the objectives. Yet, as Kahl and Witt *et al.* point out, that assumption concerning objectives is significant because it alters the optimal hedge ratio. Also lacking in the literature are direct comparisons between futures and options hedging strategies; the two markets are evaluated separately, never addressing the question of which is preferable in particular situations. However, hedgers are not indifferent to market choice. Peck points out that although commercial firms have used futures in three ways (for operational convenience, anticipatory pricing, and arbitrage), anticipatory pricing appears to be the sole commercial use of options. She notes that options are preferred when price expectations are strong, but futures are preferred over options in other situations. Finally, the literature's scant discussion of hedge ratios greater than one or less than zero mostly concerns financial markets in which the cash and futures product are not identical. These ranges have been considered speculation (Feder *et al.*, Meyer and Robison) or simply irrelevant to commodity hedgers because they represent market activities that potentially increase, rather than reduce, absolute levels of risk. This approach assumes that hedgers are highly risk averse (Bond and Thompson) and use futures (and options) markets only to reduce existing total risk exposure. A contrary opinion, based on Working's view of hedgers, is that it is rational for market participants to use futures and options markets as vehicles to adjust their total level of risk exposure up or down as dictated by their expectations of resulting utility (Cecchetti *et al.*). Therefore, the hedge ratio range relevant in a particular situation is altered by the objective chosen by a hedger.

The purpose of this paper is to demonstrate the relationship between an individual hedger's objectives, choice of hedging market, and optimal hedge ratio. Specifically, issues involved in making each of these decisions are addressed. First, a standard model is presented for the case of inventory-holding hedgers. Then, using the model, the effects of factors relevant to each decision are illustrated through evaluations of three general propositions concerning hedging. It is argued that all hedgers consider the same factors, but the conclusions reached by each person will vary with market circumstances.

I. THE MODEL

A person holding a cash market inventory who wants to maintain (or improve) the value of that inventory could do so by becoming a short futures hedger. With inventory value maintenance as the goal, the hedger's profit function at time t can be specified[1] as

(1) $$E_t(\pi_{t+i}) = K\,[E_t\,(CP_{t+i}) - CP_t] + X[FP_t - E_t\,(FP_{t+i}) - D_m]$$

where:

π = profit measured as change in wealth between time t and $t+i$,

E = the expectations operator,

K = the total number of units of inventory held for cash market sale at time $t+i$,

X = the total number of units sold short in the hedging market (or purchased if negative),

CP = cash price,

FP = futures price,

D_m = transactions costs of hedging in market *m* (these costs are defined as being zero for cash market trades; for futures trades costs include *f* [which is brokerage fees and interest expense on futures margin], and for options costs include *f* and/or *O* [options premium expense]).[2]

The variance in expected profits using the futures market to hedge is

$$\text{Var } E(\pi_f) = K^2\sigma_c^2 + X^2\sigma_f^2 - 2KX\sigma_{cf} \tag{2}$$

where σ_c^2 and σ_f^2 are the variance in price changes of the cash and futures markets, respectively, and σ_{cf} is the covariance between cash and futures price changes.

Recent research concerning hedging has focused on determining the optimal hedge ratio, defined as X/K. Studies which have assumed that the objective of hedging is to minimize risk have set the partial derivative with respect to X in Equation 2 equal to zero and solved for X/K to find

$$\frac{X}{K} = \frac{\sigma_{cf}}{\sigma_f^2}. \tag{3}$$

If utility maximization is assumed to be the objective, the focus moves to the certainty equivalent of $E(\pi)$, which Freund shows is

$$E(U) = E(\pi) - \frac{\lambda}{2}\,[\text{Var } E(\pi)] \tag{4}$$

where U is utility and λ is a risk-aversion parameter which is positive for risk-averse hedgers. The first order conditions for Equation 4 gives the utility-maximizing hedge ratio:

$$\frac{X}{K} = \frac{\sigma_{cf}}{\sigma_f^2} + \frac{FP_t - E(FP_{t+i}) - D}{K\,\lambda\,\sigma_f^2} \tag{5}$$

II. THE OBJECTIVE OF HEDGING

If hedgers are viewed as investors, the motive for all market activities is to earn a return. Risk associated with an investment is considered by risk-averse investors to be a by-product of market activity, not the object. The incentive for any action is the expected return, risk is a disincentive. With this perspective it is unrealistic to assess business decisions using risk as the only criterion. Both return and risk must be considered when investment decisions are made. Therefore, objective functions must focus on risk-adjusted investment return (utility) to realistically reflect the decision framework of hedgers. Equation 4 is an example of such a function.

This view of all investors being utility maximizers does not preclude hedgers from acting like risk minimizers at some points in time. The objectives of utility maximization and risk minimization are not mutually exclusive, as portrayed in much of the hedging literature. In fact, it is argued here that risk minimization is a subset of the utility maximization objective of all hedging activity.

PROPOSITION 1: The two alternate objectives of risk minimization and profit maximization are both pursued by every hedger. The choice of which objective is pursued at any point in time is made based on expectations concerning factors reflecting the relationship between cash and futures prices.

General Proof: If the second portion of Equation 5,

$$\frac{FP_t - E(FP_{t+i}) - D}{K\lambda\sigma_f^2}$$

equals zero in some circumstance, then Equation 5 is identical to Equation 3, implying that a utility-maximizing hedger will adopt a trading strategy aimed at minimizing risk in that situation.

Q.E.D.

REMARK 1: Any factor which causes the second expression in Equation 5 to equal zero is a sufficient condition to change a hedger's behavior temporarily from that consistent with an objective of utility maximization to that of risk minimization. For example, if stable (unbiased) prices are expected [$FP_t = E(FP_{t+i}) + D$], all hedgers behave as risk minimizers during that period even though they are still maximizing utility.

To maximize utility, $E(\pi)$ from Equation 4 must be known. If $E(\pi)$ is not known, maximizing utility requires minimizing Var $E(\pi)$ in Equation 4. This means that factors affecting profit expectations affect which objective guides hedging strategies over each period of time. Since profit is defined in Equation 1 as a function of cash and futures prices, significant factors include a hedger's risk aversion, confidence in a forecast of expected prices, correlation between cash and futures prices, the nature of basis, and the ratio of basis variance to cash price variance. Specific proofs demonstrating how each of these five variables influence the choice of hedging objective follow. Interactions between the variables are summarized in Exhibits 1a, b, c, and 2a.

Proof 1a: *Risk aversion*. As $\lambda \rightarrow \infty$, the second expression in Equation 5 approaches zero, making $E(\pi)$ less significant in Equation 4 and making price expectations [as they affect Var $E(\pi)$] more sensitive in the choice of objective.

Q.E.D.

REMARK 2: It is unrealistic to use $\lambda = \infty$ because this means that returns are never significant to the hedger. Such an assumption is inconsistent with any investment activity other than those offering "risk free" returns.

Proof 1b: Confidence in forecast of $E(FP_{t+i})$. The effective price used in calculating profits is [$E(FP_{t+i}) \pm \varepsilon$], where ε is a confidence interval. As a hedger's confidence in $E(FP_{t+i})$ increases, ε decreases and the value of the second expression in Equation 5 becomes more clear (statistically significant), enabling its effect to be included in that hedger's decision process regarding the hedging objective. Defining $[FP_t - E(FP_{t+i}) - D]$ as $\Delta\pi_f$, it is clear that if $\Delta\pi_f = 0$, Equation 5 reduces to Equation 3 and risk minimization is the objective of hedging activities . Therefore, the probability of a profit (or loss) from a short hedge affects the choice of objective. That probability can be estimated as 1-z, where z is the probability from a normal (or other) distribution table using *n* (the number of standard deviations of futures prices) from

$$(6) \qquad \mathrm{Prob}_t \mid \Delta\pi_f \mid \geq n(\sigma_f) \quad \text{or} \quad \mathrm{Prob}_t \mid \frac{\Delta\pi_f}{\sigma_f} \mid \geq n.$$

Q.E.D.

REMARK 3: Having high confidence of a forecast profit (or loss) from a price change in the short term is consistent with the "random walk with drift" description of futures prices. If $E_t(FP_{t+i}) = FP_t + i(d)$, prices are expected to fluctuate randomly around a trend line with slope of *d*, which is the drift per time period and *i* is the number of periods covered by the forecast. Profits (losses) will occur for short hedges if $d < 0$ ($d > 0$).

Proof 1c: Correlation between cash and futures prices. This correlation, ρ_{cf}, (standardized covariance) directly affects $E(\pi)$ in Equation 1 and Var $E(\pi)$ in Equation 2, which both affect utility (Equation 4). For example, if $X = K$ and $\rho_{cf} = 1$, then $\mathrm{Var}(\pi) = 0$ and $\pi = -D(X)$. Since $\sigma_{cf} = \rho_{cf}\,\sigma_c\,\sigma_f$, ρ_{cf} is positively related to the size of X/K in Equation 5. Therefore, as $\rho_{cf} \to 0$, the potential for risk reduction declines. Q.E.D.

Proof 1d: The nature of basis. Basis, B, is defined as $FP_t - CP_t$, with variance: $\text{Var B} = \sigma_c^2 + \sigma_f^2 - 2\,\sigma_{cf}$. If basis is systematic (predictable) during the *i* periods over which a hedge will be held, $E(\pi)$ becomes more important in Equation 4. For example, if X = K = 1, Equation 1 can be rewritten as

$$E_t\,(\pi_{t+i}) = [FP_t - CP_t] - [E_t\,(FP_{t+i}) - E_t\,(CP_{t+i})] - D_m$$

which indicates that profit (loss) per unit equals the amount that basis narrows (widens), adjusted for hedging costs. If basis is not expected to change, expected profit is zero and risk minimization becomes the hedging objective. The probability of a profit (or loss) from a basis change, 1-z, is calculated just as for a price change, explained in Proof 1b:

$$\text{Prob}_t\ |\ \Delta\pi_b\ |\ \geq n(\sigma_b) \qquad \text{or} \qquad \text{Prob}_t\ |\ \frac{\Delta\pi_b}{\sigma_b}\ |\ \geq n.$$

Q.E.D.

REMARK 4: It is possible that the direction of futures price changes may not be known with sufficient confidence, yet basis changes may be predictable with sufficient confidence to make the second expression in Equation 5 significantly different than zero[3], enabling a hedger to behave like a utility maximizer. For numerous commodities, historical basis patterns have identifiable trend (drift) lines for some time periods.

Proof 1e: Ratio of basis and cash price variances. This new measure directly reflects the risk-reducing potential offered by hedging. The Variance Ratio (VR) is defined as basis variance divided by cash price variance.

$$VR \equiv \frac{\sigma_c^2 + \sigma_f^2 - 2\sigma_{cf}}{\sigma_c^2}$$

$$(7) \qquad \equiv 1 + \frac{\sigma_f^2 - 2\sigma_{cf}}{\sigma_c^2}$$

The ratio will always be positive. If $1 > VR \geq 0$, hedging reduces risk and either of the two hedging objectives may be chosen. If $VR > 1$, hedging increases total risk so the focus of any hedging activities must be utility maximization.

Q.E.D.

REMARK 5: This measure also identifies which market (cash or futures) is relatively more volatile. Combined with knowledge of correlation and basis predictability, this aids in forming an estimate of E (π) because it indicates which of the two expressions in Equation 1 may be larger.

III. HEDGING MARKET SELECTION

Once a hedging objective is chosen, a specific hedging strategy must be developed. A major factor in such a strategy involves selecting the type of market in which to hedge: forward cash, futures or options. It is argued here that all three markets should be evaluated jointly, if all are available for the product held in inventory; no market will always be the best choice for any hedger.

The cash market considered as a hedging alternative here is the market through which the product will move as part of the firm's normal business operation. It is assumed that forward cash contracts are always available to hedgers in that market.

The futures and options markets are assumed to both be operated by exchanges and highly liquid so pricing is efficient. The product deliverable on the futures contract is identical to the firm's inventory. Also, the options being evaluated here are assumed to cover futures contracts for the product held, therefore futures prices directly affect the value of the related options.[4]

When hedging using options, profits are calculated using futures prices in Equation 1 (since the options contracts available to a hedger are assumed to cover the purchase or sale of futures contracts), but with the added constraint that the maximum loss from hedging activities is $X(D_o)$. This constraint implies that a sell hedger's strategy would involve only the purchase of (at-the-money) put options, and those options would be exercised only if a favorable futures price change (a decrease) occurred $[(FP_t - FP_{t+i}) > 0]$. If an unprofitable change occurred (in this case, a futures price increase), the options would not be exercised, thereby allowing the hedger to avoid taking a trading loss on those futures contracts. The affect of this constraint is to alter the variance of expected profits from that in Equation 2 to

(8) $$\text{Var E}(\pi_o) = K^2 \sigma_c^2 + \frac{X^2\sigma_f^2}{2} - KX\sigma_{cf}.$$

PROPOSITION 2: A hedger's optimal strategy can involve using either the futures, options, or cash market. The choice between futures and options markets does not depend on the hedging objective; it is a function of the hedger's risk aversion and price (profit) forecast.

Proof: If a hedger's objective is to maximize utility (minimize risk), proof requires only that a comparison of futures and options results from Equation 4 (Equations 3 and 8) vary for that hedger in different circumstances.

First, the case of utility maximizing, ignoring the effect of λ: If the futures price is expected to decrease or remain stable, futures are preferred over options because $E(\pi_f) > E(\pi_o)$ (since $D_f < D_o$) and Var $E(\pi_f) <$ Var $E(\pi_o)$. If an expected change in futures price $[E(\Delta FP)]$ is an increase greater than the time value expense of holding the relevant option, options will become the preferred market; $E(\pi_f) < E(\pi_o)$ because the options are not exercised, and at some point (depending on risk aversion) will compensate for the fact that Var $E(\pi_f) <$ Var $E(\pi_o)$.

Adding the affect of λ: If lower futures prices are expected, risk aversion has no affect on the choice of markets; futures hedging is always preferred. If stable prices are expected, the $E(\Delta FP)$ required for use of options, rather than futures, approaches zero as $\lambda \rightarrow 0$, but $E(\pi_o)$ must always exceed the initial profit level or futures will be preferred. Selective futures strategies may have expected profits equaling those of options strategies, but possibly lower D_{sf}.[5] When higher prices are expected, cash markets replace options as $\lambda \rightarrow 0$ because $E(\pi_c) > E(\pi_o)$ (since $D_o > D_c = 0$) and higher profits eventually overcome the higher variance in cash market profits. Speculative futures, rather than call options, are used when the optimal hedge ratio is below zero because $D_o > D_f$, making $E(\pi_f) > E(\pi_o)$.

Finally, in cases where risk minimization is the objective: Setting the partial derivative of Equation 8 with respect to X_o equal to zero and solving for X/K gives σ_{cf}/σ_f^2. Since this result is identical to Equation 3, the hedger is indifferent to the choice of market; K is given so the risk-minimizing X is the same whichever market is used. Therefore, since the risk objective is met with either market, the choice is made based on other factors. Those factors must be the same as in the utility maximizing case: expected price and risk aversion.

Q.E.D.

REMARK 6: It appears that a highly risk averse hedger would not use options due to the higher Var $E(\pi_o)$, compared to futures hedging, even though all the increase in variance is due to <u>increases</u> in profit opportunities. It is counter-intuitive that such hedgers would avoid markets which offer higher potential profits, lower maximum losses which are known at the time of a trade, and no change in the probability of suffering a trading loss. Therefore, when selecting a hedging market, it is more realistic to define "risk" as adverse affects on profit: semivariance (see Holthausen).

COROLLARY: Options are always preferred to futures by highly risk averse hedgers when favorable cash prices are expected. This preference is due to options offering a higher expected profit[6] and equal variance in expected profits, when measuring returns and risk using the semivariance approach.

REMARK 7: Exhibit 2b summarizes the interactive affects of a hedger's risk preference and price forecast on the choice of hedging market. Using futures is a rational choice in more of the scenarios than either options or cash, and futures is the only rational choice when a hedger expects significant adverse cash price movements during the period a hedge is to be held, regardless of risk preference.

IV. HEDGE RATIO RANGE

The final hedging strategy decision to be made concerns the size of the hedge trade, usually expressed as the hedge ratio, $HR = X/K$. In the past, X was usually expected to be $100\% \geq X \geq 0\%$ of the cash position, giving a hedge ratio of $1 \geq HR \geq 0$ using Equation 3, implying a risk minimization objective for all hedging activities. The traditional limitation on hedges of $HR < 0$ and $HR > 1$ is based on the idea that such hedges raise absolute levels of risk (as risk is usually defined) and, therefore, are inconsistent with the (traditional) objective of hedging.

The main characteristics of the definition of hedging used by the CFTC are that to be considered a "hedge," futures transactions must be related to a firm's cash position and reduce that firm's level of business risk. It is argued here that this definition of permissible hedge trades must be interpreted to include transactions which lower relative risk levels (risk versus return) to be consistent with the concept of utility maximization.

When utility maximization is the objective of hedging, the results of trading are added to the risk-minimizing HR, as shown in Equation 5, either raising or lowering the HR, possibly beyond the traditional limits of one or zero. As will be shown below, factors which influence the size of the optimal HR are a hedger's futures price (profit) forecast and confidence level.

PROPOSITION 3: The utility maximizing hedge ratio can be greater than one or less than zero.

Proof: Equation 5 is used with simplifying assumptions that $\sigma_c = \sigma_f = K = \lambda = 1$. This gives

$$X = \rho_{cf} + [FP_t - E(FP_{t+i}) - D].$$

The only unknown in this equation is $E(FP_{t+i})$, so the profit affects on HR depend on the hedger's price forecast and confidence interval (ε) for that forecast. The effective price used in the calculations is $[E(FP_{t+i}) \pm \varepsilon]$. If confidence is low, ε is large and $E(\pi_h)$, which equals $[FP_t - E(FP_{t+i}) - D]$, will not be significantly different than zero and the optimal HR is at the traditional risk-minimizing level between one and zero (ρ_{cf} in this case). Over time (i.e. as the forecast period becomes shorter[7]), the confidence interval shrinks and HR approaches one (zero) if $E(\pi_h)$ is greater (less) than zero. If confidence is high, the affect of $E(\pi_h)$ will be significant in the calculation of HR. In this case utility maximization requires that

$$\text{when } E(\pi_h) > 1 - \rho_{cf}, \quad \text{then } X > 1, \text{ and}$$

$$\text{when } E(-\pi_h) > \rho_{cf}, \quad X < 0$$

(noting that the simplifying assumptions have made X = HR).

Q.E.D.

REMARK 8: The interactive affects of a hedger's price forecast and confidence interval for that forecast on the size of the utility-maximizing hedge ratio are summarized in Exhibit 2c. In general, if a hedger is very confident that prices are going to change during the hedging period, the optimal hedge ratio shifts away from the risk-minimizing level. That relationship is illustrated graphically in Figure 1 (see the appendix for an explanation of the figure).

V. CONCLUSIONS

This paper demonstrates the relationship between an individual hedger's objectives, choice of hedging market, and optimal hedge ratio. It is shown that risk minimization is a subset of the utility maximization objective of all hedging activity. As a result, the two alternate objectives are each pursued by every individual hedger at some time. Also, all hedgers behave as risk minimizers during periods in which stable prices are expected, which may explain why early hedging studies assumed risk reduction to be the objective of hedging.

In a joint analysis of futures, options, and cash markets, it is found that a hedger's optimal strategy can involve using any of the markets. The choice between futures and options markets does not depend on the hedging objective. Also, futures are found to be the best hedging market in more circumstances than are options, which is consistent with the relative level of usage commercial firms make of those markets.

The optimal hedge ratio range depends on the hedging objective. If risk minimization is the chosen objective, the ratio will be between zero and one. The utility maximizing hedge ratio can be greater than one or less than zero if a hedger is confident of expected price levels.

APPENDIX: Graphical Analysis of Optimal Hedge Ratios

In Figure 1 the expected return calculated from equation 1 is plotted on the left vertical axis and the variance in returns (measured using equation 2) is plotted on the horizontal axis. Note that the exact scale and location of the zero point on the expected returns (left vertical) axis is determined by the expected cash and futures market prices. Values on that axis increase for points closer to the top of the figure if favorable futures and unfavorable cash prices are expected. Values increase for points closer to the bottom of the figure if unfavorable futures and favorable cash prices are expected. The right vertical axis represents the hedge ratio and has a fixed scale and zero point at all times. The location and shape of the curve YMZ is determined by the historical cash and futures price data used in the calculations. Different data sets would, therefore, be expected to produce different curves.

The YMZ curve in Figure 1 represents the opportunities available to a hedger. Point F represents the expected return and variance of returns associated with a fully hedged inventory (HR = 1). In the case of this traditional, full hedge, the variance in returns is the basis risk. The size of this risk depends on the level of correlation between the relevant cash and futures prices (as explained earlier). The expected return and variance associated with an unhedged inventory is represented by point U. Notice that in this example, neither point F or U represent the minimum risk hedge. Point M is the hedge position which offers the lowest level of variance in returns for these markets. Notice also that the variance at point M is not zero. If the cash and futures prices were perfectly correlated (either positively or negatively), point M would be on the left vertical axis, indicating zero variance in returns.[8]

Only half of the opportunity curve is relevant to a hedger at any time. For example, if favorable futures and unfavorable cash prices are expected, the left vertical axis is scaled for increasing returns as one moves toward the top of the figure. This means that moving from point M toward point Y identifies opportunities with increasing expected returns and variance. For each of those points, there is a point with equal variance on the segment of the curve from M to Z. However, no point on the MZ segment would be chosen given current price expectations because they offer expected returns lower than those offered by points making up the MY segment. Therefore, only the most positive half of the opportunity curve (beginning with the minimum-risk hedge) is considered by a hedger.[9]

The hedge ratio chosen depends on the indifference curve of the hedger. In Figure 1 an indifference curve, labeled I, is drawn as an upward sloping convex curve, indicating that the hedger is risk averse to some degree. The point of tangency, H, identifies the utility-maximizing hedge ratio of a hedger that has no uncertainty about expected prices. However, normally some uncertainty will exist, meaning that the hedger has less than 100% confidence in his price forecast, requiring the opportunity set to be adjusted. In this case, a confidence band is placed around curve YMZ to get curve Y*M*Z*.[10] The optimal hedge in the uncertain case is found at point H*.

Figure 1 illustrates that the effect of increased (decreased) price uncertainty is a widening (narrowing) of the confidence band, shifting the opportunity curve to the right (left), which causes the utility-maximizing optimal hedge ratio to move toward (away from) the risk-minimizing hedge ratio. This implies the need to adjust hedge ratios as confidence levels change over time. Also, wider confidence bands cause the curve to collapse such that the relevant range of hedges may not include ratios greater than one or less than zero in those circumstances.

Finally, it is also clear from Figure 1 that confidence in price forecasts affects a hedger's certainty-equivalent return. That return is the point at which the indifference curve intersects the expected return axis. As uncertainty increases (decreases), the confidence band widens (narrows), and the certainty-equivalent return declines (rises).

FOOTNOTES

1. This base specification is for futures hedging. Hedging with options requires a constraint on the equation, as discussed in section III.
2. The option premium expense equals the change in an option's time value. Increases (decreases) in intrinsic value are considered options profits (losses). Expenses incurred when exercising an option are *f* plus *O*. If an option expires worthless, or if it is sold back to the market, D_o expenses include only *O*.
3. Although "basis" does not explicitly appear in the second (speculative) expression in Equation 5, it would if Equation 1 was rewritten in the basis version (discussed in Proof 1d). In that case, basis change would replace futures price change in the expression (Batlin). This illustrates the notion that "hedging is speculating on the basis."
4. This means that price changes on the futures contract underlying an option which is "in-the-money" will change the option's intrinsic value by an amount equal to the resulting change in futures contract value.
5. Selective futures trading costs will be lower than options trading costs if no futures position is actually held or if the size of positions (the hedge ratio) are reduced by circumstances. Either of these situations are possible only if futures price volatility proves to be low during the hedging period. In volatile markets, selective futures strategies may result in numerous transactions being made, which raises D_{sf} significantly, but may also raise actual profits received, yet the level of those profits cannot be estimated *a priori*.
6. Options have a maximum loss of the premium paid, but potential futures losses are not limited (they are a function of the probability distribution of expected prices).

7. Merrick shows that the amount of time before futures contract maturity affects the minimum-risk hedge ratio such that underhedging and upward revisions of the ratio are required as expiration approaches.
8. It is expected that point M would be the farthest from the left vertical axis if the cash and futures prices were uncorrelated, and would be increasingly close to the axis as correlation between the prices increased. If the correlation is positive (negative), a short (long) futures position would be used to hedge a long cash position. If the prices are perfectly correlated and an identical product is being traded in the cash and futures markets, point M will touch the left vertical axis at the point for a hedge ratio of 1.0 (or –1.0 for negatively correlated prices). If the products are not identical, a zero-variance hedge is much less likely, but could occur with hedge ratios other than (–)1.0.
9. If favorable cash and unfavorable futures prices are expected, the "bottom" half of the opportunity curve is used. In this case, Figure 1 can be "flipped over" so that the positive end of the left vertical axis is at the top. In doing so, the scale of the right vertical axis (the hedge ratio) is reversed: the negative end of the scale would be at the top of the figure.
10. Although confidence bands exist on both sides of YMZ, only the band "to the right" of the original curve is relevant because it represents adjustments due to increased risk (in the semivariance definition), while the band "to the left" of YMZ represents the high end of the range of possible returns. The relevant confidence level is that chosen by a hedger.

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Figure 1. Confidence in Expected Prices and the Optimal Hedge Ratio Range

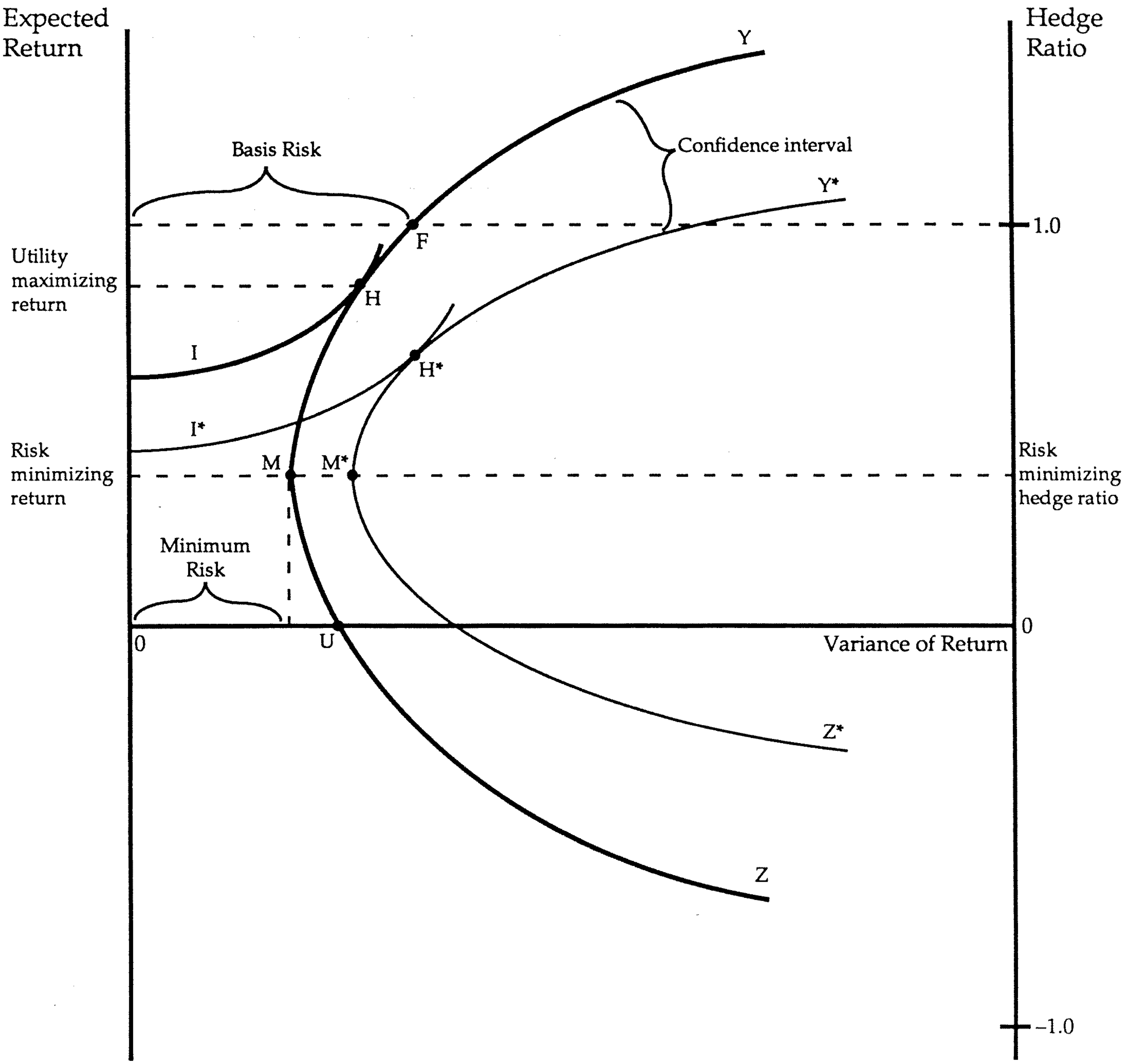

Exhibit 1c: Relationship Between the Variance Ratio and Price Correlation

	Large Basis Variance to Cash Price Variance Ratio	Small Basis Variance to Cash Price Variance Ratio
Low Correlation	Hedging may be feasible. Implies basis patterns dominate correlation between futures and cash, therefore util max may be best. Futures market is making adjustments.	Unlikely case because it is possible only when absolute prices in prices in futures and (especially) cash markets are highly volatile. There is limited hedge potential because risk and returns could both suffer in the short run.
High Correlation	Possible only when cash and futures price trends are both flat (especially cash). There is little potential for profits from hedging, and price risk is low, so risk could increase if hedged (assuming a confident forecast of continued flat prices).	Risk minimization could dominate because basis variation is small.