A Note on First-Price Sealed-Bid Cattle Auctions in the Presence of Captive Supplies

John M. Crespi and Tian Xia

The authors present an analytical model of a first-price sealed-bid cattle auction in which a spot and coordinated markets are interconnected. The model reveals that the conventional wisdom that market coordination negatively affects the bid price in the spot market is an oversimplification. The relationships between key market variables impact bids and bid shading in complex ways. While captive supplies can lead to lower spot prices, the price reductions do not necessarily stem from an increase in market power due to contracting. The model emphasizes the importance of several variables for future empirical studies.

Key Words: auction, captive supplies, cattle markets, contracting

Crespi and Sexton (2004, 2005) argued that U.S. spot markets for cattle ought to be contemplated in the purview of a first-price sealed-bid auction because of how such cattle are sold. Generally, packers submit spot market bids to feedlots without knowing the amount of the bids made by other packers, and the highest bidder obtains the lot. Crespi and Sexton also argued that important idiosyncrasies in these markets make analytical models of bidding behavior difficult to derive. One impediment to the analysis is the existence of non-spot-market cattle obtained by packers via contracting and other forms of vertical coordination. The terms contracting, captive supplies, and packer ownership are not entirely interchangeable, but captive supplies is used by producers as a catch-all term for any coordination that creates a separate, private market between a packer and a supplier. This is how we shall use the term captive supplies as well.

There is no dispute among producers or economists that such non-spot markets influence the spot market for fed cattle. A relatively traditional oligopsony model of two conjoined markets has been derived (Xia and Sexton 2004), but an analytical model of bidding in a first-price sealed-bid auction in the presence of captive supplies has not. Rather than developing an analytical model, Crespi and Sexton (2004, 2005) focused on structural simulations of equilibrium bidding and empirical estimations of a probabilistic supply function. Thus, there is a significant gap in this literature, and incorporation of captive supplies is essential since it has a significant effect on returns for cattle producers and beef processors. Evidence of the grave importance of captive supplies and contracting for spot markets is found in numerous

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comments by producers and some economists in public hearings in 2010 on livestock market power conducted by the U.S. Departments of Agriculture (USDA) and Justice in Fort Collins, Colorado, and Washington, D.C. (U.S. Department of Justice 2010).

The conventional story for the impact of the secondary market on spot markets has been that bids in the spot market are driven downward by the presence of a separate packer-owned inventory (see discussion and references in Crespi, Saitone, and Sexton (2012)). The economic story that fits this view is simple enough. A cattle buyer will bid less in the spot market when its demand for cattle is split between the spot market and a contract market. The presence of the contract market shifts the buyer's valuation of the marginal product so that it intersects the marginal acquisition curve at a lower point. Rogers and Sexton (1994) warned the profession against the misuse of simple theorems in not-so-simple agricultural markets. While the type of textbook shift just described may in fact occur, we demonstrate in this brief note that the relationship is not so straightforward when the spot market is best described as an auction. We present an analytical model of first-price sealed-bid strategies in the presence of captive supplies that can be used as the basis for future research of such markets.

The equilibrium bidding strategy for a first-price sealed-bid auction without incorporation of captive supplies is well known. It was first described by Vickrey (1961) and later expanded by Riley and Samuelson (1981) into its textbook form (see, for example, Krishna (2002) and Menezes and Monteiro (2005)). The claim of producers is that captive supplies lower the amounts of bids made in the spot market. This seems like common sense, but we find that the mechanism is more complicated. Consequently, a cursory understanding of how to adapt the auction framework to incorporate captive supplies, which has not been done for cattle auctions, is important for the present discussion.

The Model

In our model, M packers obtain a supply of cattle from contracts and purchase cattle in a spot market via a competitive process in which bids are submitted as private information to a seller, which chooses the highest bid. The spot-market supply for a given packer is stochastic; it depends on how many cattle are left unpurchased through the contract markets and on the outcome of bids placed in the spot auction.

Following the first-price sealed-bid approach, we choose a format in which bidders know the distribution function of each other's values but draw their own values independently (independent private values). We do not incorporate learning, updating of information, or correlation in the valuations and do not allow updating of the bids, which does occur in some livestock auctions. We assume that the buyers (packers) are risk-neutral. Because only the spot market is stochastic over the bids, the supply of spot market cattle, q^s, is taken as the residual from the contracted supply. With total supply set to 1, the spot market supply is represented by $q^s = 1 - q^c$ with s and c denoting spot and contract supplies.

Let R_i denote the independent private valuation that packer i has for cattle for any $R \geq 0$. One could consider R_i as packer i's valuation of its marginal product of the cattle. It could be that the boxed-beef price less the packer-specific marketing cost varies by packer. Or the output price or marketing cost alone could vary by packer. Regardless, when the cost and/or price vary by packer, our assumption is that it is due to firm-specific production features or otherwise differentiated outputs. This valuation is the key to the equilibrium bidding strategies of the packers in the model. Each packer knows its value of R and the cumulative distribution for any value less than or equal to some number, x, which is denoted as $F(x)$, and the density of the distribution, $f(x) = F'(x)$. We are interested in identifying the symmetric Bayesian-Nash equilibrium bidding strategy, $p(R)$: the strategy that the packer believes all of the other packers in the market are following and for which it must determine the best response. Because all of the packers follow the same strategy, we begin by examining the perspective of a single packer.

Consider packer $i = 1$. If packer 1 bids p_1 in the spot market, it procures the cattle only if $p_1 > \max\{p_2, \ldots, p_M\}$ and receives no cattle otherwise. In the case of a tie, we assume there is some method for determining the winner, such as a coin flip or that the cattle go unsold into another round; we focus only on rounds in which an unequivocal sale is made. Let packer 1 have a valuation for the cattle of $R = R_1$ with similar notation for packers 2 through M such that the expected profit for packer 1 is

$$(1)\quad \pi(p_1) = \pi(R, p_1, p(R)) = (R - p_1)\,\mathrm{Prob}(p_1 > \max\{p(R_2), \ldots, p(R_M)\})q^s + (R - p_1)q^c = (R - p_1)\,\mathrm{Prob}(p_1 > p(R_2), \ldots, p_1 > p(R_M))q^s + (R - p_1)q^c.$$

Supply in the spot market is the residual of total supply minus the contract supply, which makes the contract supply a function of the spot bid: $q^c \equiv q^c(p_1)$ and $q^s = 1 - q^c(p_1)$.

When $p_1 \in [p_l, p_h]$ and $x \in [0, R_h]$ where l and h denote the lowest and highest prices/valuations and the bidding function is strictly increasing and differentiable, then some $p_1 = p(x)$ must exist. Therefore, instead of choosing the optimal bid price (as in a Bertrand model), we can instead maximize expected profit in terms of the bidders' valuations. The expected profit is

$$(2)\quad \hat{\pi}(x) = \pi(p(x)) = (R - p(x))\mathrm{Prob}(p(x) > p(R_2), \ldots, p(x) > p(R_M))q^s + (R - p(x))q^c.$$

Because the bids are increasing in x and all players follow the same strategy in equilibrium, we can rewrite equation 2 as

$$\hat{\pi}(x) = (R - p(x))\,\mathrm{Prob}(x > R_2, \ldots, x > R_M)q^s + (R - p(x))q^c.$$

Finally, independent private-value auctions assume that all bids are independent and identically distributed. This convention allows us to rewrite the expected profit function yet again as

$$(3)\quad \hat{\pi}(x) = (R - p(x))\,F(x)^{M-1}q^s + (R - p(x))q^c.$$

Deriving the first-order condition of equation 3 yields

$$(4)\quad \hat{\pi}'(x) = (R - p(x))(M - 1)f(x)F(x)^{M-2}q^s - p'(x)F(x)^{M-1}q^s - q^{c\prime}(\cdot)p'(x)(R - p(x))F(x)^{M-1} - p'(x)q^c + (R - p(x))q^{c\prime}(\cdot)p'(x) = 0.$$

Because of the monotonic transformation from expected profit in equation 1 to the profit function in terms of valuations in equation 2 and because bid prices are increasing and continuously differentiable in x, any x that satisfies the first-order condition of equation 4 will correspond with a value of R that likewise maximizes equation 3 in a symmetric equilibrium: $\hat{\pi}'(x=R)=0$. Using this, we rearrange equation 4 into

$$(5)\qquad p'(R)F(R)^{M-1}q^s + p'(R)q^c = (R - p(R))(M-1)f(R)\,F(R)^{M-2}q^s + (R - p(R))q^{c\prime}(\cdot)p'(R) - q^{c\prime}(\cdot)p'(R)(R - p(R))F(R)^{M-1}.$$

To simplify the exposition, let $\Theta = p(R)F(R)^{M-1}q^s + p(R)q^c$. Next, we differentiate Θ with respect to the underlying packer valuation as in equation 6:

$$(6)\qquad d\Theta / dR = p'(R)F(R)^{M-1}q^s + p'(R)q^c + (M-1)p(R)f(R)F(R)^{M-2}q^s - p(R)F(R)^{M-1}q^{c\prime}(\cdot)p'(R) + p(R)q^{c\prime}(\cdot)p'(R).$$

Combining equations 5 and 6 yields

$$d\Theta / dR = (M-1)p(R)F(R)^{M-2}f(R)q^s - p(R)F(R)^{M-1}q^{c\prime}(\cdot)p'(R) + p(R)q^{c\prime}(\cdot)p'(R) + (R - p(R))(M-1)f(R)F(R)^{M-2}q^s + (R - p(R))q^{c\prime}(\cdot)p'(R) - q^{c\prime}(\cdot)p'(R)(R - p(R))F(R)^{M-1}.$$

After cancelling like terms and rearranging, we derive equation 7.

$$(7)\qquad d\Theta / dR = R[M-1]f(R)F(R)^{M-2}q^s + Rq^{c\prime}(\cdot)p'(R)[1 - F(R)^{M-1}]$$

According to the fundamental theorem of calculus, if u is continuous on $[a,b]$, the function v defined by $v(x) = \int_a^x u(t)dt$ is continuous on $[a,b]$ and differentiable on (a,b) and $v'(x) = u(x)$. Using this result yields

$$p(R)F(R)^{M-1}q^s + p(R)q^c = \int_0^R x[M-1]f(x)F(x)^{M-2}q^s + xq^{c\prime}(x)p'(x)[1 - F(x)^{M-1}]dx$$

and the equilibrium bidding strategy in the presence of contracted cattle is given by the implicit relationship defined in equation 8:

$$(8)\qquad p(R) = \frac{\int_0^R x(M-1)[f(x)F(x)^{M-2}q^s]dx + \int_0^R xq^{c\prime}(x)p'(x)[1 - F(x)^{M-1}]dx}{(1 - q^c)F(R)^{M-1} + q^c}.$$

Though implicitly defined, equation 8 identifies the necessary components of the bid. For example, a researcher who wants to consider the empirical impact of captive supplies on spot prices must include (at a minimum) the number of packers, M; some proxy for valuation, R, such as the boxed-beef price; the quantity of cattle comprising the spot (q^s) and captive (q^c) supplies;

the likelihood and distribution of packer wins, $f(\cdot)$ and $F(\cdot)$; and the slope of the captive supply curve, $q^{c\prime}(\cdot)$.

An informal proof of the mathematical procedure is easy to demonstrate. Consider a case in which there is no contract market so that $q^c = 0$ and $q^s = 1$. In that case, equation 8 becomes the well-known equilibrium bid price for a model of an independent private-value first-price sealed-bid auction (Riley and Samuelson 1981):

$$p(R) = \frac{\int_0^R x[M-1][f(x)F(x)^{M-2}]dx}{F(R)^{M-1}}.$$

Discussion

Because we have derived the bid function of equation 8 in terms of the underlying value, R, we cannot further solve the strategy without additional information about the auction's density function and the contract-supply curve. Such information is obtainable. All that would be needed to solve equation 8 either explicitly or numerically is the form for both of those functions or reasonable assumptions. Therefore, equation 8 provides a basis for a fuller investigation of the impact of captive supplies on spot market bids. And even without such additional information, we can discern some of the impacts of the presence of captive supplies.

We find that the presence of contracts can lower the equilibrium bid price. The numerator in equation 8 becomes smaller and the denominator becomes larger as the contract quantity, q^c, increases ($F(\cdot) \leq 1$) when assuming that the supply curve for captive supplies is non-negative ($q^{c\prime}(\cdot) \geq 0$) and the elasticity is more or less constant. These are not unreasonable assumptions.

What about the hypotheses that captive supplies increase packers' market power? In this case, the appearance of contract supplies in both the numerator and the denominator identifies an intricacy that goes beyond the notion that captive supplies simply lead to fewer cattle in the spot market. A closer examination of equation 8 shows that the contracts are tied in less obvious ways with the "normal" market power that oligopsony bidders wield in the spot market.

To elucidate this relationship, we use integration by parts in which

$$\int_a^b u\,dz = uz\Big|_a^b - \int_a^b z\,du$$

to further explore equation 8. Letting

$$dz = (M-1)f(x)F(x)^{M-2}(1-q^c)dx,$$

$z = F(x)^{M-1}(1-q^c)$, $du = dx$, and $u = x$ yields equation 9.

$$\begin{aligned}(9)\qquad &\int_0^R x(M-1)f(x)F(x)^{M-2}(1-q^c)dx\\ &= xF(x)^{M-1}(1-q^c)\Big|_0^R - \int_0^R F(x)^{M-1}(1-q^c)dx\\ &= RF(R)^{M-1}(1-q^c) - \int_0^R F(x)^{M-1}(1-q^c)dx\end{aligned}$$

Replacing the numerator of equation 8 with equation 9 gives an implicit function for the amount of bid shading due to market power: $R - p(R)$.

$$(10)\quad p(R) = \frac{RF(R)^{M-1}(1-q^c) - \int_0^R [F(x)^{M-1}(1-q^c)]dx + \int_0^R xq^{c\prime}(x)p'(x)[1-F(x)^{M-1}]dx}{(1-q^c)F(R)^{M-1} + q^c}.$$

If the contract supplies are zero, we can explicitly solve for the bid shading that would normally occur in the spot market:

$$(11)\qquad p(R) = R - \frac{\int_0^R [F(x)^{M-1}]dx}{F(R)^{M-1}}.$$

Comparing equation 10 to equation 11 shows that not all of the bid shading is due to the presence of contracts. The ratio to the right of the minus sign in equation 11 is the amount of bid shading in the auction for spot cattle if there are no captive supplies. Hence, equation 10 provides the basis for further investigation of a rather complicated relationship between captive supplies and market power that previously has been discussed mostly in terms of the conventional wisdom that market power increases in the presence of captive supplies. Equation 10 shows that increased bid shading is one possible outcome but certainly not the only one. Thus, it appears that an empirical investigation of the effects of captive supplies might be preferable and that the relationships in equations 8 and 10 should be considered when modeling such studies.

This brief note provides the foundation for a formal, analytical model of a first-price sealed-bid cattle auction in the presence of captive supplies. It is offered in an effort to provide researchers with a means by which to analyze such markets more fully and thus better understand the intricate relationship between captive supplies and spot-market cattle prices.

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