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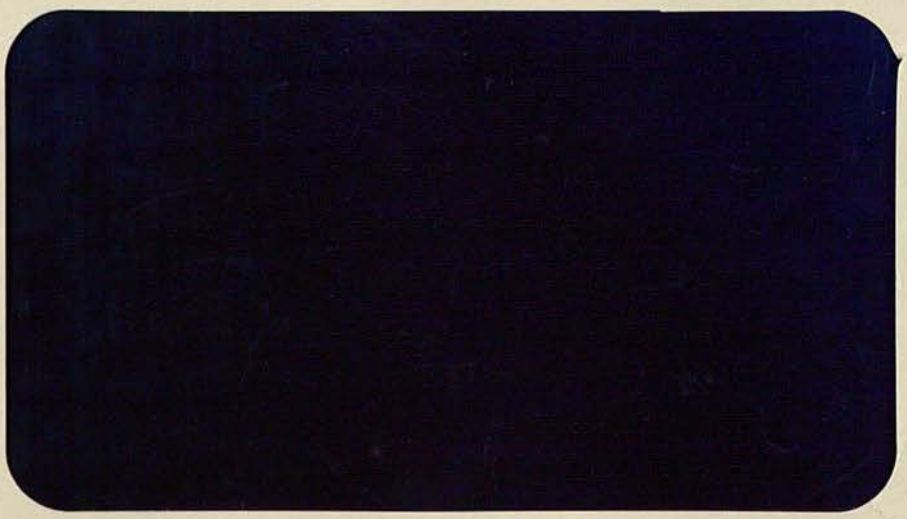

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A TEST OF NUTRIENT NONSUBSTITUTION IN CROP RESPONSE AND FERTILITY CARRYOVER

by

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A TEST OF NUTRIENT NONSUBSTITUTION IN CROP RESPONSE AND FERTILITY CARRYOVER

Introduction

For many years agricultural economists have clung to the idea that the best form of crop response might be a second degree polynomial function. As can be verified from the literature, they even succeeded in convincing many agronomists to adopt it, although few of them would ever subscribe to its implications. A critical one concerns the possibility that macronutrients such as phosphorus (P), potassium (K) and nitrogen (N) substitute for each other in the process of plant growth. Although this idea was denied as early as 1840 by von Liebig in his "law of the minimum," it was propounded by Iowa scientists during a series of renowned seminars in the 1950's. A second degree polynomial was proposed then as a function capable to measure substitution coefficients. No doubt, a second degree polynomial with cross-product variables provides the framework for nutrient substitution. What it cannot provide is the possibility of nutrient nonsubstitution. In other words, a second degree polynomial function builds in substitution simply by the form of its structure. The cross-product variable terms, which in general represent interaction effects, have been interpreted almost exclusively as substitution terms. It is important to realize that substitution implies interaction while the converse does not. Surprisingly, the assumption whether or not macronutrients can and do substitute for each other in crop production was never put to a rigorous statistical test.

Another crucial aspect of a second degree polynomial function is its symmetric shape which prevents the possibility of representing with sufficient accuracy a rather generalized phenomenon in crop response: the yield plateau maximum. Many crop response experiments, including that one analyzed by Iowa

scientists in their original proposal, reveal by simple inspection an extended yield plateau maximum which contrasts sharply with the point maximum of a second degree polynomial. As a consequence, the adoption of such a function for representing crop responses results in an overestimate of the yield maximum and, perhaps more importantly, produces a serious overestimate of the optimal dose of fertilizer.

Recently, a functional form has been proposed for crop response analysis which allows the representation of von Liebig's "law of the minimum" and a yield plateau maximum (Anderson and Nelson; Waggoner and Norvell; Lanzer, Paris, and Williams). Therefore, the objective of this study is to conduct a rigorous statistical test of the nutrient nonsubstitution hypothesis in crop response. Specifically, second degree polynomial functions will be contrasted with a minimum and plateau function in their ability to represent the principal aspects of crop response. The test is a complex one for at least two reasons. First of all, it involves non-nested hypothesis procedures. Secondly, the estimation of the minimum function requires a mathematical programming approach. Finally, the complete specification of a crop response model requires the accounting of residual fertility by means of a carryover function expressed as a difference equation of soil test levels of nutrients.

The Response and the Carryover Functions

Numerous soil and weather variables affect crop response together with fertilizers' levels. To avoid the necessity of explicitly accounting for the multitude of soil type and weather variables (rarely available), we follow Lanzer, Paris, and Williams in their assumptions of weak separability between these variables and fertilizer nutrients. In this way, the general form of the response function can be specified as

(1) $Y = g(s, w)f(b + x) = A_{ws}f(b + x)$

where b represents a vector of nutrient levels initially available in the soil, x is a vector of fertilizer levels added to the soil, Y represents yield levels, while s and w are vectors of soil and weather variables. When knowledge of w and s is unavailable, the parameter A_{ws} can be thought of as representing the yield plateau in a given set of experimental data determined by soil type and weather conditions at that site. In this way, soil and weather variables at the site do not need to be measured. The maximum yield for a given experiment can be used to measure the parameter A_{sw}, leaving only the problem of specifying the form of the function $f(b + x)$.

The specification of the crop response model is incomplete without the carryover function of residual fertility. The ith component of the b vector in equation (1) is interpreted as the amount of the ith nutrient already in the soil. It is the flow component of the stock of the ith nutrient carried forward from past growing seasons. No direct measure of this variable is possible. Chemical soil analyses (soil tests) at the beginning of the season are used as an index of the potential amount of available nutrient in the soil, a part of which may be made available to the current crop. Soil tests have been traditionally assumed to be proportional to the actual level of nutrient available to the crop, that is $b_i = \lambda_i b_i^*$, where b_i^* is the soil test for the ith nutrient and λ_i is the corresponding factor of proportionality.

Assuming furthermore that the λ-values are stable over time, a fertility carryover function for the ith nutrient can, therefore, be specified as a difference equation such as

(2) $b_{it}^* = g(b_{it-1}^* + \lambda_i^{-1} x_{it-1}, Y_{t-1})$

where Y_{t-1} is the crop yield of the preceding period prior to soil sampling, and i = P, K. Where nutrient losses through either leaching or oxidation are insignificant, or alternatively, for immobile nutrients such as phosphorus and potassium, crop yields represent a major source of nutrient loss for the soil.

The Rival Empirical Response Functions

The empirical analysis of this study has attempted to contrast, by means of a rigorous statistical test, two families of mathematical specifications of crop response functions. Among the polynomial family, the quadratic and square root functions were considered in particular. With the available data resulting from an experiment with P and K, the quadratic polynomial function can be indicated as

$$(3) \quad Y = A_{ws}[\gamma_0 + \gamma_1 X_{PT} + \gamma_2 X_{KT} + \gamma_3 X_{PT}^2 + \gamma_4 X_{KT}^2 + \gamma_5 X_{PT}X_{KT}] + \varepsilon$$

where Y is the crop yield, A_{ws} is the experiment's maximum yield, X_{PT} and X_{KT} are, respectively, the total amount of adsorbable phosphorus and potassium available in the soil, that is, $X_{PT} = \lambda_P^* b_P^* + x_P$ and $X_{KT} = \lambda_K^* b_K^* + x_K$.

The square root specification is

$$(4) \quad Y = A_{ws}[\gamma_0 + \gamma_1 X_{PT} + \gamma_2 X_{KT} + \gamma_3 X_{PT}^{1/2} + \gamma_4 X_{KT}^{1/2} + \gamma_5 (X_{KT}X_{PT})^{1/2}] + \varepsilon.$$

The only difference between these two specifications is that the square root function is flatter than the quadratic formulation. These two functions have been regarded for many years as the most flexible and suitable response functions to fit the data from fertilizer experiments. Functions of the type (3) and (4) can be readily estimated by ordinary least squares if the appropriate assumptions can be made about the associated error term, ε.

As a rival specification the following nonsubstitution and plateau response function will be analyzed:

(5) $Y = \min[f_P(Z_{PT}), f_K(Z_{KT})] + \varepsilon,$

where the f_P and f_K are linear splines with plateau in P and K, respectively. Specifically

$$f_P(Z_{PTt}) = \sum_{j=0}^{I_P} \beta_{Pj} Z_{Pjt} - S_{Pt}$$

where t=1,...N, is the observation index, $Z_{P0t} = A_{ws}X_{P0t}$, $Z_{Pjt} = \hat{A}_{ws}(X_{PTt} - X_{PTj})D_{Pj}$ for j=1,...I_P, X_{PTj} is the jth knot, and D_{Pj} is set to 1 if $A_{ws}(X_{PTt} - X_{PTj}) > 0$ and is zero otherwise. The knots X_{PTj} are fixed and known. A similar spline is specified for the potassium function $f_K(Z_{KT})$. The parameters S_{Pt} and S_{Kt} are slacks which must satisfy the constraint $S_{Pt}S_{Kt} = 0$, t=1, N. Finally, the following constraints on the β coefficients $\beta_{P0} > 0$, $\beta_{Pj} \leq 0$, $j > 1$ and $\sum_{j=0}^{I_P} \beta_{Pj} = 0$ are imposed to assure the concavity and the plateau of the response spline function. The estimation of the function (5) requires the use of mathematical programming methods as outlined below.

Equation (5) can be estimated directly by maximum likelihood techniques if a particular probability distribution is assumed for the error term ε. Assuming that $E(\varepsilon_t) = 0$, $E(y_t)$ is given by $\text{Min}\{f_{Pt}, f_{Kt}\}$ which will be denoted by μ_t. Equation (5) may then be written as

(6) $y_t = \mu_t + \varepsilon_t$

where (i) $\mu_t = f_P(Z_{PTt}) = Z_{PTt}\beta_P - S_{Pt}$

(ii) $\mu_t = f_K(Z_{KTt}) = Z_{KTt}\beta_K - S_{Kt}$

(iii) $0 = S_{Pt}S_{Kt}$; all t=1, n

and (iv) $S_{Pt} \geq 0$, $S_{Kt} \geq 0$.

The symbols S_{Pt} and S_{Kt} represent slack variables, and because they are restricted to be nonnegative, the mean yield, μ_t, will be equal to f_{PTt} whenever Z_{PTt} is limiting response. For instance, when Z_{PTt} is the limiting nutrient, $S_{Pt} = 0$ and $\mu_t = Z_{PTt}\beta_P$.

With the additional assumption that $\varepsilon_t \sim N(0, \sigma^2)$, equation (6) may be viewed as the standard regression model to which the technique of maximum likelihood estimation (MLE) could be applied in a straightforward manner. The only distinction is that in this case, there are the nonlinear and inequality restrictions stated in (i) through (iv) above. These restrictions together with the joint density functions of $\varepsilon = (\varepsilon_1,\ldots,\varepsilon_N)'$ could be used to construct a Lagrangean objective function whose first-order conditions could then be used to obtain estimates of the parameters of (6).

The residuals (ε_t) are assumed to be independently and identically distributed with a common mean and variance. Thus, directly minimizing the error sum of squares, $S(\varepsilon)$, from (6) subject to the given restrictions is equivalent to maximizing the log likelihood function of ε subject to the same restrictions.

Let $Z_i = (Z_{i0},\ldots,Z_{iI_i})$ i=P,K, be an $N \times (I_i + 1)$ matrix of the X_i^T vector transformed as indicated in (6). Also let $\beta_i = (\beta_{i0},\ldots,\beta_{iI_i})'$ be the vector of spline coefficients. In compact form, μ, the vector of expected yields, is now given by $Z_i\beta_i$. If $\hat{\mu}$ is the estimate of μ and e, the estimate of ε, the vector of residuals from (6), then S(e), the error sum of squares, is given by $e'e = (Y - \hat{\mu})'(Y - \hat{\mu})$ where Y is the N x 1 vector of yields. The problem of estimating β_i for i = P, K therefore becomes

(7) $\text{Min } \frac{1}{2}\varepsilon'\varepsilon$

subject to $S_{Pt}S_{Kt} = 0 \qquad \text{all } t=1,N$

$$A_1 B_{NL} + A_2 B_L = Y^*$$

$$S_{it} \geq 0 \quad i = P,K$$

ε unrestricted

where

$$A_1 = \begin{bmatrix} 0 & 0 & I \\ -I & 0 & 0 \\ 0 & -I & 0 \end{bmatrix}; \qquad A_2 = \begin{bmatrix} I & 0 & 0 \\ -I & Z_{Pt} & 0 \\ -I & 0 & Z_{KT} \end{bmatrix}$$

$$B_{NL} = \begin{bmatrix} S_P \\ S_K \\ \varepsilon \end{bmatrix}; \qquad B_L = \begin{bmatrix} \mu \\ \beta_P \\ \beta_K \end{bmatrix}$$

$$Y^* = \begin{bmatrix} Y \\ 0 \\ 0 \end{bmatrix}; \qquad \beta_i' = (\beta_{i0},\ldots,\beta_{iI_i})$$

$\beta_{i0} \geq 0$; $\beta_{ij} \leq 0$, $j \geq 1$, I_i and $i = P, K$.

The problem in (7) has a nonlinear objective function and one set of nonlinear constraints, $S_{Pt}S_{Kt} = 0$ for all t, in addition to linear and inequality constraints. The problem is solved by employing a nonlinear programming algorithm developed by Murtagh and Saunders. The algorithm is code named MINOS/AUGMENTED and is designed to solve large-scale optimization problems involving sparse linear and nonlinear constraints.

A Test of the Nutrient Nonsubstitution Hypothesis

The use of quadratic and square root polynomials for crop response analysis gained popularity largely on grounds of simplicity and ease of computation. They also seem to fit the data on crop production rather well. Hence, they appear attractive whenever measures of relative performance are the only means for discriminating among competing mathematical forms.

The most common measure of relative performance of models is the coefficient of multiple determination R^2. The R^2, however, is not a powerful tool for selecting the best specification particularly when the competing forms are performing equally well. It may also be an ambiguous statistic when the models are estimated under unequal transformations.

Selection procedures employing measures of relative performance are generally concerned with the subject of discrimination. There are times, however, when the interest is in hypothesis testing rather than discrimination. In the former case, the null hypothesis (H_0) is tested against an alternative (H_1). H_0 is either rejected or not rejected at a prescribed probability level of a type I error (significance level). The decision process is restricted to only two possibilities since the truth of one hypothesis implies falsity of the other. The researcher must, therefore, be willing to be committed to one of the models being tested. Such a commitment implies that (only) one of the models is the true specification in terms of obeying prescribed assumptions.

In the regression case, a test of $H_0 : f_0(Y|Z, \beta, \varepsilon_0)$ against an alternative $H_1 : f_1(Y|X, \gamma, \varepsilon_1)$ can employ the classical F-statistic based on the Neyman-Pearson likelihood ratio method if either X is orthogonal to Z or $M_X Z = 0$. The symbols X and Z refer to the sets of regressors, β and γ are the

associated parameter vectors and ε_0 and ε_1 are the error terms of the respective models. M_X is the principal idempotent matrix under the alternative hypothesis and is given by $I - X(X'X)^{-1}X'$. Given that all the classical assumptions of the regression model are obeyed under the respective models, the two hypotheses are said to be nested. This means that one can be obtained as a limiting case of the other.

When the above conditions are not met, as in the case when X is not a subset of Z (or vice versa) or Z = logX, the hypotheses are said to be non-nested and the classical F-test is inappropriate [Cox, Pesaran].

In this study the interest is in testing the nonsubstitution specification presented in equation (5) against an alternative such as the quadratic or square root polynomial specification.

Polynomials have been used in many studies intended for comparing the performance of different mathematical formulations. This tradition of comparing polynomial approximations with models based on biological principles of plant growth will be upheld here. In the present case, the thesis is that polynomials are used to approximate a biological relationship which is better explained, at least theoretically, in terms of the law of the minimum. The polynomials allow substitution among the essential nutrients P and K contrary to theory of plant nutrition. Equation (5), which is a generalization of the law of the minimum, does not allow substitution among nutrients. A polynomial portrays a symmetrical surface around a unique maximum yield. It would, therefore, fail to capture sharp bends and plateaux in a crop response surface. A polynomial fitted to a set of data showing a significant plateau can lead to overstated optimal yields and costly positive biases in fertilizer recommendations to producers.

The problem at hand is to statistically test the nonsubstitution model against a polynomial formulation. The null and alternative hypotheses are, therefore, given as

(8) $H_0 : f_0(Z, \beta) = \mu + \varepsilon_0;\ \mu = Z_P\beta_P - S_P;\ \mu = Z_K\beta_K - S_K;\ S_{Pt}S_{Kt} = 0,$ all t;

$S_P \geq 0;\ S_K \geq 0;\ \beta_{i0} \geq 0;\ \beta_{ij} \leq 0,\ j \geq 1,\ i=P,K.$

$H_1 : f_1(X^T, \gamma) = X^T\gamma + \varepsilon_1$

The symbols in (8) represent variables already defined in the preceding equations and $X^T\gamma = \gamma_1 X_1^T + \gamma_2(X_1^T)^{1/2} + \gamma_3(X_1^T X_2^T)^{1/2} + \gamma_4(X_2^T)^{1/2} + \gamma_5 X_2^T$ for the square root formulation. In the case of the quadratic formulation, $X^T\gamma = \gamma_1 X_1^T + \gamma_2(X_1^T)^2 + \gamma_3 X_1^T X_2^T + \gamma_4(X_2^T)^2 + \gamma_5 X_2^T$.

The null and alternative hypotheses in (8) are non-nested and, hence, the classical F-test is invalid. The literature on fertilizer use presents numerous comparisons of polynomials and other mathematical forms, but no empirical study in which the non-nested nature of hypotheses such as those in (8) was statistically recognized.

Studies have shown that simplicity and performance of polynomials can be matched by models whose parameters are readily interpreted in biological terms. Waggoner and Norvell, for example, stated that "In fact, the law of the minimum fits these yields as well or better than any of the empirical functions used by Heady et al., (Table 1)." Only the R^2 was used as the criterion for comparison between the law of the minimum and those used by Heady et al. Obviously, the authors' concern was whether their model was better or worse than the empirical alternatives previously suggested. The selection procedure was restricted to the models at hand. There was no room for the possibility that an alternative not considered together with the data could be used to reject all the hypotheses tested.

The problem of testing the hypotheses in (8) can be tackled by first constructing a comprehensive model. For the sake of brevity, the following notation will be adopted: f_i $(i = 0, 1)$ are to be viewed as probability density functions (pdf's), $f_0 = f_0(Z, \beta)$ and $\hat{f}_0 = f_0(Z, \hat{\beta})$ while $f_1 = f_1(X_1\gamma)$ and $\hat{f}_1 = f_1(X, \hat{\gamma})$.

Linear nesting of models was discussed by Quandt. The comprehensive model is obtained as

(9) $f_\theta(y|\beta, \gamma) = (1 - \theta)f_0 + \theta f_1$

where y is the dependent variable and θ is the nesting parameter to be tested for zero and unity. In (9) parametric identification may fail but tests on θ may still determine departures from the null hypothesis in the direction of the alternative or away from it. The identification problem can be circumvented in a number of ways, some of which will be mentioned shortly. A major weakness of the comprehensive model shown above is that it may not in itself constitute a viable theory regarding crop response. The two functions f_0 and f_1 may be such that the comprehensive model constructed from them does not make sense. Yet the latter acts as a third alternative whenever θ is significantly different from zero and lies somewhere between zero and one. Another problem that may arise from (9) is that of multicollinearity.

An alternative way of overcoming the identification problem is by using prior information. If, for instance, β and γ are known, then θ in (9) becomes both a nesting as well as a testing parameter. The null hypothesis is rejected if θ is significantly greater than zero but less than one, while the alternative hypothesis is rejected when θ is less than zero. Values of θ outside [0, 1] are interpreted as a movement beyond H_1 if $\theta > 1$ and away from both H_0 and H_1 if $\theta < 0$ [Fisher and McAleer]. The possibility of ending up with an artificial model such as (9) still exists even with prior knowledge

of β and γ. Furthermore, such prior information is hardly available. One may, therefore, have to employ numerical techniques of identification mentioned below.

An estimate of θ may be obtained by estimating a comprehensive model in which $\hat{\beta}$ and $\hat{\gamma}$ are substituted for β and γ giving $Y = (1 - \theta)f_0(\hat{\beta}) + \theta f_1(\hat{\gamma}) + \varepsilon$. Using the notation of (9) this would imply estimating

$$(10) \quad Y - \hat{\mu} = \theta(X^T\hat{\gamma} - \hat{\mu}) + \varepsilon.$$

H_0 is then tested on the basis of the t-ratio statistic for θ, not on the basis of its absolute value. The estimate of θ is conditional on those of β and γ, and, hence, Davidson and MacKinnon referred to the test based on (10) as the C-test. The C-test is a simple way of testing the nonsubstitution hypothesis. Davidson and MacKinnon stated that the t-statistic from (10) provides a test the asymptotic size of which is smaller than its nominal size. The C-test, therefore, has a higher probability of type I error. It follows that if H_0 is rejected by H_1, this would be strong evidence against the hypothesis under test.

Other alternative procedures for testing non-nested hypotheses are available. Such procedures will not be discussed here in detail. A good summary can be found in Gaver and Geisel, who also present Bayesian techniques.

The procedure to be discussed next was first proposed by Cox and has since been elaborated by Pesaran for linear regression models and Pesaran and Deaton for nonlinear regression models. The procedure will simply be referred to as CP. The test statistic derived from the CP procedure is given by

$$(11) \quad T_0 = \hat{L}_{01} - N[\underset{N\to\infty}{\text{Plim}}\ (\hat{L}_{01}/N)]_{\beta=\hat{\beta}}$$

which is shown by Cox to be asymptotically normally distributed with zero mean and variance $V_0(T_0)$, given that the null hypothesis is true. $\hat{L}_{01} = L_0(\hat{\beta}) - L_1(\hat{\gamma})$ where $L_0(\hat{\beta})$ and $L_1(\hat{\gamma})$ are the log likelihood functions of samples of size N under the null and alternative hypotheses. $\hat{\beta}$ and $\hat{\gamma}$ are MLE of β and γ, respectively. It follows that

$$(12)\quad D_0 = T_0/[V_0(T_0)]^{1/2}$$

is a standardized normal variate that can be used in conjunction with regular statistical tables of the normal distribution to test the truth of H_0.

Pesaran demonstrated that the statistic in (11) for linear regression models is given by

$$(13)\quad T_0 = \frac{N}{2} \log (\hat{\sigma}_1^2/\hat{\sigma}_{10}^2)$$
$$= \frac{N}{2} \log[\hat{\sigma}_1^2/(\hat{\sigma}_0^2 + \frac{1}{N} e_{10}'e_{10})]$$

where σ_{10}^2 is the asymptotic expectation of $\hat{\sigma}_1^2$ under H_0. The respective sample variances $\hat{\sigma}_i^2$ for i = 0, 1 are given by $e_i'e_i/N$, e_i being the residual vector of the ith hypothesis. In the case of the hypothesis testing problem in (13), the vector e_{10} is obtained from the following regression:

$$(14)\quad \hat{\mu} = X^T b_0 + \varepsilon_{10}$$

where $\hat{\mu}$, the estimate of $E(y) = \mu$ for the nonsubstitution model, is obtained from (7) and X^T is the set of regressors for the polynomial model.

The variance of T_0 is given by

$$(15)\quad V_0(T_0) = \frac{\hat{\sigma}_0^2\, e_{100}'e_{100}}{\hat{\sigma}_{10}^4}$$
$$= \frac{\hat{\sigma}_0^2\, e_{100}'e_{100}}{(\hat{\sigma}_0^2 + \frac{1}{N} e_{10}'e_{10})^2}$$

where the vector e_{100} is obtained from the following regression:

(16) $e_{10} = \mu + \varepsilon_{100}$

To test the truth of H_0 using D_0 in (12), one would, therefore, require the auxiliary regressions in (14) and (16). The regression in (14) simply requires substituting $\hat{\mu}$ on the left hand side of the program package used for estimating the polynomial model. This regression yields e_{10} which is then used as the right hand side column vector in the MINOS/AUGMENTED program employed in estimating (7). The final estimate of D_0 in (12) is obtained by substituting $\overset{-2}{\sigma_0} = e_0'e_0/(N - K_0)$ and $\overset{-2}{\sigma_1} = e_1'e_1/(N - K_1)$ for $\hat{\sigma}_0^2$ and $\hat{\sigma}_1^2$, where $N - K_0$ and $N - K_1$ are the respective degrees of freedom under H_0 and H_1. This final value of D_0 will be referred to as $\overline{D}_0$ to distinguish it from the one defined in (12).

Since the parameter spaces for the two models being tested are disjointed, a two-tailed test will be employed. For a given level of significance, α, if the tabulated value of the statistic is given by $D\alpha$, then H_0 is not rejected if $|\overline{D}_0| < |D\alpha|$. H_0 is rejected in favor of H_1 if $|\overline{D}_0| > |D\alpha|$ and $\overline{D}_0$ is negative. Finally, if $|\overline{D}_0| > |D\alpha|$ and $\overline{D}_0$ is positive, the null hypothesis is rejected but in favor of some alternative H_2 differing from H_0 in some sense opposite to that in which H_1 differs from H_0.

The statistic $\overline{D}_0$ is only valid for testing the truth of H_0. In order to test the truth of H_1 the roles of the hypotheses are reversed in (8). A new statistic is then computed estimating two more auxiliary regressions. The new statistic will be referred to as $\overline{D}_1$. The regression which corresponds to (14) is now given by

(17) $X^T\hat{\gamma} = \mu + \varepsilon_{01}$.

Equation (17) provides the residuals e_{01}. It requires replacing the right

hand side of (7) by $X^T\hat{\gamma}$. Next, the residuals are substituted on the left hand side of the program package used for estimating the polynomial model to obtain the equation

(18) $e_{01} = X^T b_1 + \varepsilon_{011}$

Equation (18) provides the residuals e_{011}. The new statistic which tests the truth of the polynomial specification is then computed as originally done for $\overline{D}_0$.

Equation (11) can be written alternatively as

(19) $T_0 \equiv T_0(\hat{\Gamma}) = (\hat{L}_0 - \hat{L}_1) - E_0(\hat{L}_0 - \hat{L}_1)$

where $\hat{\Gamma} = (\hat{\gamma}' \ \hat{\sigma}_1^2)$. This leads to the relationship in (13). $\hat{L}_0 \equiv L_0(\hat{\beta}) = N/2 \log (2\Pi\hat{\sigma}_0^2) - N/2$ is independent of Γ. It does not matter, therefore, what value is assigned to $\hat{\Gamma}$, provided it is consistent for Γ. In (19) MLE $\hat{\Gamma}$ is used. One may, however, replace Γ with a consistent estimate of Γ_0, the asymptotic expectation of $\hat{\Gamma}$ under H_0. This leads to a different numerator for D_0 and is given by

(20) $TA_0 = T_0 + (\hat{L}_1 - \hat{L}_{10})$

Atkinson shows that TA_0 and T_0 are asymptotically equivalent under H_0. Fisher and McAleer derived TA_0 as

(21) $TA_0 = \frac{N}{2} \{(\hat{\sigma}_1^2/\hat{\sigma}_{10}^2) - 1\} + 1/2\hat{\sigma}_{10}^2\{(Y - \hat{f}_{10})'(Y - \hat{f}_{10}) - \hat{e}_1'\hat{e}_1\}$

where $\hat{f}_{10} = f_1(\hat{\gamma}_0, \hat{\sigma}_{10}^2)$ and γ_0 and σ_{10}^2 are the asymptotic expectations of $\hat{\gamma}$ and $\hat{\sigma}_1^2$ under H_0, respectively.

Since the expression of T_0 in (13) is approximated by

$$(22)\quad TL_0 = \frac{N}{2}\{(\hat{\sigma}_1^2/\hat{\sigma}_{10}^2) - 1\}$$

$$\geq \frac{N}{2} \log (\hat{\sigma}_1^2/\hat{\sigma}_{10}^2)$$

the following relationships must hold:

$$(23)\quad TA_0 \geq TL_0 \geq T_0.$$

All three variations of the Cox-Pesaran statistic are asymptotically equivalent under H_0, and if a common variance $V_0(T_0)$ in (15) is assumed, the relationship in (23) implies that

$$(24)\quad DA_0 \geq DL_0 \geq D_0$$

Fisher and McAleer concluded that "When the alternative, H_1, is fitting much better (worse) than it ought, relying solely on D_0 (DA_0) will more likely lead to rejection of H_0 than would otherwise be the case." The linearized statistic DL_0 is more conservative at rejecting H_0 than is D_0 (DA_0) when H_1 is fitting much better (worse) than might be expected.

Since DL_0 requires only a slight modification of the numerator for D_0, it will also be computed. Thus, to recap, the techniques to be employed are the C-test as a preliminary test of H_0 and the two variations (D_0 and DL_0) of the CP procedure. It is also worth noting that CP procedures involve computing the test statistic under only one of the hypotheses, and, hence, there is no choice involving an artificial (comprehensive) model. The test, however, indicates whether there is a more appropriate specification beyond H_1 or away from both H_0 and H_1. The test may, therefore, reject both hypotheses while indicating a direction in which to search for alternatives.

The Empirical Data[1]

The experiment considered in this study was started in 1952 on the Agronomy Farm at Purdue University on a Raub silt loam, an imperfectly drained prairie soil. The crops initially used were corn (Zea mays L.), soybean (Glycine max L.), wheat (Triticum vulgare L.) and hay (a mixture of alfalfa (Medicago sativa L.), red clover (Trifolium pratense L.), and brome grass (Bromus sp.)). The applied nutrients were phosphorous (P) and potassium (K) in the form of superphosphate and potassium chloride, respectively.

There were 22 treatments of P and K randomized within each of the eight blocks representing two replicates. Crops were randomized within each replicate. The rotation sequence was corn-1-soybean-wheat-hay. In 1963 the hay crop was replaced by a second crop of corn designated here as corn-2.

All the straw was plowed back into the field and soil samples were taken only from the hay plots prior to broadcasting P and K in fall. Corn-1 was planted on these same plots in May of the following year. This means that for a given block, both soil sampling and broadcasting of P and K took place only once in four years. Soybean, wheat, and hay did not receive direct broadcast applications of P and K. Only corn-1 and wheat received row applications of fertilizer. Nitrogen was plowed under for corn-1 plots and top-dressed to the wheat plots at nonlimiting levels. The soil Ph was adjusted to 6.5 by applying suitable amounts of lime.

Experimentation continued through 1980. The soil sampling and fertilization procedures are presented schematically on Table 1. Application rates for P and K are those indicated by Barber.

Table 1 is only a stylization of the operations involved in the experiment. Some essential details such as how and specifically when fertilizer was braodcast have been omitted. The important fact to remember is

the sequence in the rotation as it relates to soil sampling and the fertilization routines. This fact is crucial to the development and interpretation of subsequent mathematical formulations.

The soil sampling procedure involved taking 15 cores in the plow layer, 0-15 cm, from the central portion of each of the hay plots. The plots measured 4.3 m wide and 19.8 m long.

The available phosphorous and potassium was extracted at the Purdue Soil Testing Laboratory by shaking 5 g of soil with 15 ml of 0.7 HCl in a shaker for two minutes. In 1968, the phosphorous extraction procedure was changed to Bray P1. At a soil Ph of about 5.8, the average initial soil tests, before P and K applications in 1952, were 18 and 45 μg/g, respectively.

The Estimation of the Phosphorous Carryover Function

The fertility carryover function (2) for P and K was given a distributed lag specification resulting in the following autoregressive form:

$$
\begin{aligned}
(25)\quad B^{*}_{ij} &= \rho_i B^{*}_{ij-1} + \gamma_i (B^{*}_{ij-1} - \rho_i B^{*}_{ij-2}) + \gamma_i \lambda_{i4}^{-1} (X^{C}_{ij-1} - \rho_i X^{C}_{ij-2}) \\
&\quad + \gamma_i \lambda_{i2}^{-1} (X^{W}_{ij} - \rho_i X^{W}_{ij-1}) + \alpha_{i1} (Y^{H}_{j} - \rho_i Y^{H}_{j-1}) + \alpha_{i2} (Y^{W}_{j} - \rho_i Y^{W}_{j-1}) \\
&\quad + \alpha_{i3} (Y^{S}_{j} - \rho_i Y^{S}_{j-1}) + \alpha_{i4} (Y^{Cl}_{j} - \rho_i Y^{Cl}_{j-1}) + W_{ij}
\end{aligned}
$$

where B^{*}_{ij} = soil test level of the ith nutrient in the jth rotation;

X^{k}_{ij} = total fertilizer (row and broadcast) applied to kth crop in the jth rotation; k = corn-1 (C), wheat (W);

Y^{h}_{j} = yield level of the hth crop in the jth rotation, h = H, W, S, Cl for hay, wheat, soybeans, and corn;

ρ_i = autoregressive coefficient;

γ_i = geometric declining coefficient of the Koyck lag distribution;

λ_{i2}, λ_{i4} = constants of proportionality between soil nutrient levels and soil tests; λ_{i4} refers to fertilizer applied four periods back and λ_{i2} to fertilizer applied two periods before soil sampling;

$W_{ij} \sim N(0, \sigma^2 I)$, $i = P, K$.

The estimated phosphorus equation assumed the following specific form

$$(26)\ SP_j = b_1 + b_2 + b_3 + b_4 + (\rho + \gamma)\, SP_{j-1} - \rho\gamma SP_{j-2} + (1 - \rho)\gamma\lambda_4^{-2} CP_{j-1}$$
$$+ (1 - \rho)\gamma\lambda_2^{-1} WP_j + \alpha_1 HAY_j - \rho\alpha_1 HAY_{j-1} + \alpha_2 WHEAT_j - \rho\alpha_2 WHEAT_{j-1}$$
$$+ \alpha_4 CORN_j - \rho\alpha_4 CORN_{j-1} + W_j$$

where j refers to a rotation and the dependent variable SP is the soil test phosphorus measured in μg/g by Bray P1. The error term is assumed to be white noise with a distribution $W_j \sim N(0, \sigma^2 I)$ for all j.

Soil sampling was not done for the years 1970, 1971, and 1972. Estimation of both P and K carryover functions was, therefore, restricted to the period 1953-1969. The period consists of four complete rotations and one quarter of the fifth rotation (1969). During this period of experimentation, the application rates for P in wheat (WP) and corn-1 (CP) were constant. WP_j and WP_{j-1} are, therefore, identical and so are CP_{j-1} and CP_{j-2}. The term $(1 - \rho)$ in equation (26) arises from collection of like terms on the basis of this fact as can be deduced from equation (25).

The block dummy variables denoted by b_i $(i = 1, 4)$ were included in equation (26) to account for the influences of the different blocks. Such differences may be due to block to block variations in seasonal availability of P. The inclusion of the dummy variables (all the four since there was no natural constant in the equation) led to a significant improvement in the fit as judged from the residual sum of squares (SSR).

Except for the soil test, the values for all the other variables were transformed to kg/ha. The results are given in column (a) of Table 2.

The primary purpose for estimating the carryover function in (26) is to obtain a consistent estimate of the parameter λ. This is the proportionality constant required in the calibration of soil test values. Subsequent estimations leading to the test of the nonsubstitution hypothesis will be concentrating on the response of corn-1 to total nutrient supply. The objective here is to derive the relevant value of λ to be used for constructing total P for the response function. The total P, PT, available for the corn-1 crop is given by $(\hat{\lambda}_p SP + CP)$ where the soil test variable SP is in $\mu g/g$ and CP is in kg/ha of applied P.

The basic time framework in equation (26) is the rotation which consisted of four growing seasons. The coefficient estimate in Table 2, therefore, refer to four seasons. The response function for corn-1 will, however, be estimated using a single growing season as the basic time framework. It follows that the relevant λ-value for P must be extrapolated from those given in column (a) of Table 2.

The estimate of the proportionality constant λ_4 given in Table 2 is relevant for P applied four seasons back while the value for λ_2 refers to P applied two seasons prior to soil sampling. The (linear) extrapolation of a single season λ-value denoted by $\hat{\lambda}_p$ from $\hat{\lambda}_4 = 33.11$ and $\hat{\lambda}_2 = 20.16$ resulted in a value of $\hat{\lambda}_p = 12.26$. Recall that λ_p is the amount of applied P required to raise the soil test level by one unit. Barber reported, for the same set of data used here, that Bray P1 levels increased by one $\mu g/g$ for every 17.0 kg/ha of P added. He obtained this figure as the reciprocal of a slope coefficient of the regression of Bray P1 analyses on calculated net change in P over a period of 25 years. There are many differences, both statistical and

technical, between Barber's approach and that used in this study, hence, it is not surprising that the two results differ. The estimated λ-values are, however, of the same magnitude.

The advantage of the carryover function over other techniques of estimating total nutrient supply is that it provides useful additional information. Table 2, column (a), for example, presents estimates of the autoregressive and distributed lag parameters ρ and γ. The reported values are on the basis of four period rotations. Single period estimates of the absolute values of these parameters are obtained as $|\hat{\rho}_p| = \sqrt[4]{|\rho_p|} = 0.6068$ and $|\hat{\gamma}_p| = \sqrt[4]{|\gamma_p|} = 0.9113$.

The autoregressive coefficient $\hat{\rho}_p$ is negative implying a negative influence of a given season on the contiguous seasons. The estimate of the distributed lag parameter is positive and has an absolute value less than one as required. The results show that for the Indiana soil investigated here, only about 10 percent of the applied P is taken up by the crop in a single season, the remainder being carried over. This would explain the high soil test values recorded in the plots which received high doses of applied P.

Finally, results for the estimation of the carryover function show the influence of the various crops on soil phosphorus buildup, information which may prove to be useful in designing a rotation sequence. Soybean did not seem to have any significant effect on the carryover of P from one growing season to the other. Wheat had an unexpected but insignificant positive coefficient as can be seen from column (a) of Table 2. Hay/corn-2, the crop harvested just prior to the soil sampling period, had a significant negative effect on phosphorus carryover; the higher the yield the lower is the succeeding soil test value. Corn-1 planted four seasons prior to the soil sampling had a significant positive effect on the soil test value.

The Potassium Carryover Function

In estimating the phosphorus carryover function, equation (26), it was assumed that W_j was white noise. There was no indication for the need to simplify the estimated equation on the basis of an assumption that $\gamma = \rho$. In the case of potassium, there was no clear-cut distinction between these two hypotheses.

The alternative assumption that $\gamma = \rho$ appeared to be more plausible, especially in terms of t-ratios and the proportion of the applied K carried over from one season to the next. The latter information is obtained as the seasonal estimate of the distributed lag parameter (γ). The values of γ reported in column (b) of Table 2 are not the seasonal values since the basic time framework in the analysis was a rotation consisting of four growing seasons. The seasonal value is obtained as $\gamma_k = \sqrt[4]{|\gamma|}$ which turns out to be about 77 percent. The single season value of λ_k was extrapolated to be equal to 1.60.

The Polynomial Crop Response Functions

Among the family of polynomials, the two commonly employed for analyzing crop response to fertilizer are the square root and the quadratic forms. Over the last three decades the application of these two functions for analyzing fertilizer experiments has been extensive.

The crop selected for the response analysis was the first crop of corn designated earlier as corn-1 but which will now be referred to simply as corn. The corn crop was planted in May and harvested in October or November.

The results discussed below are based on relationships in which the response of corn was considered to be a function of total P and total K. These variables should be viewed as the total supplies of the respective

nutrients available to the corn crop during the growing season. In either case, the total nutrient supply was computed as the sum of the applied and residual (carryover) amounts. For each nutrient the carryover was obtained by multiplying the soil test index by the λ-value derived from Table 2. The yield and applied fertilizer observations have been converted to quintals/ha and the soil tests are in $\mu g/g$. Due to the presence of gaps in the soil test data, only the observations for six years were used in analyzing the response of the corn crop. The corn data span the years 1960 through 1966. The corresponding soil test values are for the years 1959 through 1965.

As in the case of the carryover functions, the effects of the different blocks were accounted for by four dummy variables S_i ($i = 1, 4$). The estimated square root function was of the form

$$(27)\quad CORN = S_1 + S_2 + S_3 + S_4 + \gamma_1 PT + \gamma_2 (PT)^{1/2} + \gamma_3 (PTKT)^{1/2} + \gamma_4 (KT)^{1/2} + \gamma_5 KT + \varepsilon_1$$

where PT is total supply of P given by 0.1226 SP + CP and KT is the total supply of K given by 0.016 SK + CK. SP and SK are the soil test values for P and K whereas CP and CK are the applied amounts for the respective nutrients.

The observations used in estimating equation (27) were the 44 treatment values for one growing season (22 for each replicate). The individual error terms ε_{1t} (t denoting treatment) were assumed to be identically and independently distributed with zero mean and a common variance σ_1^2.

Table 3 summarizes the results for the estimation of the square root response functions for the years 1960 through 1966. The table is self-explanatory but a few comments are in order. First of all, no interaction effects between P and K were significantly different from zero during the years of investigation. Secondly, the coefficients of the linear

and the square root terms had the expected signs. Thirdly, the table reveals a variation in response of corn from year to year. Of particular interest is the fact that although the linear and the square root terms for PT had significant coefficients in the pooled function, phosphorus had a significant contribution in only three out of the seven years reported in Table 3. The t-ratios for the coefficients of the linear and the square root terms for potassium on the other hand suggest that this nutrient had a significant contribution to the response of corn in each of the seven years investigated.

The results for the estimation of the quadratic polynomial summarized in Table 4 indicate a seasonal response pattern similar to that obtained from the square root formulation. The quadratic equation estimated can be obtained from equation (27) by replacing the square root terms with the quadratic terms $(PT)^2$, PTKT and $(KT)^2$. The symbols used for the block dummy variables are Q_i ($i = 1, 4$) in the case of the quadratic function. As in the square root form, the estimated coefficients for the block dummy variables were highly significant at any reasonable level of probability for a type I error.

In terms of data "fit," as judged from the R^2 and the residual variance estimate, the square root specification appears to have a slight edge over the quadratic form. No definitive statements can be made on the subject of discrimination between these two specifications without carrying out proper statistical tests.

The Nutrient Nonsubstitution Model

The nutrient nonsubstitution model was estimated in the form given in (7). Before doing so, however, estimates of the yield plateaux for the individual years had to be provided. In addition, the knots needed in

estimating the spline functions had to be estimated. The realized yield plateau or maximum, A_{ws}, for a given crop variety and location depends, among other things, on the prevailing weather. Under idealized conditions, the biological maximum yield, M, for a given crop variety will be a constant depending on the variety's genetic potential. In this study, the realized maximum was estimated as the average of the top five corn yields.

The nonsubstitution model was estimated by a maximum likelihood procedure subject to a set of linear and nonlinear constraints. The computer program used was MINOS/AUGMENTED written by Murtagh and Saunders, which will be referred to simply as MINOS. The knots were treated essentially as unknown and had to be searched for by repeated estimations of the model.

The requirement of a plateau surface implies the following restriction on the spline coefficients

$$(28)\quad \beta_i = \sum_{\tau=0}^{j} \beta_{i\tau} = 0$$

where i represents the two nutrients and j+1 is the total number of knots. The summation in (28) gives the slope of the spline function at the last knot.

For the spline function approximations to the single nutrient response curves, only one knot was used. This amounted to estimating simple models of the linear and plateau type and the spline function approximations had only two segments. The knot was considered as an unknown variable to be chosen so as to minimize the value of the objective function given in model (7). The simplistic nature of the linear and plateau type approximations adopted here made this task rather easy. First, the scatter diagrams were used to select

initial ranges of the knots for PT and KT. For example, the range for the 1960 data for both nutrients was taken as 100-350 kg/ha. Values of the objective function corresponding to these extreme points were computed and then different knots falling within that range were tried. The points which miniminized the error sum of squares were taken as the final knots. The final knots for the 1960 data were 198 kg/ha of PT and 196 kg/ha of KT.

The estimated spline function parameters for the year 1960 are summarized in Table 5 and the estimated nutrient nonsubstitution model for corn response is, therefore, given by

$$\text{(29) CORN} = 103.0 \text{ Min } \{0.6829 + 0.1615KT,\ 0.8197 + 0.0909PT\}:$$

$$KT \leq 1.96,\ Pt \leq 1.98 \text{ quintals/ha}$$

$$= 103.0:\quad KT > 1.96,\ PT > 1.98 \text{ quintals/ha}$$

where KT and PT represent total nutrient supply for K and P, respectively, and the estimate of the maximum yield ($\hat{A}$) is given as 103 quintals/ha.

In spite of its simplicity, equation (29) contains the basic information that may be required by most farmers. The estimated model can be used to derive long term optimal fertilization strategies. Such optimization techniques tailored to specific soils and weather conditions may be beyond the budget of most agricultural extension systems, especially in the less developed economies. All that an extension agent may need, therefore, are the soil test results and a simple formula such as (29) relevant for a given soil type while remembering that KT = K + 0.0159SK and PT = P + 0.1226SP where K and P are the required application rates and SK and SP are in soil test units of potassium and phosphorus, respectively. The knots given at 1.96 and 1.98 quintals/ha of K or P, respectively, mark the critical level of total supply of these nutrients. Supplies beyond the critical level are considered as overfertilization of the crop and, hence, a waste.

From the point of view of calculating optimal fertilizer applications, the basic differences between the polynomials and the simple model in (29) are the more unfamiliar computations encountered in the linear and plateau function and the risk that polynomial functions overestimate the critical level of nutrient supply corresponding to the maximum possible yield. The estimated square root function for 1960, for example, gives the levels of K and P corresponding to the maximum yield at 6.319 quintals/ha and 5.509 quintals/ha, respectively. The corresponding figures for the quadratic function are 5.727 and 5.368. These two sets of figures are for all practical purposes, similar and are more than two times the optimal levels of K and P in the nonsubstitution model. The estimated maximum yields for the two polynomial models are, however, not of the same magnitude. The quadratic tops at a yield level of 121.487 quintals/ha whereas the square root form is flatter, reaching its peak at 101.083 quintals/ha. The largest recorded corn yield for the 1960 data was 105.34 quintals/ha. It is, therefore, easy to see that, in comparison to the estimated nonsubstitution model and the information on actual crop yield, the quadratic has a tendency to overstate not only the region of positive response to total nutrient supply, but also the corresponding yield. The yield maximum estimated by the quadratic was more than four standard deviations in excess of the yield maximum ($\hat{A}$) used in the nonsubstitution model and more than three standard deviations above the maximum recorded yield of corn in 1960.

The square root formulation, in comparison to the nonsubstitution model, exaggerated the region of positive crop response, but otherwise had a yield maximum close to the $\hat{A}$ approximation used in the latter specification.

As can be deduced from Table 5, the standard deviation for corn yield estimated by the nutrient nonsubstitution model was 428.9 kg/ha which, using a

conversion factor of 25.4 kg for every bushel of shelled corn, is equivalent to 16.9 bu/ha. The corresponding figures for the square root and quadratic polynomials were 17.2 bu/ha and 17.6 bu/ha, respectively. These figures show that, purely on the basis of the standard error of the regression, the simple linear and plateau model performed no worse than the popular polynomials.

In spite of the apparent similarity as judged from data fitting, the three models imply different fertilization strategies in view of the differences already mentioned above. Economic criteria as a basis of discriminating among these models should, therefore, be precluded not only because of the pervasive manner in which the biases inherent in the models under comparison are manifested in such criteria, but also because such a procedure is not statistically appropriate. It would not be statistically appropriate because the models are not different transformations of a more general specification and, hence, do not lend themselves easily to direct comparison. Furthermore, as with the case of using RVE and R^2, there is the presumption that the real specification is known. In crop response analysis, comparisons which presuppose knowledge of the actual functional relationship may not be justified.

In this study, each model is viewed as constituting a hypothesis, the validity of which has to be tested against specified alternatives. A priori, there will be no maintained hypothesis. However, there will be a special interest on the performance of the nutrient nonsubstitution hypothesis in view of its conformity to conventional theories of plant growth. Hence, this hypothesis will be tested against the polynomial approximations. The latter will in turn be tested against the nutrient nonsubstitution hypothesis. Tests will be done not only for the models estimated from the 1960 data, but also for those derived from the years 1963 and 1965. Non-nested hypothesis testing procedures will be employed. Three different test statistics will be used:

the Cox-Pesaran (CP) statistic, the linearized CP statistic and the C-statistic, whose descriptions are given in a previous section.

The estimated spline functions for the other selected years are presented in Tables 6 and 7. In terms of RVE, the 1963 and 1965 results indicated that the nonsubstitution model fitted the data better than either the square root or the quadratic specifications. This reinforces the 1960 results which indicated only a marginal advantage of the nonsubstitution model over the other specifications.

Results and Discussion of Hypothesis Tests

Three specifications were tested: the nonsubstitution model and the square root and the quadratic polynomials. The polynomials were not tested against each other since the basic interest was to test them against the nonsubstitution formulation. Therefore, the results presented in this section are for the following hypothesis tests, abbreviated as indicated:

H_0 Tested Against H_1	Abbreviation
Nonsubstitution (NS) against Square Root (SR)	NS/SR
Square root against Nonsubstitution	SR/NS
Nonsubstitution against Quadratic (Q)	NS/Q
Quadratic against Nonsubstitution	Q/NS

For the three years studied, this gives 12 tests; 6 against the polynomial specifications (NS maintained) and 6 against the nonsubstitution specification (either SR or Q maintained). Only the abbreviations are used in subsequent tables.

As stated in a previous section, the linearized CP statistic is more conservative at rejecting the null hypothesis than the CP statistic when the alternative is fitting much better than expected. It is, therefore, used as an adjustment to the CP statistic to counteract the unexpected superiority of the alternative hypothesis.

The results discussed first are for the C-test. For a pair of hypotheses, the desired statistics were obtained by estimating the two equations:

$$(30i) \qquad Y - \hat{\mu} = \theta_0(X^T_{\underset{\sim}{\gamma}} - \hat{\mu}) + \varepsilon_0$$

$$(30ii) \qquad Y - X^T_{\underset{\sim}{\gamma}} = \theta_1(\hat{\mu} - X^T_{\underset{\sim}{\gamma}}) + \varepsilon_1.$$

The first equation is a reproduction of equation (10) and is used to test the nonsubstitution model against a polynomial formulation. All the symbols in (30i) are vector valued and retain the same interpretation given in previous sections. It is, however, worth noting that while $\hat{\mu}$ is the expected yield estimated by the nonsubstitution model, $X^T_{\underset{\sim}{\gamma}}$ symbolizes the expected yield as estimated by either the square root or the quadratic, depending on which is in the alternative hypothesis.

When the nonsubstitution hypothesis is maintained, as in (30i), the interest is on an estimate of θ_0 and its standard error. These two estimates give the t-ratio which is the C-statistic used to test the validity of the maintained hypothesis.

The second equation (30ii) tests the polynomial approximations (SR or Q) with the nutrient nonsubstitution hypothesis as an alternative.

The expected yield estimated by the nonsubstitution model was given by $\hat{\mu} = ZK\hat{\beta}_K$ or $\hat{\mu} = ZP\hat{\beta}_P$ depending on whether potassium (K) or phosphorus (P) was the growth limiting nutrient and where the right-hand expressions are the estimated spline functions. In (30i), θ_0 is a nesting parameter, this role being obvious when the equation is written out as $y = (1 - \theta_0)\hat{\mu} + \theta_0 X^T\hat{\gamma}$. Substituting $\hat{\beta}' = (\hat{\beta}_K\ \hat{\beta}_P)$ for $\beta' = (\beta_K\ \beta_P)$ and $\hat{\gamma}$ for γ is just one way of parametric identification of the resulting comprehensive model. Any other estimators could have been used provided they were consistent for β and γ. The resulting estimate of θ_0 is, therefore, conditional on $\hat{\beta}$ and, hence, not unique. Its t-ratio is, however, unique and can thus be used to test the validity of the tested hypotheses.

Asymptotically, the estimate of θ_0 in (30i) will converge to unity when the polynomial specification is valid. Alternatively, it will converge to zero when the tested hypothesis is true. These facts may be used to conclude that if θ_0 is significantly different from one then the alternative hypothesis, in this case the polynomial, is not supported by the data. However, in view of the foregoing discussion about the estimated (absolute) value of $\hat{\theta}_0$, it was necessary to reverse the roles of the hypotheses under test and, hence, the need for the second equation in (30). The tests were then based on the estimated t-statistics for $\hat{\theta}_0$ and $\hat{\theta}_1$.

The C-tests are easy to perform and the resulting statistics will be used in this study both as preliminary crude tests and also for the purposes of comparison with the Cox-Pesaran procedure. The results for the C-tests are given in Table 8. The abbreviations in the table are those explained at the beginning of this section. The first element in each group of entries in

the table is the estimate of the nesting parameter, θ. The second element is the standard of error of $\hat{\theta}$ and the C-statistic appears last.

Due to the fact that the concepts used in the tests are true only asymptotically and they are applied here to a relatively small sample size (N =44), a relatively large probability of type I error (α) will be allowed. The selected level is $\alpha = 0.025$ corresponding to a critical level of $Z_{\alpha/2} = 2.24$ for a two-tailed test. The two-tailed tests will be used because interest is on hypothesis tests rather than discrimination and because of the disjointed nature of the parameter spaces of the hypotheses under test.

Looking at the columns of Table 8 labeled NS/SR and NS/Q which give the results of testing the nutrient nonsubstitution hypothesis against the polynomial formulations, it can be seen that the estimated C-statistic is lower than the critical value of 2.24 for all the three years. Even when the significance level is increased to $\alpha = 0.05$, the nonsubstitution hypothesis cannot be rejected at the resulting critical level of 1.96.

The estimate of the variance of $\sqrt{N}\,\hat{\theta}_0$ is given by $N\hat{\sigma}_0^2(X'\,X)^{-1}$ where $X = (X^T_{\hat{\gamma}} - \hat{\mu})$ and $\hat{\sigma}_0^2$ is the consistent estimator of σ_0^2 in (30i). This variance estimate for $\sqrt{N}\,\hat{\theta}_0$ can be shown to be asymptotically biased upwards (Davidson and MacKinnon, p. 787). The resulting t-statistic used in the C-test will, therefore, have a tendency of not rejecting the null hypothesis more often than the CP statistic. This may explain why the nonsubstitution hypothesis could not be rejected in all the three years studied. However, a look at the results for the tests in which the polynomial formulations were tested against the nonsubstitution hypothesis (columns labeled SR/NS and Q/NS), reveals that using a critical level of 2.24 leads to the rejection of the polynomial formulations in five out of six cases.

The quadratic formulation was rejected in all the three years, whereas the square root could not be rejected in 1960. When the significance level is increased from 2.5 percent to 5 percent, the estimated C-statistics indicate that both the square root and the quadratic formulations should be rejected in all the three years.

The fact that the C-statistic has a tendency of not rejecting the tested hypothesis, further erodes the credibility of the polynomials since, in this case, the indications are that they should both be rejected and at a rather high level of significance.

No conclusive statements can be made solely on the basis of the C-tests, especially in view of the fact that the small sample size properties of the statistic used are unknown. It is also apparent from these results that the C-statistic is unable to decisively break the deadlock which existed between the nonsubstitution and the square root models for 1960 when comparisons were based on RVE. However, when the significance level is increased to 5 percent, the tie is broken and the edge goes to the nutrient nonsubstitution hypothesis.

In order to carry out the non-nested hypothesis tests on the basis of the Cox-Pesaran procedure, some auxiliary estimations must be done, in addition to estimating the pair of models contained in the hypothesis under test. The relations to be estimated and the desired residual sums of squares were presented in a previous section and will, therefore, not be discussed here again.

Three different formulations were compared and, since the polynomials were not tested against each other, only four pairs of hypotheses were tested in each year. Using the abbreviations introduced earlier, the four hypotheses tested were NS/SR, SR/NS, NS/Q, and Q/NS. The first two were used to test the

critical (rejection) points. Large negative values of the estimated statistic suggest that the null hypothesis (H_0) should be rejected in favor of the alternative (H_1). Large positive values on the other hand suggest that the null hypothesis should be rejected but in favor of a third hypothesis (H_2) differing from H_0 in some sense opposite to that in which H_1 differs from H_0. Once again, since interest is on hypothesis tests as opposed to discrimination among a set of specifications, two-tailed tests will be employed.

When a critical value of 2.24 corresponding to a 2.5 percent significance level is used, the estimated CP statistics in Table 9 suggest that both polynomial specifications must be summarily rejected in favor of the nonsubstitution hypothesis. In fact, the figures for SR and Q tested against NS are such that the polynomial approximations cannot be accepted at any reasonable level of probability for a type I error.

Table 9 also reports the estimated linearized CP statistics (figures in parentheses). This statistic is supposed to be more conservative at rejecting the tested hypothesis when the alternative is performing better than expected. This can be seen from the fact that the reported linearized CP statistics are slightly smaller than the corresponding CP statistics. The absolute values of the latter for SR/NS and Q/NS were, however, such that the linearization procedure still left the polynomial hypotheses in the rejection region.

The CP test results confirm those based on the crude C-statistic discussed earlier. The only exception was the 1960 square root form not being rejected by the C-statistic at the 2.5 percent significance level. The overall picture, however, indicates no inconsistencies between the results obtained from the C-tests and those from the CP tests.

The C-tests are relatively easy to perform and if the correct asymptotic standard error for the estimated nesting parameter ($\hat{\theta}$) is available, may be sufficient as a basis for testing non-nested hypotheses. Judging from the results discussed above, it does not appear as if the power of the C-statistic was significantly diminished by the fact that use was made of a t-statistic employing an asymptotically biased estimate for the standard error for $\hat{\theta}$. The inability of the statistic to reject the nonsubstitution hypothesis does not appear to be due to a problem inherent in the statistic. It may, however, be due to the inability of the polynomial hypotheses to reject the nonsubstitution hypothesis for the given sets of data. The C-tests may, therefore, be preferred to the more complex CP procedure if its power can be ascertained.

The CP statistic is invariant to the relative performance measures such as RVE and R^2 for the hypotheses under test. This attribute was demonstrated here by the fact that it was able to reject the 1960 square root specification whereas the C-statistic and the individual RVE's indicated no clear superiority of the nonsubstitution model.

No definite statements can be made about the power (the probability of a statistic failing to reject a false hypothesis) of any of the tests discussed above, especially for small samples. However, asymptotically, the CP statistic will reject the tested hypothesis with a probability of one whenever the alternative is true. The same cannot be said of the C-statistic in the form used, that is, without adjusting the variance of the estimated nesting parameter, $\hat{\theta}$. What can be said, though, is that for the small samples

analyzed here, the C-statistic seems to have a performance similar to the CP statistic and is, therefore, worthy of consideration in non-nested hypothesis tests.

Conclusions

The lopsided nature of the outcome of the hypothesis tests discussed above is hardly surprising in view of the fact that most crop response surfaces display significant plateaux with respect to major nutrients. The polynomials, with their intrinsic symmetric nature, are incapable of accounting for such plateaux or any sharp bends on the response surface.

For the data analyzed in this study, the presence of plateaux was evidenced not only by the fact that the nonsubstitution hypothesis, which explicitly accounted for them, could not be rejected in most of the cases, but also by the fact that the quadratic hypothesis was rejected at significance levels higher than those of the square root hypothesis.

The nutrient nonsubstitution model differs from the polynomial approximations in respects more basic than just accounting for the plateau surface. The most important of these differences is that the nutrient nonsubstitution model, as the name suggests, does not admit substitution among the major nutrients, although it allows for interactions. The fact that the major nutrients do not substitute each other in plant nutrition is now widely acknowledged. It is also true that the results of this study indicate that the nutrient nonsubstitution hypothesis performs better than the alternative polynomial approximations for the given data sets. In the absence of alternative models with assumptions more suitable for established biological principles than those implicit in the nutrient nonsubstitution model, advocating its adoption in crop response analysis seems scientifically sound.

The polynomial approximations do not seem to be supported by the hypothesis test results of this study, and even on the basis of measures of relative fit, such as RVE, the nonsubstitution model is highly competitive.

Footnote

[1]The data set used in this study was kindly provided by Professor S. A. Barber of the Department of Agronomy, Purdue University. Additional information regarding experimental procedures can be gleaned from Barber's publications.

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Table 1. Summary of Field Operations at the Purdue Experiment

Rotation (J)		1					2				3				4				5				. . .
Calendar Year		52	53	54	55	56	57	58	59	60	61	62	63	64	65	66	67	68	69	70	71	72	. . .
Period In Rotation (K)		0	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	1	2	3	4	. . .
		B	R		R	B	R		R	B	R		R	B	R		R	B	R		R	B	
Block	I		Cl	S	W	H	Cl	S	W	H	Cl	S	W	C2	Cl	S	W	C2	Cl	S	W	C2	. . .
		* b_0				* b_{14}				* b_{24}				* b_{34}				* b_{44}				* b_{54}	
	II	* b_0	H * b_{11}	Cl	S	W	H * b_{21}	Cl	S	W	H * b_{31}	Cl	S	W	C2 * b_{41}	Cl	S	W	C2 * b_{51}	Cl	S	W	. . .
	III	* b_0	W	H * b_{12}	Cl	S	W	H * b_{22}	Cl	S	W	H * b_{32}	Cl	S	W	C2 * b_{42}	Cl	S	W	C2 * b_{52}	Cl	S	. . .
	IV	* b_0	S	W	H * b_{13}	Cl	S	W	H * b_{23}	Cl	S	W	H * b_{33}	Cl	S	W	C2 * b_{43}	Cl	S	W	C2 * b_{53}	Cl	. . .

R = Row application of P and K shown here only for Block I in order to avoid clutter.

B = Broadcast application of P and K done after soil sampling from the hay plots, shown here only for Block I.

b^*_{jk} = Soil sampling, after harvesting hay/corn-2; b^*_0 represents initial soil sampling.

Other letters represent the four crops: corn-1, soy, wheat, hay, and corn-2.

Table 2. Estimates of the Fertility Carryover Function

	Coefficient	Estimate (t-ratio) Phosphorus (a)	Potassium (b) $\rho=\gamma$
Block Dummy	b_1	5.7427(2.0)	60.1460(7.6)
	b_2	3.2865(1.1)	44.5680(5.2)
	b_3	6.6034(2.2)	60.2060(7.2)
	b_4	5.3739(1.9)	64.4970(8.0)
Autoregressive	ρ	-0.1356(2.0)	--
Distributed Lag	γ	0.6898(10.4)	0.3513(5.3)
Proport. Const.	λ_4	33.1389(4.7)	6.3635(4.1)
	λ_2	20.1593(3.7)	3.1898(2.8)
Hay/Corn-2	α_1	-0.0011(7.4)	-0.0030(5.5)
Wheat	α_2	0.0002(0.5)	-0.0002(0.1)
Soybean	α_3	--	-0.0061(2.8)
Corn-1	α_4	0.0005(2.5)	-0.0001(0.2)
Extrapolated	λ_i	12.26	1.60
Log Likelihood Fn.		-1013.25	-1373.37
RVE*		18.53	143.36
Sample Size	N	352	352

*RVE is the residual variance estimate obtained as SSR/N-K where N = 352 is the sample size, SSR is the residual sum of squares and K is the number of coefficients estimated.

Table 3. Estimation of the Square Root Response Function

Coefficient	Year							
	1960	1961	1962	1963	1964	1965	1966	Pooled
S_1	48.5102 (4.7)[a]	--	--	--	52.8821 (4.8)	--	--	46.5294 (7.7)
S_2	--	44.5219 (4.4)	--	--	--	35.8657 (3.5)	--	40.1861 (6.6)
S_3	--	--	43.1531 (2.3)	--	--	--	25.6395 (1.8)	37.1160 (6.1)
S_4	--	--	--	58.4693 (7.1)	--	--	--	52.6375 (8.6)
PT	-2.3631 (1.2)	-1.2786 (0.5)	-4.9496 (1.5)	-6.0523 (3.4)	-3.6345 (1.5)	-6.1185 (2.9)	-7.3137 (3.2)	-5.3055 (4.4)
$(PT)^{1/2}$	14.3080 (1.8)	4.0661 (0.4)	9.6857 (0.7)	21.7762 (3.3)	15.9070 (1.8)	24.3660 (3.0)	23.1012 (2.5)	18.7972 (4.1)
$(PTKT)^{1/2}$	-1.3446 (0.9)	1.1143 (0.7)	3.6615 (1.4)	0.8623 (0.6)	-0.1284 (0.1)	1.0626 (0.7)	1.2923 (0.6)	0.9978 (1.1)
$(KT)^{1/2}$	30.2544 (3.5)	36.3546 (4.3)	46.7651 (3.3)	21.9894 (3.1)	24.5651 (2.6)	28.5159 (3.2)	30.1458 (2.6)	30.4536 (6.2)
KT	-5.7149 (3.0)	-8.5847 (4.3)	-12.3266 (4.2)	-5.1114 (3.1)	-5.8625 (2.7)	-6.6167 (3.3)	-6.8904 (2.7)	-7.1355 (6.4)
R^2	0.5388	0.6097	0.5714	0.6148	0.3522	0.6230	0.5380	0.4828
RVE	19.1451	32.6245	58.2853	18.8084	34.0171	23.8852	41.2274	522.1684

[a]Figures in parentheses are t-ratios.

Tabel 4. Estimation of the Quadratic Response Function

Coefficient	Year							
	1960	1961	1962	1963	1964	1965	1966	Pooled
Q_1	74.4564 (17.3)[a]	--	--	--	73.4647 (14.8)	--	--	72.1627 (26.4)
Q_2	--	62.9909 (12.6)	--	--	--	63.4734 (13.8)	--	65.5730 (23.9)
Q_3	--	--	72.7921 (9.1)	--	--	--	53.3757 (8.8)	62.7537 (22.9)
Q_4	--	--	--	80.1309 (20.8)	--	--	--	77.8333 (27.0)
PT	3.4964 (2.3)	1.0906 (0.5)	0.8486 (0.3)	4.9988 (3.5)	2.6600 (1.4)	4.6778 (2.7)	4.6047 (2.4)	3.9255 (4.1)
$(PT)^2$	-0.2751 (1.6)	-0.1190 (0.5)	-0.2976 (0.9)	-0.5897 (3.4)	-0.2427 (1.1)	-0.4527 (2.4)	-0.6137 (3.0)	-0.4619 (4.2)
PTKT	-0.0948 (1.0)	-0.0614 (0.4)	0.2531 (1.3)	0.0457 (0.4)	-0.0089 (0.1)	0.0271 (0.2)	0.0612 (0.4)	0.0527 (0.8)
$(KT)^2$	-0.5514 (3.2)	-1.0771 (4.5)	-1.2344 (3.9)	-0.5803 (3.3)	-0.7140 (3.1)	-0.8012 (3.8)	-0.7075 (2.8)	-0.7660 (6.7)
KT	6.8245 (3.6)	10.8524 (4.7)	11.8918 (3.5)	6.1036 (3.4)	6.9921 (3.0)	8.4741 (4.0)	7.7125 (2.9)	8.0087 (6.8)
R^2	0.5183	0.5537	0.4944	0.5758	0.3177	0.5705	0.5229	0.4583
RVE	19.9980	37.3045	68.7537	20.7131	35.8294	27.2166	42.5729	546.9579

[a]Figures in parentheses are t-ratios.

Table 5. Estimated Spline Response Function - 1960

Description of Variable	Coefficient	Estimate[a]
Yield Maximum	A	102.9900
Potassium		
Block dummy variable	ZK1	0.6829
Relative yield function slope for KT = [0-1.96] quintals/ha	ZK2(β_{K0})	0.1615
Change in slope at KT = 1.96 quintals/ha	ZK3(β_{K1})	-0.1615
Phosphorus		
Block dummy variable	ZP1	0.8197
Relative yield function slope for PT = [0-1.98] quintals/ha	ZP2(β_{P0})	0.0909
Change in slope at PT = 1.98 quintals/ha	ZP3(β_{P1})	-0.0909
RESIDUAL VARIANCE[b]	RVE	18.3914

[a]Measurements are in 100 kg/ha.

[b]The residual variance estimate, RVE = SSR/(N - K). In the nonsubstitution single year models, N = 44 and K = 6.

Table 6. Estimated Spline Response Function - 1963

Description of Variable	Coefficient	Estimate
Yield Maximum	A	105.6600
Potassium		
Block dummy variable	ZK1	0.5409
Relative yield function slope for KT = [0-0.98] quintals/ha	ZK2(β_{K0})	0.4679
Change in slope at KT = 0.98 quintals/ha	ZK3(β_{K1})	-0.4679
Phosphorus		
Block dummy variable	ZP1	0.6383
Relative yield function slope for PT = [0-0.77] quintals/ha	ZP2(β_{P0})	0.4711
Change in slope at PT = 0.77 quintals/ha	ZP3(β_{P1})	-0.4711
RESIDUAL VARIANCE	RVE	14.1018

Table 7. Estimated Spline Response Function - 1965

Description of Variable	Coefficient	Estimate
Yield Maximum	A	93.6500
Potassium		
Block dummy variable	ZK1	0.5071
Relative yield function slope for KT = [0-1.05] quintals/ha	ZK2(β_{K0})	0.4667
Change in slope at KT = 1.05 quintals/ha	ZK3(β_{K1})	-0.4667
Phosphorus		
Block dummy variable	ZP1	0.7502
Relative yield function slope for PT = [0-1.64] quintals/ha	ZP2(β_{P0})	0.1521
Change in slope at PT = 1.64 quintals/ha	ZP3(β_{P1})	-0.1521
RESIDUAL VARIANCE	RVE	18.4974

Table 8. C-Statistic Test Results[a]

Hypothesis Tested		NS/SR	SR/NS	NS/Q	Q/NS
YEAR	1960	0.4289	0.5711	0.3717	0.6284
		0.2797	0.2797	0.2582	0.2582
		1.5334	2.0418	1.4394	2.4335
	1963	0.2614	0.7386	0.2475	0.7525
		0.1800	0.1800	0.1555	0.1555
		1.4521	4.1025	1.5916	4.8385
	1965	0.1149	0.8851	0.1015	0.8985
		0.2502	0.2502	0.2000	0.2000
		0.4593	3.5368	0.5073	4.4922

[a]For each year, the first line gives the coefficient estimate for θ of model (30) and the second line its standard error. The third line gives the t-ratio for θ which is referred to in the text as the C-statistic.

Table 9. The Estimated CP Test Statistics[a]

Hypothesis Tested		NS/SR	SR/NS	NS/Q	Q/NS
YEAR	1960	-0.553 (-0.541)[b]	-3.729 (-3.305)	-0.515 (-0.502)	-3.796 (-3.284)
	1963	-1.340 (-1.244)	-5.194 (-3.983)	-1.393 (-1.288)	-6.126 (-4.499)
	1965	0.001 (0.001)	-5.377 (-4.261)	0.010 (0.010)	-6.824 (-5.117)

[a]The entries in the table are for $\overline{D}_i = T_i/[V_i(T_i)]^{1/2}$ where the subscript i = 0 refers to the hypothesis being tested (H_0) and i = 1 to the alternative (H_1). Testing H_0 against H_1 is summarized as H_0/H_1.

[b]Figures in parentheses are the linearized CP statistics.