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A Positive Approach to Microeconomic Programming Models

by

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Introduction

Solutions to regional programming models show far greater specialization of production by region than actually occurs. This is the result of various factors including simplifying specifications as to: (1) the variability of fixed inputs facing individual farmers such as soil types, specialized machinery inventory, capital and loan arrangements, labor, etc.; (2) variations in known production technology and managerial skills for particular crops; (3) processing plant location and contractual agreements for seasonal crops; and (4) risk and other institutional constraints.

Policy analysis based on normative models that show a wide divergence between base period model outcomes and actual production patterns result in less than full acceptance of the simulated scenarios. Previous writers such as Day [1961] have attempted to provide added realism by imposing upper and lower bounds to production levels as constraints. This problem limits the value of linear models for policy purposes since models that are poorly calibrated and unbelievable will not be used. But models that are tightly constrained can only produce that subset of normative results that the calibration constraints dictate. The policy conclusions are thus bounded by a set of constraints that are expedient for the base year but often inappropriate under policy changes. This problem is exacerbated when the model is built on a regional basis with very few empirical constraints but a wide diversity of crop production. Among the most recent authors to discuss this problem is McCarl [1982] who advocates a decomposition methodology to reconcile sectoral equilibria and farm level plans. Referring to sectoral models (SM) and representative farm models (RFSM) he states:

> There is much appeal in both of these systems. However, both have major shortcomings. The SM models are highly aggregate, adequately capturing the overall markets while not capturing the full factor-product substitution possibilities which would exist in an RFSM model. In cases, this can lead to quite misleading results. . . On the other hand, the RFSM models are cumbersome and difficult to manipulate into a simultaneous equilibrium.

Meister, Chen, and Heady [1978] in their national quadratic programming model, specify 103 producing regions and aggregate the results to ten market regions. Despite this structure, they note the problem of overspecialization:

> If all producing activities are defined by single product activities, as assumed by most theoretical analyses, . . . the tendency of the programming model to produce only one type of commodity in a region or area increases.

The authors suggest the use of rotational constraints to curtail the overspecialization and reflect the agronomic nature of production. However, it is comparatively rare that agronomic practices are absolutely dictated, but more commonly reflect net revenue maximizing trade-offs between yields, costs of production, and externalities between crops. In this latter case, the rotations are themselves a function of relative resource scarcity, output prices, and input costs. This point is demonstrated by farm output response to the recent volatility of crop and energy prices which indicates that, at the margin, farmer's actions are not often rigidly constrained by rotational constraints.

In the early work in quadratic programming with nonlinear objective functions (Takayama and Judge [1964]) and risk terms in the objective function (Freund [1956]), it was recognized that the linearity of the objective function in output had to be avoided to circumvent the constraint/calibration problem. With the recent development of nonlinear optimization algorithms and modern computers, the dimensionality constraint on practical nonlinear problems has been considerably relaxed, and more realistic nonlinear specifications of the microeconomic problem can be envisaged.

A serious drawback to the implementation of separable programming and risk specifications is that they require, respectively, detailed local (stagewise) knowledge of the expected production or cost function faced by the farmer, or regional estimates of unknown indices of risk and monetary measures of risk aversion. While this data has been estimated at the individual farm level, aggregation to a regional policy relevant level has not often resulted in empirically verified models.

This paper proposes a method to amend normative linear microeconomic models by a positive measure of the nonlinear part of the cost function based on the actual actions of the farmers. Using this positive approach the linear model can be exactly calibrated to observed outputs for a single year or calibrated with a least-squares criterion if actual crop acreages for several years are known. The resulting optimization problem incorporates a quadratic cost term for each regional crop grown and is constrained only by those fixed input constraints that can be empirically justified. The problem is solved as a quadratic programming problem, and being only moderately constrained, the model reaction to policy changes is a smooth trade-off based on changed comparative advantage.

In the first section we show that the assumptions of perfect competition and rational expectations imply the existence of a nonlinear component in some regional crop cost functions. A quadratic nonlinearity results from a quadratic production function and/or a mean-variance risk specification. In the second section we prove that a partially constrained program with a quadratic cost function can yield the identical output as a fully constrained linear program and that the best estimates of the coefficients of the unknown quadratic cost function are based on the dual values associated with the artificial calibration constraints. The principal steps of the method proposed here which we call positive quadratic programming (PQP) are outlined. In section three the PQP approach is applied to a regional quadratic programming model of California crop production. This model has 14 regions and over 350 empirically observed regional crop activities. The results show that a model can be exactly calibrated for a single time period without additional constraints in a two-step procedure and at moderate computer cost. Section four presents the results of a time series/cross section regression of the derived nonlinear coefficients on their lagged values and exogenous variables. Even with the short five year time series available, the mean deviation of predicted regional crop acreage is encouraging.

The Microtheory Basis of Positive Quadratic Programming

Since the introduction of linear programming for economic analysis, it has been recognized that models with single nonseparable production technology for each output the linear constraint set imply a Leontief linear production technology. In this section, a common situation is specified in which the cost functions which satisfy the first order conditions for profit maximization differ from those resulting from linear production functions. The positive quadratic programming (PQP) specification is based on the discrepancy between the linear cost function and the cost function implied by the farmer's actions. In addition, the PQP objective function specification is shown to result from a quadratic production function and be consistent with the widely used quadratic "risk" specification.

Specifying a multi-output linear programming problem as

(1) Max $\bar{r}^T x$

Subject to $Ax \leq b$

where x is an n x 1 vector of output acres, $\bar{r}$ an n x 1 vector of net returns per acre, b an m x 1 vector of inputs, and A an m x n matrix of linear production function coefficients.

The optimal solution of k outputs $\bar{x}$ will be associated with the optimal basis matrix B and the vector of constraining resources $\bar{b}$ as:

(2) $B\bar{x} = \bar{b}$

$B = k \times k \quad \bar{x} = k \times 1 \quad \text{and} \quad \bar{b} = k \times 1, \; k \leq m$

It follows directly from the linear independence of B that the vector dimension of optimal LP outputs is equal to the number of binding constraints at the optimum which has an upper bound of m.

In regional studies of farms, the number of empirically justifiable constraints are comparatively few. Land area and soil type is clearly a constraint, as is water in some irrigated regions. Crop contracts and quotas, building capacities, breeding stock, finance, managerial skills, and perennial crops are others. However, it is rare that some other traditional programming constraints such as labor, machinery, or crop rotations are truly restricting to short-run production decisions. These inputs are limiting, but only in the sense that once exceeded, the cost per unit output increases due to overtime, increased probability of disease, or machinery failure.

The empirical situation in which PQP is an appropriate technique is when the number of crop outputs that the farmers actually produce exceeds the number of truly inflexible short-run constraints on factor inputs. We think that the majority of regional programming models and some representative farm models fall into this category. If the farmers are producing more crops than the number of binding constraints, they must be producing the more profitable crops at a level where the marginal expected profit is zero and the profit function for that crop, conditional on the binding constraints, has an interior solution. To reiterate, if farmers are observed to produce ℓ crops but there are only k ($k < \ell$) measurable constraints binding at the optimum, then farmers must expect ℓ-k unconstrained interior solutions for the most profitable crops. This in turn implies that the expected profit function must be concave in land for the region of the observed acreages of ℓ-k crops.

This conclusion is based on the assumption of optimizing behavior, inherent in all programming models, and Muth's [1961] concept of rational expectations in which "expectations, since they are informed predictions of future events, are essentially the same as the predictions of the relevant economic theory."

We make the common assumption that farmers are price takers in inputs and outputs and maximize expected net income. Since we employ a linear quadratic specification we can invoke the certainty equivalence principle and avoid more complex expectations structures. The revenue is linear in output and thus the concavity of the profit function in land must be contained in the cost function for those crops with interior solutions, hereafter termed nonmarginal crops. The increase in the cost per unit output as additional acres are allocated to a crop may arise from both increased variable inputs/acre and decreased yields/acre as crops are grown an increasingly less suitable soil types. If nonjointness of the multi-output multi-input farm production function is assumed the farm production can be represented by individual crop production functions.

Lau [1978] shows that the quadratic production specification satisfies the seven basic properties of a production function if the inputs are constrained to a rational subset. The single crop production function is:

$$(3)\quad Y_i = \alpha_i^{'}\bar{x}_i + \bar{x}_i^{'}\beta_i\bar{x}_i$$

where Y_i is output of crop i and $\bar{x}_i$ is an m x 1 vector of variable inputs with land as x_{1i}. Given the multi-output farm production function, land is a variable input to a nonjoint individual crop production function. The farm acreage constraint, representing an inelastic supply, yields the imputed price of land.

The usual programming practice of imposing Leontief priors on all the variable inputs except land allows the multi-output production function subject to linear constraints to be simplified to an input vector of land acres allocated to alternative crops. Equation (3) becomes the scalar form:

$$(4)\quad Y_i = \alpha_i x_i - \beta x_i^2$$

The quadratic term in the profit function for nonmarginal crops can be totally attributed to mean-variance risk costs if land quality is assumed uniform and the production function linear. At the other extreme, risk can be omitted and the entire nonlinear effect attributed to a nonlinear production function. The distribution of the effect is an empirical question. Since the mean-variance quadratic specification is well-known (Freund [1956], Weins [1976])[2], we will concentrate on the relationship between the quadratic production function and the PQP programming specification.

Proposition. Given the certainty equivalence assumption on stochastic revenues, price taking and maximizing behavior by farmers, the quadratic production function in land (4) is a necessary and sufficient condition for the addition of the PQP quadratic term in land to the usual LP specification.

Necessity. Defining some general multioutput production function in land $q(x)$ and representative acreages as $\bar{x}_i$, $i = 1 \ldots n$, the output price normalized LP objective function is:

$$(5) \quad \sum_{i=1}^{n} \frac{1}{\bar{x}_i} q(\bar{x}_i)\, x_i - w_i x_i$$

Defining the linear yield coefficient as r^* and the diagonal PQP quadratic matrix as E, the normalized PQP objective function is:

$$(6) \quad r^* x - x' E^* x - w^* x .$$

Equating (5) and (6) we obtain

$$(7) \quad q(\bar{x}_i) = [r_i^* x_i - e_{ii}^* x_i^2] \frac{\bar{x}_i}{x_i}$$

Thus the PQP specification implies a nonjoint quadratic production function in land. If x_i deviates significantly from $\bar{x}_i$, the linear LP term needs to be scaled by $\bar{x}_i/x_i$.

Sufficiency. Defining a single crop production function as quadratic in land (4) and omitting subscripts for simplification, the output of x acres ($Y = q(x)$) can be expressed by a Taylor series expansion around the representative land input $\bar{x}$:

$$(8) \quad Y = q(x) = \alpha\bar{x} - \beta\bar{x}^2 + (\alpha - 2\beta\bar{x})(x - \bar{x}) - 2\beta(x - \bar{x})^2$$
$$= \alpha x - 2\beta x^2 + 2\beta\bar{x}x - 2\beta\bar{x}^2$$

Defining the linear average yield coefficient r^* as

$$(9) \quad r^*|_{\bar{x}} = \frac{1}{\bar{x}} (\alpha\bar{x} - \beta\bar{x}^2) = \alpha - \beta\bar{x} ,$$

therefore the LP total output for x acres of land is

$$(10) \quad r^* x = \alpha x - \beta\bar{x}x .$$

If the true production function is quadratic, the LP objective function should be modified by the term

$$(11) \quad Y - r^* x = \alpha x - 2\beta x^2 + 2\beta\bar{x}x - 2\beta\bar{x}^2 - \alpha x + \beta\bar{x}x$$

$$= -2\beta x^2 - 2\beta\bar{x}^2 + 3\beta\bar{x}x$$

if $\bar{x} = x$

$$(12) \quad Y - rx = - \beta x^2$$

exactly equal to the PQP term. If $\bar{x} \neq x$, the linear part of the LP objective function should also be modified and $Y - r^* x = -2\beta x^2 + k_1 x - k_2$, where $k_1 = 3\beta\bar{x}$ and $k_2 = 2\beta\bar{x}^2$.

Taking a positive approach to the model, it is desirable to know the proportional contributions of declining land productivity and risk to the quadratic cost term, but not essential to short-run analysis of changing comparative advantage resulting from specified policy shifts. The quadratic cost term implicit in the observed production pattern of farmers is accordingly termed the implicit cost component. However, the implicit cost considerably improves the model in that it enables the nonmarginal crops to be at interior solutions and the full range of crops actually produced by farmers to be represented by the model without the introduction of specious constraints that distort policy analysis. This specification has the additional advantages in that it can be easily estimated from dual values in the standard linear program and solved by readily available quadratic programs. These two steps are shown in the following section.

Calibration and Solution of the P.Q.P. Problem

In this section we prove that an LP (or QP) problem which requires additional constraints to realize the empirically observed output levels can be reformulated as a quadratic program that only contains the empirically measured resource constraints, but exactly reproduces the vector of constrained and unconstrained output levels observed in the calibration period.

Miller and Millar [1976] introduced a method of adjusting the linear cost terms to satisfy the first order conditions at observed output levels for models that are quadratic in output level. This approach was implemented by Fajardo, McCarl, and Thompson [1981] in a national model of Nicaraguan agriculture by adjusting the miscellaneous cost item in each budget. This method (MM) differs from PQP in both philosophy and implementation. Briefly, the MM approach can only be used in national or very large region models where the demand flexibilities provide the nonlinearity in the sectoral profit function. The crop supply functions remain linear as output is increased, and the implied production technology remains linear. Clearly, the PQP approach addresses a different problem, since it concentrates on the crop supply function and works with or without a quadratic revenue function and at any level of disaggregation.

To reiterate, the PQP approach uses the information contained in the empirical observations of crop acreages actually grown, to derive a quadratic cost term. The cost function now satisfies the unconstrained profit maximizing conditions for nonmarginal crops at the output levels that farmers chose on the average in the district. That is, the equilibrium marginal cost that results from the PQP approach is the one that rational profit maximizing farmers would have expected for the nonmarginal crops in that year and region, in order to have decided on the acreages that they did in the absence of constraints.[3]

In the preceding section, we have shown that the additional calibration constraints needed to produce reasonable results for the more profitable nonmarginal crops are approximations to compensate for the absence of a specific nonlinear cost term in the objective function. The marginal conditions will undoubtedly change under different policy scenarios, thus representing them by constraints greatly reduces the policy value of results from these models. If the policy scenario dictates an increase in the comparative profitability of a given nonmarginal crop in a region, the calibration constraints will restrict expansion of the crop acreage and consequent policy prescriptions may be determined by arbitrary constraint relaxation by the analyst. A formal extrapolative method for constraint relaxation is found in "Recursive Programming," Day [1962]. The fundamental hypothesis of the Day approach is that the rate of response to comparative advantage is more accurately determined by historical extrapolation rather than the degree of change in comparative advantage. In times of rapid change for the agricultural sector, this would seem to be a difficult assumption to substantiate.

Empiricial validation of programming models requires that the analyst has observations on the regional output levels for one or more years. The central thrust of this paper is that this source of empirical data is most usefully used not to constrain the final model, but to estimate the missing quadratic cost term implicit in the first and second order conditions. A quadratic coefficient is estimated for each nonmarginal crop activity in each region in a way that is consistent with the linear data and priors in the model and the truly binding resource and management constraints. Fortunately, this can be achieved by a straightforward two-step procedure.

The following theorem proves that if linear transformations of the optimal dual values associated with the binding calibration constraints are used as the coefficients in a quadratic cost term, the resulting optimal solution to the quadratic program, without any calibration constraints, will be identical to the fully constrained linear program. That is, the transformed dual variables are the optimal estimates of the quadratic cost coefficients that achieve the observed interior solutions. The term estimate is used generally, since most programs are calibrated against a single year's data.

Given a time series of base runs and resulting calibration duals, the optimal expected implicit cost can be estimated by two alternative methods. For small dimension base run models, the mean implicit cost can be estimated endogenously by a simultaneous self dual specification. Where the latter approach is precluded by model dimensions or the length of the time series, a time varying stochastic parameter approach (Duncan and Horn [1972]) can be employed to estimate the systematic change in the expected implicit costs. This analysis will be addressed in a subsequent paper.

The P.O.P. Theorem

We define two problems P1 and P2 whose constraint structure is shown for two nonmarginal activities in Figures 1 and 2, respectively. The revenue component of the objective function $f(x)$ can be thought of as linear or nonlinear. Problem P1 is the usual specification with a set of empirically justified resource constraints b and an inequality calibration constraint for each regional crop activity observed. The right hand side of the calibration set is the actual acreages $\bar{x}$ plus a very small but critical perturbation factor ε,[4] without which the Fritz John constraint qualification (Aoki [1971]) is violated (unless there are no binding resource constraints).

Problem P2 has the same (nx1) vector of possible activities x, the same revenue function f(x) and the true resource constraints $Ax \leq b$, but the offending calibration constraints have been removed and a concave but unknown function of calibration constraint set has been added to the objective function.

<u>Problem</u> <u>P1</u>. Max $f(x)$ $(f(\cdot): R^n \to R^1)$

(13) Subject to $Ax \leq b$

$$Ix \leq \bar{x} + \varepsilon \qquad x \geq 0$$

where $x = nx1$ $b = mx1$. Rewriting in Lagrangian form and representing the two sets of binding constraints in (13) by the vector functions $g_1(x)$ and $g_2(x)$, we have:

(14) $\text{Max } L(x, \lambda_1, \lambda_2) = f(x) - \lambda_1 g_1(x) - \lambda_2 g_2(x).$

Define an arbitrary concave vector function of the calibration constraint set $g_2(x)$ as $h(g_2(x))$.

<u>Problem P2</u>. Max $f(x) + h(g_2(x))$

(15) Subject to $Ax \leq b \qquad x \geq 0$

where x, f(x), A, b are the same as in P1.

<u>Proposition</u>. If the optimal solution to P1 is characterized by x^*, λ_1^*, λ_2^*, the problem P2 has an optimal solution $x°$ equal to x^* if and only if

(16) $\nabla_g h(g_2(x^\circ)) = -\lambda_2^*$

<u>Necessity</u>. Since f(x) is concave and continuous,

(17) $x^* = x^\circ \Longrightarrow \nabla_x f(x^*) = \nabla_x f(x^\circ)$.

Defining the Jacobian matrix of the set of constraint vectors $g_1(x)$ with respect to x as $J_x(G_1)$, the first order conditions for P1 require that:

(18) $\nabla_x f(x^*) = J_{x^*}(G_1)^T \lambda_1^* + J_{x^*}(G_2)^T \lambda_2^*$. 5/

The first order conditions for P2 are:

(19) $\nabla_x f(x^\circ) = -J_{x^\circ}(G_2)^T \nabla_g h(g_2(x^\circ)) + J_{x^\circ}(G_1)^T \lambda_1^*$

Equating (14) and (19) implies that:

(20) $J_{x^\circ}(G_2)^T \nabla_g h(g_2(x^\circ)) = - J_{x^*}(G_2)^T \lambda_2^*$.

Since the calibration constraint function $g_2(x)$ is linear, the Jacobian is constant and satisfaction of the constraint qualification implies that $J_x(G_2)^{-1}$ exists. Therefore

(21) $\nabla_g h(g_2(x^\circ)) = -\lambda_2^*$ if $x^\circ = x^*$

<u>Sufficiency</u>. Substituting (16) into (19) and equating (18) to (19), the two revenue function gradients are equal at their respective optimal solutions.

(22) $\nabla_x f(x^\circ) = \nabla_x f(x^*)$

Since f(x) is continuous and concave, equality of the gradients implies equality of their arguments.

Figure 1 L.P. Problem P1

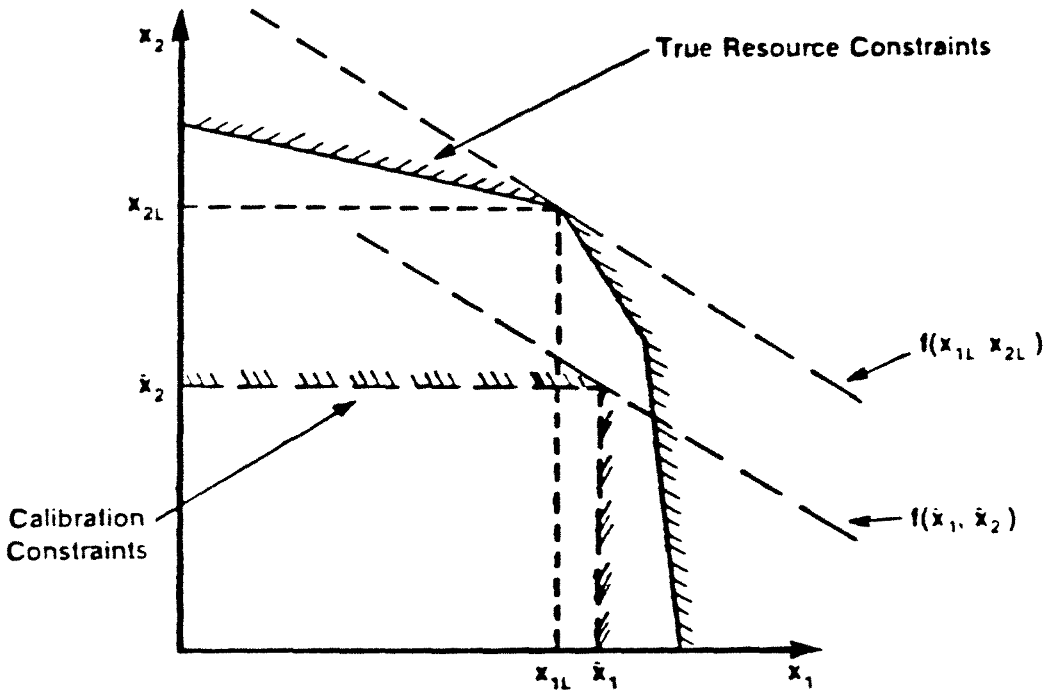
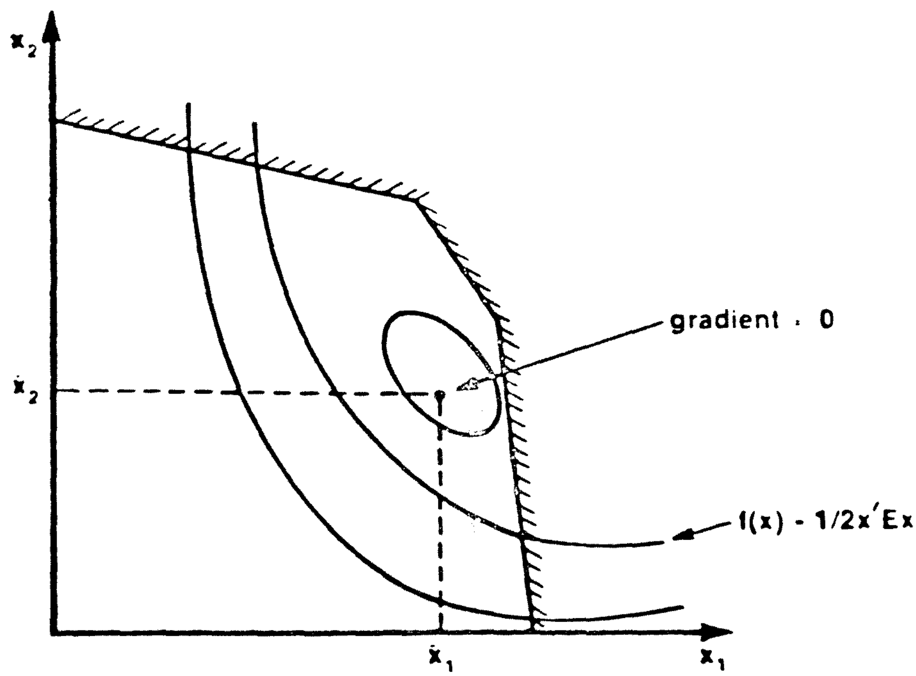


Figure 2 PQP Problem P2



(23) $\therefore\ x^{\circ} = x^{*}$ if $\nabla_g h(g_2(x^{\circ})) = -\lambda_2^{*}$

Implementation of the P.O.P. Approach

Empirical implementation of positive programming is achieved in two stages. The first stage starts with the data and specification of a conventional LP (or QP) problem. The actual regional crop acreages ($\tilde{x}$) are increased by a small perturbation ε say (.005) $\bar{x}$ and are formulated as inequality constraints. The constrained LP problem is now run to obtain the dual values on the calibration constraints for the nonmarginal crops. The ε perturbation of the calibration constraint right hand side ensures that relevant resource constraints will be binding on the marginal crops in the basis. The absence of a quadratic cost coefficient for the marginal crops is not a problem as they are constrained by the active resource constraints.

The vector of dual values from P1 for the nonmarginal crops is multiplied by the negative reciprocal of the observed acreage $\bar{x}_1$ and used as the diagonal coefficients of the quadratic cost function in problem P2. Problem P2 is then solved for the optimal base period solution. The principle steps are:

a Given a standard LP or QP and the vector of actual acreage grown $\bar{x}$. Perturb $\bar{x}$ by ε and add the calibration constraints.

b Run problem P1. If $\bar{x}$ is ℓx1 (ℓ<n) problem P1 will result in k,(k<m) binding resource constraints and ℓ-k values of $\lambda_{21}{}^{*}$ corresponding to the binding calibration constraints.

c From the PQP specification in equation (9) we know that the function $h[g(x^{\circ})]$ is quadratic in x. Therefore, $h[g(x^{\circ})]$ has the form $1/2\bar{x}^T E\bar{x}$ where E is a $\ell x \ell$ positive semidefinite matrix. By the PQP theorem

(24) $\nabla_g h[g(x^{\circ})] = -\lambda_2{}^{*} = E\bar{x}$

Given the minimal data set $\bar{x}$, cross cost effects are restricted to zero, and thus for the single period calibration case considered here E is a diagonal matrix with nonzero elements e_{ii} where:

(25) $e_{ii} = -\lambda_i^* / \bar{x}_i$

corresponding to the nonmarginal cropping activities.

d Using the values e_{ii}, the problem P2 is specified as

(26) Max $f(x) + 1/2x'Ex$

Subject to $Ax \leq b \quad x \geq 0$

Can also incorporate linear term in $c'x + 1/2x'Ex$ where $c_i = -\lambda_i^*$ and $e_{ii} = (c-\lambda$

The problem P2 calibrates exactly with the base year vector $\bar{x}$ without spurious constraints and is available for policy analysis in the knowledge that the model response will be determined by economic comparative advantage and resource constraints that have a clearly demonstrated empirical basis.

An Empirical Test of PQP

Over the past two years, the PQP approach has been used on four models of agricultural production, two small linear programs, and two large regional quadratic programming models. In demonstrating the practical aspects of the PQP approach, we will only discuss results from the California Agricultural and Resource Model (CARM) model since it is the largest and most general of the PQP models run to date.

Readers familiar with empirical quadratic programming problems will immediately question whether the greatly increased dimensionality of the quadratic coefficient matrix leads, first, to exhorbitant solution costs and, second, to scaling and convergence problems. This section shows that with modern algorithms the PQP specification can be run on small computers for an acceptable increase in cost over linear program specifications of the same problem.

Adams *et al.* [1978], developed a quadratic programming model of California's crop production that included 37 cropping activities in 14 regions. The production regions were delineated on the basis of climate, soil characteristics, and water availability. Adams *et al.* reported deviations of model results from the actual state acreages in the base year; the use of crop revenue variability coefficients improved the results somewhat.

Subsequently, the Adams model was updated, extended to the CARM specifications. The statewide crop pattern closely approximated actual acreages, however, the cropping pattern in the 14 production regions had to be constrained to avoid excessive regional crop specialization. Thus, despite endogenous crop prices the upper bounds on regional production levels imposed constraints on the whole model. These flexibility constraints were particularly confining when the model was used to derive supply elasticities or derived demands for irrigation water and energy. Consequently, an alternative approach based on microeconomics was sought, which resulted in the PQP concept.

Calibration

Calibration of a programming model is usually based on a single year or aggregate data set. Given seasonal variability, the CARM crop yield coefficients were based on three-year averages. Resource constraints on short-run cropping decisions are only specified when empirically justified. Thus, while the CARM model has input coefficients for land, labor, water, fuel, power, and regional crop processing capacities, in the short run only land, irrigation water, and some regional crop processing capacities can be empirically justified as potentially binding at the margin. In addition, the acreage of perennial crops is specified as fixed in the short run.

Following the procedure outlined in the previous section, the first calibration step involved constraining the model to the empirically observed 1978 crop acreages in the production regions and running the resulting quadratic program. The program had 65 quadratic variables for the endogenous seasonal crop demands and a 786 x 1081 linear constraint matrix with 4743 nonzero elements. The computer package used was Minos [Murtagh and Saunders 1977], which achieved optimality in 717 iterations and 44.2 seconds of central processor (CPU) time on a Control Data 7600. The cost was $10.77.

Clearly to satisfy the second order conditions for the PQP problem P.2, the matrix E in equation (26) has to be negative definite. This implies that the dual values of the empirical calibration constraints on those crops that are nonmarginal in the base run must be positive. This requirement provides a useful test of the internal consistency of the production constraints and variable costs in the base model before the PQP terms are added. If empirically observed outputs are not present in the base model solution, one of three conditions must prevail. Either positive externalities are missing from the production technology constraints or the variable costs in the model of growing the crop in that district exceed those actually expected by the average operator. Or, lastly, the average farmer is economically irrational in producing a crop in which expected revenues do not equal variable costs.

Rejecting the last possibility of mass irrationality by farmers, the omission of observed crops from the base year solution indicates that the base model is misspecified. Correction of minor misspecifications of the linear cost terms in CARM resulted in all the calibration duals being positive in sign.

The PQP version of the CARM model contained 370 nonlinear variables consisting of the original 65 revenue variables and 305 nonlinear regional crop cost variables. The model calibrated itself to the actual regional acreages with an error of less than one-tenth of a percent in every case. The only constraints on regional crops were land and water availabilities. The problem was run on a CDC 7600 and took 105.9 seconds of CPU time at a cost of $20.27.

Subsequently, the same PQP problem was run on a small $100,000 "Midi" computer, a DEC VAX750 which solved the problem in one hour and 53 minutes of CPU time and a comparable cost of $33.80. This demonstrates that not only can the procedure be implemented for large problems at a reasonable cost, but that with modern software and virtual memory machines, the analyst is not dependent on large mainframe computers to solve this class of PQP problems.

Econometric Estimation of Implicit Costs

While the ability to develop unconstrained and exactly calibrated models for a single year is an advance, the policy value of such a model depends on the degree of variability of future years from the base year, or more importantly, the ability of the model to be systematically updated. Since the PQP formulation claims to represent farmers' expectations on net revenues through the implicit cost coefficients, the implicit costs should have a statistically significant relationship with previous implicit costs if growers update expectations adaptively or with exogenous price and cost factors if they have a very simple rational expectations scheme. This hypothesis is tested using the implicit cost results for five years for 209 regional crop acreages each year. The regression results show that the variability in implicit costs over the years can be satisfactorily explained. Given the volatility of farm prices and effects of a two-year drought in California, the hypothesis that the implicit costs are a systematic response by growers to endogenous and exogenous factors is supported by the results.

Regressing the implicit costs or duals/acre from the calibration constraints raises two additional methodological points. First, is this an appropriate specification for the PQP model, and second, do the duals per acre that result from the calibration runs have justifiable stochastic properties?

The PQP model based on implicit costs derived from the regression equation becomes a model that maximizes expected net revenue from a stochastic parameter objective function subject to deterministic inequality production constraints. The expected value of the implicit cost is hypothesized to be systematically related to crop price and input cost changes that are exogenous to the model. Clearly, the implicit cost dual estimate is conditional on the linear cost coefficient, the demand function (if any) and the right-hand side factor resource constraints. However, taking a Bayesian view, the original linear model should have contained the best available data based priors on these parameters, and thus consistent estimates based on empirical production decisions by the entrepreneurs being modelled can only add to the precision of the information set.[6]

A perennial problem in applying econometric techniques to results from programming models is that the regression may be a good fit but no statistical properties can be claimed for the output of a deterministic normative model. In the PQP specification, however, the right-hand side of the calibration constraints $\bar{x}$ are random variables. The implicit cost duals associated with the calibration constraints are shown to be linear functions of the constraint values $\bar{x}$ under the following conditions. From equation (24), λ_2^* is a linear function of $\tilde{x}$. But $\tilde{x}$ is a vector of random variables of observed regional or firm production levels. Thus, if $\tilde{x}$ is attributed the usual assumption of a normal distribution

(27) $\tilde{x} \sim N(\bar{x}, \Sigma)$ and $\lambda_2 = E\tilde{x}$

since E is an orthogonal full rank matrix

(28) $\lambda_2 \sim N(E\bar{x}, E'\Sigma E)$

Thus, the per acre values of the duals associated with the calibration constraints are random variables and have the normal statistical properties.

Values for the vector λ_2 were obtained for five annual PQP solutions over 209 regional/crop output levels. Given the paucity of data for some crops, only a simple regression specification was attempted. If the implicit costs have a basis in regional comparative advantage and net revenue expectations, the implicit costs should show a constant regional comparative advantage between regions over different years and a systematic response over regions and between years to adaptive or rational expectations. Accordingly, the time series and cross section of implicit costs/acre for a given crop were regressed against regional dummy variables, the lagged implicit cost and a lagged index of net crop revenue.

Given k crops, j regions that grow crop k, and year t, k = 1, . . ., 28; j = 2 - 14; t = 1974, . . ., 1978.

(29) $\lambda_{kjt} = C_1 + \beta_1 D_1 + \ldots + \beta_{j-1} D_{j-1} + \beta_j D_j + \beta_{j+1} \lambda_{kjt-1} + \beta_{j+2} NRI_{k,t-1} + \varepsilon_t$

$D_1 \ldots D_{j-1}$ regional dummy variables

D_j post-drought dummy variable for 1978

NRI_{kt-1} price index less cost index for crop k and year t-1.

The results for the single equation regressions are tabulated in Table 1. For brevity, the coefficients on the regional dummy variables were omitted, but the majority of the coefficients were significant at the 95 percent level or above supporting the hypothesis of regional comparative advantage. The post-drought dummy variable was particularly significant for wheat, barley, and sugar beets.

Table 1 shows the significance of the lagged dependent variable and lagged net revenue. The lagged dependent variable representing an adaptive expectations structure was significant at the 95 percent level or above for half of the crops. The lagged net revenue index representing a crude indication of rational expectations was significant at the 95 percent level for 54 percent of the crops. Twenty-one percent of the crops did not have either coefficient significant, but they comprised a small proportion of the total output with the exception of sugar beets, which is grown on contract.

Three conclusions can be drawn from these initial regressions. First, a large amount of the variability of the implicit costs can be explained by the regressions. Second, many cropping regions exhibit systematic comparative advantage over the five years. Third, 79 percent of the crops showed a significant relationship to the lagged dual value, a net price index or both.

From a positive viewpoint a more important test of the model is the extent to which the vector of duals generated by the regression equations can reproduce the output mix of a given year in the PQP model. The 1977 implicit cost duals and net revenue index were used in the regression to generate the vector of 1978 regional implicit costs. It should be noted that these implicit costs are not predictions in the conventional sense, since 1978 was included in the sample used to estimate the regression. However, they do

Table 1. Regression of Implicit Cost-Duals by Crop/Region/Year
Years 1975-1978

NOTE: Regional Dummy Variables Omitted

Crop Dual by Region/Yr.	Lagged Dual Value	Lagged Net Revenue Index	R^2	DW	df
ALFALFA	0.5379 (0.0732)**	0.1416 (0.0822)*	.924	2.72***	36
ALFALFA SEED	0.3155 (0.4872)	32.49 (43.26)	.831	1.34	10
ASPARAGUS	0.4855 (0.2987)	-3.4961 (1.9885)*	.954	1.85	15
DRY BARLEY	0.0687 (0.1319)	-0.7853 (0.2345)**	.951	2.34***	33
IRRIGATED BARLEY	0.1829 (0.1752)	-1.5999 (0.5751)**	.888	1.86	33
BEANS	0.2506 (0.6809)	-7.9495 (2.2473)**	.750	2.97	21
BROCCOLI	0.5985 (0.3741)	-2.5428 (0.8953)*	.987	1.60	6
CANTALOUPES	0.6455 (0.4817)	-8.4653 (6.7251)	.798	2.91	6
CARROTS	-0.7050 (0.3302)*	-14.168 (2.562)**	.893	1.78	12
CAULIFLOWER	0.7152 (0.2465)	5.9001 (4.462)**	.892	1.72	5
CELERY	1.2044 (0.9435)	21.4411 (7.7088)*	.878	2.51	3
CORN	-0.3511 (0.7246)	-3.2600 (3.8270)	.838	2.77	12
COTTON	1.5006 (0.6617)*	7.5234 (10.409)	.776	1.91	10
GRAIN HAY	0.2783 (0.1949)	0.0346 (0.0531)	.831	2.07	33

(continued)

Table 1 (continued)

Crop Dual by Region/Yr.	Lagged Dual Value	Lagged Net Revenue Index	R^2	DW	df
SORGHUM	0.6381 (0.3603)*	1.7785 (1.0299)*	.952	2.88	18
LETTUCE	0.5465 (0.1655)**	-0.1928 (1.7032)	.890	1.89	18
ONIONS	0.6692 (0.1029)**	1.0895 (0.6823)	.952	2.03	24
IRRIGATED PASTURE	-0.3119 (0.1533)**	0.5686 (0.0393)	.887	1.66	39
POTATOES	0.9671 (0.2091)**	22.458 (4.820)**	.882	2.51	15
RICE	1.728 (0.5026)**	3.4211 (0.6374)**	.935	2.53	12
SAFFLOWER	0.08155 (0.1076)	-0.0089 (0.0080)	.980	2.72***	15
SILAGE	0.3296 (0.2552)	0.1981 (0.1109)*	.906	2.02	27
STRAWBERRIES	0.6482 (0.2613)*	32.393 (16.813)	.985	2.71***	6
SUGAR BEET	0.2174 (0.1594)	-0.3037 (0.2128)	.948	1.91	27
FRESH TOMATOES	0.5563 (0.1528)**	2.3043 (5.869)	.957	1.74	18
PROCESSED TOMATOES	0.7606 (0.1627)**	0.7889 (0.3369)*	.811	2.92***	27
DRY WHEAT	0.2868 (0.1568)*	-0.1902 (0.2726)	.928	2.43***	30
IRRIGATED WHEAT	0.2973 (0.1491)*	-2.002 (0.6267)**	.912	1.99	24

* Denotes significance at 5 percent level.
** Denotes significance at 1 percent level.
*** Autocorrelation is rejected at the 5 percent level by the Durbin-H statistic.

With an updated data set through 1982 there should be sufficient degrees of freedom to test the truly predictive nature of the model and its ability to track a volatile agricultural sector.

Additional Applications of PQP

One of the advantages of the PQP specification is that the optimal solution responds smoothly to changes in comparative advantage induced by parameterizing one or more of the revenue, technical, or constraint parameters.

This paper has posed the PQP specification in the traditional asymmetric form. However, the approach is equally applicable to the symmetric specification (Paris [1979]) in which regional and aggregate derived demands for input resources can be derived.

Paris [1982] shows that quadratic programming results are generally characterized by multiple optimal solutions. However, a necessary condition for multiple optima is that the quadratic coefficient matrix E in equation 22 is negative semidefinite. It appears that the PQP formulation avoids the problem of multiple optima since E is strictly negative definite.

Conclusions

The paper has proposed a modification of the conventional linear production constraints in microeconomic linear and quadratic programming models without increasing the data requirements of the model. The PQP specification is not an *ad hoc* modification but is derived directly from the quadratic production function in land and motivated by the common case in which the number of crop activities exceeds the number of empirically binding resources.

The PQP procedure is shown to be readily implemented by a two-step procedure at a reasonable cost using modern algorithms.

For policy analysis, the PQP results have the desirable features that they are optimally calibrated to a base year or years but are only constrained by constraints whose changes can be empirically measured. The nonmarginal, more profitable crop activities are free to respond to changes in comparative regional advantage that may result from policy changes. The PQP specification satisfies the problem posed by McCarl [1982] of reconciling the sectoral equilibrium of crop prices and output levels while yielding unconstrained regional crop activity mixes that exactly replicate the actual aggregated (or representative) farm outputs for the base year. These regional cropping patterns are free to respond to changes in local comparative advantage.

The positive quadratic programming specification advocated in this paper for aggregate microeconomic policy models appears to have considerable advantages over the conventional specifications with only linear cost or quadratic risk terms in the objective function. Given additional data, the estimates of the systematic change of the implicit cost parameters suggests a natural extension to a forecasting capacity.

Footnotes

[1]There are some special cases where regional and seasonal specialization could cause some price effect, but, given the collective nature of the effect, a rational individual will not act on it.

[2]The most common specification that yields a cost of risk that is quadratic in output levels is the mean variance approach based on Freund [1956]. There have been many modifications and applications of the mean variance concept which generally improves the diversification and reality of model output, but has not led to claims of complete validation or precise predictions.

Wicks [1978] shows that linear specifications of alternative risk formulations do not yeild good predictive results. Weins [1976] used the Kuhn Tucker conditions and the resulting duals to estimate an aggregate risk aversion coefficient, but his results were hampered by the need for a single risk aversion coefficient implicit in approaches that specify risk as the only nonlinear effect on the regional or individual revenue function.

[3]The PQP approach is related to the penalty function approach to programming solutions with nonlinear constraints (S.U.M.T) (Fiacco and McCormick [1968]); however, the economic problem has two important differences. First, the penalty function approach uses nonlinear costs in the objective function to approximate the effect of nonlinear constraints. Whereas in PQP the artificial calibration constraints are used to impute the real, but unknown costs. Second, sequential unconstrained minimization techniques (S.U.M.T.) use arbitrarily high penalty costs to achieve the constraints, while the POP implicit cost is based on marginal conditions and only equals the constraint for the calibration year or years.

[4]Note the perturbation factor ε is in no way related to the flexibility ranges inherent in the flexibility constraint approach.

[5]The notation $\nabla_x f(x^*)$ denotes the gradient function of $f(x)$ with respect to the vector x at the optimal values x^*, and T superscript denotes the transpose.

[6]The stochastic parameter specification need not stir up Classical versus Bayesian arguments on specification since it can be represented as a particular generalized least squares problem.

References

Adams, R. W., W. E. Johnston, and G. A. King, September 1978, "Some Effects of Alternate Energy Policies on California Annual Crop Production," Giannini Foundation Monograph, No. 326, University of California, Davis.

Aoki, M., 1971, Introduction to Optimization Techniques, MacMillan.

Bessler, D., 1977, Foresight and Inductive Reasoning: Analysis of Expectations on Economic Variables with California Field Crop Farmers, unpublished Ph.D. dissertation, Department of Agricultural Economics, University of California, Davis.

Day, R. H., 1961, "Recursive Programming and the Production of Supply," Agricultural Supply Functions, Heady et al., Iowa State University Press.

Duncan, D. B. and S. D. Horn, December 1972, "Linear Dynamic Recursive Estimation from the Viewpoint of Regression Analysis," Journal of the American Statistical Association 67:815-821.

Farjado, D., B. A. McCarl, and R. L. Thompson, February 1981, "A Multicommodity Analysis of Trade Policy Effects: The Case of Nicaraguan Agriculture," American Journal of Agricultural Economics 63(1):23-31.

Fiacco, A. V. and G. P. McCormick, 1968, Nonlinear Programming, Sequential Unconstrained Minimization Techniques. John Wiley: New York.

Freund, R. J., July 1956, "The Introduction of Risk Into A Programming Model," Econometrica 24:253-263.

Lau, L. J., 1978, "Applications of Profit Functions," in Vol. 1 Production Economics: A Dual Approach to Theory and Applications, M. Fuss and D. McFadden, editors. North- Holland.

McCarl, B. A., 1982, "Cropping Activities in Agricultural Sector Models: A Methodological Proposal," American Journal of Agricultural Economics 64:768-772.

Meister, A. D., C. C. Chen, and E. O. Heady., 1978, Quadratic Programming Models Applied to Agricultural Policies. Iowa State University Press.

Miller, T. A. and R. H. Millar, 1976, "A Prototype Quadratic Programming Model of the U.S. Food and Fiber System," USDA ERS CES and Department of Economics, Colorado State University.

Muth, J. F., July 1961, "Rational Expectations and the Theory of Price Movements," Econometrica 29:315-335.

Murtagh, B. A. and M. A. Saunders, Feb. 1977, "Minos User's Guide," Systems Optimization Laboratory, Dept. of Operations Research, Stanford University Technical Report No. 77-9.

Paris, Q., Oct. 1979, "New Economic Interpretations of Complementarity Problems," Southern Economic Journal 46(2):568-579.

Paris, Q., 1982, "Multiple Solutions in Complementarity Problems," Working Paper No. 82-7. Department of Agricultural Economics, University of California, Davis.

Shumway, C. R. and Anne A. Chang, May 1977, "Linear Programming versus Positively Estimated Supply Functions: An Empirical and Methodological Critique," American Journal of Agricultural Economics 59(2):344-357.

Takayama, T. and G. Judge, 1964a, "Equilibrium Among Spatially Separated Markets: A Reformulation," Econometrica 32:510-524.

Weins, T. B., 1976, "Peasant Risk Aversion and Allocative Behavior: A Quadratic Programming Experiment," American Journal of Agricultural Economics 58:629-635.

Wicks, J. A., 1978, "Alternative Approach to Risk in Aggregative Programming: An Evaluation," European Review of Agricultural Economics 5(1):161-173.