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A LINEAR PROGRAMMING SOLUTION TO SOME MARKET ALLOCATION PROBLEMS

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Certain market allocation problems involving linear average net revenue functions can be solved by linear programming. The technique can be applied to an objective function derived from linear marginal net revenue functions, the objective being to force each marginal net revenue as near to zero as possible given the constraint set. If a particular problem is suited to a linear programming solution, researchers may prefer to use this technique rather than more sophisticated optimization methods.

Introduction

Since Waugh *et. al.* [5] demonstrated the revenue gains from price discrimination, a considerable amount of research effort has been devoted to solving 'market allocation' problems.¹ These problems involve finding the allocation of a product among different markets such that the revenue net of allocation costs from the sale of the product is maximized. The markets might be separated on the basis of the product form (e.g. fresh and processing markets), time (summer and winter markets), location (domestic and export markets), or by a combination of these dimensions. The problem might involve constraints, e.g., a lower or an upper limit on the quantity to be allocated to a particular market.

The first step in this type of analysis is to estimate (usually by econometrics) demand or average revenue functions for each market. These functions, together with estimates of allocation costs (e.g. grading and packing in the case of fresh fruit), are used to derive an aggregate net revenue function. This function is then maximized, subject to the constraint set.

Some of these problems can be solved using the Lagrange multiplier (LM) procedure. Advances in mathematical programming techniques have provided solution procedures such as quadratic programming (QP) for relatively complicated problems.² This paper demonstrates the use of a familiar technique, linear programming (LP), in solving certain types of market allocation problems.

A necessary condition for solving a market allocation problem by

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¹Most of the published research has been in relation to United States agriculture, e.g., the study by Weisenborn *et al.* [6] concerning the allocation of Florida oranges among fresh and processing markets. Most agricultural marketing texts devote some attention to market allocation problems. Two good references which include examples are [1] and [4].

²In particular, the work of Takayama and Judge [3] deserves mention in this respect.

LP is that the marginal net revenue (and hence, average net revenue) function for each market and the constraint functions are linear. This is the same requirement that needs to be met in order to use QP. Any problem that can be solved by LP could also be solved by QP. However, researchers may feel that LP is a useful and computationally inexpensive alternative to QP and other more sophisticated solution procedures.

The paper proceeds by first discussing simple allocation problems in which the only constraint (other than non-negativity constraints on the quantities allocated to each market) serves to limit the total quantity allocated. This is followed by a discussion of problems involving more complex constraints sets and then some examples.

Problems Involving One Constraint

Consider a problem involving k independent markets with revenue functions of the form:

$$\begin{aligned} TNR_i &= a_i Q_i - b_i Q_i^2 && \text{(total net revenue)} \\ ANR_i &= a_i - b_i Q_i && \text{(average net revenue)} \\ MNR_i &= a_i - 2b_i Q_i && \text{(marginal net revenue),} \end{aligned}$$

where a_i and b_i are positive constants, Q_i is the quantity allocated to market i and $i = 1, \dots, k$. The aim is to:

$$\text{Max } Z = \sum_{i=1}^k TNR_i$$

subject to

$$\sum_{i=1}^k Q_i \leq T$$

$$\text{and } Q_i \geq 0 \quad (i = 1, \dots, k),$$

where T is the total quantity available for allocation.

The LM and QP solution procedures operate directly with Z , the quadratic aggregate net revenue function. However, maximizing Z subject to the constraints on the Q_i is equivalent to equating the MNR 's, subject to the same constraints. The rationale of the LP solution procedure stems directly from this fact.

The rationale can be explained with the aid of Figure 1 which depicts linear MNR functions for $k = 2$ real markets and a dummy market. Markets 1 and 2 are 'real' in the sense that MNR is positive up to a certain allocation level. Market d is a 'dummy' market in the sense that MNR is zero for all positive allocations. Allocations to this market represent abandonment or surplus disposal. The numbers are hypothetical.

The optimal allocation patterns among the three markets for different values of T are:

T	Q_1	Q_2	Q_d
$x(x < 2)$	x	0	0
2	2	0	0

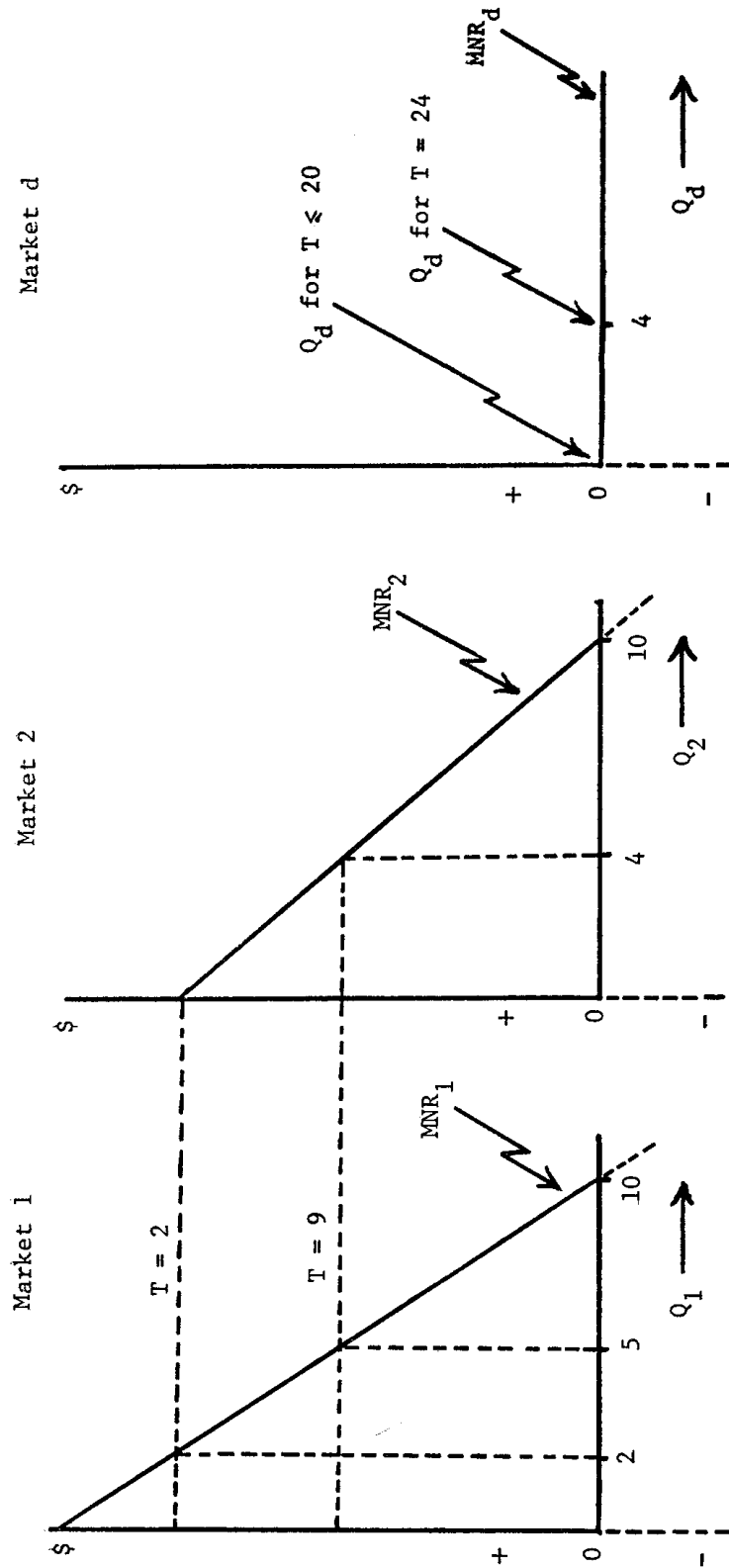


FIGURE 1
Marginal Net Revenue Functions for Two Real Markets and a Dummy Market

9	5	4	0
20	10	10	0
$y(y > 20)$	10	10	$y-20$

The allocation process (i.e. the maximization of Z) involves forcing MNR_1 and MNR_2 towards equality at the smallest non-negative value possible, given T .³ However, if $T = x$, MNR_1 and MNR_2 cannot be equated. The difference between them would be minimized by allocating all of T to market 1. If $T = y$, MNR_1 and MNR_2 are equated at zero and tonnage in excess of 20 is allocated to market d .

This allocation process is simulated by the following LP problem:

$$\begin{aligned} \text{Min } Z^* &= MNR_1 \\ \text{subject to} & \\ & MNR_1 \geq MNR_2 \quad (\text{constraint 1}) \\ & MNR_2 \geq MNR_d \quad (\text{constraint 2}) \\ & Q_1 + Q_2 + Q_d = T \quad (\text{constraint 3}) \end{aligned}$$

After substitution and rearranging the constraints (bearing in mind that $MNR_d = 0$), the problem becomes:

$$\begin{aligned} \text{Min } Z^* &= a_1 - 2 b_1 Q_1 \\ \text{subject to} & \\ & 2(b_2 Q_2 - b_1 Q_1) \geq a_2 - a_1 \quad (\text{constraint 1}) \\ & 2b_2 Q_2 \leq a_2 \quad (\text{constraint 2}) \\ & Q_1 + Q_2 + Q_d = T \quad (\text{constraint 3}) \end{aligned}$$

The intercept term for MNR_1 , i.e. a_1 , would be omitted from the objective function in an actual problem since it is a constant.

If $T = x$, Z^* would be minimized by allocating all of T (i.e., x) to market 1), and in the optimal solution, constraints 1 and 2 would not be binding. If $2 < T < 20$, MNR_1 and MNR_2 would be equated at a positive value equal to the solution value of Z^* , constraint 1 would be binding and constraint 2 would not be binding. If $T = y$, MNR_1 and MNR_2 would be equated at zero and constraints 1 and 2 would both be binding.

If it is known *a priori* that the optimal solution gives strictly positive allocations to all real markets, then any one of the real market MNR 's could be used in the objective function. In this case, the real market MNR 's would be constrained to equal each other and one of the real market MNR 's would be constrained to equal or exceed MNR_d .

Now consider the case where abandonment is not permitted. The allocation process in this case simply requires forcing the real market MNR 's toward equality at the lowest value possible, given T . Using the example in Figure 1, the LP problem is:

$$\begin{aligned} \text{Min } Z^* &= MNR_1 \\ \text{subject to} & \quad \text{—} \\ & MNR_1 \geq MNR_2 \\ & Q_1 + Q_2 = T \end{aligned}$$

The optimal Z^* would be positive for $T < 20$, zero for $T = 20$ and negative for $T = y$.

³ Given that the markets are assumed to be independent and that the real market MNR functions are negatively sloped, this allocation procedure ensures a true maximum of aggregate net revenue (see [4, pp. 88-89]).

Some allocation problems may involve markets which are related in the sense that the *ANR* functions are of the form

$$\begin{aligned} ANR_i &= a_i - b_i Q_i - c_i Q_j \\ \text{and } ANR_j &= a_j - c_j Q_i - b_j Q_j, \end{aligned}$$

where the *a*, *b* and *c* terms are positive constants, $b_i > c_i$ and $b_j > c_j$ (reasonable relationships according to economic logic). The corresponding *MNR* functions are:

$$MNR_i = a_i - 2b_i Q_i - (c_i + c_j) Q_j$$

and

$$MNR_j = a_j - 2b_j Q_j - (c_i + c_j) Q_i .$$

These problems are formulated for solution by LP in exactly the same way as problems involving unrelated markets (where c_i and c_j are implicitly zero). The objective pursued in the allocation process is the same as in problems involving unrelated markets, but it is difficult to demonstrate the allocation process graphically.⁴

There is an alternative way of viewing the allocation process. Suppose the problem involves three markets and that $a_1 > a_2 > a_3$. In the optimal solution, $MNR_1 \geq MNR_2 \geq MNR_3$. However, the difference between MNR_1 and MNR_3 should be made as small as possible, given *T*. Hence, the problem could be formulated for an LP solution as follows:

$$\begin{aligned} \text{Min } Z^+ &= MNR_1 - MNR_3 \\ \text{subject to } & MNR_1 \geq MNR_2 \\ & MNR_2 \geq MNR_3 \\ & Q_1 + Q_2 + Q_3 = T . \end{aligned}$$

If market 3 is a dummy market ($MNR_3 = 0$), then $Z^+ = Z^*$. If the problem does not allow for abandonment, then in general, optimal $Z^* \neq$ optimal Z^+ , the exception being the case where *T* coincides with the quantity necessary to reach the point of zero *MNR* in each market. For problems involving just a crop-size constraint, it is immaterial whether Z^* or Z^+ is used in the objective function. Presumably, one would opt for the simplest alternative, Z^* .

Although the examples given involve only three markets, the procedures generalize to any number of markets. In the author's opinion, the LP procedure becomes increasingly more favourable than the LM procedure as the number of markets increases. For example, Bressler and King's problem [1, pp. 249-253] of determining the optimal weekly allocation of Californian avocados would be better suited to an LP rather than an LM solution. Moreover, in view of the number of markets involved, the LM solution to their problem may give some negative 'optimal' allocations. The LP procedure would automatically restrict the *Q*'s to be non-negative.

Problems Involving More than One Constraint

Market allocation problems might involve a combination of inequality and equality constraints other than the crop size constraint. Because the

⁴ In general, if the coefficient of Q_i exceeds the coefficient of Q_j in MNR_i , then equating *MNR*'s ensures a true maximum of the aggregate net revenue. This relationship should generally hold. (See [4, pp. 88-89] for verification of this and how one proceeds should the relationship not hold).

The allocation process is difficult to demonstrate graphically because changes in the allocation to market *j* shift the intercept for MNR_i , and vice-versa.

LP procedure allows for the specification of certain inequality constraints, it has an operational advantage compared to the LM procedure for these types of problems.

It should be apparent by now that in order to use the LP procedure, one needs to determine the relationship among the *MNR*'s in the optimal solution. If these relationships cannot be determined from visual inspection of the *MNR* functions, simple arithmetic will often serve the purpose. In general, however, one cannot determine the relationships among *MNR*'s in the optimal solution if a single constraint (other than the crop size constraint and the constraints on the *MNR*'s) involves more than one of the *Q*'s. Hence, these problems generally cannot be solved by LP.⁵ However, this criticism can be discounted insofar as past allocation problems have generally not included constraints of this form. Usually, the constraints place upper and/or lower limits on the individual *Q*'s.

Even though the problem to be solved may contain constraints other than crop size, it is generally worthwhile computing the optimal allocation pattern omitting these constraints. For one thing, this allocation pattern could be used as a basis for comparison. Moreover, the solution to the less constrained problem may be of help in determining the relative sizes of the *MNR*'s in the solution to the more constrained problem.⁶

The LP approach to problems involving lower and/or upper limits on the *Q*'s draws on a few general principles which help determine the relationships among *MNR*'s in the optimal solution. The principles assume that the coefficients associated with the *Q* terms in the *MNR* functions are negative.

(1) A lower (upper) limit on Q_i implies an upper (lower) limit on the *MNR*'s determined by Q_i . Furthermore, lower limits can be satisfied by adjusting the intercept terms in the *MNR* functions and the value of *T* in accordance with these limits.

(2) If the problem allows for abandonment and if the *Q*'s determining MNR_i do not have upper and/or lower limits, then MNR_i will have a lower limiting value of zero.

(3) If the problem allows for abandonment, MNR_i will have a lower limiting value of zero if MNR_i for $Q_i = m$ is zero and m lies between the lower and upper limits on Q_i . The same will be true if Q_i has either a lower limit less than m or an upper limit exceeding m .

(4) If the *Q*'s determining MNR_i and MNR_j do not have lower or upper limits, and if the optimal allocation will require that these *Q*'s exceed zero, then $MNR_i = MNR_j$ in the optimal solution.

The first principle should be readily apparent. The remaining principles are derived from an intuitive interpretation of the allocation

⁵ If the problem does contain constraints involving linear combinations of more than one of the *Q*'s, one does have the option of computing the optimal distribution without the constraint to determine whether the solution satisfies the constraint. If so, the problem is solved. This is in the spirit of Hillier and Lieberman's proposal [2, pp. 504-505] for simplifying LP problems.

⁶ In fact the solution to the less constrained problem may allow one to determine the solution to the more constrained problem without further computations. Referring to the example in Figure 1, it is easy to see that if the constraint $Q_2 \leq 8$ was introduced and $T = 20$, the optimal solution would be $Q_1 = 10$, $Q_2 = 8$ and $Q_4 = 2$.

process, namely, that the allocation should be executed so as to force *MNR*'s toward zero while at the same time maintaining equality among *MNR*'s unaffected by upper and lower limits on the *Q*'s.

In using the LP procedure, one studies the constraint set to determine the relationship among *MNR*'s in the optimal solution. The problem is then set up using an objective function and constraint set consistent with the relationships so determined. This procedure is best illustrated by examples.

Examples

(1) Bressler and King provide an example based on the temporal, spatial and product-form dimensions of the aggregate market for lemons [1, pp. 253-571]. The *MNR* functions are:

$$\begin{aligned} MNR_1 &= 7.50 - 3.68 Q_1 && \text{(Fresh winter market)} \\ MNR_2 &= 7.35 - 2.10 Q_2 && \text{(Fresh summer market)} \\ MNR_3 &= 3.75 - 1.00 Q_3 && \text{(Fresh export market)} \\ MNR_4 &= 3.55 - 1.20 Q_4 && \text{(Processing market)}. \end{aligned}$$

A total of 17.2 million boxes of lemons have to be allocated among these markets (i.e. no abandonment allowed). Although they solve the problem using the LM procedure (solving five equations in five unknowns), the problem is readily solved by LP. It is easy to determine that the optimal solution requires strictly positive allocations to each market. Hence, one could solve the problem by minimizing any one of the *MNR*'s subject to the constraint that the *MNR*'s are equal and that the total allocation equals 17.2.

Waugh [4, p. 76] suggests that a more useful piece of analysis would be the determination of the optimal allocation pattern for a number of values of *T*. In the context of the LP solution, one would derive these results by parameterizing the solution with respect to *T*. With this in mind an inspection of the intercept terms of the *MNR* functions indicates that one of the following relationships should hold in the optimal solution, depending on the size of *T*:

- (a) $MNR_1 > MNR_2 > MNR_3 > MNR_4$
- (b) $MNR_1 = MNR_2; MNR_2 > MNR_3 > MNR_4$
- (c) $MNR_1 = MNR_2 = MNR_3; MNR_3 > MNR_4$
- (d) $MNR_1 = MNR_2 = MNR_3 = MNR_4$.

The following LP format would cover all these possibilities:

$$\begin{aligned} & \text{Min } Z^* = MNR_1 \\ \text{subject to } & MNR_1 \geq MNR_2 \\ & MNR_2 \geq MNR_3 \\ & MNR_3 \geq MNR_4 \\ & Q_1 + Q_2 + Q_3 + Q_4 = T. \end{aligned}$$

In order to parameterize with respect to *T*, one simply changes the value of *T*. This should be simpler than re-solving the set of simultaneous equations, given that most LP computer programmes have a parameterization option available.

(2) Suppose that in a problem involving three real markets and a dummy market (i.e. abandonment allowed), the *MNR* functions are:

$$\begin{aligned} MNR_1 &= 50 - 5Q_1 \\ MNR_2 &= 40 - 4Q_2 \\ MNR_3 &= 30 - 5Q_3 \end{aligned}$$

and the constraints are:

$$\begin{aligned} Q_1 &\geq 8 \\ Q_2 &\geq 5 \\ Q_1 + Q_2 + Q_3 + Q_d &= T \quad (T \geq 13). \end{aligned}$$

Because the constraints imply upper limits on Q_1 and Q_2 of 10 and 20, respectively, one of the following will be true in the optimal solution, depending on the value of T :

- (a) $MNR_3 > MNR_2 > MNR_1 > MNR_d$ ($T < 15$)
- (b) $MNR_3 = MNR_2; MNR_2 > MNR_1 > MNR_d$ ($15 \leq T < 19.5$)
- (c) $MNR_3 = MNR_2 = MNR_1; MNR_1 > MNR_d$ ($19.5 \leq T < 26$)
- (d) $MNR_3 = MNR_2 = MNR_1 = MNR_d$ ($T \geq 26$).

Hence, irrespective of the value of T , the problem should be formulated as:

$$\begin{aligned} \text{Min } Z^* &= MNR_3 \\ \text{subject to } & MNR_3 \geq MNR_2 \\ & MNR_2 \geq MNR_1 \\ & MNR_1 \geq MNR_d \\ & Q_1 \geq 8 \\ & Q_2 \geq 5 \\ & Q_1 + Q_2 + Q_3 + Q_d = T. \end{aligned}$$

Once the constraints on Q_1 and Q_2 are satisfied, the solution would proceed by allocating to market 3. However, the constraint set prevents MNR_3 from being forced below any of the other MNR 's and it is, therefore, consistent with the relationships that should hold among the MNR 's in the optimal solution.

Now suppose that the lower limit on Q_1 is increased to 11. This implies an upper limit on MNR_1 of -5 . Clearly, the allocation to market 1 should not exceed 11. But minimizing MNR_3 would not guarantee this. There may be some tonnage left to allocate after MNR_3 and MNR_2 have reached zero. Although this additional tonnage should be allocated to market d, it could be allocated to market 1 without penalizing Z^* .

In this case it would be appropriate to use the Z^+ -type objective function, i.e.,

$$\begin{aligned} \text{Min } Z^+ &= MNR_3 - MNR_1 \\ \text{subject to } & MNR_3 \geq MNR_2 \\ & MNR_2 \geq MNR_d \\ & Q_1 \geq 11 \\ & Q_2 \geq 5 \\ & Q_1 + Q_2 + Q_3 + Q_d = T. \end{aligned}$$

This formulation is consistent with the relationships among MNR s that should hold in the optimal solution. Moreover, allocations to market 1 would be kept to the minimum consistent with the constraint set, since these allocations penalize Z^+ . Note that it is unnecessary to include the constraint $MNR_d \geq MNR_1$, since $MNR_d = 0$ and MNR_1 has an upper limit of -5 .

(3) The author recently confronted a market allocation problem⁷ in which the *MNR* functions were:

$$\begin{aligned}MNR_1 &= 243.14 - 7.16 Q_1 \\MNR_2 &= 177.73 - 3.82 Q_2 - 1.40 Q_3 \\MNR_3 &= 113.03 - 1.40 Q_2 - 2.57 Q_3 \\MNR_4 &= 65.85 - 2.26 Q_4\end{aligned}$$

and the constraints were:

$$\begin{aligned}Q_1 &\geq 41.2 \\Q_2 &\geq 30.14 \\Q_3 &\geq 34.95 \\Q_3 &\leq 39.21 \\Q_1 + Q_2 + Q_3 + Q_4 &\leq 153 \quad (\text{i.e. abandonment allowed}).\end{aligned}$$

The lower limits on the Q 's imply that $MNR_1 \leq -51.85$, $MNR_2 \leq 13.66$ and $MNR_3 \leq -18.99$. Clearly, the upper limit on Q_3 is superfluous since any increase in Q_3 beyond its lower limit reduces TNR_3 . Hence $Q_3 = 34.95$ in the optimal solution. Furthermore, increases in Q_1 beyond its lower limit decrease TNR_1 and therefore $Q_1 = 41.2$ in the optimal solution. Hence, the problem reduces to finding the optimal allocation among markets 2, 4 and d subject to $Q_2 + Q_4 + Q_d = 76.85$ (i.e. $153 - 41.2 - 34.95$). This problem could be readily solved using LP by decreasing the intercept term of MNR_2 by 48.93 (i.e. 1.4×34.95) and formulating the problem as:

$$\begin{aligned}\text{Min } Z^* &= MNR_4 \\ \text{subject to } & MNR_4 \geq MNR_2 \\ & MNR_2 \geq MNR_d \\ & Q_2 \geq 30.14 \\ & Q_2 + Q_4 + Q_d = 76.85.\end{aligned}$$

Actually, this formulation would be appropriate irrespective of the value of T .

This particular problem is relatively easy to solve because abandonment is allowed. Suppose now that abandonment is not allowed. This complicates the problem insofar as the optimal allocation may result in values for Q_1 and Q_3 in excess of their lower limits.

If there were no upper limit on Q_3 , one of the following relationships would hold in the optimal solution, depending on the size of T :

- (a) $MNR_4 > MNR_2 > MNR_3 > MNR_1$
- (b) $MNR_4 = MNR_2; MNR_2 > MNR_3 > MNR_1$
- (c) $MNR_4 = MNR_2 = MNR_3; MNR_3 > MNR_1$
- (d) $MNR_4 = MNR_2 = MNR_3 = MNR_1$.

Hence, one could formulate the problem as:

$$\begin{aligned}\text{Min } Z^* &= MNR_4 \\ \text{subject to } & MNR_4 \geq MNR_2 \quad (\text{constraint 1}) \\ & MNR_2 \geq MNR_3 \quad (\text{constraint 2}) \\ & MNR_3 \geq MNR_1 \quad (\text{constraint 3}) \\ & Q_1 \geq 41.2 \quad (\text{constraint 4})\end{aligned}$$

⁷ The problem dealt with the allocation of the United States apple crop among four markets, viz: January-June fresh (market 1), July-December fresh (market 2), canning (market 3) and juice (market 4). Demand (average revenue) functions for each market were estimated as part of a simultaneous econometric model. These, together with estimates of allocation costs, were used in deriving the *MNR* functions.

TABLE 1

Matrix of Coefficients for LP Solution to Market Allocation Problem

Row	Q_1	Q_2	Q_3	Q_4	RHS
Obj	0	0	0	-2.26	—
Con. 1	0	3.82	1.40	-2.26	≥ 111.88
Con. 2	0	-2.42	1.17	0	≥ 64.70
Con. 3	7.16	-1.40	-2.57	0	≥ 130.11
Con. 4	1.00	0	0	0	≥ 41.20
Con. 5	0	1.00	0	0	≥ 30.14
Con. 6	0	0	1.00	0	≥ 34.95
Con. 7	1.00	1.00	1.00	1.00	$= 153.00$

$$\begin{aligned}
 Q_2 &\geq 30.14 && \text{(constraint 5)} \\
 Q_3 &\geq 34.95 && \text{(constraint 6)} \\
 Q_1 + Q_2 + Q_3 + Q_4 &= T && \text{(constraint 7)}.
 \end{aligned}$$

If $Q_3 > 39.21$ in the optimal solution, the problem could be re-solved with market 3 eliminated, and the intercept of MNR_2 and the value of T reduced in accordance with a value for Q_3 of 39.21.⁸

The matrix of coefficients for this problem is shown in Table 1. Note that the objective is to minimize $-2.26 Q_4$. Hence, the value of MNR_4 in the optimal solution is found by adding 65.85 to the solution value of the objective function in Table 1.

In the optimal solution, Q_1 and Q_3 attain their lower limiting values of 41.2 and 34.95, respectively, $Q_2 = 38.92$ and $Q_4 = 37.93$. The objective function attains the value -85.72 which implies a value for MNR_4 of -19.87 (i.e. $-85.72 + 65.85$). Substitution of the optimal Q 's into the MNR functions yields $MNR_1 = -51.85$, $MNR_2 = -19.87$ (i.e. $MNR_4 = MNR_2$ and constraint 1 is binding) and $MNR_3 = -31.27$. The shadow prices associated with the optimal solution are deemed to be of little economic interest, since they represent changes in MNR_4 rather than changes in aggregate net revenue.

Conclusion

The LP procedure is suited to the solution of a wider range of market allocation problems than the LM procedure and it can be used to solve certain types of problems which hitherto have been solved by QP or some other technique. Hence, given that LP is a familiar optimization technique and that LP computing facilities are widely available, the LP solution method should be a useful alternative to the more sophisticated methods such as QP.

While this paper has been specifically concerned with the solution of market allocation problems, there may be scope for the application of the ideas presented here to other optimization problems where the optimal solution requires equating marginal values and the marginal values and constraint set involve only linear functions.

⁸ The computational time could be reduced by satisfying the lower limits on the Q 's before submitting the problem to the computer.

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