ADVERSE SELECTION AND TAX EXTERNALITIES
IN A MODEL OF ENTREPRENEURIAL INVESTMENT

by

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Working Paper No. 87-9
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June 1987

ABSTRACT

This paper models a financial market in which entrepreneurs are endowed with private information about the quality of their return distributions, but must raise investment funds from external agents. Using the equilibrium concept developed originally by Wilson and Miyazaki, the entrepreneurs are shown to self-select by choice of financial contract, with high quality agents over-investing relative to the perfect information outcome. This equilibrium is known to be constrained efficient in the absence of "outside" agents who are affected by market outcomes but not parties to market transactions. However, as a recipient of taxes, government represents such an "outside" agent. In the presence of government, this paper shows that Pareto-improving interventions are not only possible, but take a form which is frequently observed in practice, namely, credit subsidies. Moreover, this constrained inefficiency cannot be attributed to tax-induced distortions; rather, it is due to the existence of taxation and consequent external affects on taxpayers.
I. Introduction

Several economists have investigated the nature and efficiency of equilibrium in models with adverse selection problems. One product of this research has been an equilibrium concept which avoids existence problems and which achieves outcomes that are constrained efficient in the following sense: given informational constraints (i.e., the ability of an agent to masquerade as a "quality" type different than his own), there is no other set of feasible outcomes that Pareto dominates the equilibrium. However, this efficiency property has only been derived for models which incorporate participants in the particular market under investigation, ignoring outside agents who may also be affected by contractual arrangements in this market. This paper is concerned with a particular set of outside agents, taxpayers. Since government may have tax claims which can be altered by a change in the equilibrium, the following question arises: Is constrained efficiency robust to the presence of a taxation mechanism?

The answer given here is: No. Specifically, this paper models a financial market in which entrepreneurs are endowed with private information about the quality of their return distributions, but must raise investment funds from external agents. It first characterizes the equilibrium without taxation, deriving an "over-investment" result. The analysis then introduces the simplest possible taxation mechanism, a proportional profit tax. This mechanism is particularly interesting because its presence does not alter the equilibrium. Thus, inefficiencies cannot be attributed to tax-induced distortions. Nevertheless, its presence leads to constrained inefficiency in most interesting cases; government can then generate a Pareto improvement by
directly intervening in financial markets. This intervention takes a form which is frequently observed in practice, namely, debt subsidies. Notably, these results are not attributable to a constraint on the choice of tax mechanism. Though a proportional tax policy is simply assumed at the outset, this policy is shown to be optimal in an example which is consistent with the earlier analysis.

These conclusions are of more than theoretical interest. Government intervention in credit markets is widespread, both in the United States and elsewhere. In the United States, examples include subsidies to small businesses, via the Small Business Administration, and to farmers, explicitly via the Farmers Home Administration and implicitly through government support of the Farm Credit System. Economists often criticize these programs as leading to an over-allocation of credit to the subsidized sectors (e.g., Barry (1981), Duncan (1983)). In the following analysis, this view is called into question.

Several other recent papers have also recognized the importance of asymmetric information in credit markets and the attendant implications for government intervention. Notable in this regard are papers by DeMeza and Webb (1987) and Mankiw (1986). In both of these analyses, as here, there is an informational asymmetry which relates to characteristics of the borrower's profit distribution. Moreover, both find a positive role for government; in Mankiw, debt subsidies can correct under-investment; in DeMeza and Webb, over-investment can be corrected by a credit tax. However, neither paper permits borrowers to signal their type by choice of contract.

In striking contrast, the economic processes driving present results center on information transfer. This distinction explains the coincidence
here of over-investment and an optimal policy of credit subsidy. Essentially, over-investment by high quality entrepreneurs is a necessary cost of signaling. By subsidizing debt, government can reduce the low quality agent's incentive to lie, thereby enabling the high quality agent to choose an investment level closer to his perfect information choice. Via its tax receipts, government gets a share of the social gain from this move. Thus, provided the cost of the subsidies is not too large—and it is not when the subsidies are sufficiently small—a Pareto improvement can be achieved.

The balance of the paper is organized as follows. Section II describes the model (and equilibrium). Section III characterizes the equilibrium without taxes. Section IV introduces a common proportional tax, deriving the key inefficiency result of the paper. Finally, Section V presents an example demonstrating that the earlier results do not emanate from constraints on tax and contractual forms made at the outset. Section VI provides some concluding observations.

II. The Model

A. Entrepreneurs.

Consider a population of risk neutral entrepreneurs who are of two types: high quality (H) and low quality (L). Each knows his own quality but others do not. Further, each produces a stochastic end-of-period net worth (also called profit) \( \pi = \pi(q, A, \theta) \), where \( q \) represents quality type, \( A \) denotes the initial asset value of the firm and \( \theta \) is a random variable. Higher quality and larger asset bases both give rise to "better" net worth distributions in the sense of first order stochastic dominance. Assets, \( A \), are financed by external funds equal to \( I \) dollars and a given amount, \( (A - I) \), of the
entrepreneur's initial wealth. Without loss of generality, each member of the population is assumed to commit a common amount of his own funds to the firm.6

B. Investors.

Investors are risk neutral and competitive in a sense to be made clear later. While ignorant of any particular entrepreneur's quality, they are assumed to know the structure of the system. In other words, they are aware of the choice problem which faces an entrepreneur of quality q, the distribution of qualities in the population, the equilibrium behavior of other investors (if relevant) and the rules of the market game. If different quality types choose different financial contracts (or different menus of contracts), investors will be able to infer quality from this choice. In addition, the usual small sector assumption will be made; that is, changes in entrepreneurial investment do not alter the opportunity cost of funds to investors.

C. Financial Contracts.

At the outset, the set of available financial instruments will be limited to standard debt and equity forms. Debt yields the minimum of a promised fixed payment (z), and the net worth of the firm. Equity gives a fixed share of net worth after debt payments. In earlier work (Innes (1986)), I have shown that this setting yields equilibrium capital structures which are all-debt. The latter "all-debt" result will be taken as an assumption for this analysis. Later (in Section V), the foregoing debt/equity constraints on contractual forms will be removed in order to demonstrate that the paper's results do not hinge on this construction.
D. Market Equilibrium.

Following Miyazaki (1977), equilibrium will be defined as a set of high and low quality contracts, \((I_H, Z_H)\) and \((I_L, Z_L)\), which maximize the high quality entrepreneur's net profit subject to the following constraints:

(M1) Incentive Compatibility: Neither quality type prefers the other's situation to his own.

(M2) Investor Rationality: Expected investor profits on the two contracts, net of the opportunity cost of funds, is zero.

(M3) Low Quality Rationality: The low quality entrepreneur receives a net expected profit which is at least as high as he would receive with the perfect information contract.

This equilibrium concept is chosen because of its well-known efficiency, existence and uniqueness properties, as well as its consistency with two behavioral specifications which are prominent in the asymmetric information literature. That the equilibrium exists and is unique follows from a trivial restatement of Miyazaki's proofs. Further, since the equilibrium solves a constrained maximization problem, any other feasible set of contracts must make at least one of the three affected agents worse off, whether it be the high quality entrepreneur, the low quality type or the investor. This property, namely, the inability to make Pareto improvements, has been called constrained efficiency (Harris and Townsend (1981), Crocker and Snow (1985)), and will be adopted here as the relevant efficiency concept.

The two behavioral specifications which lead to a Miyazaki equilibrium are as follows:
(1) Wilson (1977) Responses. Suppose that investors (rather than entrepreneurs) make the first move in the market game, offering a menu of contracts to prospective loan applicants. In addition, let investors be competitive in the following sense: due to free entry, they will offer any set of contracts which is expected to yield nonnegative mean profit (i.e., expected returns not less than $\rho$, the riskfree rate of return); however, in contrast to Rothschild and Stiglitz (RS) (1976) price-quantity competition, they are permitted to have nonstatic (i.e., non-Nash) response expectations. As in RS, investors do not only compete over a single attribute, price, but rather over multiple attributes, which here include the fixed payment $(z)$ and the investment level $(I)$. Unlike in RS, when investors consider making a new set of contract offers, competitors will be expected to respond by withdrawing all contract offers that were in their original menu but become unprofitable in the presence of the new offers. In this setting, a Miyazaki equilibrium will satisfy the following conditions: (i) it will earn investors zero expected profits and (ii) given response expectations, no investor will want to offer a nonequilibrium set of contracts. To understand the equivalence between the two equilibrium definitions given so far, suppose that the set of contracts satisfying the latter conditions does not solve the maximization problem stated earlier. Then an investor can offer the set of contracts which does solve this maximization problem, with a slight modification—infinitesimally higher fixed payments. Since the proposed
equilibrium must satisfy the same three constraints as the maximization solution, the high quality entrepreneur will prefer the new offer. Moreover, since the perfect information low quality contract earns investors zero expected profits, the third constraint (M3) implies that investors earn nonpositive profits on any equilibrium low quality contract and thus (given constraint (M2)), nonnegative profits on any equilibrium high quality contract. Hence, the proposed new contract will always be profitable to investors, contradicting condition (ii).

(2) Myerson (1983) Bargaining. Suppose now that the entrepreneur (rather than the investors) makes the first move in the market game, proposing a menu of contract offers which the investor can either accept (by agreeing to offer this menu) or reject. Further, let investors be competitive in the sense that they will accept any menu proposal which they expect to earn them nonnegative expected profit. As before, all agents are rational and, thus, any equilibrium set of contracts must satisfy constraints (M1) - (M3). In addition, assume that any menu proposal which is not eligible for an equilibrium by this criterion is simply rejected (by investors) as infeasible. High quality agents will then propose the menu which gives them highest net profit among those satisfying (M1) - (M3), namely, the solution to the above maximization. Hence, if low quality agents propose any other feasible menu proposal which they prefer to the high quality proposal, they will be revealing their type to the investor. In this event, they cannot do better than with the high
quality menu. Thus, low quality types are also forced to propose the high quality menu choice and the Miyazaki equilibrium emerges.

E. Government.

Government, which is also assumed to be risk neutral, can levy taxes on both entrepreneurs and investors. However, since this paper is not concerned with the effects of taxation on general equilibrium in capital markets, tax policy for investors is taken as fixed and, hence, investors have a fixed pre-tax expected return requirement equal to the return on a taxable riskfree bond. For entrepreneurs, government is thought to set tax parameters in the following way: Knowing the market equilibrium which each tax policy induces, it will choose a set of parameters which yields a given expected revenue and for which the equilibrium is not Pareto dominated by that produced with any other eligible tax policy. However, to illustrate the main point of this paper as clearly as possible, I will first impose a given tax policy, one with a common share tax, $\alpha$. An example will then show that this policy can be optimal in the sense just described.

Note that the central limitation placed on government here is the lack of direct intervention in the loan market. The object of this paper is to explore the prospects for welfare-enhancing interventions of precisely this type.

F. A Key Assumption.

Define, respectively, expected firm profit and expected firm profit net of debt payments as follows:

\[
\tilde{\pi}_q(1) = \int_0^\infty \pi \, f(\pi; q, 1) \, d\pi
\]
where \( f(\cdot) \) denotes the profit density function, \( \pi'(I) > 0, \pi''(I) < 0 \)

\[
\lim_{\pi \to \infty} \pi'(I) = \infty, \quad \text{and} \quad \lim_{\pi \to 0} \pi'(I) = 0.
\]

The latter constraints ensure positive and finite equilibrium investment levels for both quality types. In addition, the following assumption will be made in order to establish a correlation between quality and net firm payoffs which is central to the analysis.

**Assumption:**

\[
(3) \quad MRS_H(I, z) = \frac{-\pi^*_H(I, z)}{\pi^*_H(z)} > \frac{-\pi^*_L(I, z)}{\pi^*_L(z)} = MRS_L(I, z)
\]

for all \((I, z)\), where \(\pi^*_q(\cdot)\) denotes the partial derivative with respect to variable \(x\). This assumption is quite natural; it implies that high quality entrepreneurs' marginal investment returns are sufficiently large relative to those of low quality entrepreneurs that, in exchange for additional investment funds, the high quality agents can give up larger fixed payments and still preserve their net expected profit.

**III. Equilibrium Without Taxes**

To begin, consider the case of no entrepreneurial taxes. This case can be illustrated graphically as in Figure 1. Here, \(OC_H\) and \(OC_L\) represent the sets of \((I, z)\) combinations which give the investor zero expected profit when the borrower is high and low quality, respectively. Formally, these separating offer curves are defined as follows:

\[
(4) \quad OC_q: \quad R(I, z; q) = \pi_q(I) - \pi^*_q(I, z) = (1 + \rho)I
\]
where \( R(\cdot) \) denotes the gross expected investor return on the loan and \( p \) is the riskfree rate of return. \( \text{OC}_p \) represents a pooling offer curve, namely, the set of \((I, z)\) combinations which give zero expected investor profits when both quality types take the contract. Formally,

\[
(5) \quad \text{OC}_p: \quad \nu R(I, z; L) + (1 - \nu) R(I, z; H) = (1 + p)I
\]

where \( \nu \) is the proportion of low quality entrepreneurs in the applicant population.

The indifference curves, \( \text{IC}_H \) and \( \text{IC}_L \), correspond to \((I, z)\) pairs which give the entrepreneur a given net profit level. Formally,

\[
(6) \quad \text{IC}_q: \quad \frac{\pi^*_q(I, z)}{q} = V = \text{constant}.
\]

Note that condition (3) implies that high quality indifference curves are steeper than the low quality curves. Further, since lower indifference curves are associated with lower fixed payment commitments, they give rise to higher net entrepreneurial profit levels. Hence, the high (low) quality contract which will emerge in a perfect information world is the point on the separating high (low) quality offer curve which is on the lowest possible indifference curve, namely, \( e^*_H (e^*_L) \).

To find the equilibrium contracts in a world of asymmetric information, first consider the following observations (see Appendix A for proofs):

**Lemma 1**: The equilibrium contracts are never pooling (i.e., the same contract for both quality types).

**Lemma 2**: The incentive compatibility constraint for high quality entrepreneurs, \( \pi^*_H (I_H, z_H) \geq \pi^*_H (I_L, z_L) \), is never binding in
equilibrium. Thus, either the corresponding low quality constraint is binding or the equilibrium is the set of perfect information contracts.

**Lemma 3:** The equilibrium low quality contract always entails the perfect information optimal investment level, $I_L^*$. With these Lemmas, the relevant high quality entrepreneur opportunity set, $OC^*$, can be expressed. Above $e_H^0$, high quality agents can choose contracts on $OC_H$ without any risk of low quality masquerading. However, below $e_H^0$, the incentive compatibility constraint (M1) is binding and $OC^*$ is the set of $(I_H, z_H)$ points satisfying the following equation:

\begin{equation}
OC^*:
\nu(\bar{\pi}_L(I_L^*) - \bar{\pi}_L^*(I_H, z_H) - (1 + \rho)I_L^*) + (1 - \nu)(\bar{\pi}_H(I_H) - \bar{\pi}_H^*(I_H, z_H) - (1 + \rho)I_H) = 0.
\end{equation}

Equation (7) expresses the zero investor profit condition (M2), with the first term representing net profits (losses) on the low quality contract and the second term representing net profits on the high quality contract. Note that separation is assumed (Lemma 1), $I_L = I_L^*$ (Lemma 3), and the incentive compatibility condition (M1), $\bar{\pi}_L^*(I_L^*, z_L) = \bar{\pi}_L^*(I_H, z_H)$, has been substituted. In terms of Figure 1, $e_H^0$, $e_H^m$, and $e_H^1$ are all contracts which satisfy (7). The equilibrium high quality contract will be the point on $OC^*$ which is on the lowest high quality indifference curve, namely, $e_H^m$.

Note that, whenever $e_H^m$ is below $e_H^0$, investors make positive expected profits on high quality applicants and losses on the low quality types. This
phenomenon has been called cross-subsidization. The key result of this section can now be stated:

**Proposition 1:** In equilibrium, the high quality entrepreneur's investment level ($I_H^m$) is never less than the optimal perfect information level ($I_H^*$). Moreover, whenever the equilibrium is characterized by cross-subsidization, $I_H^m$ is strictly greater than $I_H^*$.

**Proof:** Consider the second statement first. When there is cross-subsidization, the high quality entrepreneur's net profits are $\bar{\pi}_H^*(I_H, Z_{tt}(I_H))$, where $Z_{tt}(I_H)$ solves (7). Differentiating (7) gives:

$$\frac{dz_{tt}}{dI_H} = \frac{\nu \bar{\pi}_L^* + (1 - \nu)\bar{\pi}_H^* - (1 - \nu)(\bar{\pi}_H^* - (1 + \rho))}{\nu \bar{\pi}_L^* + (1 - \nu)\bar{\pi}_H^*}$$

where all functions are evaluated at $(I_H, Z_{tt}(I_H))$. Differentiating $\bar{\pi}_H^*$, substituting from (8) and evaluating at $I_H = I_H^*$ where $\bar{\pi}_H' - (1 + \rho) = 0$,

$$\frac{d\bar{\pi}_H^*}{dI_H} = -\bar{\pi}_H^* \left( \frac{\bar{\pi}_H^*}{\bar{\pi}_H^*} + \frac{\bar{\pi}_L^* + (1 - \nu)\bar{\pi}_H^*}{\bar{\pi}_H^*} \right) \cdot \frac{\bar{\pi}_L^* + (1 - \nu)\bar{\pi}_H^*}{\bar{\pi}_H^*}$$

Using (3), express $\bar{\pi}_L^*$ and $\bar{\pi}_L^*$ as follows:

$$\frac{d\bar{\pi}_L^*}{dI_H} = k(1 + \epsilon)\bar{\pi}_H^*$$

where $0 < k < 1$ and $\epsilon < 0$. Rewriting (9):
where the inequality follows from $\tilde{\pi}_{HI} > 0$. Proof of the first statement is now trivial. When the equilibrium high quality contract is above $e_0^0_H$, it is the optimal perfect information contract (i.e., $I_0^m_H = I_0^* _H$). If the equilibrium contract is $e_0^0_H$ itself, $I_0^m_H < I_0^* _H$ implies that the equilibrium is above $e_0^0_H$, a contradiction: thus, in this case, $I_0^m_H \geq I_0^* _H$. For $e_0^0_H$ below $e_0^0_H$, there is cross-subsidization and the proposition follows. Q.E.D.

This "over-investment" result is analogous to conclusions in labor market models. Essentially, the cost of entrepreneurs signaling quality here is the excessive investment, just as the cost of good workers signaling their type is excessive education (e.g., Spence (1974), Riley (1975)) or work intensity (e.g., Akerlof (1976), Miyazaki (1977)). This outcome will be central to the following analysis of tax externalities.

IV. Common Proportional Taxes and Constrained Inefficiency

To investigate the implications of taxation for the optimality properties of equilibrium, this section begins with the simplest possible tax regime, a share tax, $0 < \alpha < 1$, common to both quality types. Thus, an entrepreneur of quality $q$ will pay taxes equal to $\alpha \cdot \tilde{\pi}_q^*( )$. This regime is chosen not only for its simplicity, but also for the following noteworthy property:

Lemma 4: Market equilibrium contracts are unaltered by the presence of a common proportional tax on entrepreneurs. Hence, these contracts will also be invariant to the level at which the tax parameter $\alpha$ is set.
Recalling that tax policy for investors, and thus $p$, are being held fixed, Lemma 4 follows directly from the analysis in Section III. This Lemma indicates that any constrained inefficiency in the post-tax equilibrium is not due to a distortion caused by taxation. Nor is it due to any constrained inefficiency in the no-tax equilibrium: as noted earlier, the Miyazaki equilibrium is constrained efficient in this case. Rather, any such inefficiency is attributable to the existence of a third affected party, government, who is not a player in the market and, therefore, whose ability to compensate other agents for contract changes is not considered in the process of equilibrium determination. In other words, any inefficiency is due to an externality created by the existence of taxation.

To determine whether or not there is such an inefficiency, consider the following question: Given $\alpha$, can government compensate investors to deviate from the Miyazaki equilibrium, preserving (or increasing) entrepreneurs' net profits and increasing its own expected tax revenues, net of investor compensation? More specifically, suppose that the Miyazaki equilibrium differs from the perfect information equilibrium. Now consider the policy depicted in Figure 2, namely, moving southwest along the high quality indifference curve through $e^m_H$ (the market equilibrium high quality contract) toward $e^l_H$. The high quality agents' net profits will be preserved with this policy, while low quality agents' net profits will rise (due to condition (3)). The question remains: can government compensate investors for its losses with the new contracts and still receive higher net revenues?

To answer this question, let us write down the net government payoff with this policy:
where $\bar{\pi}_L^*$ and $\bar{\pi}_H^*$ must satisfy the constraints:

\begin{align}
(12) \quad & \bar{\pi}_L^*(I_L^*, z_L) = \bar{\pi}_L^*(I_H^*, z_H) \\
(13) \quad & \bar{\pi}_H^*(I_H^*, z_H) = \bar{\pi}_H^*(I_M^*, z_M^*).
\end{align}

The first major term in (11) is the expected government tax revenue from entrepreneurs. The second term represents the net losses to investors, which government must cover. Equation (12) gives the low quality incentive compatibility constraint, while (13) gives the high quality profit-preservation constraint. Note that (11) does not include a term for investor tax effects. Given the small sector assumption, the expected taxable asset base for investors is $(1+\rho)$ per dollar invested, regardless of whether funds are allocated to an entrepreneurial firm or a taxable bond. Thus, since the representative investor marginal tax rate is also unaltered by this allocation, outcomes in the entrepreneurial capital market do not affect government expected tax revenue from investors.\(^{11}\)

Substituting (12) and (13) into (11):

\begin{align}
(11') \quad & G = \alpha\{\nu \bar{\pi}_L^*(I_H^*, z_H(I_H)) + (1 - \nu) \bar{\pi}_H^*(I_M^*, z_M^*)\} \\
& \quad + \{\nu[\bar{\pi}_L^*(I_L^*) - \bar{\pi}_L^*(I_H^*, z_H(I_H))] - (1 + \rho)I_L^*\} \\
& \quad + (1 - \nu)[\bar{\pi}_H^*(I_H) - \bar{\pi}_H^*(I_M^*, z_M^*) - (1 + \rho)I_H^*]\end{align}
where $z_H(I_H)$ solves (13). Differentiating (11'):

$$
\frac{dG}{dI_H} = \pi^*_L \left[ \frac{\pi^*_L}{\pi^*_L} - \frac{\pi^*_H}{\pi^*_L} \right] (\alpha - 1)\nu + (1 - \nu)[\pi'_H - (1 + \rho)]
$$

where functions are evaluated at $(I_H, z_H(I_H))$.

To evaluate $\frac{dG}{dI_H}$ in (14) at the market equilibrium, consider a case of cross-subsidization. In this case, the equilibrium is at a point of tangency between $OC^*$ and a high quality indifference curve. Thus, $\frac{dz}{dI_H}$ in (8) is set equal to $\text{MRS}_H(I_H, z_H)$. With a little rearrangement, this equality can be stated as follows:

$$
\nu \pi^*_L \left[ \frac{\pi^*_L}{\pi^*_L} - \frac{\pi^*_H}{\pi^*_L} \right] = (1 - \nu)(\pi'_H - (1 + \rho))
$$

Substituting (15) into (14) gives:

$$
\frac{dG}{dI_H} \bigg|_{I^*_H} = \alpha(1 - \nu)(\pi'_H(1^*_H) - (1 + \rho)) < 0
$$

where the inequality follows from Proposition 1. The following Proposition expresses (16) in words:

**Proposition 2:** When government levies common proportional taxes and the Miyazaki equilibrium is characterized by cross-subsidization, the latter equilibrium is constrained inefficient. Government can subsidize both high and low quality contracts, with the new high quality contract characterized by investment which is lower than in the Miyazaki equilibrium, and it will thereby achieve the following distributional effects: (1) preserve high...
quality entrepreneurs' and investors' net expected profits; (2) increase low quality entrepreneurs' net expected profits; and (3) increase net government expected revenues.

While Proposition 2 is the key result of this section, equation (14) gives rise to two other observations:

1. Evaluated at the optimal perfect information high quality investment level \( I_H^* \), \( \frac{dG}{dI_H} > 0 \) (due to condition (3)). Now note that the constrained optimal tax/intervention policy with common proportional taxes will simultaneously set \( \frac{dG}{dI_H} \) in (14) equal to zero and satisfy a government revenue requirement constraint. Thus, the former observation implies that the optimal intervention will never achieve first best investment choices.

2. When there is no cross-subsidization at the equilibrium and \( e_H^* = e_H^0 \), the sign of \( \frac{dG}{dI_H} \) at \( I_H^m \) is ambiguous, despite "over-investment."

Figure 2 permits a graphical interpretation of these results. When making a small change in \( I_H \) away from \( I_H^m \) and along \( I_C^m \), the government cost of investor losses is approximately zero because it is approximately moving along \( OC^* \). Thus, it gets the full benefit of the extra net high quality firm profits (equation (16)). However, the costs of investor compensation become strictly positive as the government subsidized contracts move further away from \( I_H^m \). Thus, benefit of further subsidy will require strictly positive net gains from lower high quality investment; since the latter gains approach zero as \( I_H \) approaches \( I_H^* \), the optimal intervention will never go this far (observation (1)). In addition, consider an equilibrium at \( e_H^0 \). Since the high quality entrepreneur's opportunity set kinks at \( e_H^0 \), an equilibrium at
this point need not be characterized by tangency between $OC^*$ and $IC^M_H$. Therefore, the government's cost of covering investor losses need not be negligible as it moves along $IC^M_H$ from this point, leading to the ambiguity described above.

V. General Tax and Contract Forms: An Example

The preceding sections have shown that, in the presence of taxation, direct government intervention in entrepreneurial firm financial markets can lead to Pareto improvements, even when the equilibrium is constrained efficient in the absence of taxes. However, the economic structure examined above was constrained in two important ways: (1) financial contracts were limited to standard debt and equity forms, and (2) taxes were assumed to be proportional to firm income net of debt obligations. While these assumptions have some casual empirical basis, their consistency with optimizing behavior is not obvious. Hence, the objective of this section is to demonstrate that the results above can persist even when the most general contractual and tax forms are permitted. Specifically, an example with the following four properties will be presented:

Property 1: For a given investment level ($I$), there does not exist a payoff function, with observable variables as arguments, such that the expected payoff is invariant to quality.

Discussion: If this property were violated, investors could offer a contract for each investment level which would have an expected investor payoff of $(1 + \rho)I$, regardless of quality. Entrepreneurs would then choose the perfect information investment levels and information would be irrelevant.
Property 2: Without any constraints on contractual forms, debt is the equilibrium financial arrangement.

Property 3: Condition (3) is satisfied (i.e., \(MRS_H(I, z) > MRS_L(I, z)\)).

Property 4: A proportional tax is optimal in the following sense: No other entrepreneurial firm tax structure will yield an equilibrium which Paretodominates the equilibrium with a proportional tax.

The Example

For any investment level, there are two possible firm profit outcomes: good and bad. When the entrepreneur has bad luck, the profit is \(R(I) \ll (1 + \rho)I\), regardless of quality. When the entrepreneur has good luck, the profit is \(f(I)(1 + q)\) if the entrepreneur is high quality and \(f(I)\) if he is low quality, with \(q > 0\). The probability of a good outcome is \(P_H\) for the high quality entrepreneur and \(P_L\) for the low quality type, with \(P_H > P_L\). Further, the following informational assumption will be made in order to ensure the satisfaction of Property 1: investors can observe the outcome, good or bad, but cannot distinguish lower from higher profits. Finally, to ensure that information is important in the example, assume that the no-tax equilibrium is not the perfect information equilibrium.

The next step is to prove that the four properties stated above are satisfied:

Proof of Property 1: Given the informational assumption just made, the investor payoff function for a given investment level can only depend on the outcome, good or bad. Since \(P_H > P_L\), the expected payoff with a high quality entrepreneur must, therefore, be greater than with a low quality entrepreneur.

Q.E.D.
Proof of Property 2: With debt contracts (and Property 3), Lemmas 1 and 2 establish that \( \pi^*_H(I_H, z_H) > \pi^*_H(I_L, z_L) \) and, so long as information problems are relevant, \( \pi^*_L(I_H, z_H) = \pi^*_L(I_L, z_L) \). Taking this all-debt equilibrium as a starting point, consider changing the capital structure. Specifically, consider an arbitrary set of payments to investors such that the expected investor payoffs are \( \bar{z}_H \) and \( \bar{z}_L \), the same as with the all-debt fixed payments, \( z_H \) and \( z_L \). These payoffs can be written:

\[
\bar{z}_H = P_H z_{HG} + (1 - P_H) z_{HB}, \quad \bar{z}_L = P_L z_{LG} + (1 - P_L) z_{LB}
\]

where \( z_{qG} \) and \( z_{qB} \) denote payments for quality \( q \) with good and bad outcomes.

With all-debt, \( z_{qB} = R(I_q) \). A change from this financial structure can break the proposed equilibrium only if it permits the high quality agent to increase his net expected profit. Moreover, since \( \bar{z}_H \) is held fixed, this increase will only be possible if the change relaxes one or more of the binding constraints. Since the high quality incentive compatibility condition (M1) is slack, there are only two relevant constraints, (M1) for low quality entrepreneurs and (M3):

\[
\begin{align*}
(17a) \quad & \pi^*_L(I_L, z_{LG}, z_{LB}) = P_L(f(I_L) - z_{LG}) + (1 - P_L)(R(I_L) - z_{LB}) \\
& = P_L(f(I_H) - z_{HG}) + (1 - P_L)(R(I_H) - z_{HB}) = \pi^*_L(I_H, z_{HG}, z_{HB}) \\
(17b) \quad & \pi^*_L(I_L, z_{LG}, z_{LB}) \geq \pi^*_L(I_L) - (1 + \rho) I_L
\end{align*}
\]

Changing the low quality capital structure (preserving \( \bar{z}_L \)) does not alter either of these conditions. Thus, \( z_{LB} \) can be set equal to \( R(I_L) \) without loss of generality. However, changing the high quality capital structure alters (17a). Consider reducing \( z_{HB} \), preserving \( \bar{z}_H \) by increasing \( z_{HG} \). Differentiating the right hand side of (17a):
Therefore, reducing $z_{HB}$ tightens the constraint and the all-debt equilibrium cannot be broken.\textsuperscript{12} The derivation of Property 3 below completes the proof.

Q.E.D.

**Proof of Property 3:** Since the promised fixed payment is $z > R(I)$,

$$MRS_H(I, z) = f'(1 + q) > f' = MRS_L(I, z).$$

Q.E.D.

**Proof of Property 4:** The "optimal tax" problem can be posed in the following way: With two quality types and two outcome possibilities, there are four tax payments, $T_{qs}$, $q = H, L$ and $s = G, B$. Let these tax payments and the financial contracts solve a constrained high quality entrepreneur profit maximization analogous to the Miyazaki problem but with an added constraint: expected government revenues reach a given target, $\bar{T}$. Any eligible tax structure which gives rise to an equilibrium different from the solution to this maximization will be inferior from the high quality agent's point of view and, hence, will not Pareto dominate. Therefore, the solution to this problem will imply an optimal tax structure in the sense of Property 4.

In stating the maximization, several earlier results will be useful: Property 2 and Lemmas 1-3, all of which are easily extended to the present setting (proof left to the reader).\textsuperscript{13} The Lemmas imply that $I_H \neq I_L = I_L^*$. Property 2 implies that $z_{qB} = R(I_q) - T_{qB}$. Formally, the maximization can now be stated as follows:
(21) \[
\max_{I_H, (z_G), (T_q)} f(I_H) \cdot (1 + q) - z_G - T_G 
\]

Subject to:

\begin{align*}
(21a) & \quad \nu\{P_L(z_G) + (1 - P_L)(R(I_L^*) - T_LB) - (1 + \rho)I_L^*\} \\
& \quad + (1 - \nu) \{P_H(z_G) + (1 - P_H)(R(I_H) - T_HB) - (1 + \rho)I_H\} = 0 \\
(21b) & \quad f(I_L^*) - z_G - T_G = f(I_H) - z_G - T_G \\
(21c) & \quad \nu\{P_L(z_G) + (1 - P_L)T_LB\} + (1 - \nu)\{P_H(z_G) + (1 - P_H)T_HB\} = T 
\end{align*}

(21a) gives the zero investor profit condition, (21b) the incentive compatibility constraint and (21c) the government revenue requirement.

Substituting the constraints into (21) gives:

\begin{align*}
(21') & \quad \max_{I_H, T_LB, T_HB, T_G} f(I_H)(1 + q) + \\
& \quad \left[\frac{1}{\nu P_L + (1 - \nu)P_H}\right] \cdot \{\nu P_L(f(I_L^*) - f(I_H) + T_G) - T \\
& \quad + (1 - \nu)[P_H(z_G) + (1 - P_H)T_HB] + \nu(1 - P_L)T_LB \\
& \quad + \nu(1 - P_L)(R(I_L^*) - T_LB) - \nu(1 + \rho)I_L^* \\
& \quad + (1 - \nu)[(R(I_H) - T_HB)(1 - P_H) - (1 + \rho)I_H]\} - T_G. 
\end{align*}

Differentiating (21'):
Now note that the high quality investment level in the no-tax Miyazaki equilibrium, \( I_H \), sets (22) equal to zero.\(^{14} \) Thus, any tax structure which yields no-tax Miyazaki equilibrium investment levels is optimal in the sense of Property 4. As we know from Sections III and IV, a common proportional tax satisfies this condition. Q.E.D.

In summary, the taxation-induced inefficiency of Miyazaki equilibria is not attributable to the restrictions on contractual and tax forms assumed earlier. Rather, the inefficiency is a general phenomenon attributable to the existence of asymmetric information and a third agent (government or "the taxpayer") who is affected by equilibrium market arrangements but not a party to them.

VI. Conclusion

The public finance literature has dealt extensively with the following question (e.g., see Atkinson and Stiglitz (1980)): how can a given amount of government revenue be raised so as to cause a minimum of deadweight loss? This paper suggests that, with information problems or other deviations from Neoclassical "perfection", there is another side to the coin: in order to raise revenue, government must put itself in a position of an external party who does not participate in decisions which affect it. Thus, another question
is raised: does taxation imply that there are direct government market interventions which correct "tax externalities" and thereby increase social welfare?

In a simple entrepreneurial investment model, this paper has shown that the answer is "yes". The government gets a share of the gain due to a move from the "second-best" market equilibrium toward the "first-best" (perfect information) investment levels. Thus, it can compensate investors for losses they must incur in making the move.

Analogously, information problems are central to a wide variety of strategic (e.g., takeover), dynamic (e.g., foreclosure) and incentive phenomena not considered here. All of these phenomena admit a government tax interest, the implications of which merit investigation.
APPENDIX A

Proof of Lemma 1

Let e\textsuperscript{P} denote the contract most preferred by high quality entrepreneurs among those on OC\textsubscript{P}. Given the definition of equilibrium in Section II, e\textsuperscript{P} is the only possible pooling equilibrium. But the following two observations imply that e\textsuperscript{P} cannot be an equilibrium.

(i) Investment at e\textsuperscript{P}, I\textsubscript{P}, is less than I\textsuperscript{*}:

By definition, e\textsuperscript{P} solves the following maximization: \( \max \pi\textsubscript{H}^*(I, z(I)) \) where \( z(I) \) solves (5). Substituting from (5), the first order condition is:

\[
(A1) \quad \pi\textsubscript{H}^* \left[ \frac{\nu[\pi\textsubscript{L}^*(1+\rho)] + (1-\nu)[\pi\textsubscript{H}^*(1+\rho)]}{\nu \pi\textsubscript{L}^* + (1-\nu) \pi\textsubscript{H}^*} \right] \\
+ \frac{\pi\textsubscript{H}^*}{\pi\textsubscript{L}^*} \left[ \frac{\pi\textsubscript{H}^* - \nu \pi\textsubscript{L}^* + (1-\nu) \pi\textsubscript{H}^*}{\pi\textsubscript{L}^* \nu \pi\textsubscript{L}^* + (1-\nu) \pi\textsubscript{H}^*} \right] = 0
\]

When \( I \geq I\textsuperscript{*} \), \( \pi\textsubscript{H}^* - (1+\rho) < 0 \) and \( \pi\textsubscript{L}^* - (1+\rho) < 0 \) (since \( I\textsuperscript{*} < I\textsubscript{H}^* \)). Thus, the first bracketed term is negative. Moreover, due to condition (3), the second bracketed term is also negative (see equations (9) and (10)). Therefore, when \( I \geq I\textsuperscript{*} \), the left-hand side of (A1) is strictly negative, confirming the observation.

(ii) e\textsuperscript{P} can be broken:

Consider the following set of contracts: e\textsubscript{L} = e\textsuperscript{P} for low quality entrepreneurs and e\textsubscript{H} = (I\textsubscript{H}, z\textsubscript{H}) which satisfies the following constraints: (a) I\textsuperscript{P} < I\textsubscript{H} < I\textsuperscript{*} (possible due to observation (i));

(b) \( z\textsubscript{H} = z\textsubscript{H}^* - \epsilon \) where \( z\textsubscript{H}^* \) solves: \( \pi\textsubscript{H}^* (I\textsuperscript{P}, z\textsuperscript{P}) = \pi\textsubscript{H}^* (I\textsubscript{H}, z\textsubscript{H}^*) \). (In terms of Figure 1, e\textsubscript{H} is northeast of e\textsuperscript{P}, \( \epsilon \) below ICP.) If there exists a positive \( \epsilon \)
such that investors make nonnegative profits and incentive compatibility constraints are met, eP will be broken with this set of contracts; high quality entrepreneurs will prefer this set to eP, implying that eP cannot solve the Miyazaki maximization problem. To see that such an $\epsilon$ exists, note first that, due to condition (3), there exists an $\epsilon^*_1 > 0$ such that

$$\pi_L^*(IP, zP) = \pi_L^*(I_H, z_H^* - \epsilon_1^*).$$

Further, define $\epsilon^*_2$ such that investor expected profits are zero:

$$(A2) \quad \nu[R(IP, zP; L) - (1+\rho)IP] + (1-\nu) [R(I_H, z_H^* - \epsilon_2^*; H) - (1+\rho)I_H] = 0$$

Using (5), rewrite (A2):

$$(A2') \quad (1-\nu)[\pi_H^*(I_H) - (1+\rho)I_H] - [\pi(IP) - (1+\rho)IP]$$

$$+ [\pi_H^*(IP, zP) - \pi_H^*(I_H, z_H^* - \epsilon_2^*)] = 0$$

Constraint (a) on $I_H$ implies that the first difference in (A2') is positive. Therefore, given the definition of $z_H^*$, $\epsilon_2^* > 0$. Letting $\epsilon = \min(\epsilon_1^*, \epsilon_2^*) > 0$, the proposed set of contracts will satisfy (M1) - (M3), thus, breaking eP.

Q.E.D.

Proof of Lemma 2

To prove Lemma 2, the following claim must first be validated:

Claim: In equilibrium, $I_H > I_L$.

Proof of Claim: Suppose not. Then, given Lemma 1, $I_H < I_L$. Define $z^*_q$ as follows: $\pi^*_q(I_H, z_H) = \pi^*_q(I_L, z^*_q), q = L, H$. To satisfy the incentive compatibility constraints (M1), $z_H^* < z_L < z_L^*$. However, due to condition (3),
\[ z^*_L = z_H + \int_{I_H}^{I_L} \left( \frac{dz}{dI} \right)_{I_{CH}} dI > z_H + \int_{I_H}^{I_L} \left( \frac{dz}{dI} \right)_{I_{CL}} dI = z^*_L \]

where \( I_{CL} \) and \( I_{CH} \) represent indifference curves for net profits at \((I_H, z_H)\).

**Q.E.D. Claim.**

Proof of Lemma: Suppose that the constraint, \( \pi^*_H(I_H, z_H) > \pi^*_H(I_L, z_L) \), were binding. Define \( z^*_H : \pi^*_L(I_H, z^*_H) = \pi^*_L(I_L, z_L) \).

Equivalently,

\[ (A3) \quad z^*_H = z_L + \int_{I_L}^{I_H} \left( \frac{dz}{dI} \right)_{I_{CL}} dI < z_L + \int_{I_L}^{I_H} \left( \frac{dz}{dI} \right)_{I_{CH}} dI = z_H \]

where \( I_{CL} \) and \( I_{CH} \) represents the indifference curves for net profits at \((I_L, z_L)\), and the inequality follows from the foregoing claim and condition (3). Thus, the low quality incentive compatibility constraint \((M1)\) is slack.

There are now two cases to consider: (i) when the low quality rationality condition \((M3)\) binds, and (ii) when it does not. In case (i), let \( I_L = I^*_L \) without loss of generality. Then, due to condition (3), \( I_{CH} \) (through \((I_L, z_L)\)) rises above \( O_{CL} \) and there exists a contract between \( I_{CH} \) and \( O_{CL} \) which is preferred by the high quality agent to \((I_L, z_L)\) and, hence, to \((I_H, z_H)\).

Together with \((I_L, z_L)\), the latter contract satisfies \((M1)-(M3)\), thus breaking the equilibrium.

In case (ii), consider decreasing the high quality \( z \) by \( (z_H - z^*_H) \cdot \epsilon_1 \), \( 0 < \epsilon_1 < 1 \), and increasing the low quality \( z \) by \( \epsilon_2 > 0 \). For sufficiently small \( \epsilon \), investor profits will increase with the change, the low quality rationality constraint \((M3)\) will remain slack, as will the low quality incentive compatibility constraint \((M1)\) (due to \((A3))\), and the high quality
incentive constraint will become slack. Therefore, the new set of contracts is feasible, makes high quality agents better off and breaks the hypothesized equilibrium.

Since the high quality incentive constraint must therefore be slack in equilibrium, a slack low quality incentive constraint implies that information is irrelevant to the problem. Q.E.D. Lemma

Proof of Lemma 3

Suppose instead that the equilibrium low quality contract were \((I_L, z_L)\), \(I_L \neq I_L^*\). There are two cases to examine:

1. \(I_H < I_L^*\). Define \(z_H^0\) implicitly, \(\pi_H^*(I_H, z_H) = \pi_H^*(I_L^*, z_H^0)\). Now consider the pooling contract \((I^*, z = z_H^0 - \varepsilon)\). Investor profits at this new contract are (using the zero profit condition \((M2)\) at the hypothesized equilibrium):

\[
\nu\left\{\left[\pi_L^*(I_L^*) - (1+\rho)I_L^*\right] - \left[\pi_L(I_L) - (1+\rho)I_L\right]\right\}
+ (1-\nu)\left\{\left[\pi_H^*(I_H^*) - (1+\rho)I_H^*\right] - \left[\pi_H(I_H) - (1+\rho)I_H\right]\right\}
+ \nu\left\{\pi_L^*(I_L, z_L) - \pi_L^*(I_L^*, z)\right\} + (1-\nu)\left\{\pi_H^*(I_H, z_H) - \pi_H^*(I_L^*, z)\right\}
\]

The first two terms are positive. At \(\varepsilon=0\), the fourth term is zero and the third can be shown to be positive. Specifically, from \((M1)\), \(\pi_L^*(I_L, z_L) \geq \pi_L^*(I_L, z_H)\). Further, define \(z_{L0}^0\) implicitly, \(\pi_L^*(I_H, z_H) = \pi_L^*(I_L, z_{L0}^0)\). Now note:

\[
z_{L0}^0 = z_H + \int_{I_H}^{I_L^*} \frac{dz}{dI} |_{IC_L} \quad dI < z_H + \int_{I_H}^{I_L^*} \frac{dz}{dI} |_{IC_H} \quad dI = z_H^0
\]
where the inequality follows from (3). Thus, 
\[ \pi^*_L(I_H, z_H) \geq \pi^*_L(I_L^*, z_L^0), \]
implicating that investor profits at the new contract are strictly positive with 
\( \epsilon = 0 \), and \( \epsilon > 0 \) is feasible. Since the latter contract is preferred by high 
quality entrepreneurs, the equilibrium can be broken.

(2) \( I_H > I_L^* \). Define \( z_L^0 \) implicitly, \( \pi^*_L(I_L^*, z_L^0) = \pi^*_L(I_L, z_L) \). Now consider 
the contract proposal, \( (I_L^*, z_L^* = z_L^0 - \epsilon) \) and \( (I_H, z_H^* = z_H - \delta(\epsilon)) \) where \( \epsilon > 0 \) and 
\( \delta(\epsilon) \) is implicitly defined by the relation, \( \pi^*_L(I_L^*, z_L^*) = \pi^*_L(I_H, z_H^*) \). This 
last equality implies that the low quality incentive compatibility constraint 
is met. Further, given (3), the \( z_H^0 \) which solves \( \pi^*_H(I_L^*, z_L^*) = \pi^*_H(I_H, z_H^0) \) is 
greater than \( z_H^* \):

\[
z_H^0 = z_L^* + \int_{I_L}^{I_H} dz \int_{I_C} dz > z_L^* + \int_{I_L}^{I_H} dz \int_{I_C} dz = z_H^*
\]

where \( I_C \) and \( I_C^L \) contain \( (I_L^*, z_L^*) \). Thus, the high quality incentive 
compatibility constraint is also met. Finally, adding and subtracting 

\[
\pi^*_H(I_H, z_H) + \pi_L(I_L) - (1+\rho)I_L - \pi^*_L(I_L, z_L)
\]

and using the zero investor profit condition at \( (I_q, z_q) \), investor profits 
with the new contracts can be written:

(A5) \[ [(\pi_L(I_L^*) - (1+\rho)I_L^*) - \pi_L(I_L) - (1+\rho)I_L] \]

\[ + [\pi^*_L(I_L, z_L) - \pi^*_L(I_L^*, z_L^0 - \epsilon)] + [\pi^*_H(I_H, z_H) - \pi^*_H(I_H, z_H - \delta(\epsilon))] \]

At \( \epsilon = 0 \), \( \delta(\epsilon) = 0 \) and (A5) is positive. Moreover, since \( F(z_H) < 1 \) in 
equilibrium, \( 0 < \delta' < \infty \). Thus, there are sufficiently small \( \epsilon < 0 \) such that 
(A5) remains positive and the equilibrium can be broken. Q.E.D.
FOOTNOTES

1 For example, see Rothschild and Stiglitz (1976), Wilson (1977), Miyazaki (1977), and Crocker and Snow (1985).

2 This conclusion is analogous to a recent result of Kenneth Judd (1985). Judd examines an insurance market model in which consumers are outside agents. Specifically, consumers are affected when product market supply responds to contractual arrangements between producers and insurers. As in this paper, constrained inefficiencies result from effects of the informational equilibrium on the external parties.

3 This type of asymmetry contrasts sharply with the costly state verification construction employed by Gale and Hellwig (1985) in their derivation of an "under-investment" result (also see Williamson (1987)). The latter construction implies asymmetry of information concerning ex-post profits, not the ex-ante distribution of profits.

4 The latter property is shared by the Stiglitz and Weiss (1981) analysis from which much of this literature springs.

5 Implicit in this specification is an assumption that the functional relationship between profits and assets does not depend on equilibrium in the entrepreneurial capital market. Introducing such a dependence would add complexity, but also, as in Judd (1985), other external agents. Thus, this assumption is made to focus analysis on the role of government as an outside party.

6 The arguments of Leland and Pyle (1977) suggest that the entrepreneur's commitment of his own resources to the firm can serve as a signal of quality. If so, the entrepreneur can be thought to invest all of his wealth in the firm.
(as often observed in practice) and the relevant population will be constrained to have homogeneous wealth endowments.

7Note that \( \pi''_q < 0 \) guarantees satisfaction of second order conditions at these contracts.

8This inequality is the consequence of the first order stochastic dominance characterization of investment effects. The proof, being somewhat technical, is omitted here, though it is available from the author. Less formally, this inequality can be established by noting that \( IC_H \) must be upward-sloping at an equilibrium and \( \pi^*_H \) < 0.

9Of course, if the tax is so high as to drive entrepreneurs out of the market, equilibrium will be altered. For the present analysis this case will be ruled out by assumption.

10If the perfect information equilibrium prevails, Pareto improvements are clearly impossible.

11Drawing on Miller's famous argument (1977), the relevant investor tax rate is the marginal tax rate of the marginal investor in taxable bonds, however investment funds are allocated.

12With taxes, the proof of Property 2 is virtually identical (see Footnote 13).

13To prove these extensions, note that taxes are analytically equivalent to debt payments here. Thus, defining \( z^*_{qs} = z_{qs} + T_{qs} \), the Miyazaki maximization is modified to choose \( \{z^*_{qs}\} \) and \( \{I_q\} \), subject to incentive compatibility, low quality rationality, and adjusted investor/government return constraints. The latter constraint is altered to ensure an expected return of \( \bar{T} \), rather than zero. With debt contracts, the problem is thus placed in the context for which Lemmas 1-3 were proven above, with one
exception: the innocuous addition of a positive expected "investor" return requirement. Given the Lemmas, Property 2 follows directly from the above proof.

14To get the no-tax Miyazaki problem, set $T = T_{HG} = T_{HB} = T_{LB} = 0$ in (21'). Differentiating with respect to $I_H$ gives (22).
REFERENCES


Figure 1

\[ e^0_H = (I^0_H, z^0_H) \]
Figure 2