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HOMOGENEITY AND NON-NESTED DYNAMIC SINGLE EQUATION DEMAND MODELS

by

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Introduction

An issue facing empirical analysts of demand relations concerns whether to deflate price and income data. Those using complete demand systems often do not deflate the data but impose (or test) homogeneity through parametric restrictions. Homogeneity implies that real or relative prices and income are the relevant independent variables in the demand model. For the single commodity study, the choice is more apparent. A survey of the literature shows such relations modeled in both real and nominal prices and income, but "real" or "deflated" models appear more frequently.[1/] The abundance of studies using real prices and incomes is likely explained from the mircoeconomic justification of this procedure based upon the derived homogeneity (of degree zero) property.

Though few would argue about the plausibility of this property, it is rejected with regularity using aggregate systems approaches (Barten). The relevant question facing the researcher studying a single commodity is "Which of the deflated or the nominal models perform best in some sense?"[2/] The answer to such questions may be based upon t-ratios, goodness of fit, or some predictive ability. Yet for most problems, the structural content of the model is of prime interest and one is impressed that the above criteria may not be satisfactory. Since in the absence of homogeneity, one cannot generally derive deflated models as nested versions of a nominal one, the recent developments in non-nested hypothesis testing seem to be relevant.

In this paper, such tests are performed on a time series of consumption data often known as the meat group which has been studied by Christensen and Manser, Brown and Heien and others. The group includes prominent agricultural commodities: beef, pork, poultry and fish.

Conditional on the stock of habits, homogeneity of demand functions is often rejected. Using non-nested hypotheses testing procedures, the nominal model of consumption is generally rejected in favor of a deflated model even though homogeneity may be unwarranted. This gives support to the common practice of specifying deflated models.

The Consumption Model

Aside from functional form, two important challenges involve the choice of a dynamic representation of changing tastes and the appropriate way to enter "all other prices" into the demand relation. In this study, the state adjustment characterization of Houthakker and Taylor is used. In unrestricted reduced form, it has the advantage of having the partial adjustment and static models nested within it and thus one can easily test whether a representation more simple than the state adjustment is appropriate.

There appears in the literature numerous approaches for including the prices of "all other goods" where these are all goods which are not close substitutes or complements but should in theory enter the demand relation.[3/] In a majority of cases, demand is assumed to be a function of a price index (the deflator) and is assumed to be homogeneous. Thus by dividing all included prices by the index, relative (and real) prices are formed.

Though avoiding the homogeneity assumption, it is invariably not possible to include the index as a separate independent variable due to collinearity between the index and included prices, and income. A pragmatic alternative was proposed by Stone. Demand is assumed to be homogeneous of degree zero in real income. This gives the basic nominal structural demand equation used in this study

$$(1) \quad q_{it} = f(Y_t/P^*_t \equiv Y^*_t;\ P_{1t} \ldots P_{Nt};\ S_t)$$

$$i = 1 \ldots N;\ t = 1 \ldots T,$$

where q_i is per capita consumption of good i, Y is per capita disposable income, P* is the price index (CPI), P_i is the ith price, and S_t is the stock of habits at time t. Homogeneity, conditional on S_t, implies that

$$(2) \quad \sum_j \eta_{ij} = 0 \quad \text{where } \eta_{ij} = (\partial \ln q_i/\partial \ln P_j).$$

The "deflated" model corresponding to (1) is customarily of the form

$$(3) \quad q_{it} = \tilde{f}(Y^*_t,\ P_{1t}/P^*_t \equiv P^*_{1t},\ \ldots\ P_{Nt}/P^*_t \equiv P^*_{Nt};\ S_t)$$

and homogeneity in Y_t, $P_{1t} \ldots P_{Nt}$, P^*_t is maintained.[4/]

A remaining issue regarding the specification of (1) and (3) is the choice of functional form. Single commodity relations can be modeled with great flexibility of functional form (Chang). Yet, the most commonly used functional forms are the linear and log-linear - chosen presumably for convenience and ease of interpretation and estimation. Since the tests here would have most relevance using commonly used forms, the model tests will be conducted conditional on each of these two forms.[5/]

Using generally the state adjustment scheme, the unrestricted linear reduced form analogous to the nominal model, (1), is

$$(4)\quad q_{it} = A_o^i + A_q^i q_{it-1} + \sum_{j=1}^{N} A_j^i P_{jt} + A_y^i Y_t^* + \sum_j \bar{A}_j^i P_{jt-1} + \bar{A}_y^i Y_{t-1}^*,$$

$$= 1 \ldots N;\ t = 1 \ldots T,$$

or in matrix form,

$$(4')\quad \underset{Tx1}{q_i} = \underset{(Tx2N+4)}{X_i} \quad \underset{(2N+4x1)}{A_i}, \qquad i = 1 \ldots N,$$

where $A_i' = (a_0, a_q, a_1 \ldots a_N, a_y, \bar{a}_1 \ldots \bar{a}_N, \bar{a}_y)$ and X_i is the associated observation matrix. The corresponding linear nominal model is

$$(5)\quad q_{it} = B_o^i + B_q^i q_{it-1} + \sum_j B_j^i P_{jt}^* + B_y^i Y_t^* + \sum_{j=1}^{N} \bar{B}_j^i P_{jt-1}^* + \bar{B}_y^i Y_{t-1}^*,$$

Or analogous to (4'),

$$(5')\quad q_i = Z_i B_i,\ i = 1 \ldots N.$$

The double log form replaces each element of (4') and (5') of X and Z with its logarithm. Equations (4') and (5') shall be referred to as the state adjustment model but no attempt is made to impose the structure of the state adjustment model which customarily is written with first differences serving as independent variables in place of lagged values. The state adjustment structure is not convenient to impose (Lin) and for our purposes there is no need to do so. In the next section, procedures to test econometric models of (4) and (5) are discussed.

Non-nested Hypothesis Testing

Pesaran has developed a test statistic based upon the work of Cox. Adding random errors to (4') and (5') and maintaining (5') under the null hypothesis gives

(6) $H_o : q_i = Z_i B_i + \varepsilon_{oi}$

$$H_a : q_i = X_i A_i + \varepsilon_{ai}$$

where it is assumed that $\varepsilon_{oi} \sim N(o, \sigma_o^2 I_T)$, $\varepsilon_{ai} \sim N(0, \sigma_a^2 I_T)$, and $E(\varepsilon_{oit}\, \varepsilon_{ojt'}) = 0$ for all t, t' and $i \neq j$. The assumptions needed here due to the presence of a lagged dependent variable are plim $(1/T)\, X_i'\, X_i = \sum_X^i$, plim $(1/T)\, Z_i'\, Z = \sum_Z^i$, plim $(1/T)\, Z_i'\, X_i = \sum_{ZX}^i$, where each $\sum$ matrix is nonsingular. Further, it is assumed here that plim $(1/T)\, X_i'\, \varepsilon_{oi}$ = plim $(1/T)\, Z_i'\, \varepsilon_{ai}$ = plim $(1/T)\, Z_i'\, \varepsilon_{oi}$ = plim $(1/T)\, X_i'\, \varepsilon_{ai} = 0$. Given these assumptions, the results of Pesaran and Cox follow. That is

(7) $TC_{oi} = L_{oi}\, (\hat{B}_i) - L_{ai}\, (\hat{A}_i) - E_o\, \{L_{oi}\, (\hat{B}_i) - L_{ai}\, (\hat{A}_i)\}$

is normally distributed with mean zero and variance $V(TC_{oi})$ under H_o, where L_o and L_a are the maximized likelihood function under H_o and H_a respectively, $\hat{A}_i$ and $\hat{B}_i$ are the maximum likelihood estimates under H_a and H_o respectively and E_o is the expectation under H_o.

A consistent estimate of TC_{oi} is found to be

$$T_{oi} = \frac{T}{2}\, \frac{\hat{\sigma}_{ai}^2}{\hat{\sigma}_{oai}^2}$$

where $\hat{\sigma}_{ai}^2 = \frac{1}{T} \cdot (q_i - X_i\, \hat{A}_i)'(q_i - X_i\, \hat{A}_i) \equiv \frac{1}{T}\, e_{ai}'\, e_{ai}$

and $\hat{\sigma}_{oai}^2 = \hat{\sigma}_{oi}^2 + \frac{1}{T}\, e_{oai}'\, e_{oai}$ and where $e_{oai}'\, e_{oai}$ is the sum of squared residuals upon regressing $Z_i \hat{B}_i$ on X_i.[6/] The estimate of the variance of

T_o is found to be

$$\hat{V}(T_{oi}) = \frac{\hat{\sigma}_{oi}^{2}}{\hat{\sigma}_{oai}^{4}} \; \hat{\sigma}_{oa,oi}^{2}$$

where $\hat{\sigma}_{oa,oi}^{2}$ is T times the sum of squared residuals upon regressing e_{oai} on Z_i. Thus the test statistic is

$$(8) \quad N_{oi} = \frac{T_{oi}}{\left[\hat{V}(T_{oi})\right]^{\frac{1}{2}}}$$

and large negative values of N_{oi} are consistent with rejection of H_{oi} (the deflated model) in favor of H_{ai} (the nominal model).

By reversing the roles of H_{oi} and H_{ai}, one can likewise test H_{oi} maintaining H_{ai} is true and it may be that both models are rejected or accepted.

Recently Dastoor, using the Pesaran-Cox framework and an estimate of $\hat{A}_i$ developed by Atkinson, has analyzed a slightly different statistic with identical asymptotic properties to N_{oi}.[7/] Denoting this statistic by NA_{oi}, Dastoor shows that $NA_{oi} \geq N_{oi}$. Thus NA_{oi} and N_{oi} may yield different results with a stronger tendency for N_{oi} to reject H_{oi} in favor of H_{ai} when they are negative. For comparison purposes, tests based upon both N_{oi} and NA_{oi} will be reported.

Empirical Application

The data used to obtain parameter estimates and to perform the tests are U.S. time series observations on beef, pork, poultry and fish from 1950 to 1975. Variables used in (4) and (5) are per capita food consumption in retail weight equivalents (1970 base), per capita income deflated by the consumer price index (CPI), and nominal price indexes for the commodity groups. For the deflated model, each nominal price index is divided by the CPI.

Though tests are to be conducted on all four commodities, it would be cumbersome to report regression results for all of them. Hence, the OLS regression results for beef and poultry only are presented in Table 1 and later tables contain non-nested test results for all four commodities. Examination of Table 1 indicates the signs of coefficients are consistent with apriori expectations. Also the results of the Table reveals that the models do appear to give very different results in terms of coefficient values and t tests. For example, lagged beef quantity has both higher coefficients and t statistics in the nominal model. Note also that Durbin h tests do not generally indicate the presence of autocorrelation as presumed in the assumptions given earlier.[8/]

The restrictions implied by the partial adjustment model (H_o: $a_j = \overline{a}_y = \overline{b}_j = \overline{b}_y = 0$ for all j; H_a: H_o untrue) gave the following F statistics: beef, $F_1 = 5.798$, $F_2 = 8.357$; Pork, $F_1 = 10.01$, $F_2 = 4.97$; Poultry, $F_1 = 10.4$, $F_2 = 11.31$; Fish, $F_1 = 5.69$, $F_2 = 6.73$, where F_1 is the F statistic under H_o given the double-log model and F_2 is the F statistic for the linear model. Though there are numerous coefficients on lagged values which have low t values (particularly in the deflated cases), a comparison of the F statistics with critical values at the 1% level ($F_{.01}(5,13) = 4.86$) implies rejection of the partial adjustment model in favor of the state adjustment model. Hence, the latter model is maintained throughout for the remainder of the analysis.

Applying (2) to the nominal model, given lagged values, homogeneity may be tested. The test statistics for global homogeneity in the double-log and local (at mean prices) homogeneity in the linear form gave:[9/] beef, F_1 = 7.38, F_2 = 40.37; pork, F_1 = 1.25, F_2 = 8.13; poultry, F_1 = .16, F_2 = 31.68; fish, F_1 = .42, F_2 = .58. Comparing these statistics with $F_{.05}(1,13) = 4.67$ indicates rejection of H_o (homogeneity as given in (2)) in favor of H_a (H_o untrue) for the beef-log and linear cases, the pork-linear case and poultry-linear models. Thus, the hypothesis of homogeneity in the model does not receive support across all commodities but fares well in the double-log model.

Thus, there are two reasons indicating the need for the non-nested tests of (1) versus (3). For cases where homogeneity is unwarranted, it is unclear that any deflated model is perferred to a similar nominal model. Secondly, as customarily practiced, nominal models do not include the price index (numeraire price) as a separate independent variable.[10/] Thus, (3), the customary deflated model, cannot be obtained as a nested version of the nominal model, (1).

Turning now to the non-nested tests. In each case, the functional form is maintained and the nominal and deflated models are compared. Table 2 presents the non-nested results for the linear model. The subscripts on NA_o and N_o refer to the case where the nominal model is the null hypothesis and NA_1 and N_1 refer to the statistics calculated when the deflated model is the null hypothesis.

Comparison of the tests statistics with a standard normal table at the 5% level ($Z = 1.96$) indicates the following results:

Commodity	Pesaran (N) Nominal	Pesaran (N) Deflated	Dastoor (NA) Nominal	Dastoor (NA) Deflated
Beef	reject	accept	reject	reject
Pork	reject	accept	reject	accept
Poultry	accept	accept	accept	accept
Fish	accept	accept	accept	accept

Here, it is noted since NA > N, the null hypothesis is rejected less often in favor of the alternative using Dastoor's statistic. However, both tests give mixed results. For the beef and pork models, the nominal model is not indicated in any case. With the exception of beef and using the Dastoor statistic, the deflated model is upheld. It is also interesting to note that in most cases R^2 is higher for the nominal models even though the nominal model is generally rejected by the tests.

Table 3 presents similar results for the double log form. Comparison of test statistics with $Z(.05) = 1.96$ gives

Commodity	Pesaran Nominal	Pesaran Deflated	Dastoor Nominal	Dastoor Deflated
Beef	reject	accept	accept	reject
Pork	reject	accept	reject	reject
Poultry	reject	accept	reject	accept
Fish	accept	accept	accept	accept

Again the results based upon NA and N yield different results. Using Pesaran's statistic the "truth of the nominal model" against the deflated model is rejected for all commodities but fish. The nominal fish model is, however, rejected at the 10% level. With equal consistency, the Pesaran statistic suggests acceptance of the deflated model. Again, the Dastoor statistic gives more mixed results with the deflated model accepted for poultry and fish and the nominal model accepted for beef and fish.

Summary

To summarize the above results, the common practice of deflating data is generally supported by the data and models employed here. Only in a few cases, was there support for the nominal model over the deflated model. In such cases, the statistic explored by Dastoor and the statistic developed Pesaran give conflicting results. Based upon the Pesaran statistic, the deflated model has a significantly improved likelihood function over the nominal model and confirms the deflation practice even when homogeneity was rejected. This is in sharp contrast to the practice of choosing the model with the highest R^2 (generally, the nominal model).

TABLE 1

Maximum Likelihood Estimates of the Parameters for the State Adj. Demand Equation, 1950-1975

Commodity-Model	Explanatory variables: Const.	Beef P.	Pork P.	Poultry P.	Fish P.	Income	q_{i-1}	Beef P_{-1}	Pork P_{-1}	Poultry P_{-1}	Fish P_{-1}	$Income_{-1}$	R^2	Durbin h
1. BEEF														
a. linear-	-7.936	-0.693	0.035	0.019	0.165	0.354	0.726	0.656	-0.098	0.068	-0.203	0.082	.993	N.A.[a]
nominal	(0.39)[b]	(4.85)	(0.56)	(0.19)	(0.65)	(1.22)	(3.01)	(3.39)	(1.42)	(0.69)	(1.04)	(0.24)		
b. double log-	-0.248	-0.789	0.064	0.027	0.064	0.253	0.655	0.666	-0.058	0.036	-0.114	0.253	.990	N.A.
nominal	(0.22)	(4.41)	(0.65)	(0.22)	(0.17)	(0.66)	(2.34)	(2.91)	(0.61)	(0.28)	(0.39)	(0.58)		
c. linear-	121.461	-86.213	.309	6.583	23.87	37.238	-.232	-6.541	-2.113	-.748	-20.066	49.626	.994	N.A.
deflated	(4.57)	(12.39)	(.06)	(.92)	(1.63)	(1.93)	(.89)	(.33)	(.43)	(.119)	(1.48)	(2.03)		
d. double log-	4.956	-4.757	.005	.541	2.660	1.09	-.173	.324	.049	-.300	-2.04	2.98	.989	N.A.
deflated	(3.91)	(10.24)	(.02)	(1.22)	(2.55)	(.82)	(.64)	(.311)	(1.44)	(.727)	(2.18)	(1.88)		
2. POULTRY														
a. linear-	3.398	0.321	0.190	-0.580	-0.115	0.452	0.580	-0.051	-0.122	0.413	-0.132	0.028	.998	1.526
nominal	(0.30)	(2.66)	(2.90)	(5.84)	(0.56)	(1.54)	(3.02)	(0.44)	(1.66)	(3.06)	(0.68)	(0.08)		
b. double log-	1.046	0.271	0.229	-0.596	0.191	0.599	0.272	0.034	-0.085	0.237	-0.415	0.039	.999	-3.194
nominal	(2.23)	(3.49)	(4.41)	(9.50)	(1.16)	(3.00)	(1.57)	(0.39)	(1.22)	(1.89)	(2.68)	(0.15)		
c. linear-	13.306	32.747	13.21	-56.568	-8.053	-.801	.570	.382	-9.844	44.33	-35.554	-.801	.996	-.185
deflated	(1.17)	(3.56)	(1.70)	(5.12)	(.38)	(.03)	(2.92)	(.04)	(1.31)	(3.46)	(1.64)	(.03)		
d. double log-	1.017	1.464	.941	-2.634	.726	2.618	.242	.117	-.429	1.226	2.064	.525	.998	.937
deflated	(2.16)	(5.62)	(3.92)	(9.15)	(1.18)	(3.12)	(1.42)	(.322)	(1.49)	(2.26)	(3.27)	(.494)		

a/ The resulting statistic involved complex roots.

b/ Absolute values of t statistics are in parentheses.

Table 2

Nonnested Tests on the Linear Model

Commodity	N_o [a/]	NA_o [b/]	N_1 [c/]	NA_1	R^2 Nominal	R^2 Deflated
beef	-7.53	-1.96	1.54	3.1	.993	.994
pork	-3.42	-2.46	-.88	-.77	.974	.964
poultry	-.633	-.52	-1.67	-1.37	.998	.996
fish	-.91	-.58	-1.53	-1.12	.952	.912

a/ Subscript o refers to the case where H_o is the nominal model and the deflated model is the alternative. N is the statistic developed by Pesaran.

b/ NA is the Atkinson-Dastoor statistic.

c/ Subscript 1 refers to the case where H_o is the deflated model and the alternative hypothesis is the nominal model.

Table 3

Nonnested Tests on the Double-Log Model

Commodity	N_o [a]	NA_o [b]	N_1 [c]	NA_1	R^2 Nominal	R^2 Deflated
beef	-7.590	-.34	1.390	3.460	.990	.989
pork	1.610	2.10	-3.33	-2.33	.970	.938
poultry	-2.520	-2.08	.418	.480	.999	.998
fish	-1.770	-1.410	-.612	-.447	.935	.890

a/ Subscript o refers to the case where H_o is the nominal model and the deflated model is the alternative. N is the statistic developed by Pesaran.

b/ NA is the Atkinson-Dastoor statistic.

c/ Subscript 1 refers to the case where H_o is the deflated model and the alternative hypothesis is the nominal model

Footnotes

1/ For examples of deflated models, see Chang, Chow, Houthakker and Taylor, Johnson and Okanen. Examples of apparently nominal models are Dahl, Nelson, Hamilton, Heien, and Hassan, et. al., Green et. al.

2/ Clearly, the performance criterion differs somewhat by purpose, e.g., prediction versus a structural test.

3/ Occasionally an implicit deflator and an index of the price of "all other goods" appear together in the same model. However, the practice appears to be very rare.

4/ As earlier, P^*_t is assumed to be linearly homogeneous. Simultaneity between per capita consumption and price are not generally presumed (Chang).

5/ That is, two regimes are considered separately, the linear and double-log forms. The truth of these forms are not tested against one another. Such a test could be accomplished in the general framework of Cox but not in the Pesaran nonnested framework since it requires the dependent variable to be the same in both models.

6/ In each case, the ordinary least squares and maximum likelihood estimators of mean regression parameters (e.g., A_i and B_i) and the corresponding residuals are identical given our assumptions.

7/ In the Atkinson approach A_i is replaced by $\text{plim}_o A_i$ where plim_o is the probability limit under H_o. The new estimate, TA^o_{oi}, which is analogous to T_{oi}, is

$$TA_o = \frac{T}{2}\left(\frac{\hat{\sigma}^2_{ai}}{\hat{\sigma}^2_{oai}} - 1\right) + \frac{e'_{oi} P_{Xi} e_{oi}}{2\hat{\sigma}^2_{oai}}$$

where $e'_{oi} P_{Xi} e_{oi} = e'_{oi} e_{oi} - e'_{oi} M_{Xi} e_{oi}$ and the latter term is the sum of squared residuals from regressing e_{oi} on X_i. The asymtotic variances of T_{oi} and TA_{oi} are identical.

8/ A noted exception is the double-log nominal poultry model. It is also acknowledged that the Durbin h may have low power in finite samples (Kenkel). The Durbin-Watson statistic also did not suggest autocorrelation.

9/ The model with income undeflated but with P^*_t (all other prices) omitted gave similar results.

10/ That is, $q_{it} = f(Y_t; P_{it} \ldots P_{Nt}; P^*_t; S_t)$ is not generally estimated due to collinearity problems (and it would rarely yield a coefficient on P^*_t which was significantly different from zero).

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