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A DYNAMIC OPTIMIZATION MODEL OF FIRM BEHAVIOR IN THE CONTEXT OF UNCERTAINTY AND TECHNICAL CHANGE

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ABSTRACT

A dynamic model of firm behavior in the presence of uncertainty and technical change is built on the assumption that the firm's technology, or production method, is chosen from the set of available technologies, along with the human and nonhuman resources it employs. The firm's optimal technology choice is modeled as a problem of optimal stochastic control. Examples from agricultural and industrial production are used to show how this model contributes to the understanding of the roles of human capital and learning by doing in production, the process of technological diffusion, and the effects of economic policies and exogenous changes in relative prices on producer behavior. Implications of this model for econometric specification of production function models are also discussed.

A DYNAMIC OPTIMIZATION MODEL OF FIRM BEHAVIOR IN THE CONTEXT OF UNCERTAINTY AND TECHNOLOGICAL CHANGE

In this paper a dynamic model of firm behavior in the presence of uncertainty and technical change is built on the assumption that the firm's technology, or production method, is chosen from the set of available technologies, along with the human and nonhuman resources it employs. The firm's optimal technology choice is modeled as a problem of optimal stochastic control. The theoretical contribution of this approach is to go beyond the simplistic treatment of firm-level technical change as either a problem of investment in capital, or as an unspecified process of exogenously imposed "residual" productivity growth. In the model of optimal technology choice, a firm's decision maker is assumed to choose a production process from the technologically feasible set in each production period so as to maximize the firm's objective function, subject to both human and nonhuman resource constraints facing the firm. Being in the form of a general investment model, this model provides a framework useful for the analysis of issues such as the roles of human capital and learning by doing in production decisions, the process of technological diffusion, and the effects of economic policies and exogenous changes in relative prices on producer behavior. Examples from agricultural and industrial production are used to show how this model helps identify those factors which may constrain firms' technology choices and hence their productivity.

The paper begins with a presentation of the general model of optimal technology choice. The following sections show how the model can be used to gain insight into a variety of theoretical and applied production problems. Finally, it is shown how the model can be translated into econometric production models with "variable" coefficients.

THE MODEL OF OPTIMAL TECHNOLOGY CHOICE

In this section we consider a firm's behavior when the conventional neoclassical assumption of a fixed, known technology is replaced with the situation in which at time t a set of technologies B is available to the firm. We observe that generally a firm's production process is a stochastic relationship between inputs and output, and that the firm takes this fact into account in its input and technology choices. The quantitative relationship between inputs and output is therefore described as an output distribution, which is represented by the probability distribution function $f(Q_t \mid x_t, \beta_t)$, where Q_t is output, x_t is an input vector, and $\beta_t \in B$ is the parameter vector which determines the form of the output distribution. Some inputs, referred to as variable inputs, are purchased for use in the current production period only. Fixed or capital inputs are utilized over more than one production period. The product price is p_t, and per unit input prices are r_t, so that total revenue is $p_t Q_t$ and total production costs are the inner product $r_t x_t$ each period. The elements of r_t corresponding to the variable inputs are market input prices and the elements corresponding to fixed inputs are rental rates. Changes in input levels may also involve an adjustment cost c_t which is a convex function of the rate of input purchases I_t, the value of the current input vector x_t, and the technology parameter vector β_t. Thus we have $c_t = c(I_t, x_t, \beta_t)$.[1] Firms are assumed to attempt to maximize the expected present value of utility which is a function of profit. Profit is expressed as:

$$\pi_t = \pi(\Psi_t, x_t, \beta_t) = p_t Q_t - r_t x_t - c_t \tag{3}$$

where Ψ_t is a vector measuring prices and adjustment costs. Since in general output and prices are random variables, profit is a random variable. We assume that in each period the firm formulates expectations in terms of anticipated price distributions; such expectations may or may not be "rational" in the sense of Muth (1961).

Defining I_t as new input purchases in period t for use in period t+1 allows us to define the general adjustment equation:

(2) $$x_{t+1} = \delta x_t + I_t$$

where δ is a diagonal matrix with ith diagonal element δ_i such that $\delta_i = 0$ if the ith element of x_t is an input consumed fully in period t, and $0 < \delta_i < 1$ if it is partially consumed in period t. Thus the elements of I_t are purchases of variable inputs and gross additions to the firm's capital stock in period t. From equation (1) we have also:

(3) $$x_{t+1} = \delta^t x_0 + \sum_{j=0}^{t} \delta^{t-j} I_j$$

where x_0 represents the value of the firm's initial input vector.

In order to construct a model of technology choice, the choice set B must be ordered in an economically meaningful way. Such an ordering can be defined as follows: let $\beta^i < \beta^j$ mean that the superscript $i < j$ if and only if:

(4) $$\sup_{x_k \varepsilon X} EU[\pi(\Psi, x_k, \beta^i)] < \sup_{x_k \varepsilon X} EU[\pi(\Psi, x_k, \beta^j)]$$

Here X is the set of all feasible input vectors x_k and E denotes the mathematical expectations operator. Thus, $\beta^i < \beta^j$ if and only if the expected utility possible with any feasible input vector x_k and technology vector β^j is greater than that possible with any x_k and β^i. We note that the function:

$$EU[\pi(\Psi, \beta)] = \sup_{x_k \varepsilon X} EU[\pi(\Psi, x_k, \beta)]$$

generates an envelope of the expected utility functions. The technology choice set $B = (\beta^1, \ldots, \beta^M)$ is now ordered according to (4). It should be emphasized that the ordering of the technologies is defined in terms of the set of prices Ψ. Subsequently, we will use this ordering, and its dependence on relative prices, in the analysis of the firms' technology choices.

The problem of optimal technology choice is now stated as follows:

$$\max_{\{\beta_t\}, \{I_t\}} V_{t_0} = E_{t_0} \sum_{t=t_0} \gamma^{t-t_0} U(\pi_t), \quad 0 < \gamma < 1$$

subject to:

$$X_{t+1} = \delta X_t + I_t$$

$$\pi_t = P_t Q_t - r_t x_t - C_t$$

$$Q_t \sim f(Q_t | x_t, \beta_t)$$

$$c_t = c(I_t, X_t, \beta_t)$$

$$\beta_t \quad \varepsilon \; B$$

The optimal choices of β_t and I_t are known to be functions of x_t and the parameters of the firm's anticipated price distributions, so as to satisfy the dynamic programming algorithm:

$$(6) \qquad V_t(x_t) = \sup E_t[U(\pi_t) + V_{t+1}(\delta x_t + I_t)]$$

We can now use the ordering defined in equation (4) to study the firm's path over time to long-run equilibrium. At time t_0 the firm is producing with (x_{t_0}, β_{t_0}). In making plans for production in future periods the firm recognizes that it may achieve higher expected utility with a different input/technology combination, but to achieve these other positions may be costly. Therefore, the firm's optimal paths for I_t and β_t and hence x_t will depend on the firm's expectations, it's utility function, the perceived relative benefits of the elements of B, and adjustment costs. To obtain stronger results from this framework, it is useful to consider in more detail the concept of a firm's "technology." We define a firm's technology as a method by which it productively combines its human resources, in the form of useful knowledge and other forms of human capital, and its physical or nonhuman resources. The crucial role of human resources in this definition is emphasized by the word "method," for it seems clear from the most casual empirical observation that productivity depends not only on the kinds and amounts of physical inputs used, but also on how they are used. This simple insight provides motivation for two assumptions that we now make:

(i) For given x, the dispersion of the firm's output distribution is nondecreasing in β.

(ii) For given x, expected output $\bar{Q}(x, \beta)$ is a concave function of β.

Assumption (i) means that the dispersion of the firm's output distribution tends to increase as the firm tries to use more advanced production techniques with a given set of inputs. The motivation for this assumption follows directly from the definition of a technology as a method of producing; adjustment costs associated capital inputs, both human and nonhuman, may severely constrain the firm's ability to utilize more advanced production techniques. In this discussion "dispersion" is used in the sense of Rothschild and Stiglitz (1970), as an increase of the weight given to the tails of the probability distribution function. Rothschild and Stiglitz show that a general measure of the "riskiness" of a random variable with a given mean value is the expected utility obtained by a risk averse individual; the riskier is the random variable the less is the expected utility obtained.[2/] Therefore, since the firm's input vector is constrained by adjustment costs in each period, as it attempts to use more advanced techniques, beyond some point the effect of constraints on input choices must be to increase the dispersion of the output distribution. This increased riskiness of the more advanced technologies will eventually offset the potential benefit of greater mean productivity of the new technologies, and reduce the expected utility obtained from them. Therefore, $EU(\pi_t)$ is concave in β_t for firms whose adjustments costs are high relative to the perceived benefits of the more advanced technologies. The most advanced technology β^M is optimal only for those firms with sufficient opportunity and incentive to use it successfully.

This conclusion is based on assumption (i), and is relevant to risk averse firms. For risk-neutral firms, assumption (ii) is relevant. If $\bar{Q}(x, \beta)$ is concave in β for given x, the firm's production process exhibits diminishing marginal productivity as it attempts to use more advanced technologies with a fixed set of inputs. This effect reinforces the concavity of $EU(\pi_t)$ in the case of risk-averse firms, and also ensures that the same properties hold for risk-neutral firms.

Equation (6) shows the important adaptive behavior that rational decision processes may exhibit. The firm which behaves optimally chooses paths for β_t and I_t cognizant of the fact that current choices may influence its ability to produce currently and in the future. It is this adaptive feature of the model that will be exploited in the next section to examine in more detail the role of human capital and learning-by-doing in the firm's technology choices.

We are now prepared to study the adjustment process of firms over time. Consider first a firm at time t_0 producing with (x_{t_0}, β_{t_0}). In Figure 1 we see some possible forms of the relationship between $EU(\pi_t)$ and β_t, for given input vectors x. At point a, the firm is in a sub-optimal position and can benefit from an immediate move to b, using technology β^2. However, β^2 is optimal only if it is the firm's long-run equilibrium technology, for in that case V_t is maximized by maximizing $EU(\pi_t)$. Otherwise, equation (6) tells us that the optimal technology choice may be some $\beta^3 > \beta^2$ in order to maximize V_t. Over time, the firm will then choose input vectors $x_{t_1}, x_{t_2}, \ldots, x_{t_N}$ until the long-run equilibrium technology is reached. The major implication of this analysis is that the long-run equilibrium technology may be $\beta^N \leq \beta^M$, with equality holding only if the firm has sufficient incentive to make the necessary, but costly, adjustments.

We now have the general framework of the model of optimal technology choice. The basic elements of this model are the output distribution, the existence of adjustment costs, the hypothesis of firms who maximize the present value of expected utility which is a function of profit, the ordered set of available technologies, B, and the two behaviorial assumptions about the relationship between $EU(\pi)$ and the ordered set B. We can now use this model to gain insight into a variety of theoretical and applied production problems.

HUMAN CAPITAL, LEARNING BY DOING, AND TECHNOLOGICAL DIFFUSION

Recall that we define a technology as a method of productively combining human and nonhuman resources. The complementarities between these two types of inputs is thus an essential ingredient in our conception of the production process. To accommodate this view, we shall modify the conventional theory of the firm by including a decision-making process in the specification of the firm's production process. We define information as a flow of data about the outcome of future or uncertain events. The firm's decision maker receives information from a variety of sources such as markets, experience, and printed matter. Information pertaining to the use of technology, or that improves one's ability to organize production, is referred to as technical information (TI). The decision maker's technical knowledge, K, is defined as a net stock of accumulated TI. Given an initial stock of knowledge K_0 and a depreciation rate of knowledge $(1 - \delta)$, a stock of technical knowledge at time t can be expressed as:

(7) $$K_{t+1} = K_0\delta^t + \sum_{j=0}^{t} \delta^{t-j} TI_j$$

which in turn implies:

(8) $$K_{t+1} = \delta K_t + TI_t$$

In order to make production decisions the firm's decision maker combines time, general knowledge, education, technical knowledge, and possibly other material resources.[3/] The flow of services obtained in this manner from human capital and other inputs is defined as the decision-making input (D). Letting ι denote a vector of variable inputs, and letting ED measure the decision maker's education, the function:

$$(9) \qquad D_t = D(\iota_t, K_t, ED)$$

expresses decision making abilities, as measured by D, as a function of the inputs into the decision-making process. This function is interpreted as a production function obeying the usual regularity laws.[4/]

As the definition of a technology makes clear, there is an intimate relationship between the technology used, the decision-making input and output. When best-known production methods are used, we can define the envelope function:

$$(10) \qquad EU[\Psi, \beta]^* = \sup_{x,\, D} EU[\Psi, X, D, \beta]$$

This is simply a special case of the general model discussed above, to which the ordering of the technologies and the corresponding analysis directly apply.

To investigate the process of firms investing in technical knowledge, we must recognize that new knowledge is costly, but it may yield benefits over more than one production period. Therefore, this is a problem of investment and the decision problem is one of inter-temporal choice. Let us simplify the analysis and assume that the adjustment costs only involve investment in technical knowledge. The rate of investment in K_t is defined as TI_t, as described above. There may be several components to the cost of learning. One is the cost of physical resources used, although of greater importance may be the opportunity costs of time and effort spent in acquiring TI. It seems reasonable to assume that ED, the measure of the decision maker's educational capital, may facilitate the acquisition of TI, so the marginal opportunity cost of TI should be lower the more schooling the decision maker has. It is also plausible that the marginal opportunity cost of TI rises with the quantity of TI acquired per period, due to a rising opportunity cost of the decision makers time and other resources. Also important may be the external factors which affect the supply of technical information to firms. For example, extension programs are often a major source of TI to farmers, and in general private industry provides technical information about the innovations they sell to firms. For notational convenience, let y_t represent a vector measuring such factors as schooling and the costs of outside information; this may be thought of as a vector of shadow prices affecting the costs of acquiring TI. These considerations then suggest that the cost of acquiring TI may be expressed as the following function:

$$(11) \qquad c_t = c(TI_t, y_t)$$

This function is convex in TI_t, to reflect the assumption of increasing marginal opportunity cost, and increasing or decreasing in the elements of y_t.[5/]

Firms also acquire TI through the process of learning by doing.[6/] This learning is a function of the firms' current stock of technical knowledge and the production process being used. The possibility of learning by doing may induce the firm to choose a production method partly for the TI it acquires while using it. Therefore, the cost of acquiring TI in period t may depend on K_t and β_t:

(12) $c_t = c(TI_t, y_t, K_t, \beta_t)$

The problem of optimal technology choice can now be formulated for this model. The control variables are β_t, TI_t, x_t, and ι_t, chosen subject to the constraints described above, so as to satisfy an equation of the form of (6). With this model of optimal learning of technical knoweldge, the interpretation of the firm's adaptive learning process has strong intuitive appeal. Referring back to Figure 1, we see that point b is optimal only if learning by doing does not occur. If the firm learns by doing, it may be worthwhile for it to produce at point C if it can acquire new technical knowledge at lower cost by so doing; in the long run this benefit may offset the somewhat lower expected utility obtained in the current production period. Then as the firm learns, and K increases over time, the firm will move to higher $EU(\pi)$ curves and eventually converge to long run equilibrium, at point d, at which time it will again maximize $EU(\pi)$ in each production period. We can now examine some of the interesting behavioral implications of this model.

First let us consider the interpretation of the relationship between education and production implied by this model. Schooling of decision makers affects their ability to use productive information, as represented by the function $D = D(\iota, K, ED)$, and it affects their ability to acquire

TI, as discussed above in terms of the adjustment cost function. These roles of ED in decision making may be interpreted as comprising the "allocative" effect of education on production defined by Welch (1970). Welch argues that the value of ED is largely due to its contribution to individuals' ability to adapt to changing circumstances, and therefore the value of additional education, in particular higher education, can be maintained at high levels only by a continuing process of technical change. The discussion of the adjustment process to long run equilibrium with the model of optimal technology choice supports the view that ED affects the path of adjustment and hence should derive value from this attribute. However, we would like to mention that this does not mean that in a dynamic environment of technical change, the rate of return on investment in education could not be bid down to levels competitive with other investment opportunities. Moreover, it should be emphasized that each long run equilibrium position can be maintained only by preserving the <u>level</u> of decision-making human capital input that allowed the firm to achieve that equilibrium level. That is, while a certain human capital stock is essential to technical change, it is also necessary to the maintenance of the <u>level</u> of technical expertise achieved through technical change. Therefore, returns to schooling are derived not only from its enhancement of individuals' abilities to respond to disequilibria, but also because it provides individuals with the ability to maintain new equilibrium levels of technical advance.

Another important implication of this model is the existence of a scale effect on the incentive to learn which is due to the role of the decision-making input, D, in the production process.[7/] Firms operating on a large scale benefit much more than small firms from an investment in new knowledge because they can apply it to more resources. Therefore, for a given cost of acquiring new technical knowledge, larger firms have a greater economic incentive to learn about and use new techniques. This relationship may cast some light on the process of technical diffusion. In both manufacturing industries and agriculture, scale effects on the rate of technical diffusion have been found to exist, in studies by Mansfield (1963) and Huffman (1974). This issue is particularly relevant in the agricultural development literature on the differential effects of the introduction of certain types of new agricultural technologies on farms of different size. (See Dalrymple 1979, Griffin 1972). In general, our model indicates that large farmers and large firms generally may tend to adopt more rapidly regardless of the nature of the technology in question; this is simply a result of the scale effect on the benefits of investment in technical knowledge. Another factor may be the tendency of larger farmers, and perhaps firms generally, to be less risk averse. The discussion of the previous section suggests that more risk averse firms will tend to adjust slower to new production techniques than less risk averse firms. Also very important to the analysis is the stock of fixed human capital of the firm, as described above. In the case of farmers, schooling has been found in many studies to influence farmers' abilities to learn about and use new production techniques (Schultz, 1975). In developing agriculture, the high correlation of farm size, schooling, and access to information may play an important role in reinforcing this pattern of bias towards large scale. To our knowledge, no study has yet attempted to sort out the relative importance of these complementary effects.

Up to this point we have viewed the analysis strictly from the point of view of a single firm. To obtain a complete view of the process of technical diffusion, however, we must recognize that the introduction of new techniques will generate supply-induced price effects. Firms exhibiting some form of "rational" expectations are generally aware of this fact too, and will incorporate relevant information into their price expectations.[8/] In the case of a competitive industry, the firm is faced with the fact that quasi-rents may be earned from successful adoption, but these rents are uncertain, and in the longer run the technical information and experience they acquire thusly will usually become public knowledge.[9/] The industry supply function will then shift as all firms adopt, and rents from early adoption will be eliminated. This analysis implies that we can view early adopters as choosing from a portfolio of investment opportunities; if the expected return to adoption of new techniques is favorable, the firm will devote some of its resources to that purpose. If the industry is competitive, entry of skilled entrepreneurs will reduce the rate of return on this type of activity to what is obtainable from other investment opportunities. However, if some entrepreneurs possess unique abilities to adapt their production processes to new, less costly techniques, they may continue to earn "monopolistic" rents in the long run by continuously seeking out promising new technical processes. While we would not necessarily expect the return to education to remain high during prolonged periods of technical change, for reasons described above, it is clear that monopolistic rents of this type would be earned only during periods of continued technical change.

ECONOMIC POLICIES AND CHANGES IN RELATIVE PRICES

Another broad set of factors which can be identified as determinants of firm's technology choices are economic policies and relative prices; in particular, we are interested in looking at the effects of exogeneously imposed relative price changes on firm's technology choices. Very often, of course, economic policies have precisely such-effects, either intended or unintended. One well-documented example is government intervention in agriucltural markets. This has occurred throughout the world, but particularly in developing countries government policies have had the effect of holding agricultural product prices below market equilibrium while simultaneously keeping prices of "modern" inputs, such as fertilizers, above equilibrium levels (Schultz, 1968). This discourages farmers from attempting to learn about and use more productive new techniques because they have little or no economic incentive to do so. While conventional static production analysis indicates that such policies will discourage production and the use of high-cost new inputs, it misses the important dynamic effects of such policies. The model of optimal technology choice implies that such policies not only distort optimal factor proportions, they also inhibit the entire process of technical change, which experience in developed countries has shown to involve investments in human capital and the learning of technical expertise in the form of accumulated technical knowledge (Hayami and Ruttan, 1971). That is, successful technical change is a process of investment in both human and nonhuman resources, and this long-run process of adjustment is thwarted by distortions of relative prices against the use of new technologies. The effect of such policies is captured by the model of optimal technology choice in the ordering of techniques defined in equation (4). Recall that the ordering is a function of the relative price vector Ψ. When government policies result in, say, relatively higher input prices for the newer, more advanced technologies, there are two possible outcomes. One is that the new techniques may still be economically superior to older techniques for some farmers, but not by enough to induce farmers to bear the costs of adjustment necessary to successfully use the new techniques. The result of such policies may be to reinforce the bias of technology choice more towards the farmers who face relatively low adjustment costs because of their larger stocks of human capital or easier access to sources of technical information and credit. The other outcome may be that the entire ordering of technologies is changed by the price policies. In this case, the newer technologies are economically inferior to the older production processes regardless of adjustment costs.

This example from agriculture can also be used to illustrate another potentially important effect of government policies, namely those which affect the costs of adjustment from older to newer technologies. The literature abounds with discussion of investment in agricultural research to produce more productive new technologies (Evenson, 1968, Evenson and Kislev, 1975). However, the information produced by such research must be transmitted to farmers, and clearly the lower the cost of such information to farmers, the more beneficial will new technologies appear to them. This fact provides economic rationale for subsidization of agricultural extension programs by governments. The model of decision-making set out in the previous section also implies that extension and other programs which lower the cost of technical information to farmers may be observed to substitute for farmers' education, in the sense that either education or extension may lower the cost of acquiring technical information and hence speed the farmer's adjustment to new technologies. This substitution relationship between education and extension was found by Huffman (1974) in his study of the adoption of nitrogen fertilizer by American corn farmers.

Another set of factors which constrain agricultural development in less-developed countries are investments in transportation and communications infrastructure (Wharton, 1967; Johnson, 1980). The services produced by these components of an economy's capital stock are essential to both the low-cost transmission of technical information to farmers, as well as to the integration of farmers into markets which provide an economic incentive to adopt new production techniques. Empirical work by Antle (1980) suggests that these infrastructural factors may be as important as factors such as irrigation which are directly related to the production process.

Another very important and striking example of the effects of changes in relative prices on firm's technology choices is the rise in oil prices. While it is clear from standard production theory that this rise will induce firms and consumers to substitute in production and consumption away from oil intensive production processes and products, the dynamic investment and learning process is omitted from the conventional analysis. A good case in point would appear to be the automobile industry in the United States during the past decade. The rise in oil and gasoline prices made large, fuel inefficient automobiles produced by American manufacturers economically inferior to smaller, fuel efficient models produced by European and Japanese companies who had faced the problem of relatively higher fuel prices for many years. While many observers of this situation have essentially blamed American auto makers for their apparent technological ineptitude, and their apparent unwillingness to take the "energy crisis" seriously, the model of optimal technology choice suggests a much different explanation for what has happened. The technological expertise, or stock of technical knowledge, acquired by foreign auto makers such as Toyota over the previous decade or more is not a costless investment. Unfortunately for the American auto builders, in addition to adjustment costs specific to physical capital that must be born in changing over to the production of smaller cars, there is also the potentially very large investment in technical knowledge that must be made. This will involve a process of "re-tooling" the firm's stock of human capital as well as in the form of technical expertise with small engines, manual transmissions, light-weight materials, and so forth. Therefore, while the American auto makers are facing serious financial difficulties following the path of adjustment that has so far been observed, it remains to be proved that this path was actually sub-optimal given their limited stocks of human and nonhuman capital.

These examples were intended to show the usefulness of the model of optimal technology choice for the understanding of production problems. In the next section we show how the model can be translated into econometric models of firm behavior.

ECONOMETRIC MODELS OF TECHNOLOGY CHOICE

We have seen that the problem of optimal technology choice is stated as a problem of optimal stochastic control. In the general model, the optimal sequences of the control variables (I_t) and (β_t) are functions of the model's exogenous variables. These exogenous variables may be thought of as prices and other "policy" variables. They will now be referred to as the "technology choice variables," and denoted by the vector τ. The solution of the optimal technology choice problem, therefore, shows that the technology choice of the j^{th} firm in the t^{th} production period can be written as $\beta_{jt} = \beta(\tau_{jt-1}')$. The firm also chooses the optimal input vector I_{jt-1} and hence x_{jt} as a function of prices and other exogenous variables.

The theoretical model can be translated into a useful empirical model by using these results. Since β_{jt} is the parameter vector which characterizes the production process of the j^{th} farmer, we can interpret the model of technology choice in terms of specific production function models as variable coefficients models with the parameters specified as functions of τ. A simple example illustrates the use of this model in production function specification and estimation. Let us consider the problem of specifying a farmer's production function. Suppose, as is often done, that the production function is of the Cobb-Douglas form:

$$(13) \qquad Q_{jt} = b_{0jt} X_{1jt}^{b_1} \cdots X_{njt}^{b_n} e^{u_{jt}}, \; u_{jt} \sim (0, \sigma^2)$$

In this formulation, the multiplicative term b_{0jt} represents the firm-specific production effects of management ability (see Mundlak 1961, Mundlak and Hoch 1965). It has been suggested that farmers' schooling might be a determinant of farmers' management abilities (Schultz 1964, Griliches 1964), and of course this hypothesis is embodied in the decision-making process described in the earlier sections of this paper. The discussion above also suggested that extension work may be an important factor in the transmission of technical information to farmers. We define EX_{jt} as a measure of the extension contact between extension workers and farmer j in period t. The model of optimal technology choice suggests that the farmer's accumulated technical knowledge may be a determinant of his ability to acquire and use more productive technologies. Thus we let:

$$(14) \qquad K_{jt} = \sum_{i=0}^{t} EX_{ji}\delta^{i}$$

be a measure of the farmer's accumulated technical knowledge. The technology choice variables are then $\tau_{jt} = (ED_j, K_{jt})$. Following the form of the Cobb-Douglas production function, this analysis leads to the following hypothesis.

$$(15) \qquad b_{0jt} = b_{00}\, ED_j^{b_{01}}\, K_{jt}^{b_{02}}$$

Combining (3) and (5) we have an empirical production function model amenable to test with well known estimation methods.

While this simple and familiar model is only meant to be suggestive of the way that the model of optimal technology choice can be employed in empirical work, this example shows that the model of optimal technology choice provides a theoretical foundation for the inclusion of many types of variables, such as human capital variables, policy variables, and relative prices, in production functions to measure the effects of changes in the economic constraints which determine the type of production processes firms use. Within the class of Cobb-Douglas production functions, this example can be generalized to models with not only the multiplicative constant specified as a function of the relevant technology choice variables, but also the parameters $b_1, \ldots, b_n$ which represent the input production elasticities. For example, if we redefine b_k, $k = 1, \ldots, n$, as:

$$(16) \qquad b_{kjt} = b_{k0} + \log \tau_{1jt} + \ldots + b_{km} \log \tau_{mjt}, \; k = 1, \ldots, n$$

$$b_{0jt} = b_{00} \tau_{1jt}^{b_{10}} \ldots \tau_{mjt}^{b_{m0}}$$

then combining (3) and (6), taking logarithms of the model, and employing the convention that $\log \tau_{0jt} = \log x_{0jt} = 1$, we have:

$$(17) \qquad \log Q_{jt} = \sum_{i=0}^{m} \sum_{k=0}^{n} b_{ik} \log \tau_{ijt} \log X_{kjt} + u_{jt}$$

We thus have a function which is "bi-linear" in the logarithms of x_{kjt} and τ_{ijt}. This function is somewhat similar to the translog production function of Christensen, Jorgenson, and Lau (1973) in appearance. However, this function has an entirely different interpretation, and has the property of being globally concave for given τ_{ijt}, unlike the trans-log function which is a local quadratic approximation to a general function.

Since the Cobb-Douglas function is in wide-spread use, let us make a few comments about the use of the general model (17). First, observe that this specification is based on the hypothesis that the output elasticity of each input, b_{kjt}, is a function of the technology choice variables τ_{ijt}.

While the output elasticity represents the responsiveness of the technology to a change in one input, ceteris paribus, the reader should be aware that under the expected utility maximization hypothesis made here, the b_{kjt} are not generally equated to the factor share in competitive equilibrium. Therefore, the production process need not exhibit constant returns to scale in a competitive equilibrium; nevertheless, there may be characteristics of the production process which induce constant returns to scale. Therefore, even if the b_{kjt} sum to one, they cannot be interpreted as factor shares. They can be shown to be related to the expected factor shares, and equal to them if the farmer is risk-neutral.[10/]

Using model (17) the hypothesis of constant returns to scale in production can be subjected to statistical test. Homogeneity of degree one of equation (17) can be shown to require that the following conditions hold:

$$\sum_{k=1}^{n} b_{k0} = 1, \sum_{k=1}^{n} b_{ik} = 0 \text{ for all } i$$

These constraints have important implications for the interpretation of the model's parameters. Note that the b_{ik} show how the technology choice variables (τ_{ijt}) affect the production elasticities (b_{kjt}) and thus alter the responsiveness of the technology to changes in the x_{kjt}. Thus the b_{ik} show the effect of τ_{ijt} on the relative importance of x_{kjt} in the production process. If (19) holds a change in any τ_{ijt} must increase the relative importance of at least one input and decrease that of at least one other, because for each 'i' there must be at least one positive and one negative b_{ik}. This means, for example, that more extension services provided to the farmer will alter the responsiveness of the technology to the conventional inputs x_{kjt} but it will not increase the responsiveness to all of the x_{kjt}. However, we would expect extension to increase the farmer's overall productivity. This effect can be expressed in terms of output elasticities for the technology choice variables:

$$\varepsilon_{ijt} = \frac{\partial \log \bar{Q}_{jt}}{\partial \log \tau_{ijt}} = \sum_{k=1}^{n} b_{ikt} \log x_{kjt} \text{ (recall } EQ_{jt} = \bar{Q}_{jt})$$

We can also compute the marginal productivity of each conventional input, and the effect of the τ_{ijt} on it. The expected marginal product of x_{kj} is expressed as:

$$MP_{kjt} = \frac{\bar{Q}_{jt}}{x_{kjt}} \sum_{i=1}^{m} b_{ik} \log \tau_{ijt}$$

The following expression represents the effect of τ_{ijt} on MP_{kjt}:

$$\varepsilon_{ikjt} = \frac{\partial^2 \bar{Q}_j}{\partial x_{kj} \partial \tau_{ij}} \frac{x_{kj} \tau_{ij}}{\bar{Q}_j} = b_{ik} + b_{kjt} \varepsilon_{ijt}$$

These relationships can be utilized to interpret the parameter estimates of model (17) in terms of the implied responsiveness of production to changes in the technology choice variables.

In concluding it should be emphasized that any functional form can be adapted to this variable coefficients framework to study the effects of changes in the technology choice variables on firms' productivity. The Cobb-Douglas production function is attractive because of the simple linear-in-parameters property, but other nonlinear functional forms can be handled with the nonlinear optimization techniques that are now generally available.

CONCLUSION

The model of firm behavior presented in this paper is based on the observation that in an environment of technical change, firms are actually faced with the problem of choosing a production method or technology from many that are available to them. The definition of a technology as a method by which firms productively combine their resources emphasizes the complimentarity of human and nonhuman inputs in the production process. We have emphasized the role of decision-makers' abilities to acquire and utilize technical information to illustrate how the role of human capital in production can be analyzed with the model of optimal technology choice. One contribution of this model is the theoretical foundation it provides for the use of production models to study the effects of changes in economic constraints on the types of production processes firms use.

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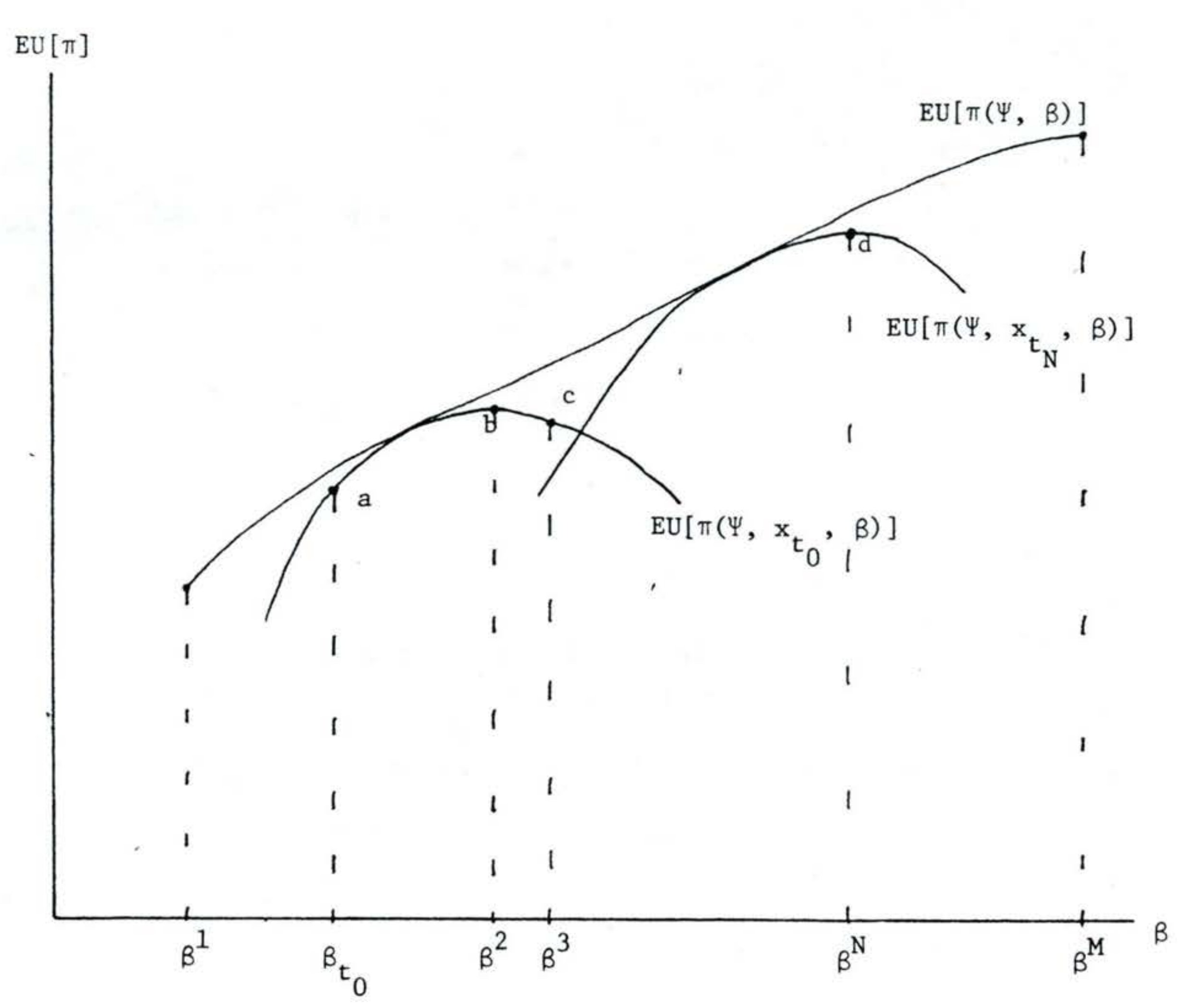

Figure 1

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FOOTNOTES

1/ Adjustment costs may be defined as separable from the production process, as is done here, or may be incorporated into the production function. See Lucas (1967), Treadway (1969), and Mortenson (1973).

2/ Formally, for a concave function U(•), if EX = EY, X is riskier than Y if EU(X) < EU(Y).

3/ In this study education represents a measure of knowledge and skills acquired through formal schooling. Technical knowledge is one specific component of the decision-maker's total stock of knowledge. The initial stock of technical knowledge K_0 may, therefore, be a function of both education and experience.

4/ The work of Ram and Schultz (1978) suggests that health capital is also an important part of productive human capital. This too could enter the specification of the decision-making process.

5/ This discussion is based on the assumption that the market structure is reasonably competitive so that information flows depend primarily on the factors described. However, it should be noted that governmental intervention or monopolistic elements may seriously impede the flow of information and require that the model be altered to account for these facts.

6/ The concept of learning-by-doing is discussed by Haavelmo (1954), Kaldor (1957), and Arrow (1962), in the context of economic growth. It is also used in the literature on technological diffusion, surveyed by Davies (1979).

7/ Welch (1979) also hypothesizes that an information-related scale effect may exist.

8/ Grossman (1975) provides an instructive analysis of market equilibrium in the context of firms with rational expectations.

9/ Davies (1979) provides a useful discussion of the effect of information diffusion on technological diffusion.

10/ Under fairly general conditions, $\frac{\partial EU(\pi_t)}{\partial x_{kjt}} = E\left[\frac{\partial U_t \partial \pi_t}{\partial \pi_t \partial x_{kjt}}\right]$, which from equation (10) implies: $b_{kjt} = \frac{E[\frac{\partial U_t}{\partial \pi_t} \; r_t x_{Kjt}].}{E[\frac{\partial U_t}{\partial \pi_t} \; P_t Q_{jt}].}$