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FOREIGN EXCHANGE RATES: A MULTIPLE CURRENCY AND MATURITY ANALYSIS

by

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ABSTRACT

Three dollar exchange rate series, the German Mark, the British pound, and the Japanese Yen, are converted to market price revisions by calculating the difference between the price for delivery at a fixed date and the preceding period's price for delivery at the same date. These price revisions are likely to meet the stationary stochastic process assumptions required for time series modeling even if the original series are nonstationary.

Using error covariance assumptions analogous to a pooled time series of cross-sections, a multivariate time series model is fitted, and tested on 168 observations out of sample. Forecasting performance is evaluated using both mean squared errors and a non parametric test of direction.

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I. Introduction

We present models for three dollar exchange rate series--the German Mark, British Pound, and Japanese Yen--that we believe are useful at both the micro and macro level. At the micro level, the models provide exchange rate forecasts useful to individual decision makers in managing their affairs, while at the macro level, the models provide what Berger and Craine have called a direct test of market efficiency, a test that requires the minimum assumptions possible.[1] Loosely, the null hypothesis is that no trading strategy beats a buy and hold rule; we reject the null when we show that our forecasts have value. Such tests abstract from pre-test difficulties in fitting time series models as well as potential inconsistencies arising from data construction (discussed below).

The basic data series employed is a pooled time-series of cross-sections of market price revisions on three maturities for contracts in each of the three currencies itemized above. We define a market price revision to be the difference between one period's price for delivery at a fixed date in the future and the preceding period's price for delivery at the same date. [Thus, we implicitly incorporate the criterion that useful forecasts must be better than forward rates, proposed in a slightly different context by Just and Rausser.] Under certain conditions these series will meet a stochastic arbitrage equilibrium condition that makes them independent of the impossibly varied factors that affect exchange rates. As a result, these series are more likely to meet the stochastic specifications required for successful multivariate time series modelling (adjusted for multiple cross sections) than are the raw data series.

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[1]There is a large literature on exchange rate forecasts; an excellent summary as well as alternative models are available in Schinasi and Swamy. See Kohlhagen and Bilson for discussions of the importance of the efficiency hypothesis in hedging, speculation, and government intervention.

In the next section the nine data series are outlined, followed by a discussion of the stochastic properties implicit in their construction and the covariance assumptions underlying the pooled time-series cross-section multivariate time series identification and estimation procedures. The third section evaluates the post sample forecasting performance, followed by the conclusions.

II. A Pooled Multivariate ARMA Model

For most major currencies, daily data are available on spot rates, and 30-day, 90-day, and 180-day forward rates.[2] For each currency, we define ${}_tF_{t+\tau m}$ with τ = 0, 1, 3, 6 as the τ month ahead forward rates (the 1m, 3m, and 6m refer to months, not time periods). These can be used to construct three separate price revision series (for each currency) in each time period

$$
\begin{aligned}
{}_tF_t - {}_{t-1m}F_t &= e_t^1 \quad \text{(spot - 30 day)} \\
{}_tF_{t+1m} - {}_{t-2m}F_{t+1m} &= e_t^2 \quad \text{(30 day - 90 day)} \\
{}_tF_{t+3m} - {}_{t-3m}F_{t+3m} &= e_t^3 \quad \text{(90 days - 180 days)} \qquad (1)
\end{aligned}
$$

where, for notational simplicity, we have renamed them e_t^1, e_t^2, and e_t^3; e will take on the values £, M, and Y for the British pound, German Mark, and Japanese Yen, respectively. We will consider such series for four different days of each month: days 7, 14, 21, and 28.

Assuming that the three series are jointly covariance stationary, by Wold's decomposition the vector $\underline{e}_{it}' = (e_{it}^1, e_{it}^2, e_{it}^3)$ has an ARMA(p,q) representation,

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[2]Data on spot rates and 30-day, 90-day, and 180-day forward rates for the British pound, the Japanese yen, the German mark were collected from the Wall Street Journal. They are selling rates among banks in amounts of $1 million and more as quoted at 3:00 p.m. on the first, seventh, twenty-first, and twenty-eighth day of each month. See Riehl and Rodriguez 1977 for a discussion of the date alignment problem.

i.e.,

$$\Phi(B)\underline{e}_{it} = \underline{\delta} + \theta(B)\underline{u}_{it}$$

$$i=7,\ 14,\ 21,\ 28;\ t=1,\ 2,\ldots,T \tag{2}$$

with

$$\Phi(B) = I - \Phi_1 B - \Phi_2 B^2 - \ldots - \Phi_p B^p, \text{ and } \theta(B) = I - \theta_1 B - \theta_2 B^2 - \ldots - \theta_q B^q,$$

where $\underline{\delta}$ is a vector of constants, Φ and θ are (3x3) matrices of constant coefficients, B is a backward shift operator, and the error $\underline{u}_{it}$ is an uncorrelated white noise process with $E\underline{u}_{it} = 0$ and $E\underline{u}_{it}\underline{u}_{it}' = \Sigma$ for all i, t.

To economize on the number of parameters to be estimated, we assume that the exchange revisions on different days of the month are generated by the same stochastic process, i.e., for each day of the month we have a model of the form (2). The assumption of a common generating process means that the error term, while different for different days, has the same distribution and so the same moments. This means that the coefficients $\underline{\delta}$, Φ, and θ and the error covariance matrix Σ are the same for each day of the month, enabling us to pool the data and thereby increase the number of observations without significantly increasing the number of parameters to be estimated.

To derive the stochastic properties of the model under the fair game - efficient market assumption, note that all series include the t-1m to t period, and e_t^2 and e_t^3 also share the t-2m to t-1m period. This means that for a monthly model with pooled data collected on days 7, 14, 21, and 28 the structure implied by the overlapping revision periods alone is

$$E e^j_{i,t}\, e^k_{i,t-sm} \neq 0 \quad \text{for} \quad j,k=1,\ 2,\ 3, \text{ and } s=0,\ 1,\ldots,\max(j,k)-1$$

$$= 0 \quad \text{for} \quad j,k=1,\ 2,\ 3, \text{ and } s \geq \max(j,k) \tag{3}$$

where j and k are series indices, and i=7,14, 21, 28 is the "cross section" or day index. This auto- and cross-covariance pattern implies that the vector $\underline{e}_{it}$ must follow a constrained second order moving average process of the form[3]

$$\underline{e}_{it} = \underline{u}_{it} - \theta_1 \underline{u}_{i,t-1} - \theta_2 \underline{u}_{i,t-2} \qquad i=7,\ 14,\ 21,\ 28;\ t=1,\dots,T \qquad (4)$$

with

$$\theta_1 = \begin{bmatrix} 0 & 0 & 0 \\ \theta_{121} & \theta_{122} & \theta_{123} \\ \theta_{131} & \theta_{132} & \theta_{133} \end{bmatrix}, \qquad \theta_2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \theta_{231} & \theta_{232} & \theta_{233} \end{bmatrix},$$

as a result of series construction alone, regardless of any additional pattern that may be present.

In order for the price revision model to be of use, the predictions must be generated in time for the underlying contracts to be entered. Thus the predictions for the first, second and third series must be based on information available at times t-1m, t-2m, and t-3m, respectively. The usual identification procedures must, therefore, be modified for two reasons: first, we have some built-in moving average terms of the form (4), and second, the autoregressive terms begin at different times, i.e., t-1m for series 1, but t-2m and t-3m for series 2 and 3, respectively. These unique features make it difficult to interpret the sample cross-correlation and partial autocorrelation matrices.

On the assumption that a relatively long ARMA model will nest the true model with a high probability, we began with an ARMA(5,5) process in which the Φ and θ matrices in (2) are constrained to the following forms:

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[3]This overlapping forecast problem has been encountered, for example, in Hansen and Hodrick (1980).

$$\Phi_1 = \begin{bmatrix} \Phi_{111} & \Phi_{112} & \Phi_{113} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \qquad \theta_1 = \begin{bmatrix} \theta_{111} & \theta_{112} & \theta_{113} \\ \theta_{121} & \theta_{122} & \theta_{123} \\ \theta_{131} & \theta_{132} & \theta_{133} \end{bmatrix}$$

$$\Phi_2 = \begin{bmatrix} \Phi_{211} & \Phi_{212} & \Phi_{213} \\ \Phi_{221} & \Phi_{222} & \Phi_{223} \\ 0 & 0 & 0 \end{bmatrix}, \qquad \theta_2 = \begin{bmatrix} \theta_{211} & \theta_{212} & \theta_{213} \\ \theta_{221} & \theta_{222} & \theta_{223} \\ \theta_{231} & \theta_{232} & \theta_{233} \end{bmatrix}$$

$$\Phi_3 = \begin{bmatrix} \Phi_{311} & \Phi_{312} & \Phi_{313} \\ \Phi_{321} & \Phi_{322} & \Phi_{323} \\ \Phi_{331} & \Phi_{332} & \Phi_{333} \end{bmatrix}, \qquad \theta_3 = \begin{bmatrix} \theta_{311} & \theta_{312} & \theta_{313} \\ \theta_{321} & \theta_{322} & \theta_{323} \\ \theta_{331} & \theta_{332} & \theta_{333} \end{bmatrix}$$

$$\Phi_4 = \begin{bmatrix} 0 & 0 & 0 \\ \Phi_{421} & \Phi_{422} & \Phi_{423} \\ \Phi_{431} & \Phi_{432} & \Phi_{433} \end{bmatrix}, \qquad \theta_4 = \begin{bmatrix} 0 & 0 & 0 \\ \theta_{421} & \theta_{422} & \theta_{423} \\ \theta_{431} & \theta_{432} & \theta_{433} \end{bmatrix}$$

$$\Phi_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \Phi_{531} & \Phi_{532} & \Phi_{533} \end{bmatrix}, \qquad \theta_5 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \theta_{531} & \theta_{532} & \theta_{533} \end{bmatrix}. \qquad (5)$$

In other words, we have augmented (4) by adding three autoregressive and three moving average terms to each of the three series, starting at t-1m, t-2m, and t-3m, respectively. These are the earliest dates in each of the constructed series, i.e., the dates at which a contrast must be entered to make use of the forecasts.

The model identification procedure[4] consisted of iteratively estimating the parameters (5), checking the "significance" of the coefficient estimates (at the 5 percent level) to eliminate the insignificant ones (including the constants) beginning with the longest lags, and then reestimating and checking significance

[4]This procedure is similar to the one employed by Tiao and Box.

again, etc. This procedure was carried out for each set of three exchange rate revisions for each of the three currencies (British Pound, German Mark, and Japanese Yen). In addition, to increase the efficiency of the parameter estimates we estimated the three models jointly permitting contemporaneous error correlation across both revisions and currencies. This joint model estimation was based on the model identification above with insignificant coefficients eliminated. All these estimates (reported in the next part) were obtained by maximizing the likelihood function conditional on the p initial value of each series and zero values for the errors for each series, over the observations i=7, 14, 21, 28 and t=p+1, p+2,...,96.[5]

III. Estimates, Forecasts, and Post Sample Performance

In this part we examine the conditional maximum likelihood pooled multivariate time series parameter estimates, and apply out-of-sample timing and magnitude tests to examine market efficiency and the usefulness of the model forecasts.

A. The Estimates

Results from four multivariate time series models are reported below. Three are models for three maturities [see equation (1)] for each currency (pound, mark, yen). The fourth is the nine-series joint model.

In describing the estimates we use the term "significant" to identify coefficient estimates more than 1.96 standard deviations away from zero. This is purely for convenience, since the identification procedure has compromised such statements far beyond any statistical meaning. We base no statistical conclusions

[5]See Hilmer and Tiao for the distinction between conditional and exact maximum likelihood parameter estimation in general ARMA models. Swamy and Mehta argue that the exact likelihood method is appropriate in situations in which the process began long ago and the coefficients are believed to have been stable, a condition at odds with the change to flexible exchange ratio that determined the beginning of our series (with some time for adjustment).

on these values, instead using only the statistically valid out-of-sample forecast performance detailed in part B below.

The coefficient and error covariance estimates for the single currency revisions appear in Tables 1 and 2. Most of the implied moving average coefficients [i.e., those introduced in the construction of the series and identified in equation (4)] are significant, confirming the role of unanticipated events in exchange rate movements.[6] All of the induced moving average coefficients except one are positive, implying that agents revise their forecasts of future exchange rates in the same direction as the change in the spot and forward rates between the time the forecasts are made and the revision dates. The one negative implied coefficient appears insignificant (the coefficient of $\underline{u}^1_{i,t-1}$ in the third equation in the Japanese Yen model). Also, the off-diagonal elements of $\hat{\Sigma}$ are all positive for all currencies implying that the contemporaneous spot and forward rates move in the same direction in response to new developments in the foreign exchange market; agents generally revise their spot and future rate predictions in the same direction.

The importance of modeling multiple maturities (as opposed to single maturity studies elsewhere in the literature) is borne out by the radically different models for different price revisions for the same currency. Consider, for example, the dollar/pound estimates. The first series contains two coefficients in excess of those induced, the second series only one, and the third series none. It is important to model all three series jointly, however, since the series are correlated and a later series represents an opportunity to exit an earlier investment--both important considerations to the investor--while at the macro level

[6] See Frankel for a discussion of the effect of news on exchange rate movements.

Table 1: Single Currency Coefficient Estimates†

Dollar/Pound Exchange Rate Revisions

$$£^{1}_{it} = \underset{(3.6)}{0.16}\, £^{1}_{it-1} + u^{1}_{it} + \underset{(6.1)}{0.26}\, u^{3}_{it-1}$$

$$£^{2}_{it} = u^{2}_{it} + \underset{(7.0)}{\overline{0.38}}\, u^{1}_{it-1} + \underset{(4.7)}{\overline{0.24}}\, u^{2}_{it-1} + \underset{(8.5)}{\overline{0.36}}\, u^{3}_{it-1} + \underset{(5.6)}{0.19}\, u^{3}_{it-2}$$

$$\begin{aligned} £^{3}_{it} = {} & u^{3}_{it} + \underset{(3.9)}{\overline{0.29}}\, u^{1}_{it-1} + \underset{(5.0)}{\overline{0.38}}\, u^{2}_{it-1} + \underset{(7.2)}{\overline{0.39}}\, u^{3}_{it-1} + \underset{{}^{*}(1.6)}{\overline{0.12}}\, u^{1}_{it-2} \\ & + \underset{(2.5)}{\overline{0.19}}\, u^{2}_{it-2} + \underset{(8.1)}{\overline{0.44}}\, u^{3}_{it-2} \end{aligned}$$

Dollar/Mark Exchange Rate Revisions

$$M^{1}_{it} = \underset{(9.9)}{0.79}\, M^{1}_{it-1} - \underset{(-4.1)}{0.12}\, M^{3}_{it-1} + u^{1}_{it} + \underset{(7.3)}{0.68}\, u^{1}_{it-1}$$

$$\begin{aligned} M^{2}_{it} = {} & \underset{(5.4)}{0.27}\, M^{2}_{it-2} + u^{2}_{it} + \underset{(5.6)}{\overline{0.27}}\, u^{1}_{it-1} + \underset{(3.7)}{\overline{0.24}}\, u^{2}_{it-1} + \underset{(5.9)}{\overline{0.39}}\, u^{3}_{it-1} \\ & + \underset{(6.3)}{0.32}\, u^{3}_{it-2} + \underset{(8.1)}{0.39}\, u^{3}_{it-3} \end{aligned}$$

$$\begin{aligned} M^{3}_{it} = {} & \underset{(2.2)}{0.20}\, M^{3}_{it-3} - \underset{(-2.1)}{0.20}\, M^{2}_{it-4} + u^{3}_{it} + \underset{{}^{*}(1.1)}{\overline{0.05}}\, u^{1}_{it-1} + \underset{(6.3)}{\overline{0.39}}\, u^{2}_{it-1} \\ & + \underset{(7.2)}{0.46}\, u^{3}_{it-1} + \underset{(6.4)}{0.25}\, u^{1}_{it-2} + \underset{{}^{*}(1.2)}{\overline{0.08}}\, u^{2}_{it-2} + \underset{(10.1)}{\overline{0.67}}\, u^{3}_{it-2} - \underset{(-3.0)}{0.17}\, u^{2}_{it-3} \\ & + \underset{(4.9)}{0.51}\, u^{3}_{it-3} - \underset{(-3.7)}{0.30}\, u^{2}_{it-4} + \underset{(6.8)}{0.42}\, u^{3}_{it-4} \end{aligned}$$

Dollar/Yen Exchange Rate Revisions

$$\begin{aligned} Y^{1}_{it} = {} & \underset{(4.9)}{0.48}\, Y^{3}_{it-1} - \underset{(-3.3)}{0.32}\, Y^{2}_{it-2} + \underset{(3.3)}{0.36}\, Y^{3}_{it-2} - \underset{(-3.7)}{0.31}\, Y^{2}_{it-3} \\ & - \underset{(-2.5)}{0.12}\, Y^{3}_{it-3} + u^{1}_{it} + \underset{(5.9)}{0.62}\, u^{3}_{it-1} - \underset{(-4.0)}{0.41}\, u^{2}_{it-2} + \underset{(6.4)}{0.91}\, u^{3}_{it-2} \\ & + \underset{(3.3)}{0.28}\, u^{3}_{it-3} \end{aligned}$$

$$\begin{aligned} Y^{2}_{it} = {} & \underset{(12.4)}{0.94}\, Y^{3}_{it-2} - \underset{(-10.9)}{1.07}\, Y^{2}_{it-3} + \underset{(4.5)}{0.51}\, Y^{3}_{it-3} + \underset{(4.3)}{0.48}\, Y^{1}_{it-4} \\ & + \underset{(7.1)}{0.73}\, Y^{2}_{it-4} + u^{2}_{it} + \underset{{}^{*}(0.34)}{\overline{0.02}}\, u^{1}_{it-1} + \underset{(3.9)}{\overline{0.26}}\, u^{2}_{it-1} + \underset{(9.3)}{\overline{0.78}}\, u^{3}_{it-1} \\ & - \underset{(-2.2)}{0.10}\, u^{1}_{it-2} + \underset{(11.1)}{1.11}\, u^{3}_{it-2} - \underset{(-6.7)}{0.50}\, u^{2}_{it-3} + \underset{(9.1)}{1.03}\, u^{3}_{it-3} \\ & + \underset{(4.9)}{0.54}\, u^{1}_{it-4} + \underset{(3.0)}{0.24}\, u^{2}_{it-4} \end{aligned}$$

$$\begin{aligned} Y^{3}_{it} = {} & - \underset{(-4.1)}{0.28}\, Y^{2}_{it-3} + \underset{(5.9)}{0.60}\, Y^{3}_{it-3} + \underset{(6.4)}{0.67}\, Y^{1}_{it-4} - \underset{(-9.5)}{0.87}\, Y^{2}_{it-4} + u^{3}_{it} \\ & - \underset{{}^{*}(-1.4)}{\overline{0.06}}\, u^{1}_{it-1} + \underset{(7.8)}{\overline{0.46}}\, u^{2}_{it-1} + \underset{(8.9)}{\overline{0.69}}\, u^{3}_{it-1} + \underset{{}^{*}(0.4)}{\overline{0.02}}\, u^{1}_{it-2} + \underset{(3.58)}{\overline{0.17}}\, u^{2}_{it-2} \\ & + \underset{(9.8)}{\overline{0.85}}\, u^{3}_{it-2} + \underset{(6.1)}{0.54}\, u^{3}_{it-3} + \underset{(6.7)}{0.71}\, u^{1}_{it-4} - \underset{(-6.1)}{0.43}\, u^{2}_{it-4} \end{aligned}$$

†Observations: 384 [96 monthly observations for each day i=7,14,21,28 from 1974(6) to 1982(5)]. Superscripts index the series. Insignificant MA coefficients at the 5 percent significance level are marked with *; numbers in brackets are t ratios. Barred coefficients correspond to those induced by construction.

asymptotic efficiency requires the error covariances be taken into account in examining market efficiency.

Table 2: Error Covariances, Single Currency Estimates

$\hat{\Sigma}^{£} \times 10^2$			$\hat{\Sigma}^{M} \times 10^4$			$\hat{\Sigma}^{Y} \times 10^8$		
.41			2.30			1.90		
.31	.41		1.81	2.22		1.56	2.11	
.29	.32	.63	1.91	2.06	2.38	1.61	1.90	2.06

The multiple currency parameter estimates and the estimated error covariance matrix, Σ^*, appear in Tables 3 and 4. The off diagonal blocks in the error covariance matrix Σ^* indicate that exchange rate revisions for the three currencies respond to unanticipated shocks in the same direction, in both the spot and forward markets: they are uniformly positive. This is an intuitively plausible result. To the extent that these cross-currency covariances are important, the coefficient estimates in the multiple currency model are more efficient asymptotically than those in the single currency models, i.e., they have a smaller actual asymptotic covariance matrix. The reductions in the estimated standard errors were generally small, however, and for some coefficients there have actually been increases in the standard errors. A comparison of the different tables reveals that the British pound is least affected by cross currency links--the number, sign, and the size of the t-ratios are very similar in Tables 1 and 3. The same is true for the dollar/mark model except for the loss of one coefficient in the third equation. The behavior of the dollar/yen exchange revision is again different from the other two. A large number of coefficients turn out to be insignificant (five in the first equation, three in the second equation, and one in the third equation). Moreover, there seem to be changes in the sizes and signs of some coefficients. One possible explanation may be related to the foreign exchange policy of the

Table 3: Multiple Currency Coefficient Estimates†

Dollar/Pound Exchange Rate Revisions

$$\pounds^1_{it} = \underset{(3.8)}{0.15}\, \pounds^1_{it-1} + u^1_{it} + \underset{(6.6)}{0.27}\, u^3_{it-1}$$

$$\pounds^2_{it} = u^2_{it} + \underset{(7.0)}{\overline{0.37}}\, u^1_{it-1} + \underset{(4.2)}{\overline{0.22}}\, u^2_{it-1} + \underset{(9.5)}{\overline{0.41}}\, u^3_{it-1} + \underset{(6.0)}{2.0}\, u^3_{it-2}$$

$$\pounds^3_{it} = u^3_{it} + \underset{(3.3)}{\overline{0.22}}\, u^1_{it-1} + \underset{(4.6)}{\overline{0.31}}\, u^2_{it-1} + \underset{(9.4)}{\overline{0.47}}\, u^3_{it-1} + \underset{{}^*(1.4)}{\overline{0.09}}\, u^1_{it-2}$$
$$+ \underset{{}^*(1.9)}{\overline{0.13}}\, u^2_{it-2} + \underset{(10.5)}{\overline{0.54}}\, u^3_{it-2}$$

Dollar/Mark Exchange Rate Revisions

$$M^1_{it} = \underset{(7.8)}{0.70}\, M^1_{it-1} - \underset{(-5.9)}{0.16}\, M^3_{it-1} + u^1_{it} + \underset{(6.0)}{0.59}\, u^1_{it-1}$$

$$M^2_{it} = \underset{(4.0)}{0.17}\, M^2_{it-2} + u^2_{it} + \underset{(6.3)}{\overline{0.28}}\, u^1_{it-1} + \underset{(5.1)}{\overline{0.29}}\, u^2_{it-1} + \underset{(6.2)}{\overline{0.38}}\, u^3_{it-1}$$
$$+ \underset{(5.7)}{0.27}\, u^3_{it-2} + \underset{(7.5)}{0.33}\, u^3_{it-3}$$

$$M^3_{it} = \underset{(2.2)}{0.20}\, M^3_{it-3} - \underset{(-2.5)}{0.24}\, M^2_{it-4} + \underset{{}^*(0.9)}{\overline{0.04}}\, u^1_{it-1} + \underset{(8.0)}{\overline{0.44}}\, u^2_{it-1}$$
$$+ \underset{(7.6)}{\overline{0.46}}\, u^3_{it-1} + \underset{(7.3)}{\overline{0.29}}\, u^1_{it-2} + \underset{(3.3)}{\overline{0.18}}\, u^2_{it-2} + \underset{(9.4)}{\overline{0.59}}\, u^3_{it-2} + \underset{(4.3)}{\overline{0.40}}\, u^3_{it-3}$$
$$- \underset{(-3.9)}{0.30}\, u^2_{it-4} + \underset{(6.7)}{0.41}\, u^3_{it-4}$$

Dollar/Yen Exchange Rate Revisions

$$Y^1_{it} = \underset{(3.0)}{0.13}\, u^3_{it-1} + u^1_{it} - \underset{(-3.6)}{0.26}\, u^2_{it-2} + \underset{(2.6)}{0.20}\, u^3_{it-2} + \underset{(2.9)}{0.11}\, u^3_{it-3}$$

$$Y^2_{it} = \underset{(5.0)}{0.49}\, Y^3_{it-2} - \underset{(-4.5)}{0.40}\, Y^2_{it-3} + \underset{(2.2)}{0.19}\, Y^3_{it-3} - \underset{(-3.2)}{0.24}\, Y^2_{it-4} + u^2_{it}$$
$$+ \underset{(2.1)}{\overline{0.10}}\, u^1_{it-1} + \underset{(3.6)}{\overline{0.24}}\, u^2_{it-1} + \underset{(9.3)}{\overline{0.75}}\, u^2_{it-3} + \underset{(5.2)}{0.58}\, u^3_{it-2} - \underset{(-3.4)}{0.26}\, u^2_{it-3}$$
$$+ \underset{(5.6)}{0.59}\, u^3_{it-3} + \underset{(2.7)}{0.13}\, u^1_{it-4}$$

$$Y^3_{it} = \underset{(2.3)}{0.17}\, Y^3_{it-3} + \underset{(3.9)}{0.38}\, Y^1_{it-4} - \underset{(-5.4)}{0.33}\, Y^2_{it-4} + u^3_{it} + \underset{{}^*(0.6)}{\overline{0.02}}\, u^1_{it-1}$$
$$+ \underset{(7.0)}{\overline{0.40}}\, u^2_{it-1} + \underset{(9.4)}{\overline{0.69}}\, u^3_{it-1} + \underset{(3.4)}{\overline{0.12}}\, u^1_{it-2} + \underset{(3.8)}{\overline{0.17}}\, u^2_{it-2} + \underset{(10.4)}{\overline{0.76}}$$
$$+ u^3_{it-2} + \underset{(4.4)}{0.33}\, u^3_{it-3} + \underset{(4.0)}{0.43}\, u^1_{it-4} - \underset{(-3.2)}{0.14}\, u^2_{it-4}$$

†Observations: 384 [96 monthly observations for each day i=7,14,21,28 from 1974(6) to 1982(5)]. Insignificant MA coefficients at the 5 percent significance level are marked with *; numbers in brackets are t ratios. Barred coefficients correspond to those induced by construction.

Table 4: Error Covariance Matrix, Multiple Currency Estimates

Block Order: Pound, Mark, Yen

$10^5\ \hat{\Sigma}^* =$

400								
300	400							
290	310	560						
.32	.32	.27	.00195					
.34	.36	.30	.00160	.00217				
.35	.36	.34	.00164	.00196	.00221			
42	33	25	.097	.101	.101	23		
46	38	37	.088	.107	.106	18	22	
46	38	35	.081	.099	.104	19	21	28

Japanese government. If the government of a country intervenes in the market in order to maintain some sort of relationship between the value of the domestic currency and other currencies, the behavior of other currencies becomes a valuable piece of information to investors and they base their forecasts of the value of that currency on forecasts of the values of other currencies. Therefore, to the extent that the behavior of other currencies in unpredictable, the errors made in forecasting the value of the one currency will be related to the errors made in forecasting the values of other currencies.

We also see that the short side of the dollar/yen maturity structure is more affected by multiple currency estimation than the long side, e.g., we lose more coefficients in the first equation than in the last one. One explanation for this result may be that agents expect the Japanese government to intervene more heavily in the spot market than in futures markets so that $({}_tF_t - {}_{t-1m}F_t)$ is affected more than other series by movements in other exchange rates. The second and third series, $({}_{t}F_{t+1m} - {}_{t-2m}F_{t+1m})$ and $({}_{t}F_{t+3m} - {}_{t-3m}F_{t+3m})$, are price revisions for more distant dates; these would be less affected by disturbances in other currencies if investors did not expect the current intervention brought about by movements in other exchange rates today to be of a permanent nature or to have a lasting effect on spot rates.

B. Out of Sample Forecasts

The iterative procedure of model specification and respecification that underlies time-series analysis makes in-sample confidence intervals virtually impossible to calculate. In order to evaluate the models presented above we withheld 168 observations for each exchange rate (14 forecasts for 4 days for 3 series) from the end of the estimation sample [1982 (5)] until 1983(7)] entirely from the identification and estimation procedure, and used them to formally test the model validity by means of out-of-sample forecasts.

Since the observations to which the model is fitted are price revisions, i.e., for each exchange rate

$$\underline{e}_{it} = \begin{bmatrix} e^1_{it} \\ e^2_{it} \\ e^3_{it} \end{bmatrix} = \begin{bmatrix} {}_{t}F_{it} - {}_{t-1m}F_{it} \\ {}_{t}F_{it+1m} - {}_{t-2m}F_{it+1m} \\ {}_{t}F_{it+3m} - {}_{t-3m}F_{it+3m} \end{bmatrix} \qquad (8)$$

the forecasts must be conditioned on different information sets to be useful. In particular, a contract based on an e^1_{it} forecast can be entered at time i,t-1m while contracts based on e^2_{it} and e^3_{it} forecasts must be entered at i,t-2m and i,t-3m, respectively. This means that the errors from the earlier maturity contracts (e^1 for e^2 and e^3, and e^1 and e^2 for e^3) are not available in forecasting later-maturity price revisions, i.e., the moving average error terms due to data construction, while important in obtaining efficient parameter estimates, are not available at the time the forecasts must be made. This makes forecast evaluation easier, since any pattern in the forecasts represents usable information and not simply (previously exploited) events in a common time interval--we do not have to remove the induced MA terms, since the forecasts are made before these terms can be utilized.

Table 5: Root Mean Squared Errors for Out-of-Sample Forecasts

	Currency								
	Dollar/Pound			10^{-4} Dollar/Yen			10^{-2} Dollar/Mark		
Maturity / Model	30	90	180	30	90	180	30	90	180
Single Currency Model	.0399	.0639	.0875	.0250	.0364	.0327	.162	.192	.241
Multiple Currency Model	.0414	.0634	.0856	.0158	.0267	.0299	.146	.186	.236
Market Price Revisions	.0439	.0668	.0875	.0152	.0239	.0283	.129	.179	.229

For each of the single currency models and the multiple currency model we calculated the conventionally used mean squared error, MSE, and a non-parametric test on the direction of forecasts, all for observations excluded from the identification and estimation procedure. In our case, the MSE took the form

$$MSE = \frac{1}{56} \sum_{i}\sum_{t} e_{it},$$

where it is understood that i=7, 14, 21, 28 and t=1, 2,...,14. The square roots of these MSE's are reported in Table 5. As expected, in almost all cases forecasting accuracy decreases as time to maturity increases, and the calculated statistic is very close to the respective market price revision numbers. It is seen that, for all maturities the dollar/pound models produce forecast errors marginally smaller than simply using the market forward rate (compare to the market price revisions row), while the dollar/yen and dollar/mark models are generally less successful. If governmental intervention does in fact underlie the elaborate dollar/yen specification, it seems not to be systematic enough to exploit through the fitted coefficients. The importance of other currency information in dollar/yen forecasts is again evident in the large decreases in MSE for the multiple currency model, however.

Root mean squared errors are routinely used in evaluating forecasting accuracy, and we have provided them for this reason. They may provide some second moment information to the policymaker investigating market efficiency or the investor making a portfolio decision, but the primary interest is in the forecasts themselves and the value of decisions based on them. Direct test of the usefulness of the model require knowledge of the utility function of the person using the forecasts, information not generally available. The relevant statistic is not as simple as a χ^2 test on the sum of squared forecast residuals, or even a test based on the far more complicated change in wealth given an optimal response to the forecasts (which already requires knowledge of the utility function); rather, a direct test of model usefulness would require a measure of the change in utility resulting from optimal reaction to the forecasts, an impossible condition in practice. Fortunately, Merton in one paper and Henriksson and Merton in another recognized and solved this problem with an ingenious nonparametric test that separates the information in the forecast from an individual's optimal response to the forecast, a ". . . test of forecasting ability which does not require any assumptions about either the distribution of returns on the market or the way in which individual . . . prices are formed."[7]

[7]Henriksson and Merton, 1981, p. 516.

To apply the test to our forecasts, let us set $\gamma_{it} = 1$ if the forecast price revision for a particular series is nonnegative, and set $\gamma_{it} = 0$ otherwise. Then, following Henriksson and Merton, under the null hypothesis of no value in the forecasts.

$$\text{prob}(\gamma_{it} = 0 \mid e_{it} < 0) + \text{prob}(\gamma_{it} = 1 \mid e_{it} \geq 0) = 1. \qquad (9)$$

The conditional probabilities above can be estimated by counting the conditionally correct forecasts. For each series from each model, define, over the out-of-sample forecast period,

N_1 = number of observations with negative price revisions,
N_2 = number of observations with nonnegative price revisions,
$N = N_1 + N_2$,
n_1 = number of successful predictions given a negative price revision,
n_2 = number of unsuccessful predictions given a nonnegative price revision,
$n = n_1 + n_2$.

Then the confidence level[8] c of rejecting the null hypothesis of no value in the forecasts is given by

$$c = 1 - \sum_{x=n_1}^{\min(N_1,n)} \binom{N_1}{x} \binom{N_2}{n-x} \Big/ \binom{N}{n}, \qquad (10)$$

see Henriksson and Merton equation (9), p. 519. Table 6 summarizes the results of this test applied to our models' forecasts. Besides the confidence levels, we have also provided the solution of equation (15) for the lower bound of the summation such that c = .95, i.e., the number of conditionally correct predictions required to reject the null hypothesis with 95 percent confidence. We have included n_1 and

[8]This is a one-tailed test, since the forecasts are judged valuable if they are systematically perverse as well as systematically correct on the grounds that a forecaster this astute would have the sense to reverse the forecasts.

N_1 with this bound to give a sense of the importance of the small sample properties of this test.[9]

We found the results in Table 6 surprising. While it is true that some of the confidence levels look more like significance levels, the remarkable thing is that there are some that do not. Recall that these are price revisions that are being forecast, not simply exchange rate changes already incorporated by the market in the forward rates; these are forecasts of the part of the change in exchange rates that the market has not anticipated. The 90 and 180 day forecasts are consistent with efficiency, having essentially no value except in the multiple currency 90 day

Table 6: Nonparametric Out of Sample Forecast Evaluation†

	Single Currency Model				Multiple Currency Model			
	c	n_1	95%	N_1	c	n_1	95%	N_1
Dollar/Pound								
30 day	.703	28	30	44	.950	32	32	44
90 day	.374	37	40	47	.000	35	40	47
180 day	*	0	*	46	*	0	*	46
Dollar/Yen								
30 day	.738	19	21	36	.159	18	23	36
90 day	.460	20	24	38	.687	19	21	38
180 day	.466	18	21	38	.460	20	24	38
Dollar/Mark								
30 day	.779	22	24	42	.882	23	25	42
90 day	.169	16	20	45	.018	12	18	45
180 day	.004	15	22	42	.022	16	22	42

†c is the confidence level defined in equation (15); n_1 is the number of successful predictions given a negative price revision; 95% is the n_1 value required for 95% confidence; N_1 is the number of observations with negative price revisions.

*The test is not defined in these cases since there were no negative forecasts at all (n=0).

[9]The small sample effects are also evident when the 90 day single and multiple currency dollar/pound forecasts are compared.

dollar/yen case. The 30 day forecasts are significantly better, however. With the one exception of the 30 day multiple currency dollar/yen forecast, we can claim with relatively high confidence (from .703 to .950) that the forecasts would have been of value in predicting the shrot-term price revisions in all cases. To the investor, these forecasts provide potential profit opportunities, and to the policy maker they suggest that, given the required assumptions, the markets may not be perfectly efficient.

Of course, a number of caveats are in order. First, we have necessarily assumed away information and transaction costs. Second, the macro conclusions depend on risk neutrality in the aggregate to identify the market price revisions as forecast errors rather than time-varying risk premia. It does seem, however, that if we are forecasting risk premia, we would be at least as likely to find the effects in the longer maturities as in the 30 day series, which we do not. Finally, the absence of value to all but one of our longer term forecasts is only evidence of smoothly functioning markets in these maturities if you have confidence in our modelling skill; in our defense, we did manage to make useful short-term forecasts.

V. Conclusion

We have reported pooled multivariate time-series models of market price revisions for all nonredundant maturities for which data on the dollar prices of the British pound, Japanese yen, and German mark are available. In contrast to other studies that attempt to model the rates themselves, we define and model market price revisions, the part of the exchange rate changes not already anticipated by the market in the forward rates. We argue that it is these series that are of primary interest to both the micro-oriented investor and the macro-oriented policy maker. Moreover, the fixed terminal dates underlying the construction of these series when combined with rational expectations and certain assumptions can make the price revisions independent of the myriad of conditioning variables throughout the world that determine exchange rates. In practice the set of important conditioning variables is impossibly large, so that this independence is of some consequence. Since it is both unnecessary and impossible to fit structural models to these price revisions, we have turned to multivariate time series analysis. Unlike more traditional time series analyses, however, which rely on time differencing to induce stationarity, i.e., remove the effects of the potentially nonstationary conditioning variables, we have used the market forecasts (under certain assumptions), the forward rates.[10] In addition, we have specified a plausible set of pooling assumptions that permits the use of a much larger (four times larger in our case) set of observations in formulating and estimating the models. Finally, we have tested the validity of the models' forecasts on 168 observations withheld from the identification and estimation process. Since both the macro and micro usefulness of the models ultimately depends on unobservable utility changes resulting from optimal responses to the forecasts, we have eschewed the customary forecast evaluation techniques in favor of a nonparametric test due to Henriksson and Merton that does not require knowledge of the optimal portfolio adjustment or the expected market return. Using this test we find surprisingly high confidence levels associated with the value of the short-term forecasts.

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[10]This is consistent with the spot and future rates for each currency and maturity being co-integrated.

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