UCD Department of Agricultural Economics

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Numerous agricultural economists have analyzed the allocative significance of risk and risk aversion (Wolgin, Just, Feder, Pope). Most of these studies have focused on the short run behavior of farm firms. Yet, little attention has been directed towards long run behavior under risk. That is, when factors of production are mobile without barriers to entry and exit, how does one conceptualize the long run equilibrium of the firm when uncertainty prevails?

The purpose of this note is to characterize a long run equilibrium for a risk averse price uncertain firm. The intent is to provide a characterization of equilibrium firm output and expected industry price. It is shown here that the representative or typical firm minimizes average cost plus an average risk premium such that the risk averse long run equilibrium output is less than that output which minimizes average cost. Also, expected output price under risk aversion exceeds the minimum of average cost such that expected profits are positive. Further, comparative static results indicate that input demands are not necessarily downward sloping but relative demands (quantity demanded of the input divided by output) are negatively sloped. Finally, output may contract or expand as an input price increases depending on production elasticities, but expected price rises.

It is hoped that the presentation here serves two purposes. First, in every intermediate and graduate core microeconomic theory class, it is argued that competition forces price to the minimum of the long run average cost. By adding risk, a more general and more descriptive alternative to the riskless presentation may be incorporated into classroom discussions.

Secondly, the long run average cost curve has been the object of many empirical studies in agriculture. The results discussed here offer increased descriptive power when compared to certainty theories. For example, equilibrium is possible even under declining average costs. Further, as Pasour and Bullock caution, normative efficiency prescriptions may be misleading when certainty is incorrectly assumed. Here, the relevant efficiency concept may be associated with minimizing average cost plus an average risk premium.

REPRESENTATIVE FIRM EQUILIBRIUM-OUTPUT PRICE UNCERTAINTY

The paradigm presented under certainty argues that firms enter and exit the competitive industry in such a way as to drive profits to zero or where output price equals minimum average cost of the representative firm. Hence, it follows that cost curves of every firm are generally assumed identical. Few would argue that this assumption is very descriptive, but it is often felt that the representative firm paradigm is a reasonable postulate in order to enhance an understanding of long run firm behavior.

In a similar spirit, it is assumed here that there is a firm with representative beliefs and preferences. The existence of risk responsive market behavior is empirically well documented (e.g., Behrman, Just, and Jensen); hence, a movement towards the recognition of risk in the characterization of long run equilibrium appears imperative. The model employed here is *ex ante* in nature. This seems to be a reasonable assumption for agriculture and most input choices.

Short Run Equilibrium

Consider initially the conventional short run representation of an expected utility maximizing firm. Let $U(\pi)$ denote the utility of profit, where U is the preference function and π is profit. Let profit be given by

$$\pi = Pq - C(q, \gamma) \equiv Pq - C(q),$$

where P is the random output price, q is the quantity of output, $C(q, \gamma) \equiv C(q)$ is the cost function and γ denotes input prices.[1/] The output price, P, is distributed with finite mean, $\overline{P}$, and variance, σ. It is assumed that the firm maximizes expected utility of profit, $E[U(\pi)]$, postulated to be concave in output.[2/]

The necessary condition for equilibrium output is given by

$$E[U'(\pi)(P - C'(q))] = 0$$

or,

$$\overline{P} - C'(q) = - \frac{Cov[U'(\pi), P]}{E[U'(\pi)]}, \quad (1)$$

where $C'(q)$ is marginal cost and Cov denotes covariance. Marginal utility, $U'(\pi)$, is assumed positive; but, Baron, Sandmo and others have shown the covariance to be negative under risk aversion. Hence, expected price exceeds marginal cost under risk aversion at the optimum.

Following Pratt and Baron, the risk premium, R, is that quantity of money which makes the entrepreneur indifferent between a sure return of $E(\pi) - R$ and the risky prospect, π. That is, $U[E(\pi) - R] = E[U(\pi)]$ and $R = 0$ for a risk neutral firm. In equilibrium, partial differentiation of this definition with respect to output, q, and using (1) gives

$$U'[E(\pi) - R]\{\overline{P} - C'(q) - R'(q)\} = \overline{P} - C'(q) + \frac{Cov[U'(\pi), P]}{E[U'(\pi)]} = 0$$

or,

$$R'(q) = -\frac{\mathrm{Cov}[U'(\pi), P]}{E[U'(\pi)]} = \overline{P} - C'(q), \tag{2}$$

where $R'(q) = dR(q)/dq$. From (2), at equilibrium output, the marginal change in the risk premium as q changes is given by $-\mathrm{Cov}[U'(\pi), P]/E[U'(\pi)]$ which is positive under risk aversion.

The question now occurs as to the existence of any meaningful long run equilibrium output for the risk averse firm. The rationale for such an equilibrium is apparent and is found elsewhere in the literature (see particularly Baron). Suffice it to say that demands are considered random and induce a probability distribution on market price where price is determined by the intersection of the market supply and the demand schedules. It is assumed that there is a representative level of risk aversion which is possessed by participants (or potential participants) in production decisions. That is, the level of risk aversion is bounded away from zero such that all active producers have a representative level of risk aversion. The characterization of and the implication of such an equilibrium on the firm's output have not been explored in the literature and are examined below.

Long Run Equilibrium

Let $E[U(\pi|q^*)]$ denote the expected utility given the optimal output, q^*. If q^* is optimal, then the expected utility given q^* must not be smaller than the expected utility from any other production plan, or, $E[U(\pi|q^*] \geq E[U(\pi|q)] \geq E[U(\pi|q = 0)]$. It is assumed that $(\pi|q = 0) = 0$ and that exit and entry occurs such that the expected utility of producing is equal to the expected utility of not producing or, $E[U(\pi|q^*)] = E[U(\pi|q = 0)]$.[3/]

It follows that $U[E(\pi) - R] = E[U(\pi|q^*)] = 0$, or,

$$R = E(\pi) - U^{-1}[E(U(\pi|q^*))] \tag{3}$$
$$= E(\pi).$$

From (3), it is clear that expected output price is forced to an equilibrium value such that $E(\pi) - R = 0$, or

$$\overline{P} = \frac{C(q)}{q} + \frac{R}{q}, \tag{4}$$

where the first right hand side term in (4) is average cost and the second term is the average risk premium. The sum of the two terms will be referred to as the subjective average cost, SUAC. It will be assumed that SUAC has a unique minimum; hence, $\overline{P}$ = min SUAC = SUAC* specifies long run equilibrium output of the firm.[4/]

From (4), the first order conditions for a firm minimizing SUAC are:

$$C'(q) - AC(q) = \frac{R(q)}{q} - R'(q), \tag{5}$$

where $AC(q) = C(q)/q$ or average cost. Second order conditions require that $C''(q) + R''(q) > 0$ at optimum. It is assumed that the cost curves are classically U-shaped (Hanoch) with $C''(q) > 0$; hence, $R(q)$ may be concave or convex, but $C''(q) > |R''(q)|$ must hold.[5/]

When the firm is risk neutral $R = 0$ and (5) reduces to the familiar conditions: marginal cost equals average cost, or,

$$C'(q) = AC^*(q), \tag{6}$$

where $AC^*(q)$ is the minimum average cost. Under risk aversion, the risk premium is positive and (5) is rewritten as

$$AC(q) + \frac{R(q)}{q} = SUAC^* = C'(q) + R'(q). \tag{7}$$

Here, it is clear that in the long run, the firm equates subjective marginal cost $[C'(q) + R'(q)]$ with subjective average cost $[AC(q) + R(q)/q] =$ SUAC*.

The question which naturally occurs concerns the relationship of long run risk averse equilibrium output and that level of output which minimizes average cost. This is examined in the next section.[6/]

Equilibrium Output

Since R is positive under risk aversion, it is clear that the risk averse equilibrium expected output price is higher than the risk neutral equilibrium expected price. However, given classically shaped cost curves and risk aversion, it is interesting to examine long run equilibrium output of the risk averse firm vis-à-vis marginal and average cost. For, if at equilibrium, marginal cost exceeds (is less than) average cost, then the risk averse output would exceed (be less than) the risk neutral equilibrium output.

For all price events associated with zero output, profit also is zero, and therefore, the risk premium is zero $(R(0) = 0)$. Hence, if R is convex (concave) in output, the average risk premium is always lower (higher) than the marginal risk premium. Note that convex (concave) R implies that the average risk premium is rising (falling). This implies that the left side of (5) is negative (positive) and the long run nonrisk neutral optimal output is lower (higher) than that output which minimizes average cost. In the Appendix it is shown that risk aversion implies that R is convex in output and hence, the average risk premium is rising with output.[7/] The result is summarized in the proposition below.

Proposition: The long run risk averse equilibrium output occurs at the minimum of average cost plus an average risk premium. The risk averse equilibrium expected output price is higher than the risk neutral equilibrium expected price. Under risk aversion, the average risk premium is increasing. Analogously, the risk premium, R, is convex in output, and the risk averse equilibrium output is smaller than the long run risk neutral equilibrium output.

Because of the widespread empirical and theoretical use of the constant risk averse utility function (Freund, Baron, Connors), the results of the proposition are illustrated in Figure 1 under the assumption of normality. In such case, $U(\pi) = - e^{-\alpha\pi}$, where $\alpha > 0$ is the Arrow-Pratt risk aversion measure ($\alpha = - U''(\pi)/U'(\pi)$). $E[U(\pi)] = - \exp[- \alpha\{E(\pi) - (\alpha/2)\sigma q^2]$, where σ is the variance of price. In fact, from the definition of R, it is easily shown that $R = (1/2)\alpha\sigma q^2$; hence, $R'(q) = \alpha\sigma q$ and R is clearly convex since $R''(q) = \alpha\sigma > 0$. In the figure, let $\overline{P}_{SR}$ represent the short run expected price. A risk neutral firm would choose output level q_{SC}. However, as Sandmo has shown, a risk averse firm would produce at $q_{SR} < q_{SC}$. The level of output, q_{SR}, is determined by the intersection of expected price, $\overline{P}_{SR}$, and the subjective marginal cost curve, SUMC. The SUMC curve is the sum of the marginal cost curve, MC, and the marginal risk premium, $R'(q) = \alpha\sigma q$. In the case of long run equilibrium for risk neutral firms, expected price would adjust such that expected profits are zero, or equivalently, the long run expected price, $\overline{P}_{LC}$, is equal to the minimum of the average cost curve. The corresponding level of output is q_{LC}. Alternatively suppose all firms are risk averse with average risk premium given by $R/q = \alpha\sigma q/2$. The subjective average cost curve, SUAC, is the sum of the average cost curve, AC, and the average risk premium, R/q. Long run equilibrium is determined by the intersection of long run expected price, $\overline{P}_{LR}$, and the minimum of SUAC. The corresponding equilibrium output is q_{LR}. Note that since R/q is increasing, $q_{LR} < q_{LC}$.

Consider now the long run adjustment process for risk averse firms in the industry. At the short run price, $\overline{P}_{SR}$ < min SUAC, firms are not receiving a sufficiently high expected price so as to self-insure against risk. Hence, firms will exit the industry driving expected price higher until $\overline{P} = \overline{P}_{LR}$ = min SUAC with a corresponding output of q_{LR}. For the example, it is assumed that adjustments in expected price do not alter higher moments of the distribution. Hence, R/q and $R'(q)$ do not shift as $\overline{P}$ changes.

INPUT PRICE CHANGES

The question arises as to the effects of input price changes on SUAC* and optimal output in Figure 1. It is shown in the Appendix that an increase in an input price raises SUAC* and hence $\overline{P}$. The remaining issues concern the effects of input price changes on factor demands and optimal output. In the Appendix, it is shown that long run factor demands may not be downward sloping. However, relative factor demands (input demand/output supplied) are always downward sloping. Finally, optimal output falls (increases) with a rise in a factor price as the output elasticity, η, exceeds (is less than) unity.

In the case of homothetic production and certainty (or risk neutrality), Silberberg has shown that when firms minimize total cost, the output elasticities are all equal and are given by marginal cost over average cost, i.e., $\eta = MC/AC$. For a risk neutral firm in long run equilibrium MC = AC or $\eta = 1$. This implies that the change in equilibrium output with respect to a change in an input price is zero ($\partial q/\partial \gamma_j = 0$). This is so because MC and AC are displaced vertically such that the output which minimizes average cost is unchanged.

For the case of risk aversion and homothetic production, the output elasticity at equilibrium is $\eta = 1 + [R/q - R'(q)]/AC$, using (5). Hence, η is smaller than one, or, marginal cost is less than average cost, as indicated in Figure 1. Now, the average risk premium function in the example is unaffected by changes in input prices. Using (A.2), an input price increase leads to increased equilibrium output if the output elasticity (η) is less than one. Since the risk averse firm is producing on the downward sloping portion of the average cost curve where the output elasticity is less than unity (or MC < AC), a rise in an input price induces increased equilibrium output.

The above discussion is illustrated in Figure 2. The technology underlying the cost function is homothetic and corresponds to the production function, $q = 4\, X_1X_2 - (X_1X_2)^2/64$ where X_1 and X_2 are input levels.[8/] The corresponding cost function is $C(q) = (32\, \gamma_1\gamma_2)^{1/2}\, (16 - (256 - q)^{1/2})^{1/2}$, where γ_1 and γ_2 are input prices. The average and marginal cost curves with input prices, $\gamma_1 = 50$, $\gamma_2 = 11.111$, are labeled AC° and MC° respectively. The corresponding subjective average cost, SUAC°, was computed using $\alpha\sigma =$ .357. The long run equilibrium output under uncertainty is q_{LC} and under risk aversion is q°_{LR}. Note that q°_{LR} is to left of q_{LC}. With a change in γ_2 from 11.111 to 44.444, the new average and marginal cost functions are AC^1 and MC^1, respectively. Since production is homothetic, AC is displaced vertically such that under certainty, equilibrium output, q_{LC}, is unchanged. With the new set of prices, the subjective cost function becomes $SUAC^1$ with an equilibrium output of q^1_{LR}. The output elasticity before the price change is approximately $\eta = MC/AC = 1.13/1.46 = .77$. Hence, as γ_2 rises, the equilibrium output under risk aversion rises as well as indicated by q^1_{LR}. This is so because SUAC is displaced to the right by the constancy of the average risk premium function.

In closing this section, it is appropriate to comment that increasing risk (σ) or risk aversion (α) has the effect of raising the average risk premium. This implies that SUAC* and expected price rises. However, unlike input price increases, an increase in risk or risk aversion implies that the long run equilibrium output falls. Though not explicitly proven in the Appendix, these results should be intuitively clear from Figure 1.

CONCLUDING REMARKS

It is of course possible to add other kinds of uncertainties. For example, when production is also random given by $q = E(q)w$, $E(w) = 1$, then corresponding to (4) is the expression $\overline{P} = R/E(q) + C/E(q) - \text{Cov}(P, w)$, where $C = C(E(q))$. Again, the curvature of R is crucial in determining long run equilibrium.

It seems fruitful to consider briefly the empirical relevance of the model. Many researchers have offered the long run theory as a basis for empirical research. Indeed, in agricultural economics, many long run average cost curves have been computed with an eye towards determining economies of scale and optimal firm size. Yet, many estimated average cost curves show little evidence of a minimum (or a unique minimum [Madden]). Without discussing the difficulties of empirical verification of this result, it is shown here that risk aversion can lead to a determinate firm size (output) even when average cost is decreasing.

In the previous section the cost curves were assumed to be classically shaped for purposes of comparison. However, second order conditions are satisfied if SUAC has a minimum. Consider an example which is particularly relevant to agriculture. Suppose the cost function is not classically shaped but is a declining average cost Cobb-Douglas, $AC = A(\gamma)q^{\lambda - 1}$ where λ is the cost function scale coefficient. For the utility function illustrated in Figure 1, it is verified that even when average cost is decreasing ($\lambda < 1$), minimization of AC + R/q implies an equilibrium output of $q_{LR} = \{(1/2\alpha\sigma)/[A(\gamma)(1 - \lambda)]\}^{\frac{1}{\lambda - 2}}$. This result is illustrated in Figure 3. Hence, a risk averter with risk premium R chooses not to expand beyond q_{LR} and $E(\pi)$(area $\overline{P}_{LR}$abc) is demanded by firms in the industry as a risk premium.

Finally, the risk premium function utilized here in long run analysis can be estimated by $E(\pi)$. The hypothesis that expected price equals average cost plus an average risk premium can be tested. Procedures to estimate the average risk premium from an econometric system are available (Pope). Hopefully, the discussion here will be helpful for both didactic and empirical purposes.

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FOOTNOTES

1/ Batra and Ullah have shown that cost minimization is consistent with the model employed here.

2/ The analyses could proceed in a similar manner if the firm maximizes $E[U(w + \pi)]$ where w is initial wealth.

3/ In the long run profits are zero when output is zero. Since expected utility is defined only up to an increasing linear transformation, $U(0) = 0$ without loss of generality.

4/ Equation (4) implies $\overline{P}$ = SUAC. Yet equations (1) and (2) imply $\overline{P}$ = marginal cost plus the marginal risk premium. This can only occur at the minimum of a well behaved function as indicated in (5).

5/ Note that $AC'(q) + d[R(q)/q]/dq = 0$ at equilibrium.

6/ It is noted that the relevant comparison is not between the risk neutral and risk averse optimal outputs for a given distribution of price as in the short run analyses of Sandmo and others. Generally, the existence of the risk premium function generates a whole family of price distributions. Here, two members of this family are examined in terms of two different behavioral rules, $R > 0$ and $R = 0$. That is, the long run equilibrium expected price will be different for $R > 0$ than the case where $R = 0$. Hence, equilibrium outputs will in general be different. This is in clear contrast to the short run analysis where optimal outputs are compared for $R > 0$ and $R = 0$ where the distribution of P (and hence, $\overline{P}$) is unique.

7/ See Pazner and Razin for a similar result. However, they do not develop the result using SUAC.

8/ Note that the elasticity of substitution for this function is unity.

REFERENCES

Arrow, K. J. Essays in the Theory of Risk Bearing. Amsterdam: North-Holland Publishing Co., 1971.

Baron, D. P. "Price Uncertainty, Utility, and Industry Equilibrium in Pure Competition," Inter. Econ. Rev., 11(1970):463-480.

Batra, R. N. and A. Ullah. "Competitive Firm and the Theory of Input Demand Under Price Uncertainty," J. Polit. Econ., 82(1974):537-548.

Behrman, J. R. Supply Response in Underdeveloped Agriculture. Amsterdam: North-Holland Publishing Co., 1968.

Connors, J. F. "A Method for Computing Optimal Decision Rules for a Competitive Firm," Econometrica, 46(1978):615-630.

Freund, R. "The Introduction of Risk into a Programming Model," Econometrica, 24(1956):253-263.

Hanoch, G. "The Elasticity of Scale and the Shape of Average Costs," Amer. Econ. Rev., 65(1975):492-497.

Jensen, M. "Capital Markets, Theory and Evidence," The Bell Journal of Economics and Management Science, 3(1972):357-398.

Just, R. E. Econometric Analysis of Production Decisions with Government Intervention: The Case of California Field Crops. University of California, Giannini Foundation Monograph No. 33, 1974.

Madden, P. Economies of Size in Farming. Agricultural Economic Report No. 107, Economic Research Service, USDA, 1967.

Pasour, E. C., Jr. and J. B. Bullock. "Implications of Uncertainty for the Measurement of Efficiency," Amer. J. Agr. Econ., 57(1975):335-339.

Pazner, E. and A. Razin. "Industry Equilibrium Under Random Demand," European Economic Review, 6(1975):387-395.

Pope, R. D. "Econometric Analysis of Risk Models - Some Explorations and Problems," Paper presented at the WAEA meetings, Bozeman, Montana, July 1978.

Pratt, J. W. "Risk Aversion in the Small and in the Large," Econometrica, 32(1964):122-136.

Sandmo, A. "Competitive Firm Under Price Uncertainty," Amer. Econ. Rev., 61(1971):65-73.

Silberberg, E. "The Theory of the Firm in 'Long Run' Equilibrium," American Economic Review, 64(1974):734-741.

Appendix

CONVEXITY OF R(q)

By definition, $R = E(\pi) - U^{-1}[E(\pi)]$ and

$$R'(q) = \overline{P} - C'(q) - \frac{E[U'(\pi)\ (P - C'(q))]}{U'[U^{-1}\ (E(U(\pi)))]}\ .$$

At equilibrium output, $R'(q)$ becomes (using (1))

$$R'(q) = -\frac{\text{Cov}\ [U'(\pi),\ P]}{E[U'(\pi)]}$$

as given in (2). Differentiating again yields

$$R''(q) = -E[U''(\pi)(P - C'(q))^2]/E[U'(\pi)]$$

which is positive under risk aversion $[U''(\pi) < 0]$ since $E[U'(\pi)] > 0$. Hence, risk aversion implies that the risk premium function is convex in output.

RESPONSE TO AN INPUT PRICE CHANGE

Output Supplied

Consider for simplicity equation (7) with the marginal and average risk premiums as given in Figure 1. The solution to (7) yields optimal output, $q^* = q\ (\sigma,\ \alpha,\ \gamma_1\ .\ .\ .\ \gamma_N)$. Differentiation of (7) with respect to the $j\underline{th}$ input price, γ_j, yields:

$$\text{(A.1)}\quad \frac{\partial}{\partial q}\ [AC(q) + R(q)/q]\ \frac{\partial q}{\partial \gamma_j} + \frac{\partial AC(q)}{\partial \gamma_j} + \frac{\partial [R(q)/q]}{\partial \gamma_j}$$

$$= \frac{\partial}{\partial q}\ [C'(q) + R'(q)]\ \frac{\partial q}{\partial \gamma_j} + \frac{\partial C'(q)}{\partial \gamma_j} + \frac{\partial R'(q)}{\partial \gamma_j}\ .$$

The first term in (A.1) vanishes from first order conditions, (7). Since $C(q) = \sum_{j=1}^{N} \gamma_j\ X_j\ (q,\ \gamma)$, where X_j is the $j\underline{th}$ input, $\partial AC(q)/\partial \gamma_j = X_j/q$.

Marginal cost is given by $\sum_j \gamma_j \, (\partial X_j(q, \gamma)/\partial q)$. Hence, $\partial C'(q)/\partial \gamma_j = \partial X_j/\partial q$.

Since R does not contain γ, it follows that $\partial R'(q)/\partial \gamma_j = \partial[R(q)/q]/\partial \gamma_j = 0$.

Therefore, (A.1) reduces to

$$\text{(A.2)} \quad \frac{\partial q}{\partial \gamma_j} = \left[\frac{1}{C''(q) + R''(q)}\right] \frac{X_j}{q} \left[1 - \eta_{jq}\right]$$

where $\eta_{jq} \equiv (\partial X_j/\partial q)\,(q/X_j) \equiv$ the output elasticity as defined by Silberberg.

Second order conditions require that $C''(q) + R''(q) > 0$. Hence, (A.2) implies

$$\frac{\partial q}{\partial \gamma_j} \gtreqless 0 \text{ as } \eta_{jq} \lesseqgtr 1.$$

<u>Factor Demands</u>

The indirect SUAC function $SUAC^* = AC(q^*) + R(q^*)/q^*$ where $q^* = q(\gamma, \sigma, \alpha)$ is optimal output (the solution to (7)). Differentiation of SUAC* with respect to γ_j yields:

$$\frac{\partial SUAC^*}{\partial \gamma_j} = \frac{\partial}{\partial q}\,[AC(q) + R(q)/q]\,\frac{\partial q}{\partial \gamma_j} + \frac{\partial AC(q)}{\partial \gamma_j} + \frac{\partial [R(q)/q]}{\partial \gamma_j}$$

$$\text{(A.2)} \quad = 0 + \frac{X_j}{q} + 0 = \frac{X_j}{q}\,.$$

Therefore, increases in an input price raises SUAC* and $\overline{P}$.

Since SUAC* is assumed concave in input prices, it follows that $\partial^2 SUAC^*/\partial \gamma_j^2 < 0$; or using (A.2)

$$\text{(A.3)} \quad \frac{\partial^2 SUAC^*}{\partial \gamma_j^2} = \frac{\partial \left(\frac{X_j}{q}\right)}{\partial \gamma_j} < 0.$$

Hence, relative demands (X_j/q) are negatively sloped. Since $\partial(X_j/q)/\partial\gamma_j = [q(\partial X_j/\partial\gamma_j) - X_j\ (\partial q/\partial\gamma_j)]q^{-2}$, it follows that (A.3) together with (A.2) may imply that long run factor demands are negatively sloped. For example, if $\partial q/\partial\gamma_j = 0$ as in the case of homothetic production and certainty, then actual and relative demands must be downward sloping.

FIGURE 1

Long Run Equilibrium and Risk

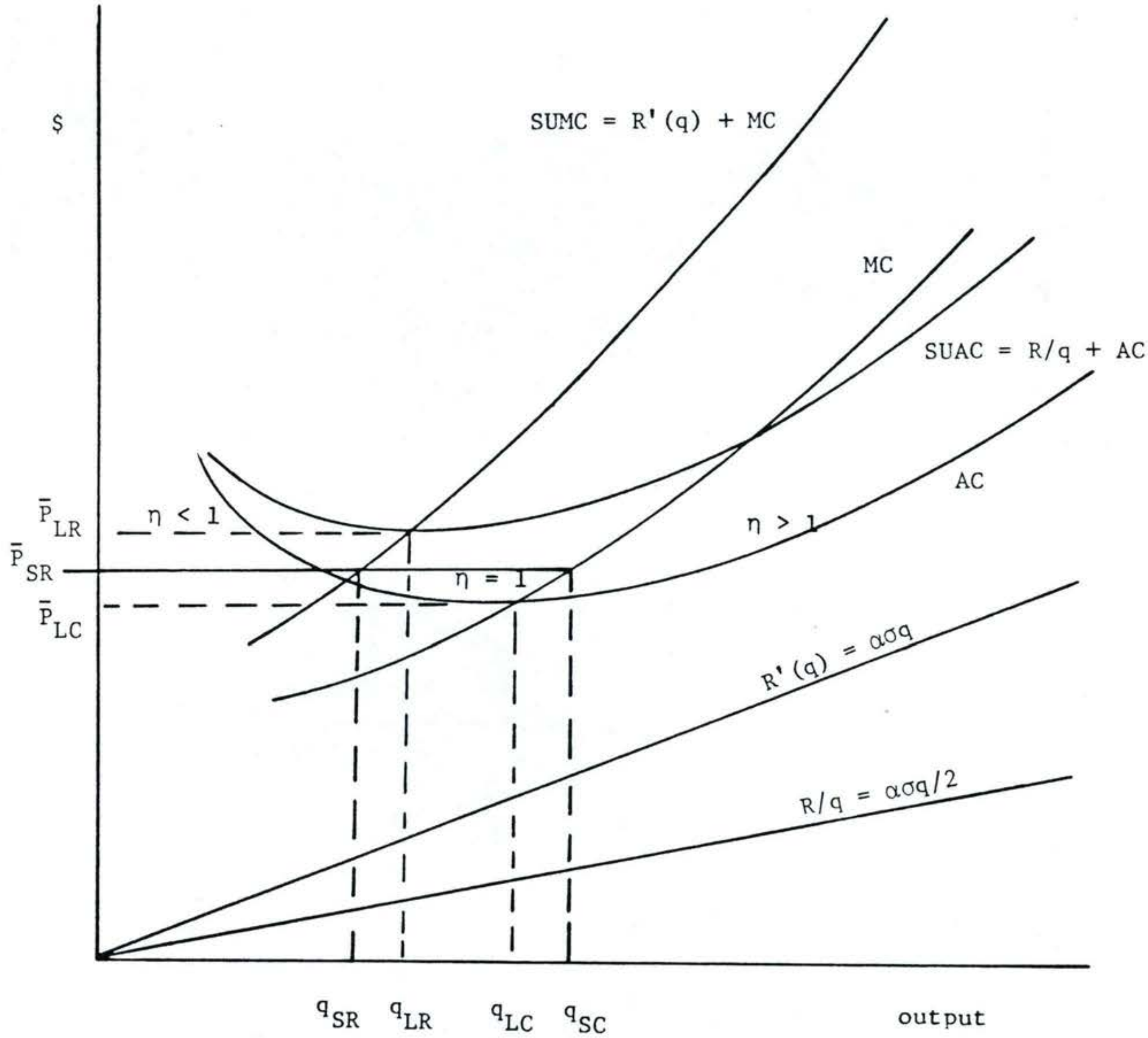

FIGURE 2

Displacement of Long Run Equilibrium Under Risk and

Homothetic Technology

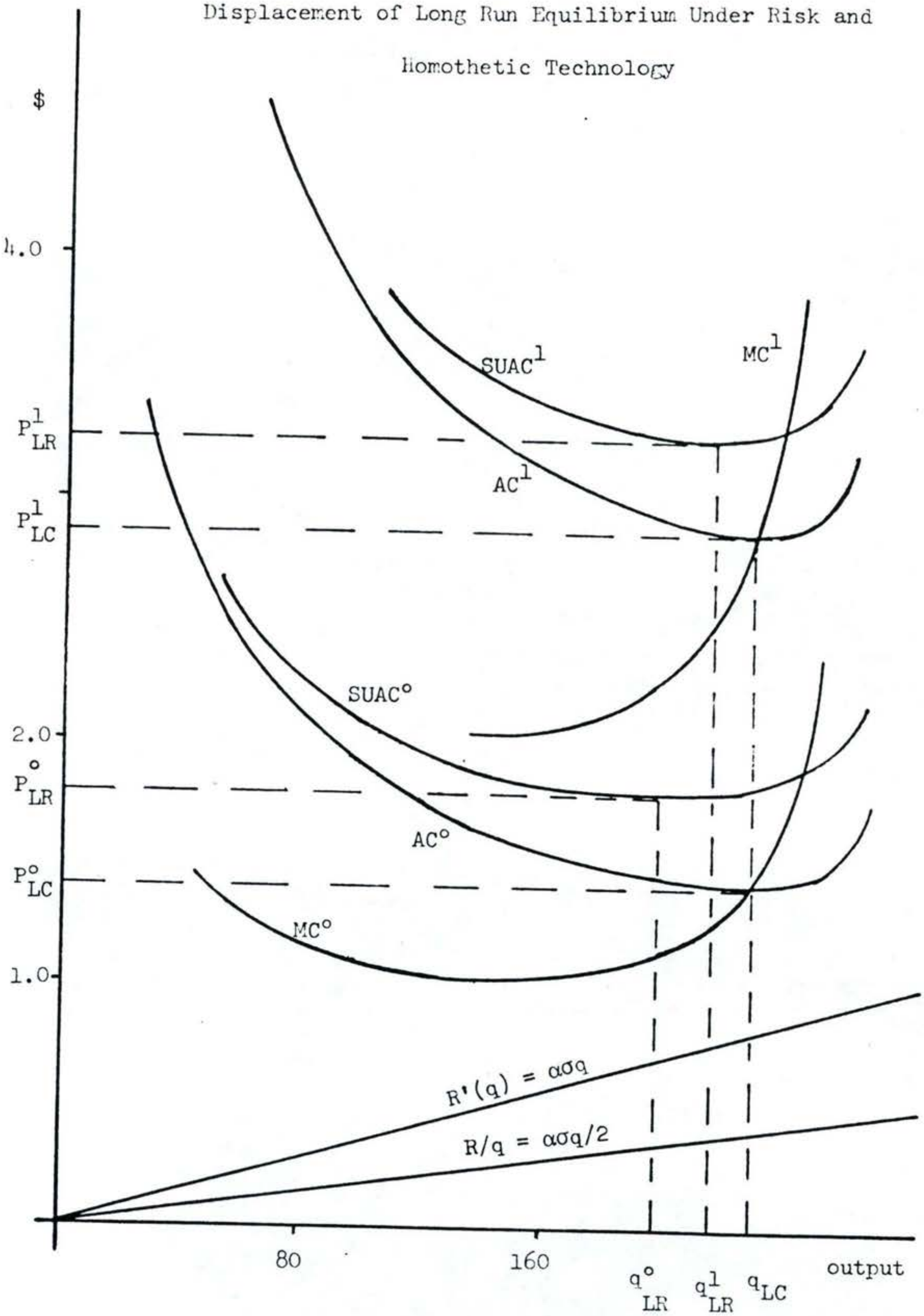

FIGURE 3

Determination of the Optimal Size with Decreasing Average Costs and Constant Risk Aversion