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ON BIASED TECHNOLOGICAL PROGRESS: COMMENT AND EXTENSION

by

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Professor Quirino Paris
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Dear Professor Paris:

On the basis of the enclosed referee comment, it appears that your paper on "On Bias Technological Progress: Comment and Extension," should not be published. I nevertheless wish to thank you for sending the paper and allowing me to consider it.

Sincerely yours,

George H. Borts
Managing Editor

GHB:dcf
encl.

Referee's Report

I believe that Takayama's method and results can be given the following interpretation. Suppose that one assumes the existence of an aggregate production function and that marginal products are equated to real factor costs. Assuming further that there are no random disturbances associated with either the technology or the factor markets, the following relationships

$$(3) \quad \hat{k} = \sigma\hat{q} + \lambda$$

$$(4) \quad \hat{x} = \sigma\hat{r} - \theta H$$

$$(5) \quad \hat{y} = \sigma\hat{w} + S$$

hold exactly at each point in time. Now, consider the last equation and distinguish two cases: (a) σ and S are constants, (b) they vary over time. Case (a) implies that (5) is linear in $\hat{y}$ and $\hat{w}$, and, since there is no stochastic element in the model, an ordinary least squares (OLS) regression of $\hat{y}$ on $\hat{w}$ will produce an R^2 of one. In case (b), $\hat{y}$ is not a linear function of $\hat{w}$, and OLS will in general yield an R^2 which is less than one. Thus, in the absence of stochastic errors, application of OLS to (5) constitutes a test of the constancy of the parameters, σ and S. This test can be applied to (3), (4), and (5) in an attempt to determine which, in any, of the Hicks, Harrod, or Solow definitions of technical change can be treated as parameters.

I believe that the above description approximates what Takayama had in mind. If so, his method seems internally consistent, but somewhat primative from an econometric standpoint. Takayama does not consider a criterion for determining when the R^2

is large enough to accept the hypothesis of constancy, and he seems to ignore random errors which are almost certainly present in the data.

On the other hand, the criticisms raised by the author of the Comment seem to go too far in their rejection of Takayama's method. As a counter example to the statements on page 4, consider a C.E.S. technology with no technical change. In the absence of random errors:

$$a\, y^{\frac{1}{\sigma}} = w, \quad b\, x^{\frac{1}{\sigma}} = r$$

and

$$\hat{y} = \sigma\hat{w}, \quad \hat{x} = \sigma\hat{r}$$

where $\hat{y} = \ln y(t) - \ln y(t-1)$, etc. Regressing $\hat{y}$ on $\hat{w}$ (or $\hat{x}$ on $\hat{r}$, $\hat{k}$ on $\hat{q}$) would clearly reproduce the exact value of σ. Furthermore, the author's equations (4') and (5') are not appropriate: with σ constant and $\lambda = 0$, either $\sigma = 1$, in which case $\hat{x} = \hat{r}$ and $\hat{y} = \hat{w}$ as above, or $\sigma \neq 1$, and the coefficients of $\hat{r}$ and $\hat{w}$ are not constant, since they depend on a variable labor share θ.

I believe that the author is right to argue that the issues raised by Takayama are more appropriately treated within the formal econometric framework of production theory. However, the author's approach really does not seem to break new ground. If I have correctly understood the author's analysis, he assumes in Section III a C.E.S. production function with a constant technological bias (pages 6 and 7). This is less general than the econometrics based on the translog production function (reported in this Review and elsewhere) since the translog permits a variable elasticity of substitution and a variable bias.

To summarize my views: (1) Takayama's method assumes that there is no stochastic disturbance and is internally consistent. The author's comments should point his out. (2) Takayama's paper is, however, not entirely clear, and readers of the Review could possibly be misled into thinking that he provides a strong econometric test of the constancy hypothesis, when he in fact does not. (3) The author's extensions seem more than necessary to comment on point (2), and less than necessary for a major contribution to the econometrics of production functions. (4) Finally, I find the tone of the Comment overly harsh and inflammatory. I, therefore, cannot recommend the Comment in its present form.

March 30, 1978

Professor George H. Borts
Managing Editor, AER
Robinson Hall, Brown University
Providence RI 02912

Dear Professor Borts:

I gave a lot of thought to your letter of January 18, 1978 and to the reviewer's comments on which you based the rejection of my paper "On Bias Technological Progress: Comment And Extension". I must confess that I found the reviewer's comments erroneous in almost all instances and, therefore, completely inadequate as a basis for rejecting the paper.

For this reason, I take the liberty of making the rather unorthodox move of appealing to you for a reconsideration of your decision. To justify my highly unusual request I will try to demonstrate the extent to which the reviewer's comments miss the point.

1. On page two of his comments the reviewer writes: "Furthermore, <u>the author's</u> equations (4') and (5') are not appropriate: with σ constant and $\lambda = o$, either $r = 1$, in which case $\hat{x} = \hat{v}$ and $\hat{y} = \hat{w}$ as above, or $\sigma \neq 1$, and the coefficients of $\hat{r}$ and $\hat{w}$ are not constant, since they depend on a variable labor share θ."

   These are not <u>my</u> equations, but <u>Takayama's</u>. In fact, they are nothing else than simple algebraic transformations of equations (4) and (5) which are indeed Takayama's (19) and (20). Surely, the substitution of the Takayama's <u>definitions</u> of indexes H and S into the equations, will not change their authorship. This point is crucial for the understanding of my criticisms of Takayama. With these two equations <u>and their estimation</u>, I have shown the source and the nature of the coefficients which Takayama claims represent estimates of σ. His "estimates" of this parameter using (4) and (5) are .6461 and .6381, respectively, which are far different from .2446 the estimate of σ obtained using (3) (his (18). How can this wide discrepancy be explained? Takayama did not explain it. It merely disregarded the results of estimating equations (3) and (4).

   I claim that--in my paper--I explained such a discrepancy by showing that Takayam's <u>estimated</u> equations (4) and (5) are nothing else than econometric misspecifications of equations (4') and (5'). Both these equations involve--by carrying Takayama's development to its logical conclusions--variables $\hat{w}$ and $\hat{r}$. Takayama has simply confined portions of each equation (4') and (5') into <u>definitorial</u> terms H and S. The econometric evidence of my assertions consists in the fact that the estimates of equations (4') and (5') show the coefficients of variables $\hat{r}$ and $\hat{w}$ (page 5) to be .62168 and .62672, respectively, matching Takayama's findings of .6461 and .6381 with remarkable precision. As shown in the paper, (page 6) the coefficients of equations (4') and (5') estimated under the hypothesis of serial correlation imply estimates of $\hat{\sigma}$ which closely match that of equation (3), thus, providing a plausible solution to the problem of multiple and widely different estimates of σ obtained by Takayama.

The conditions under which the reviewer claims that equations (4') and (5') are not appropriate are of little empirical import. Such conditions are $\lambda = o$ and σ constant. Now, the first condition $\lambda = o$ would make the entire exercise on technological progress rather meaningless. Whatever the opinion on this point, one should remember that such equations are Takayama's and not mine.

2. Throughout his comments the reviewer states that Takayama development assumes the absence of random disturbances. (page 1, line 5 and line 16; page 2, line 7). These propositions are rather peculiar since it is difficult to see how estimation can be carried out without random errors. Yet, in his "counter example", (page 2, line 7) the reviewer clearly states: "in the absence of random errors . . . ."

3. The issue of a CES technology. Takayama's sweeping claims are that his treatment of TP is carried out (1) without assuming an explicit form of the production function, (2) without assuming that the elasticity of substitution is constant and (3) without assuming any particular form of TP.

   These three promises may be considered fulfilled in the deterministic-theoretical part of his paper. When he comes to the econometric part of it, however, he renounces--perhaps without realizing it--at least two of them. The choice of estimation technique (OLS) made by Takayama in the measurement of his equations (18), (19) and (20) (corresponding to (3), (4) and (5) in my paper) imply that σ is assumed to be a constant and that the form of the underlying production function is CES. Thus, the use of essentially a CES framework in Section III of my paper is limited by the terms set by Takayama. To argue--as the reviewer does--that a translog production function would be better, indicates that the reviewer may not have grasped Takayama's intentions and methods for fulfilling them.

4. Finally, the reviewer claims that Takayama development is consistent. Of course, his mathematical-theoretical part is consistent and my criticisms are not raised against it. It is the econometric part of the paper which is marred by serious flaws. Surely, the three equations (3), (4) and (5) are linearly dependent, even when error terms are attached to each of them. In his estimation, Takayama has completely disregarded this fact, thus making his results inconsistent. Furthermore, he did not report the Durbin-Watson Statistic when in each case there appears to be considerable serial correlation.

My insistence for your reconsideration of my paper is based upon the opinion that technological progress is a crucial issue. Takayama's paper seems to claim to have disposed of it once and for all. This paper is the last appeared on the subject and as such it may be given unduly importance. Indeed, the paper is highly misleading, especially in the econometric section.

I acknowledge that my note may be cast in an "overly harsh and inflammatory" tone. But this negative aspect can be easily remedied. I am much more concerned that the substance of my comments reach the profession because I deeply believe in the inadequacy of Takayama treatments of TP.

I will await with great interest your reply. I enclose a copy of my paper and of the reviewer's comments.

Sincerely,

Quirino Paris
Associate Professor

QP:dms

Enclosures

ON BIASED TECHNOLOGICAL PROGRESS: COMMENT AND EXTENSION

Introduction

Attempts to measure technological progress abound in the economic literature. A recent paper by A. Takayama [1974] appears to be one of the last papers in this often distinguished lineage. Indeed, the article's ambitious claim could permit one to believe that the paper is to be regarded as the definite statement on the subject, achieved by these relaxing assumptions: ". . . unlike most studies on the topic, I do not assume that the production function is *a priori* of the factor augmenting type. Moreover, I do not . . . assume that the elasticity of substitution is *a priori* constant" [1974, p. 631].

The paper's conclusions are crucially dependent on a misspecified econometric model, naively estimated and interpreted. This note demonstrates that the model (1) omits relevant variables in two equations; (2) adopts an estimation procedure that--*ipso facto*--nullifies the author's claim of an unrestricted elasticity of substitution; (3) fails to account for the substantial autocorrelation induced by the moving average definition of the data series; and (4) wrongly identifies the elasticity of substitution's estimate. The purpose of this paper, however, goes beyond the mere criticism of Takayama's "method of study." It attempts to give a large amount of protection to his model by estimating it with appropriate and efficient techniques. Nonetheless, the inescapable conclusion is that the time series information of the U.S. economy does not fit a model based upon the assumptions of constant elasticity of substitution and constant rate of technological progress.

Such assumptions are those dealt with--in practice--by Takayama through the estimation technique adopted. By necessity, Takayama's inference about the direction of the technological progress' bias--whether Hicksian, Harrodian, or Solovian--is unwarranted.

The Model

The starting point is a neoclassical production function in capital and labor, $Y = F(K, L, t)$, which exhibits constant returns to scale and disembodied technological progress. Using the definition of the labor's relative share, $\theta = F_L L/Y$, where $F_L = \partial F/\partial L$, and the assumption of perfect competition, $F_L = w$, and $F_K = r$, the familiar growth equations are obtained:

$$\hat{Y} = \theta\hat{L} + (1 - \theta)\hat{K} + \phi \tag{1}$$

$$\theta\hat{w} + (1 - \theta)\hat{r} = \phi \tag{2}$$

where $\phi \equiv F_t/Y$, the rate of technological progress, and the superscript (^) indicates the percentage instantaneous rate of change of the corresponding variables. Takayama's manipulations of the various growth ingredients yield three relatively simple relations:

$$\hat{k} = \sigma\hat{q} + \lambda \tag{3}$$

$$\hat{x} = \sigma\hat{r} - \theta H \tag{4}$$

$$\hat{y} = \sigma\hat{w} + S \tag{5}$$

where $k \equiv K/L$, the capital-labor ratio; $x \equiv Y/K$, the output-capital ratio; $y \equiv Y/L$, the output-labor ratio; $q \equiv w/r$, the wage rate-interest rate ratio; σ is the elasticity of substitution between capital and labor; $\lambda \equiv -\sigma q_t/q$

is the Hicks index of TP, where $q_t = \partial q/\partial t$; $H \equiv \lambda - (1 - \sigma)\phi/\theta$ is the Harrod index of TP; $S \equiv (1 - \theta)\lambda + (1 - \sigma)\phi$ is the Solow index of TP. For each of the three TP indexes, neutral, capital-saving, and labor-saving TP is signaled by the index being equal, less, and greater than zero, respectively. The structural interdependence of the three indexes is expressed by the exact relation

$$\lambda = S + \theta H. \tag{6}$$

Using data reported in Sato [1970], Takayama proceeds to estimate system (3), (4), and (5), by applying ordinary least squares to each equation, with variables defined on a five-year moving average. Because Takayama did not report the Durbin-Watson statistic, the three equations were reestimated and found in perfect agreement with the author's estimates. The results are presented in Table 1. Very conclusively, the D-W test points to the presence of positive serial correlation in the first two equations and suggests the likelihood of it in the third.

The OLS estimates of equations (3), (4), and (5) are commented by Takayama in the following way: regression (5) involving the variables $\hat{y}$ and $\hat{w}$ exhibits a correlation coefficient, R, equal to 0.8843 and thus, it "looks like a good fit." Regression (4) shows that $R = 0.7170$ and "the fit is not so good." Finally, the fit of regression (3) "is rather poor" with $R = 0.4730$. The elasticity of substitution is around 0.64 in the second and third regressions but drops to a low 0.24 in (3). These results prompted Takayama to assert: "This [the low R in equation (3)] may suggest that λ (and possibly σ) fluctuate over the years" [1974, pp. 634-35]. He finally concludes (footnote 16) that $\sigma^* = 0.64$, seemingly confident of the close values obtained in equations (4) and (5).

It can easily be demonstrated that Takayama's estimation and inference stand on shaky ground. First of all, only two equations among (3), (4) and (5) are independent relations. Equations (4) and (5) are simply algebraic manipulations of (3) and the production function. Unfortunately, Takayama's criterion of good fit has eliminated equation (3) from serious consideration. Notice that (3) is also the only equation which gives a direct estimate of the elasticity of substitution σ. Its magnitude is considerably smaller than the "estimates of σ" obtained in equations (4) and (5). The coefficients in these equations, in fact, do not measure the elasticity of substitution, σ. Both equations are the result of an econometric misspecification (omission of relevant variable), as it will be easily shown. First, consider equation (4). By substituting the definitions of the H index and of the rate of TP, ϕ, we obtain

$$(4') \qquad \hat{x} = \sigma\hat{r} + (1 - \sigma)\phi - \theta\lambda$$

$$= \sigma\hat{r} + (1 - \sigma)[\theta\hat{w} + (1 - \theta)\hat{r}] - \theta\lambda$$

$$= [\sigma\theta + (1 - \theta)]\hat{r} + [(1 - \sigma)\theta]\hat{w} - \theta\lambda.$$

Similarly, equation (5) corresponds to the more explicit representation

$$(5') \qquad \hat{y} = \sigma\hat{w} + (1 - \sigma)[\theta\hat{w} + (1 - \theta)\hat{r}] + (1 - \theta)\lambda$$

$$= [(1 - \sigma)(1 - \theta)]\hat{r} + [\theta + (1 - \theta)\sigma]\hat{w} + (1 - \theta)\lambda.$$

In each equation, the coefficients of the $\hat{r}$ and $\hat{w}$ variables add up to unity. Takayama's "method of study" has been to apply OLS procedures to (4') and (5'), omitting the $\hat{w}$ variable in (4') and the variable $\hat{r}$ in (5'), thus obtaining biased estimates of the remaining nonlinear coefficients. One can immediately see that, in general, the application of OLS techniques to (4') and (5') is ill advised even if some definite and restrictive assumptions are postulated. The most obvious of these specifications would exploit the fact that the labor share θ has remained remarkably constant within the observation period. Thus, at first glance, it would seem that by postulating also the constancy of the elasticity of substitution, σ, and of the Hicks index of TP, λ, OLS techniques may be a legitimate procedure. It should be realized, however, that the error terms of the two equations (4') and (5') are not independent and that exactly the same parameters occur in both relations. One thing is certain: now that (4') and (5') are expressed in terms of at least two variables ($\hat{r}$, $\hat{w}$), Takayama's simplistic criterion (based upon the goodness of fit) for deciding about the constancy of σ is clearly inadmissible. Equations (4') and (5') were first estimated with OLS procedures for the sole purpose of providing numerical support to the above interpretations of Takayama's "method of study." The results are as follows:

$$(4') \quad \hat{x} = \underset{(0.05161)}{0.62168\hat{r}} + \underset{(0.06482)}{0.68949\hat{w}} - \underset{(0.00195)}{0.01074} \qquad R^2 = 0.8639, \; DW = 1.066$$

$$(5') \quad \hat{y} = \underset{(0.02642)}{0.19347\hat{r}} + \underset{(0.03414)}{0.62672\hat{w}} + \underset{(0.00103)}{0.00585} \qquad R^2 = 0.9017, \; DW = 1.155.$$

Numbers in parentheses are standard errors of the corresponding estimates. It may now be more evident that Takayama's coefficients of (4) and (5) (although biased) correspond to the coefficients of the $\hat{r}$ and $\hat{w}$ variables in equations (4') and (5'), respectively. This interpretation explains their close magnitude and establishes the fact that they cannot be taken as estimates of the elasticity of substitution. Notice also that OLS techniques applied to equations (4') and (5') imply multiple estimates of σ. From the coefficients of the $\hat{r}$ and $\hat{w}$ variables in (4'), and using the average value of the labor share $\theta = 0.663$, we obtain the following two estimates of σ: $\tilde{\sigma} = 0.4294$ and $\tilde{\sigma} = -0.0399$. Similarly, from the two coefficients of (5') we obtain: $\tilde{\sigma} = 0.4259$ and $\tilde{\sigma} = -0.1076$. The large variability of the four estimates but, above all, the negative values of σ confirm that OLS procedures as applied by Takayama for estimating his model are untenable. A first obvious attempt to improve the estimation results is simply to account for the positive serial correlation detected in all regressions. When this is done on equations (3), (4') and (5'), using a first-order autoregressive scheme and the Cochrane-Orcutt iterative algorithm, the estimates are as follows:

$$(6)\quad (\hat{k}_t - \tilde{\rho}\hat{k}_{t-1}) = \underset{(0.04526)}{0.12200}(\hat{q}_t - \tilde{\rho}\hat{q}_{t-1}) + \underset{(0.00125)}{0.00231}$$

$R = 0.3764$ $\qquad$ $\tilde{\rho} = 0.7875$ $\qquad$ iteration #4.

$DW = 1.925$ $\qquad$ $\tilde{\lambda} = 0.01086$

$$(7)\quad (\hat{x}_t - \tilde{\rho}\hat{x}_{t-1}) = \underset{(0.04988)}{0.41703}(\hat{r}_t - \tilde{\rho}\hat{r}_{t-1}) + \underset{(0.06994)}{0.55697}(\hat{w}_t - \tilde{\rho}\hat{w}_{t-1}) - \underset{(0.00105)}{0.00156}$$

$R^2 = 0.7020$ $\qquad$ $\tilde{\rho} = 0.7584$ $\qquad$ iteration #6.

$DW = 1.738$ $\qquad$ $\tilde{\lambda} = 0.00974$

$$(8)\quad (\hat{y}_t - \tilde{\rho}\hat{y}_{t-1}) = \underset{(0.02851)}{0.28031}(\hat{r}_t - \tilde{\rho}\hat{r}_{t-1}) + \underset{(0.03982)}{0.66761}(\hat{w}_t - \tilde{\rho}\hat{w}_{t-1}) + \underset{(0.00065)}{0.00151}$$

$R^2 = 0.8777$ $\tilde{\rho} = 0.6567$ iteration #8.

$DW = 2.258$ $\tilde{\lambda} = 0.01303$

As before, the coefficient of the variable $\hat{q}$ in equation (6) is an estimate of the elasticity of substitution, σ. Using $\bar{\theta} = 0.663$, the coefficients of $\hat{r}$ and $\hat{w}$ in equations (7) and (8), respectively, imply the additional four estimates of σ: $\tilde{\sigma} = 0.12071$, $\tilde{\sigma} = 0.15993$, $\tilde{\sigma} = 0.16822$, $\tilde{\sigma} = 0.01368$. The application of a first-order autoregressive scheme has greatly reduced the variability among the various estimates of the elasticity of substitution. It is also interesting to notice that, now, the estimate of σ in equation (6) tends to agree with those derived from equations (7) and (8).

Full Information Maximum Likelihood Estimates

Using all the available information, it is possible to further improve the "method of study" if we are willing to postulate *a priori* the constancy of the elasticity of substitution as well as of the Hicks bias of technological progress. This is equivalent to assume a CES production function with exponential TP. Equations (3), (4') and (5') constitute, in fact, a system of linearly dependent relations. The interrelationships of their error terms and the communality of all the parameters involved, suggest the adoption of a full information maximum likelihood (ML) approach. It is convenient to emphasize the restrictive nature of the hypotheses about the constancy of σ and λ. These are, however, the assumptions *de facto* chosen by Takayama via his estimation method, and the ML approach is eminently suitable to verify or reject the validity of them.

For absolute clarity, let $z_t' \equiv [\hat{k}_t, \hat{x}_t, -\hat{y}_t]$, $f_t' \equiv [\hat{q}_t, \hat{r}_t, \hat{w}_t]$, and $\mu' \equiv [\lambda, -\theta\lambda, -(1 - \theta)\lambda]$. Then, the stochastic representation of systems (3), (4') and (5') is the following:

$$(9) \quad z_t = Bf_t + \mu + u_t \qquad t = 1, \ldots, T$$

where $u_t' \equiv [u_{kt}, u_{xt}, u_{yt}]$ is a (3 x 1) vector of disturbance terms, μ is a (3 x 1) vector of constants, f_t is a (3 x 1) vector of exogenous variables, z_t is a (3 x 1) vector of dependent variables and B is the following coefficient matrix

$$B = \begin{bmatrix} \sigma & 0 & 0 \\ 0 & \{\sigma\theta + (1 - \theta)\} & \{(1 - \sigma)\theta\} \\ 0 & -\{(1 - \sigma)(1 - \theta)\} & -\{\theta + (1 - \theta)\sigma\} \end{bmatrix}.$$

In view of the considerable autocorrelation detected above, it is also postulated that the vector u_t follows a first-order autoregressive scheme

$$(10) \quad u_t = Ru_{t-1} + e_t \qquad t = 2, \ldots, T$$

where the e_t vectors represent purely random disturbances independently and identically distributed according to a normal density with mean zero and covariance matrix Ω, and where R is a (3 x 3) matrix of unknown parameters specifying the autocorrelation structure.

Under the specification of Takayama's model (3), (4') and (5'), we have also the adding-up condition

$$(11) \quad \iota' z_t = 0 \qquad t = 1, \ldots, T,$$

where ι is a (3 x 1) sum vector with all elements equal to unity; under the same specification we also have

$$(12) \quad \iota' B f_t = 0 \qquad t = 1, \ldots, T$$

and

$$(13) \quad \iota' \mu = 0.$$

From (9), (11), (12) and (13), it follows that

$$(14) \quad \iota' u_t = 0 \qquad t = 1, \ldots, T$$

and, therefore, from (10) and (14)

$$(15) \quad \iota' u_t = \iota' R u_{t-1} + \iota' e_t = 0 \qquad t = 2, \ldots, T$$

and, finally,

$$(16) \quad \iota' R = c'$$

$$(17) \quad \iota' e_t = 0 \qquad t = 1, \ldots, T$$

because u_{t-1} and e_t are statistically independent; c' is a vector of constants. The implication of the above specification is that the covariance matrix Ω is singular and estimation methods based upon the maximum likelihood principle must be modified to account for it. The common procedure is to drop one equation from the system. A fundamental question rises naturally: will the estimates be invariant to the equation dropped? This depends upon the further specification of the R matrix and upon the degree by which the data series actually satisfy the adding-up condition. By specifying a diagonal R matrix and taking into account the restriction (16), it follows that all the diagonal elements of R must be equal. Hence, from this standpoint, the ML estimates will be the same no matter which equation is dropped. A likelihood ratio test will be performed to verify the validity of the diagonal specification of the R matrix.[1/]

At this point we must alert the reader to a measurement problem. Theoretically, the system (3), (4') and (5') implies that the adding-up condition (11) is satisfied exactly for all t. The variables in the system represent instantaneous rates of change proportionally scaled. But, when approximating instantaneous rates of change of ratio variables using either yearly or five-year moving average observations, there is a choice of definition. For example, the proportional rate of change of the capital-labor ratio, $k \equiv K/L$, can plausibly be constructed as (i) $\hat{k}_t = (k_t - k_{t-1})/k_{t-1}$ or as (ii) $\hat{k}_t = \hat{K}_t - \hat{L}_t$, where $\hat{K}_t$ and $\hat{L}_t$ are themselves proportional rates of change defined as in (i). The same choice exists for the ratio variables $x \equiv Y/K$, $y \equiv Y/L$ and $q \equiv w/r$. Takayama chose definition (i) with the consequence that the two fundamental equations (1) and (2) expressing the rate of technological progress, ϕ_t, are largely inconsistent. The average absolute error of the estimate of ϕ_t using the (i) definition is about 25 percent per observation. Finally, notice that there is a further problem of choice involving the rate of profit, r, and as a consequence, the price ratio, q. In the original data reproduced in Sato [1970], the rate of profit is computed from the accounting identity $Y = wL + rK$ as $r = (Y - wL)/K$. Thus, a proportional rate of change series of r can be constructed as $\hat{r}_t = [1/(Y_t - w_t L_t)] [Y_t \hat{Y}_t - L_t w_t (\hat{w}_t + \hat{L}_t)] - \hat{K}_t$. With this definition of the profit rate of change, the estimates of the rate of technological progress ϕ_t computed from the two equations (1) and (2) are exactly the same. The maximum likelihood estimates of the model will be implemented with the following sets of data: those used by Takayama which, however, violate both the consistency of the rate of technological progress and of the adding-up relation, and those constructed according to definition (ii), which satisfy both conditions.

After some familiar rearrangements of equations (9) and (10), the system to be estimated can be represented as

$$(18)\quad z_t^i = R^i z_{t-1}^i + B^i f_t^i - R^i B^i f_{t-1}^i + (I - R^i)\mu^i + e_t^i \qquad \begin{array}{l} i = 1, 2, 3 \\ t = 1, \ldots, T \end{array}$$

where superscript i indicates the equation dropped and I is the (2 x 2) identity matrix. The specification in (18) embodies all the *a priori* information about the underlying economic model, the constancy of σ and λ, in addition to the first-order autoregressive scheme postulated in (10). Of course, it is possible that both the *a priori* restrictions of model (9) as well as the autoregressive restrictions (10) constitute a misspecification of the true structure. Hence, following Hendry [1971] the null hypothesis of all restrictions and of a first-order serial correlation as incorporated in (18), can be tested against any generic alternative represented by the following specification

$$(19)\quad z_t^i = Q_1^i z_{t-1}^i + Q_2^i f_t^i + Q_3^i f_{t-1}^i + q^i + v_t^i \qquad t = 1, \ldots, T$$

where the vector q^i and the matrices Q_1^i, Q_2^i, Q_3^i are completely unrestricted. Let $\mathcal{L}$ denote the logarithm of the appropriate likelihood function. A ratio test based on the likelihood function's maxima of (18) and (19), which correspond to the statistic $-2[\mathcal{L}_e - \mathcal{L}_v]$, can be used to test the null hypotheses that the model specification expressed by (18) is valid. If, however, the test fails to reject the null hypothesis, it could be because R = 0, and an explicit test is required to verify this degenerate case. This can be performed by comparing the likelihood functions of (9) and (18), or, in other words, by computing $-2[\mathcal{L}_u - \mathcal{L}_e]$. Finally, if the hypothesis of R = 0 is rejected, we can test whether R is diagonal using the statistic $-2[\mathcal{L}_{diagR} - \mathcal{L}_e]$. The three statistics are asymptotically distributed as χ^2 variables with 2, 2, and 3 degrees of freedom, respectively, which are given by the numbers of regressors in the vector f^i_{t-1}, the number of equations in the system, and by the restrictions on the matrix R^i. The computations were carried out using a full information maximum likelihood algorithm developed by Wegge [1969]. The results of the above nested testing are reported in Table 2 for three series of data: (a) the five-year moving average series constructed by Takayama (inconsistent with both the definition of ϕ and the adding-up condition); (b) the five-year moving average data and the yearly series consistent with both requirements. From the magnitude of the relevant statistics it is evident that, for any reasonable confidence level, the three likelihood ratio tests reject the null hypotheses upon which they are based. By nature of the nested sequence of tests, rejection of the first set of null hypotheses implies the rejection of the other two. We are, therefore, legitimized to derive the following conclusions: the restrictions specified in system (18) are not valid. This is equivalent to reject the first-order autoregression scheme but, more important, also the constancy of σ and λ. It is interesting to note that a specification incorporating a second-order autoregressive hypothesis was also rejected, thus reinforcing the conclusion about the variability of σ and λ. The information about the U.S. nonfarm sector is not consistent with the constancy of the elasticity of substitution and of the bias of technological progress. The same poor fit of regression (3) obtained by Takayama confirms this evidence, by his own "method of study."[2/], [3/]

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Footnotes

The author wishes to thank Leon L. Wegge for useful comments, but retains the full responsibility for any error.

1/ This approach is similar to that followed in the analysis of demand and factor shares, and clearly illustrated by Berndt and Savin [1975].

2/ The rejection of the null hypotheses underlying the ML test of Table 2 should eliminate any need of commenting upon the estimates of the elasticity of substitution. Yet, the literature abounds with references to empirical estimates of σ, obtained within a CES framework. Such estimates have been found to cluster around the value of .6 [Takayama, 1974, p. 634, footnote 14]. The danger arises that unqualified and repeated reference to such values creates the impression that the elasticity of substitution in the U.S. economy is indeed equal to .6. Thus, it is of some significance to point out that the estimate of σ obtained in this paper is substantially below the value of .6. Notice that the (CES) framework and the data are the same as those adopted in the often cited studies. But, at this point, one should recall that the CES framework has to be rejected for interpreting the growth series of the U.S. economy. Hence, neither the value of .6 nor of .15 can be chosen with any confidence to represent a proper estimate of the elasticity of substitution. There remains the alternative of estimating the model by means of a varying parameter technique. This topic may be the objective of another and proximate paper.

3/ In footnote 15, p. 634, Takayama advances the notion that his estimation framework constitutes a "sharp test" for assessing the consistency of the U.S. data series with technological progress exhibiting Solow labor-saving bias. Such a characterization seems indeed an exaggeration when applied to a simple and misspecified regression.

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TABLE 1. OLS Estimates of Takayama's Model When Data Series are Inconsistent with Equations (1) and (2)

	Equation (3)	Equation (4)	Equation (5)
σ	0.24460 (0.06790)		
"σ"[a/]		0.64610 (0.09360)	0.63810 (0.05020)
λ	0.00760 (0.00215)		
$-\theta H$		0.00504 (0.00237)	
S			0.00755 (0.00148)
R	0.47300	0.71700	0.88430
D-W	0.52700	0.76400	1.46600
n	47	47	47

a/ "σ" signifies that the corresponding coefficients are not estimates of the elasticity of substitution.

TABLE 2. Maximum Likelihood Estimates of System (18) with Diagonal R Matrix

	Takayama Data Series	Consistent Data Series	
	Five-Year Moving Averages	Five-Year Moving Averages	Yearly Data
$-2[\mathcal{L}_e - \mathcal{L}_v]$	12.68450	33.21200	37.22220
$-2[\mathcal{L}_u - \mathcal{L}_e]$	45.14900	46.32200	7.80000
$-2[\mathcal{L}_{diagR} - \mathcal{L}_e]$	11.78430	14.44440	5.71250
σ	0.13431 (0.04941)	0.14723 (0.05282)	0.23921 (0.05891)
λ	0.00989 (0.00357)	0.00893 (0.00354)	0.00585 (0.00442)
ρ	0.63688 (0.08987)	0.63520 (0.09142)	-0.12940 (0.10487)