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IMPLICATIONS OF SEQUENTIAL DECISION MAKING FOR SPECIFICATION AND ESTIMATION OF PRODUCTION MODELS

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Implications of Sequential Decision Making for Specification and Estimation of Production Models

The agricultural economics literature abounds with studies of agricultural production which are based on single equation estimates of econometric production function models. The single-equation approach has been justified by Hoch (1958, 1962) and Mundlak and Hoch (1965) under the assumption that input decisions are based on "anticipated" output, and by Zellner, Kmenta and Dreze (1966) under the assumption that input decisions are based on the maximization of the mathematical expectation of profit or some other function of output. These models are all based on the strong assumption that production inputs are chosen as part of a one-period decision problem, an assumption which appears to be inconsistent with actual production decision making. Indeed, Zellner et al., state in the conclusion of their paper that "we are fully aware of the fact that one-period maximization of expected returns is just a step in the direction of a proper treatment of stochastic elements in a firm's sequential decision-making process under uncertainty." Especially in agriculture, both short-run and long-run production decisions are not based on a one-period maximization problem but rather on a multi-period dynamic optimization problem because inputs are not all chosen or utilized simultaneously. Therefore, the farmer's optimal input choices may be intrepreted as optimal controls in a stochastic control problem.

The aim of this paper is to formulate a short-run single product production model within a stochastic control framework and to explore its implications for specification and estimation of econometric production models. The analysis demonstrates that sequential solutions to production problems generally result in input demand equations which differ from those of one-period solutions. In addition, sequential solutions may produce models which require either single equation or simultaneous equation estimation methods, depending on the assumptions made about the information the farmer uses to make input choices and on the availability of data for estimation. In particular, it is shown that simultaneous equation estimators are not required if (i) decision makers do not "feedback" information about early stages' production to later input decisions, or (ii) output and input data are available for each stage in the production process. Since both of these conditions are usually violated in agricultural production, these findings suggest that even though farmers choose inputs so as to maximize expected returns, as in the models of Hoch, Mundlak, and Zellner et al., single-equation estimates of agricultural production functions are generally subject to simultaneous equation bias. One example of how this bias occurs is the choice of inputs for harvest. Because a farmer knows how weather and other random events such as pest infestations have affected the size of his crop, his choice of harvest inputs will be a function of this knowledge. Consequently harvest input choices are likely to be correlated with output and single-equation estimates of the marginal product of labor will be biased. The harvest input bias may be particularly serious in the context of agricultural development where harvest labor is often an important input.[1] Another example is measurement of pesticide productivity. Pesticides are often applied in significant quantities only when a pest infestation occurs, so that pesticide input is associated with negative shocks to production. If a production function is estimated without accounting for the sequential structure of the farmer's decision problem, the estimated marginal product of pesticides is likely to be biased.[2] One can conclude that, as a general principle, parameter estimates with desirable properties can only be obtained by specifying and estimating empirical production models which are consistent with the sequential structure of the production process and farm managers' solutions of their input choice problems.

Using a simple two input example, the first section of the paper briefly describes the single stage Cobb-Douglas production models proposed by Marshak and Andrews and by Zellner et al. The second section extends the Cobb-Douglas example to a two-stage model, defines various sequential solutions to the input choice problem, and discusses appropriate estimation methods under the various control solutions, two stochastic specifications, and two output data assumptions. The third section shows that there is a close connection between functional separability across production stages, production function error specification, and the implied relationship between inputs and production uncertainty. These relationships have important implications for specification of multi-stage production functions.

SINGLE STAGE COBB-DOUGLAS MODELS

In this section I describe specification and estimation of the single stage Cobb-Douglas production models of Marshak-Andrews (MA) and Zellner-Kmenta-Dreze (ZKD). The Cobb-Douglas production function provides an interesting special case of the general production model because of its widespread use in theoretical and empirical research. It is also useful for illustrating issues of specification and estimation that arise in sequential models described in the following section. I shall utilize a simple crop production model defined as follows: the ith farmer chooses the amount of inputs L_{i1} and L_{i2} to use on a predetermined acreage, A_i. Output, Q_{i2}, is sold after harvest at price p_i, and input prices are w_{i1} and w_{i2}.

The MA model is based on maximization of profit in a single-period framework. The theoretical model consists of the first order conditions for profit maximization and the deterministic Cobb-Douglas production function, both in logarithmic form. The econometric model is obtained by appending random error terms to these equations. For our crop production example the structural equations with parameters α_j, j=1, 2, 3, are[3]

$$\begin{aligned} \log Q_{i2} &= \log \alpha_0 + \alpha_1 \log L_{i1} + \alpha_2 \log L_{i2} + \alpha_3 \log A_i + \varepsilon_i \\ \log L_{it} &= \log \alpha_t - \log \frac{w_{it}}{p_i} + \log Q_{i2} + u_{it}, \quad t=1,\ 2. \end{aligned} \tag{1}$$

Here ε_i and u_{it} are independent random variables with zero means, the ε_i representing random disturbances in production due to weather, pests, etc., and the u_{it} allowing for nonsystematic errors in maximization by farmers. Adding the error term ε_i to the production function transforms the deterministic theoretical model into a system of simultaneous equations with endogenous variables Q_{i2}, L_{i1}, and L_{i2}. Therefore, with a sample of i=1,...,N farms, simultaneous equation estimators are needed to obtain consistent estimates of the model's parameters. Note, too, that in the MA model prices are treated as known, nonstochastic variables.[4]

The ZKD model is also a one-period model, but in contrast to the MA model it is based on the assumption that firms recognize production is stochastic and therefore choose inputs to maximize the mathematical expectation of profit. Prices are viewed as independent random variables in the model. Writing the stochastic production function as

$$Q_{i2} = \alpha_0 L_{i1}^{\alpha_1} L_{i2}^{\alpha_2} A_i^{\alpha_3} e^{\varepsilon_i}, \quad \varepsilon_i \sim N(0,\sigma^2)$$

and letting a bar over a variable denote its expectation, the decision problem is:

$$\max_{L_{i1},L_{i2}} E[\pi_i] = \bar{p}_i\bar{Q}_{i2} - \bar{w}_{i1}L_{i1} - \bar{w}_{i2}L_{i2}$$

$$= p_i\alpha_0 L_{i1}^{\alpha_1} L_{i2}^{\alpha_2} A_i^{\alpha_3} e^{\sigma^2/2} - \bar{w}_{i1}L_{i1} - \bar{w}_{i2}L_{i2}.$$

The structural econometric model, in log form, consists of the first-order conditions and the production function:

$$\log Q_{i2} = \log \alpha_0 + \alpha_1 \log L_{i1} + \alpha_2 \log L_{i2} + \alpha_3 \log A_i + \varepsilon_i$$
$$\log L_{it} = \log \alpha_t - \log \frac{\bar{w}_{it}}{\bar{p}_i} + \log \bar{Q}_{i2} + u_{it},\ t=1,2 \tag{2}$$

u_{it} is an independent random error added to the first-order conditions to represent nonsystematic errors in maximization. For econometric estimation the important difference between models (1) and (2) is that inputs depend on actual output, Q_{i2}, in the former and expected output, $\bar{Q}_{i2}$, in the latter. Since $\bar{Q}_{i2}$ is nonstochastic, the inputs are independent of output [as long as $E(u_{it}\varepsilon_i) = 0$] and the production function can be estimated with single equation methods such as ordinary least squares.

TWO STAGE COBB-DOUGLAS MODELS AND SEQUENTIAL DECISION MAKING

We define the two-stage Cobb-Douglas production function as follows: before the first production stage labor input L_{i1} is chosen, and during stage 1 the crop is planted and grows. Random events such as weather occur during plant growth and the output of the first stage, Q_{i1}, representing the mature, unharvested crop, is

$$Q_{i1} = \beta_0 L_{i1}^{\beta_1} A_i^{\beta_2} e^{\varepsilon_{i1}} \tag{3}$$

where ε_{i1} is a $N(0,\sigma_1^2)$ random error term. In the second production stage the crop Q_{i1} is harvested by labor input L_{i2}. Adverse weather, etc., may affect the harvest, so we write the second-stage production function as

$$Q_{i2} = \gamma_0 Q_{i1}^{\gamma_1} L_{i2}^{\gamma_2} e^{\varepsilon_{i2}} \tag{4}$$

where ε_{i2} is a $N(0,\sigma_2^2)$ random error term (again, L_{i1} and L_{i2} could be interpreted as any inputs that enter production sequentially). Equations (3) and (4) comprise a system of recursive equations, a fact that is exploited below. Combining the two equations we have

$$Q_{i2} = \gamma_0 \beta_0^{\gamma_1} L_{i1}^{\beta_1\gamma_1} A_i^{\beta_2\gamma_1} L_{i2}^{\gamma_2} e^{(\gamma_1\varepsilon_{i1} + \varepsilon_{i2})} . \tag{5}$$

We note that final harvested output is a function of both ε_{i2} and ε_{i2}.

In order to discuss estimation of this model we must carefully specify the production disturbance terms. The simplest assumption is that the ε_{it} are independently distributed across both firms and time, that is

$$E(\varepsilon_{it}^2) = \sigma_t^2$$
$$E(\varepsilon_{it}, \varepsilon_{i't'}) = 0,\ i\neq i',\ t\neq t'. \tag{6}$$

These assumptions may not hold in practice, and in agricultural production as well as manufacturing and processing the ε_{it} are likely to be correlated across time. Therefore, we also consider estimation under the assumptions

$$\varepsilon_{i2} = \rho\varepsilon_{i1} + v_i,\ |\rho|<1 \qquad E(v_i\varepsilon_{it}) = 0,\ t = 1, 2$$
$$E(v_i\ v_{i'}) = 0,\ i\neq i' \qquad v_i \sim N(0, \sigma^2) \tag{7}$$

Heteroscedasticity and cross-equation, cross-firm correlation may also be present in the production errors, especially in agricultural production. These violations of assumption (5) may be introduced by making appropriate modifications of the covariance matrix and are not discussed here.

Another important factor in estimation of sequential production models is the availability of observations on the output variable Q_{it}. Often only observations of the final product Q_{i2} are possible or available. For example, in agriculture often only the quantity harvested is known and it is not known what part of output can be attributed to each farming operation. With manufacturing or processing operations, in contrast, it may be possible to disaggregate production into separate stages each of which has a measurable product. Because of this "observability" problem of intermediate products, we shall consider the properties of estimators based on the final product Q_{i2} only as well as on both Q_{i1} and Q_{i2}.

To illustrate the essential differences between the one-period and sequential solutions we continue to assume that farmers choose inputs to maximize expected returns and that prices are independently distributed.[5] The maximum problem to be solved is

$$\max_{L_{i1},L_{i2}} \quad E[\pi_i] \text{ subject to: } (3), (4)$$

$$E[\pi_i] = \bar{p}_i \, \bar{Q}_{i2} - \bar{w}_{i1}L_{i1} - \bar{w}_{i2}L_{i2} \tag{8}$$

Sequential solutions to decision problems such as (8) may be differentiated from one-period solutions in terms of the information that is utilized by the decision maker. The information pertains to three features of sequential solutions:

(a) Sequential dependence of decisions: decisions made earlier may affect decisions made later, so that the optimal choice of L_{i2} may be a function $\overset{o}{L}_{i2}(L_{i1})$ depending on L_{i1}. If the farmer takes this fact into account then his optimal input choice in period 1 may depend on how it affects the optimal input in period 2.

(b) Information feedback: information that becomes available during earlier stages may be utilized in subsequent decisions. The optimal choice of L_{i2} will depend on expected output $\overline{Q}_{i1}$ if there is no information feedback about first period production; if there is information feedback about first period production L_{i2} depends on Q_{i1}. Thus, the farmer may use his knowledge of the actual output, Q_{i1}, rather than his original estimates of production, $\overline{Q}_{i1}$ to determine the optimal amount of harvest labor to hire.

(c) Anticipated revision: decisions made earlier may be revised later as new information becomes available. If the decision maker <u>knows</u> information about Q_{i1} will become available in period 2, his choices in period 1 will depend on the conditional distribution $g_2(\varepsilon_{i2}|Q_{i1})$ rather than on the unconditional distribution $g_2(\varepsilon_{i2})$. Thus, the farmer's planting decisions may be different if he knows he can revise his harvest plans at harvest time, rather than having to base harvest decisions on his initial expectations.

We shall consider four alternative sequential solutions to the input choice problem defined in (8) which utilize different information sets.[6] We assume that at the beginning of stage 1, when L_{i1} is chosen, each farmer knows, as a minimum, wage rate w_{i1} and the probability distribution functions of ε_{i1}, ε_{i2}, p_i, and w_{i2}. This minimal information set is defined as I° in Table 1. In addition to the elements of I°, the farmer may know that the optimal input in stage 2 is a function of the input chosen in stage 1. Augmenting I° with this piece of information we have I^{a}, defined in Table 1, which incorporates the sequential dependence property (a). When choosing L_{i1} the farmer may also know that he will be able to acquire information about Q_{i1} before choosing L_{i2}, and thus be able to revise his plans for harvest labor input. This additional element of information is represented by replacing the unconditional distribution $g_2(\varepsilon_{i2})$ with the condition distribution $g_2(\varepsilon_{i2}|Q_{i1})$; making this change we obtain I^{ac} as defined in Table 1. In period 2, the farmer's choices of L_{i2} may be based only on the minimal information set I°; alternatively, the farmer's information set may be updated as additional information becomes available. When I° is updated with information about Q_{i1} and w_{i2}, we obtain I^{b} as defined in Table 1.

<u>The Open Loop (OL) Control Solution.</u> The OL solution embodies property (a) but not properties (b) or (c) of sequential solutions. The choice of L_{i1} is made with the knowledge that it may affect the optimal L_{i2}, and thus is based on I^{a}, but the information set is not updated in stage 2 and the choice of L_{i2} is conditioned on I°. Thus, the OL solution implies that the farmer does <u>not</u> use what he learns about the crop growth during the growing season to choose the optimal harvest labor input. To calculate the OL solution we proceed recursively from stage 2 to stage 1. We first solve for the optimal L_{i2}, taking L_{i1} as given, by maximizing

$$E[\pi_i|I^{\circ}] = \bar{p}_i\gamma_0\bar{Q}_{i1}^{\gamma_1}L_{i2}^{\gamma_2}e^{\omega} - w_{i1}L_{i1} - \bar{w}_{i2}L_{i2}$$

where $\omega = [\sigma_2^2 + \sigma_1^2(\gamma_1^2 + \rho\gamma_1 - \gamma_1)]/2$.[7] Note that the expectation is taken over ε_{i1}, ε_{i2}, p_i, and w_{i2} because the only information assumed to be used in choosing L_{i2} is the farmer's knowledge of the <u>distributions</u> of ε_{i1}, ε_{i2}, p_i, and w_{i2}. The solution is

$$\log \overset{o}{L}_{i2} = \frac{1}{1-\gamma_2}\,[\omega + \log \gamma_0\gamma_2] - \frac{1}{1-\gamma_2}\log \frac{\overline{w}_{i2}}{p_i} + \frac{1}{1-\gamma_2}\log \overline{Q}_{i1} \qquad (9)$$

The OL solution for L_{i1} is based on the assumption that the decision maker knows $\overset{o}{L}_{i2}$ is a function of L_{i1} through $\overline{Q}_{i1}$, so the optimal L_{i1} is obtained by maximizing $E[\pi_i|I^a]$. The solution is a complicated nonlinear function of the form,[8]

$$\overset{o}{L}_{i1} = \overset{o}{L}_{i1}\,(\overline{p}_i, w_{i1}, \overline{w}_{i2}, \sigma_1, \sigma_2, \beta_0, \beta_1, \beta_2, \gamma_0, \gamma_1, \gamma_2). \qquad (10)$$

Noting that $\overset{o}{L}_1$ and $\overset{o}{L}_2$ are independent of the production function disturbances ε_{i1} and ε_{i2}, we can conclude that the OL solution implies that a single equation estimator of the production function's parameters could be efficient and free of simultaneous equation bias. This result, which is also obtained with ZKD model, follows from the assumption that input choices are based only on information available before production begins and not on information about the random events which occur during production. Note, however, that the functional form of the input equations derived from the OL solution differ from those of the ZKD model.

<u>The Sequential Updating (SU) Solution</u>. The SU solution exhibits only property (b) of the sequential solution.[9] In each of the production stages the information set is updated with information acquired in previous stages, but the effects of the current decision on future stages is ignored. Therefore in period two labor input is chosen to maximize

$$E[\pi_i|I^b] = \overline{p}_i\gamma_0 Q_{i1}^{\gamma_1} L_{i2}^{\gamma_2} e^{\sigma_2^2/2} - w_{i1}L_{i1} - w_{i2}L_{i2}. \qquad (11)$$

Note that in (11) the expectation is taken only with respect to ε_{i2} since in stage two Q_{i1} and w_{i2} are known and this information is used to choose L_{i2}. The optimal L_{i2} satisfies

$$\log \overset{o}{L}_{i2} = \frac{1}{1-\gamma_2}\left[\frac{\sigma_2^2}{2} + \log \gamma_0\gamma_2\right] - \frac{1}{1-\gamma_2} \log \frac{w_{i2}}{p_i} + \frac{\gamma_1}{1-\gamma_2} \log Q_{i1} \qquad (12)$$

To find the optimal L_{i1} we take expectations with respect to both ε_{i1} and ε_{i2} and maximize $E[\pi_i|I^\circ]$, ignoring the fact that $\overset{o}{L}_{i2}$ is a function of L_{i1}. Solving the maximum problem gives

$$\log \overset{o}{L}_{i1} = \delta_0 + \delta_1 \log A_i + \delta_2 \log \frac{w_{i1}}{p_i} + \delta_3 \log E[\overset{o}{L}_{i2}|I^\circ] \qquad (13)$$

where δ_0, δ_1, δ_2 and δ_3 are functions of the production function parameters and σ_1 and σ_2. We conclude that when information acquired in stage 1 about Q_{i1} is used to update the decision maker's information set for the choice of L_{i2}, $\overset{o}{L}_{i2}$ becomes a function of ε_{i1} through Q_{i1} and is correlated with Q_{i2}. However, $\overset{o}{L}_{i1}$ is based on information set I° and is not a function of ε_{i1} or ε_{i2}. Therefore, when decisions are sequentially updated we obtain a simultaneous equation model consisting of equations (3), (4), (12), and (13) with properties similar to the Marshak-Andrews Model.

The Open Loop with Feedback (OLF) Solution. The OLF solution combines the properties (a) and (b) of the OL and SU solutions and is therefore generally superior to them both as an optimal solution to the maximum problem. In stage 2, L_{i2} is chosen to maximize $E[\pi_i|I^b]$ as in the SU solution; then in stage 1, L_{i1} is chosen to maximize $E[\pi_i|I^a]$ as in the OL solution. Therefore, the OLF solution, like the SU solution, has the property that $\overset{o}{L}_{i2}$ is an endogenous variable in the structural equation model. The full model consists of the production functions (3) and (4) plus the input equations (10) and (12) and therefore differs from both the OL and SU models.

The Closed Loop (CL) Solution. The CL solution utilizes properties (a), (b), and (c). It is similar to the OLF solution except that the expectation in each stage is computed with the probability density conditioned on information available at that time as well as the knowledge that more information will become available in the future so that decisions may be revised. It is this "closing" of the information loop which distinguishes the OLF and CL solutions, hence, the CL solution also possesses the simultaneity properties of the OLF and SU solutions. Thus, the CL solution is based on maximization of $E[\pi_i|I^b]$ with respect to L_{i2} and maximization of $E[\pi_i|I^{ac}]$ with respect to L_{i1}.

We may summarize the analysis of the sequential solutions to the Cobb-Douglas model by noting that sequential decision making has two distinct effects on the form of the production model. First, optimal input choices are sequentially dependent. Sequential dependence generally leads to input choice equations which are nonlinear functions of production function parameters, prices and previous inputs and outputs. Even in the case of the simple two-stage Cobb-Douglas model, one obtains the optimal first-stage input by solving a complicated ploynomial equation. One can expect this result for all but the simplest models such as linear production functions and quadratic objective functions.[10] Second, the feedback of information causes inputs chosen in later stages to depend on previous stages' outputs and thus may lead to simultaneity between inputs and outputs. To consider in greater detail the econometric properties of these models under the two error specifications (6) and (7) and under the two data availability conditions described above, we shall consider first the OL solution which does not involve information feedback, and then consider the SU, OLF, and CL solutions which do involve some degree of information feedback.

The OL model with data for both Q_{i1} and Q_{i2} consists of the two production functions (3) and (4) and the two input equations (8) and (9). The L^o_{it} are nonstochastic and the production functions may be estimated using single equation methods. Since the Cobb-Douglas functions are linear in logarithms, under error structure (6) ordinary least squares estimates will be unbiased and efficient (Q_{i1} in equation (4) is a predetermined endogenous variable). Under error structure (7), the combination of a lagged dependent variable in equation (4) with autocorrelated errors causes least squares estimates of the parameters to be biased and inconsistent. One possibility under (6) is to utilize the instrumental variables technique, although a more efficient method would be maximum likelihood estimation under appropriate distributional assumptions (see Theil, Ch. 8). An additional estimation procedure is possible due to the recursive structure of the stage production functions. Equation (5) shows that the final output can be expressed as a function of the exogenous variables alone and therefore the "reduced-form" parameters could be efficiently estimated using a single equation estimator under either error structure (6) or (7). However, it may not be possible to identify the parameters of each stage's function using this approach. Equation (5) shows that, in the Cobb-Douglas example, it would not be possible to identify γ_0, β_0, γ_1, β_1, or β_2.

The SU, OLF, and CL solutions differ from the OL solution in that information feedback from previous stages' outputs to later stages' inputs does occur, so some inputs may be endogenous variables in the structural econometric model. To illustrate, let us consider the model derived from the OLF solution to the input choice problem. The OLF model with both Q_{i1} and Q_{i2}

observed consists of the two production functions (3) and (4) and the two input equations (10) and (12). $\overset{o}{L}_{i1}$ is nonstochastic as in the OL solution but $\overset{o}{L}_{i2}$ depends on Q_{i1} and is stochastic; however, when Q_{i1} is observed (12) is an exact equation without an error term. Consequently, only the production functions need be estimated and the estimation problem is identical to the estimation problems encountered under the OL solution. Under error structure (6) Q_{i1} and L_{i2} are predetermined variables in equation (4) and ordinary least squares may be applied to both production functions in log form. Under error structure (7), the autocorrelation biases least squares estimates and must be accounted for as discussed above.

When data for Q_{i1} is not available, equation (3) may be substituted into equations (4) and (12), and the resulting "semi-reduced form" equations are

$$\log Q_{i2} = \log \gamma_0\beta_0^{\gamma_1} + \beta_1\gamma_1 \log \overset{o}{L}_{i1} + \beta_2\gamma_1 \log A_i + \gamma_2 \log \overset{o}{L}_{i2} + \gamma_1\varepsilon_{i1} + \varepsilon_{i2} \tag{14}$$

$$\log \overset{o}{L}_{i2} = \frac{1}{1-\gamma_2}(\omega + \beta_0 + \log \gamma_0\gamma_2) - \frac{1}{1-\gamma_2}\log\frac{w_{i2}}{p_i} + \frac{\beta_1}{1-\gamma_2}\log \overset{o}{L}_{i1} + \frac{\beta_2}{1-\gamma_1}\log A_i + \varepsilon_{i1} \tag{15}$$

Due to the occurrence of ε_{i1} in both equations, a simultaneous equation estimator must be utilized to obtain consistent estimates of the "semi-reduced form" parameters. Least squares estimates of equation (14) would clearly be biased in this case, in contrast to the OL solution which would allow least squares estimation of (14).

We summarize this section by observing that the sequential solutions to the production problem can yield either single or simultaneous equation models. If the decision maker is assumed to update his information set with information about output as production takes place, as in the SU, OLF, and CL solutions, simultaneity between inputs and output is introduced into the model; and if the input choices are sequentially dependent, as in the OL, OLF, and CL solutions, the form of the solution differs from the nonsequential solution. It is worth noting that when interpreted in the context of sequential decision making, the MA model is internally inconsistent, because in a one-period choice problem inputs must be chosen before production begins. Yet, the MA model shows inputs to be functions of actual output which is not known until after inputs have been chosen.[11] Interestingly, the SU solution produces a model which is similar in form to the MA model but its simultaneity is derived from an explicit sequential decision making process. It is also instructive to note that the ZKD model could be derived from a sequential solution of the input choice problem if the decision maker neither updates his information set nor takes into account the effects of first stage decisions on second stage decisions.

These qualitative results obtained using the Cobb-Douglas model can be generalized in a straightforward manner to models based on any production function and any number of production stages. Dividing the production period into T stages, and letting output of firm i in stage t be Q_{it}, with input vector x_{it}, a coefficient vector β_t, and a production disturbance ε_{it}, the stage production functions can be written

$$Q_{i1} = f_1[x_{i1}, \beta_1, \varepsilon_1] \tag{16}$$

$$Q_{it} = f_t[Q_{i,t-1}, x_{it}, \beta_t, \varepsilon_t], \; t=2,\ldots,T; \; i=1,\ldots,N.$$

Assuming the final product Q_{it} is sold in period T at price p_{iT}, and for input prices w_{it}, profit is

$$\pi_{iT} = p_{iT} Q_{iT} - \sum_{t=1}^{T} w_{ij} x_{ij} \tag{17}$$

and assuming firms maximize expected returns, the ith firm's objective is

$$\max_{x_{i1},\ldots,x_{iT}} E[\pi_{iT}] \text{ subject to (16), (17).}$$

It is worth noting that this control problem is a terminal period problem, and can be interpreted as a special case of the more general multi-period model in which output is sold in each period rather than only in the final period. Solutions to this problem will generally be nonlinear in the parameters and, as discussed further below, the probability distributions for Q_{it} are difficult to ascertain. When farmers are assumed to make decisions sequentially, and when each stage's output is not observed by the econometrician, then the structural econometric production model will be a system of nonlinear simultaneous equations. Estimation procedures for this class of models have been developed (Amemiya, Fair) but are usually very costly to implement.

ERROR SPECIFICATION, FUNCTIONAL SEPARABILITY, AND BEHAVIOR UNDER UNCERTAINTY

For over a decade production economists have studied the relationship between production inputs and the stochastic characteristics of production processes (Day, Anderson, Roumasset, Just and Pope, and Antle). The error specification of the production function is known to determine the way inputs affect the probability distribution of output, and hence the implied behavior of farmers toward production uncertainty. Dynamic production functions also introduce the added problem of tractability of the probability distribution of each stage's output. In general, when the stage functions f_t as given in (16) are nonlinear, nonseparable functions of $Q_{i,t-1}$, x_{it}, and ε_{it}, the probability distribution of Q_{it} cannot be derived analytically and one cannot utilize maximum likelihood estimation or use small sample inference procedures. However, we can show that if the production function is either additively or strongly (non-additive) separable it is possible in some cases to obtain models with tractable distributions.[12]

The production function which is additively separable in $Q_{i,t-1}$ and ε_{it} can be expressed as

$$Q_{it} = \alpha_t Q_{i,t-1} + m_t[x_{it}, \beta_t] + \varepsilon_{it}$$

where α_t is a parameter and m_t is a concave function of x_{it}. Substitution for $Q_{i,t-1}$, $Q_{i,t-2}$, etc., shows that the distribution of Q_{it} is a convolution of the errors $\varepsilon_{it}, \varepsilon_{i,t-1}, \ldots, \varepsilon_{i1}$. Therefore, if linear combinations of the ε_{it} have a known distribution, Q_{it} has a known distribution. For example, if the ε_{it} are normal $(0,\sigma^2)$ variates Q_{it} is normally distributed with a mean linear in the m_t and a variance proportional to σ^2.

Additive separability of inputs across production stages is not usually a plausible maintained hypothesis in agricultural production. Strong, nonadditive separability would appear to be a more reasonable assumption. For example, additive separability in the crop production model discussed in section would imply that the marginal product of harvest labor input is independent of the amount of crop harvested, whereas the strongly separable Cobb-Douglas function used in section 1 (see equation 4) shows that the marginal product of harvest labor L_{i2} depends on the amount of crop harvested, Q_{i1}. A production function which is strongly separable in $Q_{i,t-1}$, x_{it} and ε_{it} can be specified as:

$$Q_{it} = (Q_{i,t-1})^{\alpha_t} m_t[x_{it}, \beta_t]\varepsilon_{it}$$

Note that the logarithm of Q_{it} is linear in the logarithms of $Q_{i,t-1}$ and ε_{it} under this specification. If the ε_{it} follow a distribution such as the normal which has the property that a convolution of normal variates also has a normal distribution, the output of each stage follows the same distribution.

From these examples an important conclusion can be reached regarding error specification and functional separability of the production stages' inputs: tractable production function specifications typically must be additively separable if error specifications are additive, or must be strongly separable if error specifications are multiplicative. Otherwise, one typically obtains each stage's output as a nonlinear function of the earlier stages' error terms, and the probability distribution of output cannot be ascertained analytically (see Aoki, 1967, Ch. 2, for a discussion of this problem in the context of general solutions to stochastic control problems).

The relationship between error specification and functional separability is also relevant to the analysis of behavior towards uncertainty because additive and multiplicative error structures have different implications for the effects of inputs on the probability distribution of output. Just and Pope have shown that the standard multiplicative error specification restricts the relationship between input choice and output variance. More generally, not only the mean and variance but also higher moments of output may be functions of inputs (Day, Anderson, Roumasset). Antle shows that a general model which does not impose restrictions on the relationship between the inputs and the form of the probability distribution of output can be specified and estimated with an additive error term. The above discussion shows that a dynamic model with this error structure would have to be additively separable across production stages. However, a model with desirable properties which is strongly separable across production stages can also be specified, as follows. First, define the production function as

$$Q_{it} = Q_{i,t-1}\, m_t[x_{it}, \beta_t]e^{\varepsilon_{it}} = m_1[x_{i1}, \beta_1]\, m_2[x_{i2}, \beta_2]\ldots m_t[x_{it}, \beta_t]e^{u_{it}}$$

where

$$E(u_{it}) = 0, \qquad u_{it} = \sum_{j=1}^{t} \varepsilon_{ij}.$$

Second, assume the joint probability distribution of u_{it} is $g(u_{it}|x_{i1},\ldots,x_{it})$, a function of inputs. Then, generally, the moments of u_{it} depend on inputs:

$$\mu_{jit}\,(x_{i1},\ldots,x_{it}) = \int_0^{\infty} (u_{it})^j\, g(u_{it}|x_{i1},\ldots,x_{it})du_{it}$$

Finally, note that $e^{u_{it}} = \sum_{j=0}^{\infty} \frac{u_{it}^j}{j!}$ and, therefore,

$$E(e^{u_{it}}) = 1 + \sum_{j=2}^{\infty} \frac{\mu_{jit}}{j!}\ .$$

Using this latter expression, it can be shown that the moments of output are functions of the inputs through the m_t and the μ_{jit}. Hence, this strongly separable production function specification yields a tractable output distribution, and does not restrict the effects inputs may have on the moments of the output distribution.

CONCLUSIONS AND EXTENSIONS

It has been demonstrated that when the short-run input choice problem is solved sequentially the resulting structural econometric production model generally differs, in terms of functional form and stochastic structure, from single-stage production models. Since farm managers can be expected to utilize all available information in their decision making, they will feedback information from earlier production stages to later input choices. In addition, only the final agricultural product is usually measured. These two facts mean that agricultural production models typically are systems of simultaneous equations and single equation estimates of production function parameters will be subject to simultaneous equation bias. Estimates with desirable properties can be obtained by formulating and estimating models that are consistent with the sequential structure of farm managers' input choice problems. However, in order to implement multi-stage sequential production models, researchers must devise models which have desirable properties and which are empirically tractable.

In assessing the practical importance of these findings two points are worth emphasizing. First, the magnitude of the simultaneous equation bias due to input endogeneity remains to be ascertained. Currently, the author is conducting Monte Carlo studies to investigate the nature of the bias. Second, as any applied production economist knows, a crtical limiting factor is data availability; most available production data do not contain information on inputs by production stage or operation. An important contribution to our understanding of both the simultaneity problem and, perhaps more important, the sequential structure of farm managers' decision making and their stage-level production functions, could be made by collection of production data by stages so that stage-level production and sequential decision making could be studied.

The results presented here concerning short-run production could be extended to long-run, multi-period production problems in which farmers choose inputs over many production periods (rather than over many stages in one period) to maximize the expected present value of profit or some function of profit. If output in each period depends on outputs and inputs from previous periods then both the production functions and input demand functions are dynamic and involve lagged endogenous variables. Therefore, the long-run model generally could be expressed as a system of dynamic, recursive simultaneous equations and appropriate estimation methods could be devised along the lines pursued in this paper.

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TABLE 1

Definitions of Information Sets Used in Sequential Solutions of the Cobb-Douglas Production Model

Information Set	Input and Price Informaton					Production Information			
	$g_p(P_i)$	w_{i1}	$g_w(w_{i2})$	w_{i2}	$\overset{o}{L}_{i2}(L_{i1})$	$g_1(\varepsilon_{i1})$	Q_{i1}	$g_2(\varepsilon_{i2})$	$g_2(\varepsilon_{i2} \| Q_{i1})$
I°	X	X	X			X		X	
I^a	X	X	X		X	X		X	
I^{ac}	X	X	X		X	X			X
I^b	X	X		X			X	X	

Definitions: $g_p(p_i) \equiv$ probability distribution of product price.

$w_{i1} \equiv$ period 1 wage rate.

$g_w(w_{i2}) \equiv$ probability distribution of period 2 wage rate.

$w_{i2} \equiv$ period 2 wage rate.

$\overset{o}{L}_{i2}(L_{i1}) \equiv$ optimal labor input in period 2.

$g_1(\varepsilon_{i1}) \equiv$ probability distribution of period 1 production disturbance.

$Q_{i1} \equiv$ actual production in period 1.

$g_2(\varepsilon_{i2}) \equiv$ probability distribution of period 2 production disturbance.

$g_2(\varepsilon_{i2}|Q_{i1}) \equiv$ probability distribution of period 2 production disturbance conditional on Q_{i1}.

TABLE 2

Information Sets Used in Sequential Solutions of the Production Model

Solution	Information Set Used in Period 1	2
Open Loop (OL)	I^a	I^o
Sequential Updating (SU)	I^o	I^b
Open Loop w/Feedback (OLF)	I^a	I^b
Closed Loop (CL)	I^{ac}	I^b

Note: See Table 1 for definitions of the information sets.

Footnotes

1. Yotopoulous et al. (1976) found parameter estimates of a Cobb-Douglas production function to be very different when obtained indirectly from estimates of a profit function, and when obtained directly from estimation of the production function. They suggest the differences may be due to simultaneity of inputs and output which biases the direct production function parameter estimates.

2. For example, Hall (1977) obtained negative estimates of herbicide and insecticide production elasticities using a single-equation production function model. One might suspect that rather than being due to the over-use of these inputs, this result may be due to simultaneous equation bias. Also see Headley (1968) for a study of pesticide productivity using aggregate data. The dynamic dimension of the pesticide problem has been considered by Hall and Norgaard (1973), Talpaz and Borosch (1974), and others.

3. Throughout the paper we utilize the fact that, for a Cobb-Douglas production function, $\partial Q_{it}/\partial L_{it} = \alpha_i Q_{it}/L_{it}$, where α_i is the production elasticity of L_{it}.

4. As we shall discuss later, the MA model (1) is not internally consistent. Formally, the theoretical production model is based on single-period profit maximization and, therefore, inputs must be chosen <u>before</u> production begins. Yet, in the MA model L_{i1} and L_{i2} are specified as functions of <u>actual</u> (not expected) output!

5. A complete specification of the decision problem requires specification of the farmer's price expectation formation process as well as his sequential input choice. Modeling price expectations is a difficult

problem with no easy solution. Since our purpose is not primarily to study price expectations, we mention briefly several schemes which could be utilized. One approach is to assume prices are independently distributed and that price distributions are known to the decision maker, as we did in the previous section. Within this framework one might employ a "rational expectations" model or an adaptive expectations model. An alternative, more complex, and more theoretically satisfactory approach is to incorporate the price expectation problem into the firm's overall decision problem. By treating price information as a costly economic good, this latter approach could generate "economically rational expectations" which are consistent with the firm's dynamic behavior (see Feige and Pearce, 1976; Chow, 1981, Ch. 16) in contrast to "rational" or "adaptive" expectations which are not derived from the firm's maximizing behavior. However, this latter approach would certainly complicate the solution of an already difficult problem. In the following analysis we continue to assume price distributions are known by farmers.

6. For definitions and further discussion of alternative stochastic control strategies, see Rausser (1978), Rausser and Hochman (1978, Ch. 8), and Aoki (1976, Ch. 10). The sequential updating, open loop with feedback, and closed loop control solutions discussed in this paper involve some degree of "passive learning" by the farmer during the production process, that is, the amount of learning is not a function of the farmer's own decision making. "Active learning," in contrast, refers to the case where farmers take decisions in part for the information they generate for future decision making. The reader may note that the qualitative results derived here for passive learning models could be extended directly to fully adaptive or active learning models.

7. Note that

$$E[\pi_i|I^\circ] = \bar{p}_i\gamma_0 L_{i2}^{\gamma_2} E(Q_{i1}^{\gamma_1} e^{\varepsilon_{i2}}) - w_{i1}L_{i1} - \bar{w}_{i2}L_{i2}.$$

Since $E(Q_{i1}^{\gamma_1}) = (\beta_0 L_{i1}^{\beta_1} A_i^{\beta_2})^{\gamma_1} E(e^{\gamma_1\varepsilon_{i1}})$

$$= (\beta_0 L_{i1}^{\beta_1} A_i^{\beta_2})^{\gamma_1} e^{\gamma_1^2\sigma_1^2/2}$$

$$= \bar{Q}_{i1}^{\gamma_1} e^{(\gamma_1^2\sigma_1^2 - \gamma_1\sigma_1^2)/2}$$

and $E(e^{\gamma_1\varepsilon_{i1} + \varepsilon_{i2}}) = e^{(\sigma_2^2 + \gamma_1^2\sigma_1^2 + \gamma_1\rho\sigma_1^2)/2}$

we obtain the expresion given in the text.

8. Since

$$E[\pi_i|I^a] = \bar{p}\gamma_0\bar{Q}_{i1}^{\gamma_1} (\overset{o}{L}_{i2})^{\gamma_2} e^{\omega} - w_{i1} L_{i1} - \bar{w}_{i2}\overset{o}{L}_{i2},$$

$\overset{o}{L}_{i1}$ satisifies

$$\frac{\partial E[\pi_i|I^a]}{\partial L_{i1}} = E[\pi_i|I^a] \{\gamma_1\bar{Q}_{i1}^{-1} \frac{\partial\bar{Q}_{i1}}{\partial L_{i1}} + \gamma_2(\overset{o}{L}_{i2})^{-1} \frac{\partial\overset{o}{L}_{i2}}{\partial L_{i1}}\} - w_{i1} - \bar{w}_{i2} \frac{\partial\overset{o}{L}_{i2}}{\partial L_{i1}} = 0.$$

9. See also Zellner (1971), Ch. 11, for a discussion of sequential updating.

10. See Aoki (1967), Ch. 2, for a detailed analysis of this point.

11. This criticism of the MA model is also valid for the models of Hoch (1958), 1962) and Mundlak and Hoch (1965). Those models with endogenous input demand equations specify input demands as functions of actual output rather than expected output.

12. An additively separable function $f(x_1,x_2)$ can be expressed as $f_1(x_1) + f_2(x_2)$; a strongly (but not necessarily additive) separable function can be written $f(x_1,x_2) = F[f_1(x_1), f_2(x_2)]$.

References

Amemiya, T. "The Nonlinear Two-Stage Least Squares Estimator." Journal of Econometrics 2(1974) pp. 105-110.

Anderson, Jock, "Sparse Data, Climatic Variability, and Yield Uncertainty in Response Analysis." American Journal of Agricultural Economics 55(1974):77-82.

Antle, J. M. "Theory and Measurement of Output Distributions." University of California, Davis, Department of Agricultural Economics, Working Paper 81-3.

Aoki, M. Optimization of Stochastic Systems. Academic Press, 1967.

_______. Optimal Control and System Theory in Dynamic Economic Analysis. North-Holland, 1976.

Chow, G. C. Econometric Analysis by Control Methods. New York: Wiley, 1981.

Day, R. H. "Probability Distributions of Field Crops." Journal of Farm Economics, 47(August 1965):713-741.

Fair, R. C. "The Estimation of Simultaneous Equation Models with Lagged Endogenous Variables and First Order Serially Correlated Errors." Econometrica (May 1970):507-516.

Feige, E. L. and D. K. Pearce. "Economically Rational Expectations: Are Innovations in the Rate of Inflation Independent of Innovations in Measures of Monetary and Fiscal Policy?" Journal of Political Economy 84(June 1976):499-522.

Hall, D. C. "An Economic and Institutional Evaluation of Integrated Pest Management, with an Empirical Investigation of Two California Crops." Ph.D. dissertation, University of California, Berkeley, Department of Agricultural Economics, 1977.

Hall, D. C. and R. B. Norgaard. "On the Timing and Application of Pesticides." American Journal of Agricultural Economics, (May 1973):198-201.

Headley, J. C. "Estimating the Productivity of Agricultural Pesticides." American Journal of Agricultural Economics, 50(1968):13-23.

Just, R. E. and R. D. Pope. "Stochastic Specification of Production Functions and Economic Implications." Journal of Econometrics 7(1978):67-86.

Kushner, H. J. and F. C. Schweppe. "A Maximum Principle for Stochastic Control Systems." Journal of Mathematical Analysis and Applications 8(1964):287-302.

Marshak, J. and W. J. Andrews. "Random Simultaneous Equations and the Theory of Production." Econometrica 12 (July-October 1944):143-205.

Mundlak, Y. and I. Hoch. "Consequences of Alternative Specifications in Estimation of Cobb-Douglas Production Functions." Econometrica 33 (October 1965):814-828.

Rausser, G. C. and E. Hochman. Dynamic Agricultural Systems: Economic Prediction and Control. Amsterdam: North-Holland, 1979.

Roumasset, J. A. Rice and Risk: Decision Making Among Low Income Farmers. Amsterdam: North-Holland, 1976.

Talpaz, H. and I. Borosch. "Strategy for Pesticide Use: Frequency of Applications." American Journal of Agricultural Economics, (Nov. 1974):769-775.

Zellner, A. An Introduction to Bayesian Inference in Econometrics. New York: Wiley, 1971.

Zellner, A., J. Kmenta and J. Dreze. "Specification and Estimation of Cobb-Douglas Production Function Models." Econometrica 34 (October 1966):784-795.