PROGRAMMING FOR EFFICIENT PLANNING AGAINST NON-NORMAL RISK

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A planning methodology is developed based on Monte Carlo sampling of plans and sorting out inefficient plans according to the rules of stochastic dominance. Illustrations and methodological comparison are made in a context of farm planning under risk, and an application in income stabilization research is indicated.

Significant developments in the field of planning under risk have been made in the last two decades or so [15, 22, 25, 31, 36] and have often featured the work of agricultural economists. Attention in agricultural economics has recently been concentrated on planning methods that can be cast in a mathematical programming (particularly linear programming [7, 10, 20, 24, 31]) mould. Some of these methods make direct appeal to normality in the probabalistic specification (i.e. to the multivariate normal or Gaussian distribution) [10, 18], while others depend on the mean-variance criterion or, equivalently, a quadratic utility function [20, 32]. Others that do not presume normality or quadratic utility feature other limitations such as implied zero correlations between activity returns [7] or arbitrary criteria of the safety-first type [24, 31].

There is evidently no panacea in the field of programming for risky planning. However, the intention in this paper is to seek and, hopefully, to approach a methodology that has comparatively few limitations of both a theoretical and practical nature. Briefly, the suggestion pursued here is based on (a) a flexible means of generating feasible plans, namely Monte Carlo programming, and (b) a powerful means of identifying and ordering plans that are efficient according to the criteria of stochastic dominance.

Desiderata for Risky Programming Methodology

Users of linear programming have long bewailed the difficulties inherent in the basic assumptions of continuity, linearity, additivity and convexity that characterize the algorithm [21]. While it is sometimes possible to handle several indivisible resources in an integer programming algorithm [8] and while it may be possible to deal with various non-linearities in a separable programming algorithm [35], these algorithms may not be readily accessible to potential users. Taken together, such difficulties are not readily amenable to analytic solution. In contrast, Monte Carlo 'programming' [9, 14, 16, 33, 34] can readily accommodate restrictions to integer activity levels and non-linear production relationships—albeit at the cost of explicit optimization.

Packaged computer programs for Monte Carlo programming are now available [16, 34] and offer convenience and flexibility. However, the

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method is of such simplicity that users can readily contemplate writing their own routines. Briefly, the method consists of selecting activities by pseudo-random (Monte Carlo) sampling and expanding their levels to the limits of the available resources.[1] The feasible plans may then be subjected to some quality test and those that pass stored. Eventually, the stored plans may be sorted according to specified criteria and displayed informatively in various merit lists to aid planners in identifying the most desired plan(s).

Little attention appears to have been given to accommodating a dimension of risk in Monte Carlo programming. Many possibilities do, however, exist for such risk accounting. For instance, if a variance-covariance matrix is included to complement the mean activity gross margins, and sorting of plans is based on the mean and variance of total gross margin, then Monte Carlo programming can readily mime a quadratic programming algorithm but can still include, of course, the advantageous flexible features noted above. Alternatively, Monte Carlo programming can analogously mime alternative algorithmic risk specifications such as MOTAD programming [20] and focus-loss constrained programming [7, 24].

In considering alternative means of specifying risk in a programming formulation it is pertinent to enquire as to the realism, accuracy and adequacy of available alternatives. Perhaps the most important general considerations should be that statistical dependence between activity returns be explicitly modelled. As well as being an intuitively important element in the portfolio-like problem setting, the high correlations reported in empirical studies [18, 20, 21, 24] lead one to suspect that denying the stochastic dependencies would be tantamount to folly.

The multivariate normal is the distribution most often employed in risk specification for programming and portfolio selection models and, of course, there are good reasons for this.[2] First, a multivariate normal distribution is completely described by the (marginal) means and variances and the pairwise correlation coefficients. That is, it has a very economical parametric structure, and dependencies are captured completely in the simple correlations. Secondly, since the normal is a member of the family of stable distributions [30], the (univariate) distribution of a weighted sum or linear combination of activities from a multivariate normal distribution is a member of the same family, namely normal in this case. Thirdly, the assumption of (at least approximate) normality has been rationalized empirically by appeal to the Central Limit Theorem [26] to the extent that risk can be explained as the summation of independent additive stochastic effects.

However, (albeit limited) analysis of crop yield data [1, 13] and experience with simulation of farm production processes from climato-

[1] Anticipating discussion of the procedure for identifying risk efficient plans, it should be noted that one methodological shortcoming of the version of Risk Efficient Monte Carlo Programming (REMP) used in this paper is that it employs the conventional Monte Carlo procedure of expanding each activity level up to the maximum implied by remaining resources. However, it is possible that some feasible plans which deliberately leave some resources unused may also properly belong in the risk efficient set. Such plans cannot be identified by this version of REMP.

[2] Sengupta [31] has explored use of non-normal distributions in mathematical programming models, including the chi-square, exponential and truncated normal distributions.

logical data suggest that normality is not always a very accurate assumption. Indeed, pure intuition leads one to reject normality as a precise description of risk since activity returns do not logically range from negative to positive infinity. At best normality is a convenient approximation—at worst it may lead to risk analysis that is quite misleading.

Returning to the questions of realism, accuracy and adequacy of risk specification, the following suggestions are made to complement the earlier-noted need to account for non-independence of activity returns. Such distributions should, where applicable, be specifiable as asymmetric marginal distributions. Seemingly, empirical yield and revenue distributions are nearly always skewed to some extent. Finally, marginal distributions for activity returns and other stochastic components should, to accord with reality, have finite ranges. A thoroughgoing accounting for risk should identify such ranges explicitly if the risky planning environment is to be confronted squarely and completely.

The final desideratum contemplated here concerns the handling of the attitudes to risk held by the decision maker on whose behalf the programming is done. Modern decision theory [4, 15] has much to say about how such attitudes may be conceptualized, elicited and represented but little to say—beyond the fact that each individual's attitudes are probably unique—about precisely what they are. There are two exceptions to this generality that are increasingly finding their way into modern economic analysis, namely the now more-or-less uncontroversial presumptions that (a) individuals are generally averse to risk and (b) are decreasingly so as they become wealthier [5, 28].

These two behavioural assumptions, when taken together with the notion that rational decision makers act so as to maximize expected utility, lead to the 'stochastic dominance' rules for ordering risky prospects into unpreferred and 'risk-efficient' sets [19, 37]. Within an efficient set, it is not possible to say which prospects would be preferred without knowing more about the decision maker's attitudes than is contained in the corresponding underlying assumption. Application of these ordering rules requires complete descriptions of the probability distributions considered and may prove computationally tedious unless implemented on an electronic digital computer [3]. In the context of risk programming, the stochastic dominance ordering rules offer the means for saying as much as is possible about the efficiency of feasible plans in the absence of more specific knowledge of individuals' risk attitudes.

The stochastic dominance ordering rules imply a loosely defined objective function and, therefore, yet another reason for preferring Monte Carlo programming as the generator of feasible plans. Mathematical programming algorithms are efficient generators of plans for a particular specification of the objective function but it is most unlikely that these plans will encompass the risk-efficient set of plans. Monte Carlo programming appears to be the best and may be the only reasonable way of sampling the feasible plans without constraining the risk-efficient set. Since, in principle, the risk-efficient set always contains the plan with the highest possible average gross margin, it is probably worthwhile to supplement the Monte Carlo procedure by including, as the first plan to be reviewed, the linear programming (or the corresponding integer programming) solution based on expected activity returns.

An Approach

The desiderata discussed provide a blueprint for at least one method of programming under risk which can now be described succinctly since the rationale has already been sketched. There are two central components, namely a Monte Carlo programming routine which serves to generate feasible plans (the particular routine used was developed by Donaldson and Webster [6]) and a routine for reviewing stochastic dominance which serves to sort feasible plans into (preserved) efficient plans and (discarded) inefficient plans (the particular routine used is reported in Anderson [3]).

The missing link between these components concerns the required complete description of the probability distributions associated with the returns from generated plans and largely involves deciding how risk associated with each activity is best described. The following suggestion is offered as a pragmatic fulfilment of the stated desiderata without any further comment on its arbitrariness.

Consider a vector of activity unit net returns, $c' = \{c_1, c_2, \ldots, c_n\}$ which has some multivariate distribution and a corresponding vector of fixed activity levels, $x' = \{x_1, x_2, \ldots, x_n\}$ which together define the random variable, total gross margin,

$$T = c'x = \sum_{i=1}^{n} x_i c_i.$$

There are few parameters of c which combine simply to define informative parameters of the distribution of T. For instance, it is widely appreciated that, using E to denote the expected value operator,

$$E(T) = \sum_{i=1}^{n} x_i E(c_i),$$

or the mean of a weighted sum is the weighted sum of the means. Similarly, another widely used non-parametric result applying to linear combinations of random variables defines the variance,

$$V(T) = \sum_{i=1}^{n} \sum_{j=1}^{n} x_i x_j \operatorname{cov}(c_i, c_j),$$

where $\operatorname{cov}(c_i, c_j)$ denotes the covariance and, if $i = j$, the respective variance. If the standard deviation of c_i is σ_i and the simple correlation between c_i and c_j is ρ_{ij}, the variance equation may be expressed perhaps more familiarly, as

$$V(T) = \sum_{i=1}^{n} x_i^2 \sigma^2 + 2 \sum_{i>j}^{n} \sum_{j=1}^{n} x_i x_j \rho_{ij} \sigma_i \sigma_j.$$

Analogous simple operations cannot be performed on parameters such as the modes of the elements of c, or on fractiles of these elements such as the median and the quartiles—*excepting* on the zero and unit fractiles, namely the ranges of the marginal distributions. Denoting the lower range of c_i by a_i and the upper range by b_i, the lower and upper ranges of T are given by, respectively,

$$A(T) = \sum_{i=1}^{n} x_i a_i$$

and

$$B(T) = \sum_{i=1}^{n} x_i b_i.$$

It is thus proposed to specify the marginal distributions of each of the c_i by the four parameters a_i, b_i, $E(c_i)$ and σ_i^2 (i.e. range endpoints, mean and variance) and to capture interdependencies solely in terms of simple correlations, ρ_{ij}. The form of multivariate distribution is deliberately left unstated except to note that for finite ranges it is certainly not normal and that it is required to be other than a multivariate Beta distribution. The c_i variables cannot have Beta distributions since this assumption is being preserved for the T variable and, since the Beta is not a member of the stable family of distributions [30], the c_i and T distributions must be from different families. The outlined specification accords with the stated desiderata and leads to a description of the distribution of T in terms of the four parameters, $A(T)$, $B(T)$, $E(T)$ and $V(T)$.

To complete the link to the demanding requirements of the stochastic dominance criteria two further steps are required. First, there is a step of inference to go from just the four parameters to a complete description of the probability distribution of T. Secondly, there is a step of approximation which permits an approximate representation of the complete distribution as a linear-segmented cumulative distribution function. For both these steps, the Beta distribution serves most usefully. It is almost ideal for the inference step because the Beta family of distributions is remarkably rich and includes members with wide diversity of shapes of density functions. As U, J and L shaped densities seem out of place in a farm planning context, attention is confined here to single-humped densities where probability is concentrated in some interval within the specified range, A to B. This is equivalent to dealing only with Beta distributions having shape parameters [27], p, $q \geqslant 1$, but a great variety of combinations of spread, skewness and kurtosis is accommodated within this set.

The four parameters A, B, E, V uniquely determine a fitted Beta distribution for which the shape parameters are easily solved. First the mean and variance are standardized for the corresponding Beta distribution on the interval (0, 1) as

$$e = (E - A)/(B - A)$$
$$v = V/(B - A)^2,$$

and the shape parameters are then found directly [29] as

$$p + q = [e(\mathrm{v} - e)/v] - 1,$$
$$p = e(p + q).$$

In general, p and q so found will not be integers. This completes the first step of identifying a unique distribution for the returns from any plan.

Deployment of the stochastic dominance ordering rules requires in general, comparisons not only of entire cumulative distribution functions (CDFs) but also of other functions derived from CDFs through successive integrations. For continuous distributions, this proves extremely difficult unless the CDFs are approximated by a number of linear segments chosen to be large enough to permit a 'good' approximation [3]. The pragmatic suggestion is made here that an approximation that is 'good enough' can be obtained by fitting twenty linear segments each spanning a cumulative probability interval of 1/20. This is done by specifying the 19 'twentiles' $f_{.05}, f_{.1}, \ldots, f_{.95}$ which are conveniently tabulated [29] for standardized Beta distributions with integer shape parameters $p \leqslant 14$ and $(p + q) \leqslant 29$. For shape parameters beyond the bounds of the tabulation, a normal approximation to the standardized Beta is quite good [29] and permits use of the corresponding 19 standard normal fractiles based on a mean of $(p - 1)/(p + q - 2)$ and a variance of $[(p - 1)/(p + q - 2)]\,[q - 1)/(p + q - 2)]/(p + q - 2)$. Within the scope of the table, the non-integer solutions for p and q imply a necessity to interpolate the required fractiles from the fractiles of the four Beta with integer parameters that embrace the specified distributions. Various relatively precise methods of interpolation have been suggested for analogous interpolations [27] but linear interpolation seems quite adequate in practice, especially where the Beta is already being used as an approximating distribution. Another possibility for computer-implemented applications is to evaluate required points on the CDFs for any values of the parameters, by numerical integration. Since numerical integration is a slow process relative to looking up a table, the more accurate computation of twentiles and/or the more precise approximation of CDFs by larger numbers of fractiles (e.g. 99 percentiles) are probably not worthwhile because of the added computational burden.

With the linkage to the stochastic dominance routines now completed, it is a straightforward matter to arrange for stochastically inefficient generated plans to be sifted out and for the process to continue until an arbitrarily specified number of efficient plans is found and then described in detail. This process is now illustrated through some empirical examples. For the sake of brevity, the procedure will be referred to as 'REMP', an abbreviation for Risk Efficient Monte-Carlo Programming.

An Illustration

The example initially chosen to illustrate the REMP method is Hazell's [20] four-enterprise vegetable farm. Apart from relative simplicity, this example has the advantage that Hazell has given the quadratic programming and MOTAD solutions.

The data he uses consist of a time series of six yearly observations on returns from the four enterprises. Formal test of the normality of each set of these few observations reveals, not surprisingly, that the null hypothesis of normality cannot be rejected at conventional levels of statistical significance. However, we know from intuition that the underlying probability distributions cannot be truly normal, and this intuition is supported by plotting the empirical distribution functions on normal probability paper. The distributions for the celery and pepper returns in particular appear to be distinctly non-normal.

Discarding the assumption of normality and recognizing that the distributions must have finite ranges, estimates were made of the distributions from which the data are to be presumed to have come. The procedure used is a relatively subjective but reasonable [2] one based on hand-sketching of CDFs through plotted estimates of fractiles [1] based on the sets of six observations. This is illustrated in Figure 1 by the smooth curve and the crosses representing the ranked data or fractile estimates. In this application, the primary purpose of this procedure is to estimate the extrema of the range of the distribution of enterprise returns as these data are required for the non-normal risk specification. Clearly this is subjective and less than perfect but the reproducibility of

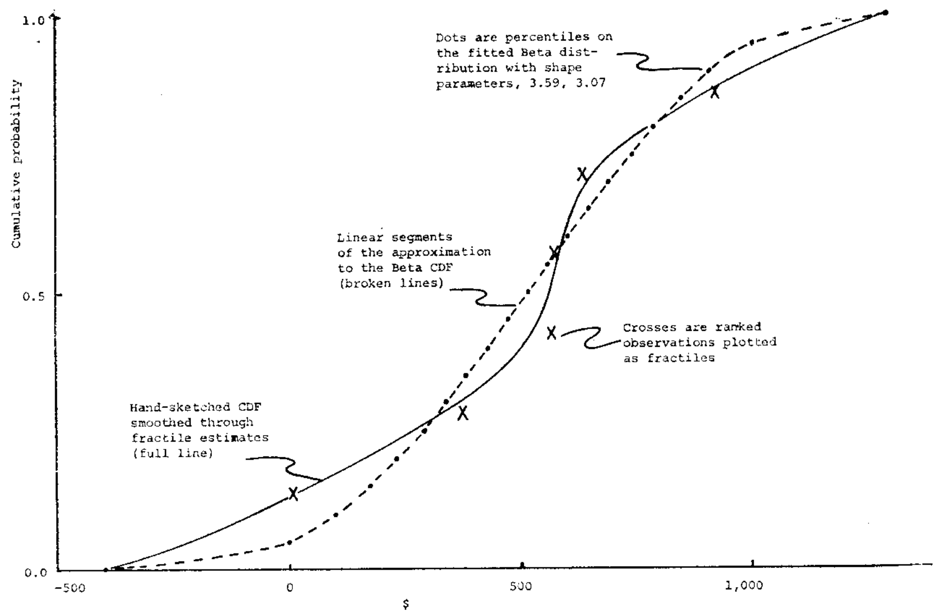

FIGURE 1—Estimation and description of Hazell's distribution for gross margin from peppers.

the results will hopefully be demonstrated to the interested reader who cares to sketch his own subjective CDF through the data (crosses) while following the guideline that such a CDF is monotonically increasing, has positive slope, finite range and is unimodal.

Such CDFs could also be used to estimate moments of the distribu-

TABLE 1

Parameters for Hazell Enterprise Revenues

	Carrots	Celery	Cucumbers	Peppers
Expected values ($/ac)	253	443	284	516
Lower extrema ($/ac)	20	—500	90	—400
Upper extrema ($/ac)	640	1,200	510	1,300
Variances & covariances ($/ac)				
Carrots	11,264	—20,548	1,424	—15,627
Celery		125,145	—27,305	29,297
Cucumbers			10,585	—10,984
Peppers				93,652

tions [1] but to deviate minimally from Hazell's analysis, his conventional unbiased estimates of the required means and covariance matrix are taken. Such estimates are especially convenient for estimating the implied correlations, as these are tedious to estimate subjectively [4]. The sketching exercise provides the only data required in addition to those used and given by Hazell [20, p. 60]. All these data are presented in Table 1.

This farm planning problem was then implemented according to the REMP method with the instruction that the generation and sorting continue until 20 plans which are stochastically efficient of degree three (risk efficient) were found. In the event, this required the generation and review of only 48 feasible plans. The efficient plans are of little intrinsic interest in this discussion but it seems relevant to make overall comparisons with those obtained by other methods. A comparison of probably most importance is with plans based only on the mean-variance (E-V) criterion of efficiency, such as are obtained with quadratic programming. It is instructive to make such comparisons in mean-variance (of total gross margin) space. This is done in Figure 2 which depicts the E-V efficient frontier (interpolated from Hazell's change-of-basis solutions found with quadratic programming and represented by crosses in Figure 2) and the mean-variance pairs associated with the 20 risk efficient plans (represented by dots).

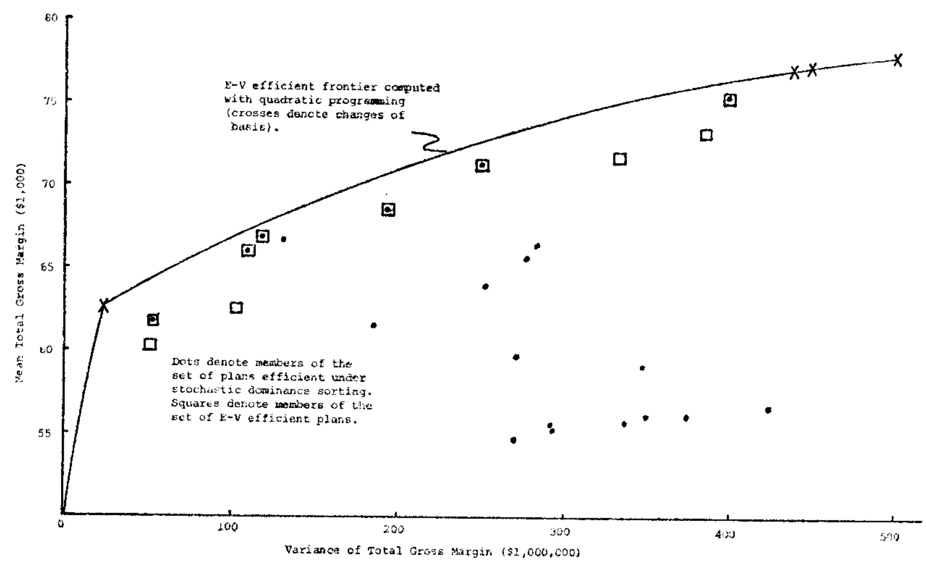

FIGURE 2—Mean-variance characteristics of two sets of "efficient" farm plans.

The version of REMP used for generating the results presented in this paper did not include the suggested early review in the considered plans of the optimal plan obtained by linear programming with mean activity returns. In terms of Figure 2, this optimal plan is denoted by the cross with the greatest mean (and variance of) gross margin. While it is a good idea to include this plan routinely (and thereby add a dot and a square to the highest cross in Figure 2), it was found that the procedure caused no significant change in the pattern of results presented, although the composition of efficient sets changed slightly.

In terms of E-V efficiency, the risk efficient plans clearly perform poorly when compared with the E-V efficient frontier. The question must be asked as to what extent this apparently poor performance is due to the very limited sample of plans considered. This is answered by reviewing the *same* 48 pseudo-randomly generated plans according to the E-V criterion rather than the stochastic dominance criteria. Only 10 of these plans were in the E-V efficient set and these are represented by squares in Figure 2. Note that there is considerable overlap between the efficient sets. It can be seen that these plans all lie relatively close to the efficient frontier, suggesting that the greater distance of the risk efficient plans from the E-V frontier is not the result of having taken only a small sample of plans.

Of course, it is inappropriate to condemn the non-normal stochastic efficiency sorting on the basis of E-V analysis. The results can be put in a more favourable light by presenting similar data with an alternative (although crude and imperfect) measure of risk such as reported in Figure 3. Here risk is measured by the maximum possible 'loss' associated with each plan in both the E-V efficient and risk efficient sets. Each loss is the lower bound of the respective gross margin distribution and the scale is constructed with the negative values rightmost to emphasize that a greater loss (a more negative gross margin) is symptomatic of a greater risk. Judged by this measure, the REMP plans appear generally less risky than the E-V efficient plans.

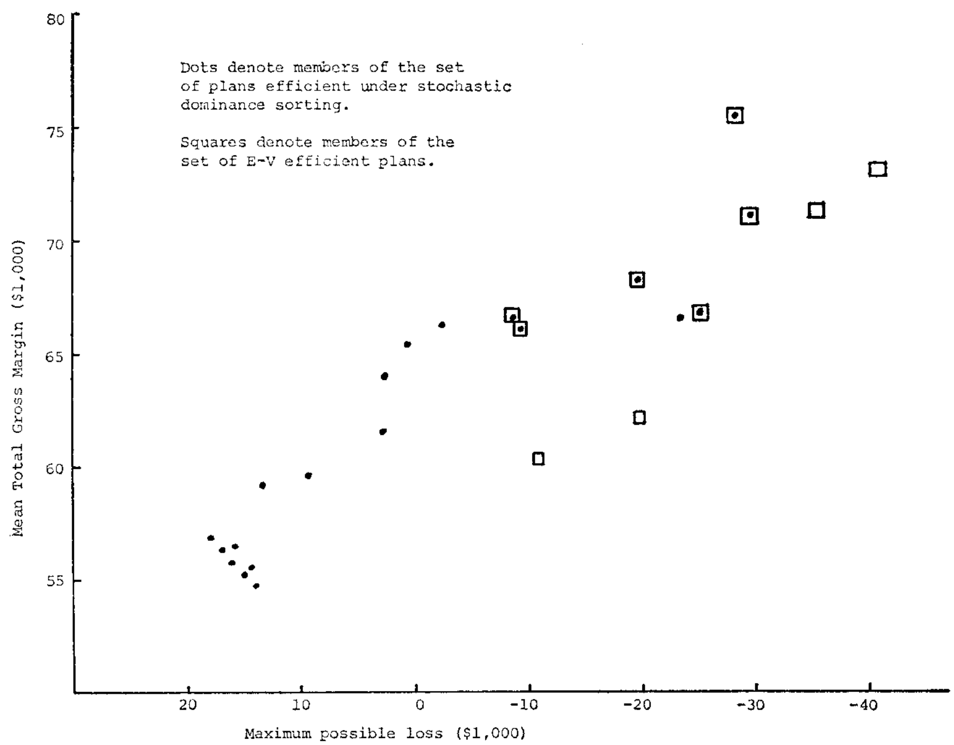

FIGURE 3—Mean-Risk characteristics of two sets of "efficient" farm plans.

An Application in Analysis of Stabilization Policy

The proposed REMP method of risky farm planning appears to have a place in analysis of stabilization policy if questions are asked such as: what effect might a specific policy instrument induce on farm organization? The example chosen to illustrate such policy analysis with the REMP method is the representative farm for the N.W. Slopes of N.S.W. as described by Kennedy and Francisco [24]. The data they present (returns for seven enterprises over five years) permit estimation of the required covariance matrix. The range extrema were estimated in the manner described for the Hazell example. The Kennedy-Francisco example is not ideal because the gross margins are regional rather than individual farm data. However, it offers convenience, simplicity and accessibility and accordingly is taken to illustrate some aspects of the impact of taxation measures as stabilization instruments.

Three situations are compared, namely (a) the most typical farm planning ploy of completely ignoring income taxation, (b) the Australian income taxation system as it presently applies to primary producers [12] (Chisholm [11] averaging Type II) and (c) an analogous marginal adjustment system (Chisholm [11] averaging Type III). The no-tax situation is a straightforward application of REMP but, in situations (b) and (c) it is necessary to convert the distributions of total gross margins for generated plans to distributions of post-tax income in order to examine the stabilizing or otherwise effects of taxation. This conversion is approximated here by sampling 105 variates from each total gross margin distribution. Assuming these variates represent 105 sequential annual gross margins, the income averaging and tax calculations were applied and the last 100 post-tax incomes were recorded. The ranges, mean and variance of each sequence of post-tax incomes then enabled the fitting of a Beta distribution so that these could be sorted in the regular REMP manner.

All three sets of risk-efficient plans included plans with lambs or merinos or cattle as the dominant livestock enterprise and pasture entered all plans at the maximum level permitted. All the efficient plans involved wheat and/or sorghum at significant levels but barley only entered a very few plans and then at trivial levels. The only distinguishing feature between the efficient sets was the level of sorghum and its relationship to the sheep enterprise. The most profitable plans in terms of expected total gross margin involve maximum levels of lambs and sorghum (370 ac.) and no wheat and merinos [24, p. 138].[3] One quarter of the 'no-tax' risk-efficient plans were of this type (with $>$ 300 ac. sorghum) whereas such plans were all eliminated from both 'tax payable' sets. The risk-efficient sets for the 'tax payable' situations were effectively identical with but one exception. Under the 'present tax system', two efficient plans appeared that were eliminated from both the other efficient sets. These two plans involved high ($>$ 300 ac.) levels of sorghum, and merinos at the maximum level permitted.

The result that these alternative schemes for averaging for purposes of calculating income tax commitments cause virtually no change in the diverse risk-efficient set of farm plans may be surprising but it is believed

[3] There is a mis-print in Kennedy and Francisco's [24, p. 138] Table 3. The three highest E(TGM) plans should all include 996 lambs.

to be an important finding. For instance, if it is generally true, it implies that policy initiatives with the taxation system which are directed at stabilizing the incomes of risk-averse farmers can be evaluated simply in terms of income stability and probably need not be complicated by attempts to account for price effects caused by policy-induced changes in aggregate levels of production.

The apparent constancy of the set of efficient plans does not, of course, mean that adjustments to the income tax provisions are ineffectual in the context of income stabilization. Table 2 reports the summary parameters defining the three probability distributions used to describe a particular plan that is common to the three efficient sets. This plan consists of 996 lambs, 830 ac. pasture, 96 ac. wheat and 274 ac. sorghum. Judging by the standard deviation and lower bound of post-tax income, the marginal adjustment system is more stabilizing than the present averaging system.

TABLE 2

Parameters of Distributions Alternatively Describing Income from a Farm Plan

Parameter of distribution of income after fixed costs and (where relevant) tax	No tax (or averaging)	Present Australian averaging system	Marginal adjustment averaging system
	$	$	$
Expected value	7,500	5,765	5,692
Standard deviation	4,957	3,586	2,284
Lower extremum	—4,718	—4,718	—3,984
Upper extremum	23,222	12,464	9,034
Shape parameters of fitted Beta distributions	2·98 3·84	2·72 1·74	3·86 1·33
Average tax payment	0	1,735	1,808

The Question of Estimational Risk

The REMP method and indeed all previously reported methods of risky farm planning (not to mention the great volume of literature on portfolio selection) proceed as if the specification of the problem involves no uncertainty whatsoever. For instance, the REMP method presumes implicitly that all the means, covariances and range extrema are known with certainty. In reality, of course, they are generally only estimates and are estimates based (at least in the elaborated examples) almost inevitably on very sparse data. Is then such risk analysis an exercise in folly because it abstracts from the inherent estimational risk?

There appear to be three broad approaches to broaching this question. First, an extreme subjectivist view would answer in the negative with the proviso that, if the specification of a risky planning problem accorded with the beliefs of the relevant decision maker, then there is no estimational risk, irrespective of the origin of the data. Secondly, a *reductio ad absurdum* type of argument would emphasize that any planning algorithm (e.g. deterministic linear programming) is essentially subject to analogous estimational risks and since decisions must be taken they may as well be based on the best available information. That is, simply assume away the inconvenience of any estimational risk!

Somewhere between these extremes a third approach of active concern for estimational risk might be identified. Some little attention has been given to this issue in E-V portfolio selection ranging from considering uncertainty about means [23] and about means and covariances [6, 17]. The latter authors' works are not strictly comparable but they reached the contrary conclusions that on the one hand estimational risk militates against identification of truly efficient portfolios [17] while on the other hand it interferes insignificantly with efficiency and only increases the riskiness of the efficient set [6]. It would be good to have some more definitive results on the impact of estimational risk in such planning models.

Farm planners seemingly have not evidenced any concern for estimational risk. In principle the question could be tackled through sensitivity analysis by systematic perturbation of all the uncertain parameters of a model. But this is anything but a trivial exercise since with n enterprises there are approximately $n\ (4 + n/2)$ parameters in the REMP method and sensitivity must be appraised by inspecting vectors of enterprise levels in efficient sets of plans. Such a foreboding task has been shirked here, as has also the testing for possible errors incurred in REMP through resort to Beta distributions and subsequent approximation with linear segmented CDFs. Figure 1 depicts, for example, one such linearized Beta CDF fitted to the pepper distribution to have the same range, mean and variance. However, to test the impact of the Beta assumption would require its substitution with some less convenient and probably no better distribution. Of course, this is a rationalization rather than a justification for using the Beta distribution.

Concluding Remarks

A method has been presented and illustrated for enterprise planning against non-normal risk. The method is conceptually simple and has some desirable flexible features. However, it is computationally demanding and provides solutions consisting of sets of efficient plans which will probably be of diverse structure and therefore difficult to interpret.

In the absence of problem-oriented applications in farming practice, conjecture about any advantages or disadvantages relative to alternative risk-programming approaches [24, 25] or, indeed, programming formulations that ignore risk, is difficult. However, when risk is non-normal or utility is not quadratic, and when the extent of farmers' risk aversion is unknown, the REMP method offers a theoretically acceptable and a practicable planning approach. The approach may be of value to policy analysts because of its facility for handling non-normal risk, since most policy instruments (e.g. crop insurance schemes, drought bonds, taxation measure and emergency relief schemes) would serve to modify any normally distributed pre-tax incomes to very definitely non-normal post-tax incomes.

References

[1] Anderson, J. R., 'Sparse Data, Climatic Variability and Yield Uncertainty in Response Analysis', *American Journal of Agricultural Economics*, 55: 77-82, 1973.

[2] Anderson, J. R., 'Sparse Data, Estimational Reliability, and Risk-Efficient Decisions', *American Journal of Agricultural Economics*, 56: 564-72, 1974.

[3] Anderson, J. R., 'Risk Efficiency in the Interpretation of Agricultural Production Research', *Review of Marketing and Agricultural Economics*, 42: 131-84, 1974.

[4] Anderson, J. R., Dillon, J. L. and Hardaker, J. B., *Decision Analysis in Agricultural Management*, unpublished, Armidale, 1975.

[5] Arrow, K. J., *Aspects of the Theory of Risk-Bearing*, Yrjo Jahnsson Saatio, Helsinki, 1965.

[6] Barry, C. B., 'Portfolio Analysis Under Uncertain Means, Variances, and Covariances', *Journal of Finance*, 29: 515-22, 1974.

[7] Boussard, J. M. and Petit, M., 'Representation of Farmers' Behaviour Under Uncertainty with a Focus-Loss Constraint', *Journal of Farm Economics*, 49: 869-80, 1967.

[8] Candler, W., Cartwright, W. and Penn, J. B., 'The Substitution of Analytic for Simulation Algorithms: A Comment', *American Journal of Agricultural Economics*, 55: 235-39, 1973.

[9] Carlsson, M., Hovmark, B. and Lindgren, I., 'A Monte Carlo Method Study of Farm Planning Problems', *Review of Marketing and Agricultural Economics*, 37: 80-103, 1969.

[10] Chen, J. T. and Baker, C. B., 'Marginal Risk Constraint Linear Program for Activity Analysis', *American Journal of Agricultural Economics*, 56: 622-27, 1974.

[11] Chisholm, A. H., 'A Comparison of Income Averaging Procedures for Income Tax Purposes', *Australian Journal of Agricultural Economics*, 15: 36-50, 1971.

[12] Commonwealth Government Printer, *Income Tax for Primary Producers*, Canberra, 1970.

[13] Day, R. H., 'Probability Distributions of Field Crop Yields', *Journal of Farm Economics*, 47: 713-41, 1965.

[14] Dent, J. B. and Thompson, S. C., 'The Application of Monte Carlo Techniques to the Feedmix Problem', *Farm Economist*, 11: 230-48, 1968.

[15] Dillon, J. L., 'An Expository Review of Bernoullian Decision Theory in Agriculture', *Review of Marketing and Agricultural Economics*, 39: 3-80, 1971.

[16] Donaldson, G. F. and Webster, J. B. G., *An Operating Procedure for Simulation Farm Planning—Monte Carlo Method*, Wye College, Department of Agricultural Economics, 1968.

[17] Frankfurter, G. M., Phillips, H. E. and Seagle, J. P., 'Portfolio Selection: The Effects of Uncertain Means, Variances and Covariances', *Journal of Financial and Quantitative Analysis*, 6: 1251-62, 1971.

[18] Freund, R. J., 'The Introduction of Risk into a Programming Model', *Econometrica*, 24: 253-63, 1956.

[19] Hadar, J. and Russell, W. R., 'Rules for Ordering Uncertain Prospects', *American Economic Review*, 59: 25-34, 1969.

[20] Hazell, P. B. R., 'A Linear Alternative to Quadratic and Semivariance Programming for Farm Planning Under Uncertainty', *American Journal of Agricultural Economics*, 53: 53-62, 1971.

[21] Heady, E. O. and Candler, W., *Linear Programming Methods*, Iowa State Univ. Press, Ames, 1958.

[22] Johnson, S. R., 'A Re-examination of the Farm Diversification Problem', *Journal of Farm Economics*, 49: 610-21, 1967.

[23] Kalymon, B. A., 'Estimation Risk in the Portfolio Selection Model', *Journal of Financial and Quantitative Analysis*, 6: 559-82, 1971.

[24] Kennedy, J. O. S. and Francisco, E. M., 'On the Formulation of Risk Constraints for Linear Programming', *Journal of Agricultural Economics*, 15: 129-46, 1974.

[25] Lin, W., Dean, G. W. and Moore, C. V., 'An Empirical Test of Utility vs. Profit Maximization in Agricultural Production', *American Journal of Agricultural Economics*, 56: 497-508, 1974.

[26] Mihram, G. A., *Simulation: Statistical Foundations and Methodology*, Academic Press: New York, 1972.

[27] Pearson, K., Ed., *Tables of the Incomplete Beta Function*, Cambridge University Press for the Biometrika Trustees, 1956 (first published 1934).

[28] Pratt, J. W., 'Risk Aversion in the Small and in the Large', *Econometrica*, 32: 122-36, 1964.

[29] Pratt, J. W., Schlaifer, R. and Raiffa, H., *Introduction to Statistical Decision Theory*, McGraw-Hill, New York, 1965.

[30] Press, S. J., *Applied Multivariate Analysis*, Holt, Rinehart and Winston, New York, 1972.
[31] Sengupta, J. K., *Stochastic Programming Methods and Applications*, North Holland, Amsterdam, 1972.
[32] Thomson, K. J. and Hazell, P. B. R., 'Reliability of Using the Mean Absolute Deviation to Derive Efficient E, V Farm Plans', *American Journal of Agricultural Economics*, 54: 503-6, 1972.
[33] Thompson, S. C., *An Approach to Monte Carlo Programming*, University of Reading, Department of Agriculture Study No. 3, November 1967.
[34] Thompson, S. C., *A User's Manual for Monte Carlo Programming*, University of Reading, Department of Agriculture Study No. 9, 1970.
[35] Thomas, W., Blakeslee, L., Rogers L. and Whittlesey, N., 'Separable Programming for Considering Risk in Farm Planning', *American Journal of Agricultural Economics*, 54: 260-66, 1972.
[36] Wagner, H. M,. *Principles of Operations Research With Applications to Managerial Decisions*, Prentice-Hall, Englewood Cliffs, 1969.
[37] Whitmore, G. A., 'Third-Degree Stochastic Dominance', *American Economic Review*, 60: 457-59, 1970.