# GAME THEORY AND A TIME-OF-MARKET DECISION

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The contribution of the theory of games to decision making has been twofold. Firstly, it has described and clarified many of those decision complexes which are reducible to a direct conflict of interests (" game ") between two or more irreconcilably opposing parties who have some control over the outcome of the conflict. Secondly, it has supplied means of obtaining solutions to some of the problems so posed. The best examples of non-trivial real-world game situations are to be found in warfare and, in the field of business, in duopolies and oligopolies.[1] But the number of applications has been limited to a large extent by some assumptions required by the theory, mainly those which define the strictness of the competition between the players and the degree of knowledge each has about the others' strategies.[2] In agriculture, too, it is rare that these assumptions can be rigorously met. However by adopting a rather liberal interpretation of these conditions, it is possible to conceive of some farm situations in terms of the theory ; landlord vs. tenant or owner vs. sharefarmer competitions[3] spring readily to mind, as do auction sales and other direct farmer-market contacts.

One of the most fruitful areas for investigation in the agricultural field is the study of games against nature. This covers a range of situations, from direct conflict between the farmer and his physical environment, (for example, raising crops subject to irregular weather conditions) to a competition between the farmer and his *economic* environment, that is an amalgamation of all his " adversaries "—marketing authorities, other farmers, etc.[4] In either case " nature " is considered as a fictitious player having no known objective and, as a starting point, no known strategy.

1. See, for example, T. E. Crawford and C. J. Thomas, " Applications of Game Theory to Fighter vs. Bomber Combat ", *J. Opns. Res. Soc. of America*, 3, 4, November, 1955, pp. 402-11 ; for a discussion of game theory in business see Martin Shubik, *Strategy and Market Structure* (N.Y. : John Wiley), 1959. For examples of game theory applied to a variety of everyday situations, see J. D. Williams, *The Compleat Strategyst* (N.Y. : McGraw-Hill), 1954, *passim.*

2. For a discussion of these points, see Charles Hitch, " Uncertainties in Operations Research ", *Operations Research*, 8, 4, July-August, 1960, p. 441.

3. See, for example, E. O. Heady, " Applications of Game Theory in Agricultural Economics ", *Canadian Journal of Agricultural Economics*, VI, 1, 1958, pp. 1-13.

4. The theoretical justification for such a decision-making approach is argued in J. L. Dillon and E. O. Heady, " Free Competition, Uncertainty, and Farmer Decisions ", *Journal of Farm Economics* (forthcoming).

In this paper we apply several criteria to a game between a particular farmer and nature, the latter being a coalition representing the weather and the market on which the farmer sells his product.[5]

## *The Problem*

The farmer in question runs about seven thousand flock merino sheep and five hundred Hereford cattle on a five thousand acre property in the Tenterfield district of Northern New South Wales. The main product of the property is wool, whilst the cattle which are kept mainly to control the coarse pasture growth (thereby making the pasture more accessible to sheep) constitute a profitable sideline. The third important source of income is the sale of sheep—both old sheep which are culled-for-age and a portion of the annual lamb drop.

Lambs have not always been bred on the property. In the initial stages of development prior to the introduction of improved pastures, the majority of the sheep flock was made up of wethers run solely for wool production. As development proceeded the number of acres topdressed and sown to pasture rose, allowing both an increase in the number of stock carried and a change in the sheep system from wethers to a breeding flock. Under this new system replacements are bred rather than purchased and, on the whole, the managerial capabilities of the farmer are taxed to a greater degree than they were under the old system.

The grazier's usual practice now is to lamb in autumn (April), to retain the ewe portion of the drop as replacements for his breeding flock and to sell the wether portion at between six and twelve months old. This group of approximately two thousand wethers may be sold at any time between October and April and it is the decision about the time at which to market these sheep which constitutes the core of the problem under consideration.

The grazier must face up to making this decision in October and the chief factor which he must take into account is the rainfall prospect for the next six months. At one extreme is the possibility that no rain whatsoever will fall over this period. If this were to be the case he would be best advised to sell the wethers in October thus avoiding being overstocked during a drought. At the other extreme is the possibility that satisfactory rain will be received early in the six month period. This would cause a prolific pasture growth over the summer months and as many livestock as possible would be required to keep it in check. Under these conditions it would be imperative that the wethers be held on the property until April ; in so doing the grazier would be enabled not only to achieve better utilization of the summer pasture by all his stock but

5. A short glossary covering game theory terms used in this paper is presented at this stage. A *game* is a set of rules and procedures which describes a competitive situation. Each participant or *player* has a finite or infinite set of *strategies* or *acts*. A participant may choose to play a *pure strategy*, i.e. the same strategy at each play of the game, or a *mixed strategy*, i.e. different strategies at different plays of the game. As a result of a single *play* of the game, there is a *payoff* which represents in some convenient units the amounts which the players win or lose. The *value* of the game is the expected value of the payoff if the players use their optimum strategies. The *payoff matrix* is a convenient way of portraying a game, the vectors of the matrix representing in an orderly fashion the strategies of the players. In a two-person game the usual convention is for the rows of the matrix to represent the strategies of the *maximizing player*, (i.e. the one who aims to maximize the payoff, since it represents a gain to him), whilst the columns specify the strategies of the *minimizing player*, (i.e. the player aiming to minimize the payoff, since it represents a loss to him). A *two-person zero-sum game* is one between two players in which the gain of the maximizing player equals the loss of the minimizing player after any play of the game (i.e. the payoffs sum to zero).

also to send his wethers to market in good fat condition. Between these two extremes is a large number of possibilities. If, for instance, the farmer were to hold on to the wethers until January or February waiting for rain, finally selling them in desperation on a depressed market, considerable losses could occur.

In the next section this decision problem is formulated as a game against nature.

### *The Strategies and the Payoff Matrix*

The number of alternative courses of action, or strategies, available to the farmer is quite large. For example the farmer could formulate a strategy denoting " Sell wethers now " for each of the two hundred-odd days between October and April. It would be possible also to define an indefinitely large number of alternative states of nature. In order to keep the problem within analytical and solvable limits it is desirable to abstract from reality by reducing the real-world situation outlined in the previous section to a fairly simple model via the following assumptions:

(a) The farmer must decide by the first day of October when, out of the ensuing six months, he will sell the wethers. He may only sell on the first day of any month between October and April and he must sell all the wethers at once. He thus has seven alternative strategies, numbered 1 to 7, as follows :

| *No.* | *Player I (Farmer) Strategy* |
|---|---|
| 1 | Sell wethers on 1st October |
| 2 | Sell wethers on 1st November |
| 3 | Sell wethers on 1st December |
| 4 | Sell wethers on 1st January |
| 5 | Sell wethers on 1st February |
| 6 | Sell wethers on 1st March |
| 7 | Sell wethers on 1st April |

(b) Rainfall for any of the months between October and April is *unsatisfactory* if less than three inches fall in that month ; it is *satisfactory* if three inches or more fall in that month. A *drought* is considered to continue until the first month in which satisfactory rain is received which is called the *critical* month. The rainfall pattern prior to October, and following the critical month, is not taken into account as affecting the outcome. There are thus eight alternative states of nature.

| *No.* | *Player II (Nature) Strategy* |
|---|---|
| 1 | Send satisfactory rain by 1st October (i.e. critical month September) |
| 2 | Send satisfactory rain by 1st November (i.e. critical month October) |
| 3 | Send satisfactory rain by 1st December (i.e. critical month November) |
| 4 | Send satisfactory rain by 1st January (i.e. critical month December) |
| 5 | Send satisfactory rain by 1st February (i.e. critical month January) |
| 6 | Send satisfactory rain by 1st March (i.e. critical month February) |
| 7 | Send satisfactory rain by 1st April (i.e. critical month March) |
| 8 | Send no satisfactory rain. |

The payoff matrix, with row and column vectors representing the strategies of farmer and nature respectively, may now be constructed. Any element $a_{ij}$ of the payoff matrix A represents the payoff from Player II to Player I as a result of a single play of the game in which I and II use their $i$-th and $j$-th strategies respectively. Hence a negative element in the matrix represents a loss to the farmer.

In our present model let us define any element of the payoff matrix as :

$$a_{ij} = x_i - \sum_{k=1}^{i} y_k - \sum_{k=i}^{m} z_k \qquad (1)$$

where $a_{ij}$ = payoff per wether to the farmer using his $i$-th strategy when nature uses her $j$-th.

$x_i$ = price per wether received by selling on the first day of the $i$-th month.

$y_k$ = amount lost per wether in $k$-th month through having sheep on hand during a drought; (for $k \geqq j$, $y = 0$).

$z_k$ = amount lost per wether in the $k$-th month through not having sheep on hand during a flush time; (for $k < j, z = 0$).

Simplicity on the numerical side is the main reason for including these values on a per wether basis ; it should not be inferred that this indicates that total payoffs are linear functions of the number of sheep in the flock to be sold. However, the total payoff after a single play of the game may be taken as $wa_{ij}$ where $w$, the number of wethers, lies within a reasonable range from the average of two thousand head.

It is necessary now to quantify the variables cited above and to construct the numerical payoff matrix relevant to our model. It has been noted already that there would be, in the real-world situation under study, a considerable number of strategies which the two players could employ ; it follows that there would be a similarly large number of corresponding payoff matrices. Even within the simplifications of the present analysis many payoff matrices could be drawn up corresponding to various assumed values of the variables involved. An additional problem is presented by the severe lack of data on which price and cost assumptions could be based ; to derive such data would require a considerable amount of experiment and/or observation beyond the scope of this study. Thus, the price and cost models used in constructing the payoff matrix have been built in a manner which seems intuitively reasonable on the basis of existing knowledge, rather than on experimentally derived data. Such an approach is justified since the bias of this paper is towards methodology rather than towards deriving farmer recommendations.

Moreover it was found that given the assumed pattern of prices and costs, the model was not particularly sensitive to reasonably large numerical changes in the coefficients used. Hence the subsequent discussion uses as a basis one payoff matrix constructed from the following assumptions :

(a) Price : The total amount received from the sale of wethers depends chiefly on their mean body weight, their fleece cover and the market conditions at the time of the sale. The rainfall pattern influences all these quantities and they are combined to give the hypothetical schedule of prices shown in Figure 1.

(b) The monthly costs of holding the wethers over a period of drought are assumed to be as follows :

| | | |
|---|---|---|
| 1st month | .. .. | £0.1 per head |
| 2nd month | .. .. | £0.2 per head |
| 3rd and 4th months | .. | £0.3 per head |
| 5th and 6th months | .. | £0.4 per head |

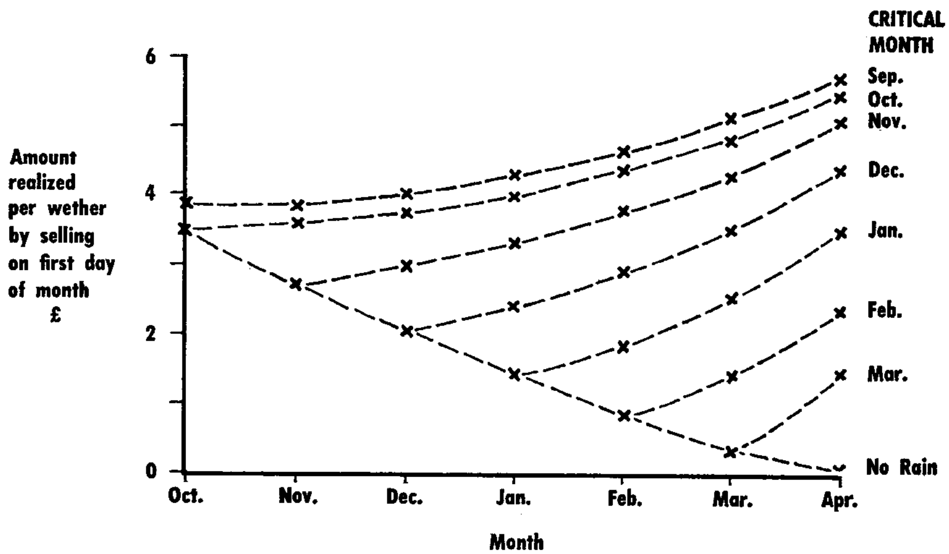



Fig. 1. Hypothetical Price Schedule for Wethers.

(c) The monthly loss due to being understocked during a flush period is much more difficult to assess. This situation is mainly costly through lost opportunities, the succulent pasture runs to seed and becomes rank and coarse if stock are not grazing it. The possibility is usually open to the farmer to cut it for hay if it is of reasonable quality, and if the feed is plentiful enough, it may even prove profitable, even though he has just sold his own wethers, to buy in store sheep for fattening over the summer months. This should be sufficient to show that the situation of being understocked in a flush period may perhaps lead to a profit rather than a loss, depending on the farmer's initiative. In any case, any loss from this cause is unlikely to be as great as the loss from being overstocked in a drought. In view of these factors we shall assume a constant loss of £0.1 per head per month when these conditions prevail.

The payoff matrix constructed from these data is shown in Table I. Payoffs throughout are measured in pounds(£).

TABLE I

*Payoff Matrix for Time-of-Market Problem*

| | | Nature Strategy | | | | | | | | Row Minima |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| Farmer Strategy | 1 | 3.2 | 2.9 | 3.0 | 3.1 | 3.2 | 3.3 | 3.4 | 3.5 | 2.9 |
| | 2 | 3.3 | 2.9 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.6 | 2.1 |
| | 3 | 3.5 | 3.2 | 2.2 | 1.4 | 1.5 | 1.6 | 1.7 | 1.8 | 1.4 |
| | 4 | 3.9 | 3.5 | 2.6 | 1.4 | 0.5 | 0.6 | 0.7 | 0.8 | 0.5 |
| | 5 | 4.3 | 4.0 | 3.2 | 2.0 | 0.6 | −0.3 | −0.2 | −0.1 | −0.3 |
| | 6 | 4.9 | 4.5 | 3.8 | 2.7 | 1.4 | −0.1 | −1.1 | −1.0 | −1.1 |
| | 7 | 5.6 | 5.3 | 4.7 | 3.7 | 2.5 | 0.9 | −0.4 | −1.7 | −1.7 |
| Column Maxima | | 5.6 | 5.3 | 4.7 | 3.7 | 3.2 | 3.3 | 3.4 | 3.5 | |

### *The Criteria*

Given the matrix of possible outcomes the decision maker's problem is to select an optimum pure strategy or mixture of strategies, using in his choice the algorithm provided by one of several criteria. The definition of " optimum " depends on the criterion used, as will be seen.

Of the many means of resolving a decision problem which have been put forward in the literature, four are examined here in relation to our time-of-market decision.[6] These criteria could be used to derive optimum strategies for both players in the game. However, since we are considering a game against nature, it is meaningless to formulate an optimum course of action for the minimizing player. In fact, whatever the empirical values contained in the payoff matrix, nature's strategy for any single play of the game may be considered as being essentially predetermined and unalterable. In the absence of knowledge about which strategy nature will play, we may use the principle of insufficient reason and assume that all states of nature are equally likely ; i.e. the probability $q_j$ that the $j$-th state will prevail is assumed to be $1/n$, where $n$ = number of possible alternative states. On the other hand, if an *a priori* or *a posteriori* probability distribution over the various states of nature is available, the decision problem under uncertainty is converted to one under risk. When weather is the major component of nature, the estimation of this distribution is usually a simply matter. Using 91 years' rainfall records for Tenterfield, the $q_j$ for the present problem were calculated, and are shown in Table II. Both assumptions about the distribution of $q_j$ are used in the discussions which follow.

6. Characteristics of the criteria used are listed in J. Milnor, " Games against Nature ", in R. M. Thrall *et al., eds., Decision Processes* (N.Y. : John Wiley), 1954, pp. 49-60 ; also in R. D. Luce and Howard Raiffa, *Games and Decisions*, (N.Y. : John Wiley) 1957, Ch. 13. An empirical application of these criteria to a seasonal production decision problem is to be found in J. L. Dillon and E. O. Heady, *Theories of Choice in Relation to Farmer Decisions*, Iowa State University, Res. Bull. 485, October, 1960 ; see also the same authors' " Decision Criteria for Innovation ", this *Journal*, **2**, 2, December, 1958, pp. 113-20.

TABLE II

*Nature Strategy Probabilities*
(for Tenterfield, N.S.W.)

| Nature Strategy | Number of occurrences since 1870 | $q_j$ |
|---|---|---|
| 1 | 18 | 0.20 |
| 2 | 27 | 0.30 |
| 3 | 19 | 0.21 |
| 4 | 13 | 0.14 |
| 5 | 9 | 0.10 |
| 6 | 4 | 0.04 |
| 7 | 0 | 0 |
| 8 | 1 | 0.01 |

(*i*) *Maximum Average Expected Outcome*

If the farmer were continually to play his $i$-th strategy, or row, then his average expected payoff $E(a)_i$ would be given by :

$$E(a)_i = \sum_{j=1}^{n} q_j a_{ij} \tag{2}$$

Using this criterion, the farmer should play the row for which $E(a)_i$ is at a maximum. The values of this quantity for the payoff matrix in Table I are shown below for the two assumptions about $q_j$ :

| Farmer Strategy | Average expected payoff where : | |
|---|---|---|
| | $q_j$ unknown and assumed equal | $q_j$ assumed known |
| 1 | 3.2* | 3.1 |
| 2 | 2.5 | 2.6 |
| 3 | 2.1 | 2.6 |
| 4 | 1.8 | 2.7 |
| 5 | 1.7 | 3.1 |
| 6 | 1.9 | 3.6 |
| 7 | 2.6 | 4.5* |

* Indicates optimum strategy.

Thus a farmer who uses this criterion under ignorance[7] would choose always to sell his wethers in October. If the problem is transformed to one of risk, the maximum average expected outcome criterion leads to the selection of strategy 7, i.e. selling the wethers in April.

(*ii*) *Minimax Play* (*The Wald Criterion*)

The minimax principle is fundamental to the theory of games. Its application to general decision theory is due mainly to Wald[8] whilst it

7. When used thus it is usually known as the Laplace criterion.
8. A. Wald, *Statistical Decision Functions* (N.Y. : John Wiley), 1950. Although the general principle is termed " minimax ", the " Wald criterion " discussed here, which applies the principle, is usually called the " maximin criterion ", and the strategies it recommends, the " maximin " pure or mixed strategies. See Luce and Raiffa, *op. cit.*, p. 279.

was von Neumann who developed its use in the context of two-person zero-sum games.

If the farmer were continually to play his $i$-th strategy or row, then his expected payoff would at least be equal to the smallest element in the $i$-th row. Using the minimax principle, the decision maker attempts to maximize this smallest possible payoff ; i.e. he selects that row for which $\min_j a_{ij}$ is maximized. Inspection of the row minima shown in Table I reveals that the first row has the largest entry. Thus, by this criterion, the safest pure strategy for the farmer to play is to sell the wethers in October, because he will always be assured of a payoff of at least 2.9.

If mixed strategies are allowed, this criterion states that the maximizing player should select row $i$ with a frequency $p_i$ in such a way that the quantity $\min_j \sum_i p_i a_{ij}$ is maximized. The solution to this can be found by solving the two-person zero-sum game, which entails finding $p_i$ and $q_j$ in the following relation :

$$\max_i \min_j \sum_i p_i a_{ij} = \min_j \max_i \sum_j q_j a_{ij} = v \qquad (3)$$

where $v$ = value of the game.

There are a number of methods available for solving two-person zero-sum games, one of the most readily usable for larger matrices being conversion of the game to a linear programming problem and solving the programme. This yields $p_i$, $q_j$ and $v$ in the one operation.[9] Whichever method is used to solve the game, a preliminary inspection of the payoff matrix for dominance may lead to a considerable reduction in the computational burden by eliminating dominated strategies.[10] The 7 × 8 payoff matrix in Table I is reducible to a 2 × 2 matrix by elimination of dominated rows 2 to 6 inclusive and columns 1 and 3 to 7 inclusive. The resultant matrix A′ has the form :

$$\begin{matrix} 2.9 & 3.5 \\ 5.3 & -1.7 \end{matrix}$$

The solution of the game using matrix A′ is a simple matter[11] and provides the solution for the original matrix A :

$$\begin{array}{lll} p_1 = 0.92 & q_1 = 0 & v = 3.0 \\ p_2 = 0 & q_2 = 0.68 & \\ p_3 = 0 & q_3 = 0 & \\ p_4 = 0 & q_4 = 0 & \\ p_5 = 0 & q_5 = 0 & \\ p_6 = 0 & q_6 = 0 & \\ p_7 = 0.08 & q_7 = 0 & \\ & q_8 = 0.32 & \end{array}$$

9. See, for example, E. O. Heady and Wilfred Candler, *Linear Programming Methods* (Ames : Iowa State College Press), 1958, Ch. 15.

10. *Dominance* is the superiority of a strategy, or of a group of strategies combined in a special fashion, over another strategy to such an extent that the latter may be neglected in the analysis.

11. See Williams, *op. cit.* ; an extensive yet simple account of methods of solution of finite two-person zero-sum games is to be found in H. H. Goode and R. E. Machol, *System Engineering* (N.Y. : McGraw-Hill), 1957, Chs. 24-5.

Thus, solving the payoff matrix as a two-person zero-sum game leads to the recommendation that the farmer should mix his strategies 1 and 7 in the ratio 9 to 1, thereby assuring himself of an average payoff of at least 3.0.

*(iii) Minimax Regret (The Savage Criterion)*

Savage has suggested that the decision maker attempt to minimize the " regret " caused by the fact that the actual payoff after a play of the game is less than that which he could have obtained, had the true state of nature been known to him beforehand.[12] Thus the " regret matrix " is constructed, in which any element $r_{ij}$ is defined as the amount which has to be added to $a_{ij}$ to equal the maximum payoff in the $j$-th column. The decision maker should then choose that row for which $\max_j r_{ij}$ (or $\max_j \sum_i p_i r_{ij}$) is minimized. The regret matrix derived from the payoff matrix in Table I is shown in Table III.

TABLE III

*Regret Matrix*

| | | Nature Strategy | | | | | | | | Row Maxima |
|---|---|---|---|---|---|---|---|---|---|---|
| | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | |
| Farmer Strategy | 1 | 2.4 | 2.4 | 1.7 | 0.6 | 0 | 0 | 0 | 0 | 2.4* |
| | 2 | 2.3 | 2.4 | 2.6 | 1.5 | 0.9 | 0.9 | 0.9 | 0.9 | 2.6 |
| | 3 | 2.1 | 2.1 | 2.5 | 2.3 | 1.7 | 1.7 | 1.7 | 1.7 | 2.5 |
| | 4 | 1.7 | 1.8 | 2.1 | 2.3 | 2.7 | 2.7 | 2.7 | 2.7 | 2.7 |
| | 5 | 1.3 | 1.3 | 1.5 | 1.7 | 2.6 | 3.6 | 3.6 | 3.6 | 3.6 |
| | 6 | 0.7 | 0.8 | 0.9 | 1.0 | 1.8 | 3.4 | 4.5 | 4.5 | 4.5 |
| | 7 | 0 | 0 | 0 | 0 | 0.7 | 2.4 | 3.8 | 5.2 | 5.2 |

* Indicates optimum strategy.

Using this criterion the farmer should choose pure strategy 1 to minimize his maximum regret. Solving the problem as in (ii) to determine the farmer's optimum *mixed* strategy it is found that he should use strategies 1 and 7 with frequencies 0.54 and 0.46 respectively in order to minimize his maximum average regret.

*(iv) The Hurwicz Criterion*

Using this criterion, the decision maker constructs an index for each of his strategies which considers both the best and worst possible outcomes from using each strategy, weighted by a constant which measures the degree of his pessimism or optimism. Let $b$, a constant between 0 and 1, measure the player's level of pessimism. The Hurwicz criterion leads the player to select that row for which the " pessimism–optimism index " :

$$b \min_j a_{ij} + (1 - b) \max_j a_{ij}$$

is maximized. Consider two farmers, one very conservative ($b = 0.8$) and one optimistic ($b = 0.2$). The pessimism–optimism indices for each are shown below :

12. See L. J. Savage, " The Theory of Statistical Decision ", *J. Amer. Stat. Assoc.*, **46**, 253, March, 1951, pp. 55-67.

| Farmer Strategy | Pessimism–optimism index for : $b = 0.2$ | $b = 0.8$ |
|---|---|---|
| 1 | 3.3 | 3.0* |
| 2 | 3.1 | 2.3 |
| 3 | 3.1 | 1.8 |
| 4 | 3.2 | 1.2 |
| 5 | 3.4 | 0.6 |
| 6 | 3.7 | 0.1 |
| 7 | 4.1* | −0.2 |

* Indicates optimum strategy.

Thus, the conservative farmer should sell his wethers in October, whilst the optimistic one should retain his until April. The farmer who can be indifferent between strategies 1 and 7 (i.e. where the pessimism–optimism indices for these two strategies are equal) is found to be the one whose $b$ value approximates 0.3.

*Discussion*

At this point it is appropriate to consider some results obtained in the last section.

One conclusion is obvious. A decision maker who has undertaken an analysis such as the above in order to resolve a particular decision is still faced with a major problem ; which criterion should he adopt ? The previous section showed that the different criteria lead to different " optimum " courses of action resulting in different outcomes. Comparisons between criteria are difficult because there is no fully satisfactory standard of comparison. To compare merely on the basis of, say, average expected payoff, may be to neglect important subjective factors. For example, a farmer may profess to aim for profit maximization, but in fact his risk aversion may be such as to deter him from trying to achieve it in specific circumstances (such as this time-of-market decision). The use of utiles in place of monetary payoffs would be a step forward ; however, with utility, problems of estimation become paramount. Before returning to the problem of choice between criteria, some general observations on their characteristics are made.

The results illustrate the conservatism of the minimax rule.[13] Both the Wald and Savage criteria recommend strategy 1, the " safest " pure strategy, despite the demonstrably greater average expected payoff obtainable from strategy 7. In other words, the latter is rejected by the minimax principle because of the possibility of incurring a substantial loss if nature is adverse. Strategy 7 is rejected no matter how large the other elements of that row and also despite the fact that on the basis of probabilities, its use will lead to a loss only once or twice in a hundred years.

It might be argued that the supposed general conservatism of farmers would make the minimax principle applicable to their actions.[14] How-

13. Some criticisms of minimax theory are put forward in L. J. Savage, *The Foundations of Statistics* (N.Y. : John Wiley), 1954, Chs. 9-13, and some objections to minimax regret are to be found in H. Chernoff, " Rational Selection of Decision Functions ", *Econometrica*, **22**, 4, October, 1954, p. 425f.

14. It is interesting to note that Dillon and Heady *op. cit.* (1960) found the Wald and Laplace criteria were the only ones out of seven tested to have any significant descriptive value for farmers' actions. Whilst some of their sample of 77 appeared to use the Wald, and some the Laplace exclusively, the majority tended to switch from one to the other according to the nature of the decision.

ever, it could equally well be pointed out that for a considerable number of payoff matrices with which farmers are faced, minimax theory applied either generally in decision problems or specifically in games, is so " ultra-conservative " as to render it almost useless as a descriptive or normative device. Only further experiment and observation will resolve this argument.

The pessimistic outlook of minimax play is further illustrated if we allow procrastination in our decision model. Suppose the farmer can put off deciding on his selling policy until 1st January. If satisfactory rain has already fallen, the decision will have been resolved for him. If it is still dry he will be faced with the payoff matrix :

| | | *Nature* | | | | |
|---|---|---|---|---|---|---|
| | | 4 | 5 | 6 | 7 | 8 |
| *Farmer* | 4 | 1.4 | 0.5 | 0.6 | 0.7 | 0.8 |
| | 5 | 2.0 | 0.6 | −0.3 | −0.2 | −0.1 |
| | 6 | 2.7 | 1.4 | −0.1 | −1.1 | −1.0 |
| | 7 | 3.7 | 2.5 | 0.9 | −0.4 | −1.7 |

which is a submatrix of A. His maximin pure strategy would be to sell immediately, whilst his maximum average expected payoff, assuming either ignorance or known risk, is obtained by playing strategy 7.[15]

A further observation on the results is that a recommendation of mixed strategies for annual games such as this has serious drawbacks. Besides the general objections to mixed strategies,[16] when the game is only played once annually, the number of years required before the overall outcome approximates " optimum " (as defined by the criterion used) could be considerable. It was shown above that the farmer playing the time-of-market problem as a game should mix strategies 1 and 7 in the ratio 9 : 1. If he decides that since the odds for 7 are so small he will play strategy 1 continuously, and if nature were an intelligent minimizing player, then he would know that he would be " wrong " one year in ten. However if approximately equal frequencies were recommended for two or more strategies (such as the outcome of using the Savage criterion above), the farmer would be in a dilemma as to which pure strategy to play in a given year. Even if he conscientiously tried to randomize his choice according to the precepts of game theory, he might meet short-run non-optimum results sufficient to deter him from continuing.

This is a general criticism which may, in some specific cases, be overcome. For instance, if the restriction dictating that all the wethers be sold at the one time were relaxed in our present model, a recommended mixed strategy could be interpreted as being the proportions of the total flock to be disposed of in different months of the same season. For a number of models, however, this " way out " would not be available.

Another comment concerns the interpretation of nature's " optimum " strategy, derived when the problem is solved on a two-person zero-sum game. This represents the " most malevolent way nature could act ",

15. The probability that he will fare *worse* by retaining the wethers until April rather than selling in January is calculated at 0.19.
16. See Luce and Raiffa, *op. cit.*, pp. 74-76.

and in the present problem is given by a 65 : 35 combination of her strategies 2 and 8. Any departure by nature from her "optimum" strategy will allow the farmer an average payoff greater than the value of the game, if he uses his own optimum strategy mixture. In this problem the chances that nature will act "optimally" are so remote that the farmer using his maximin mixed strategy can be *assured* of a greater average payoff than 3.0, the calculated value of the game.

We return finally to the unsolved problem of choice between decision criteria. Although there appear to be no objective grounds for preferring one criterion over another,[17] there are several ways of viewing this question which may lead us to a partial solution. One of these is discussed here.

Decision criteria presume certain goals and psychological attitudes on the part of the decision maker. Hence it might be argued that when confronted with an array of "optimum" strategies, the decision maker should not just accept that course of action which *appears* to give the best outcome, but rather that he should follow that course of action recommended by the criterion which seems most to characterize his goals and his own subjective values. For a simplified model it is reasonable to postulate that farmers in general aim to achieve maximum income with minimum income variability, with different farmers placing differing degrees of subjective weight on each of these goals. Thus, if the perform-

TABLE IV

*Expected size and variability of average payoff resulting from the use of various criteria*

| Criterion | Recommended Strategies | Frequency | Average Expected Payoff | Expected Variance of Payoff | "Stability" of Payoff (3.00 — Variance) |
|---|---|---|---|---|---|
| A. Laplace | 1 | 1 | 3.06 | 0.02 | 2.98 |
| B. Maximum average expected outcome under risk | 7 | 1 | 4.48 | 2.09 | 0.91 |
| C. Wald—Pure strategy | 1 | 1 | 3.06 | 0.02 | 2.98 |
| D. Wald—Mixed strategy | 1<br>7 | 0.92<br>0.08 | 3.17 | 0.31 | 2.69 |
| E. Savage—Pure strategy | 1 | 1 | 3.06 | 0.02 | 2.98 |
| F. Savage—Mixed strategy | 1<br>7 | 0.54<br>0.46 | 3.71 | 1.36 | 1.64 |
| G. Hurwicz—$b < 0.3$ | 7 | 1 | 4.48 | 2.09 | 0.91 |
| H. Hurwicz—$b = 0.3$ (indifferent between strategies 1 and 7) | 1<br>7 | 0.50*<br>0.50* | 3.77 | 1.43 | 1.57 |
| J. Hurwicz—$b > 0.3$ | 1 | 1 | 3.06 | 0.02 | 2.98 |

*Assuming choice made in any year on the toss of a coin.

17. See *ibid*, pp. 286-98 ; c.f. also a discussion in C. B. Baker, *Decision Making and Financing Farm Assets* (Ontario Agricultural College, J. S. McLean Memorial Lecture), February, 1960 (Mimeo), p. 8.

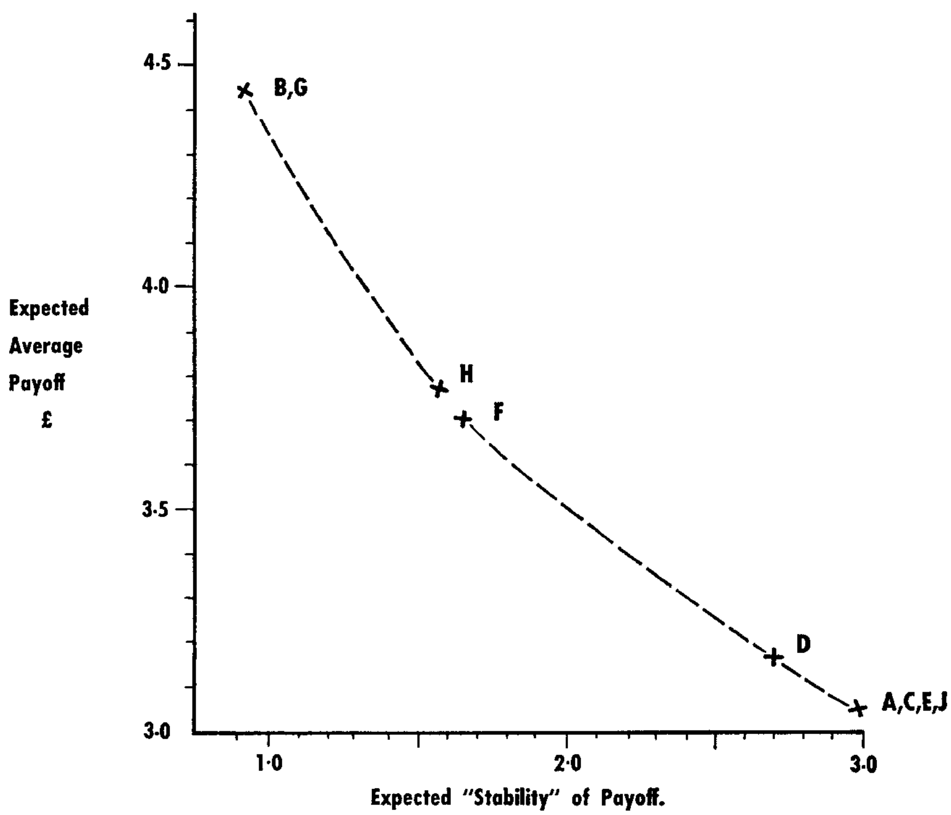
ance of each game criterion were evaluated in terms of the size and variance of the expected average payoff resulting from its use, then an individual decision-making farmer could consider it in relation to his own subjective risk-income indifference schedule.

Applying these principles, we list in Table IV the size and " stability " (measured by the variance subtracted from some arbitrarily selected constant, 3.00) of the average payoffs which could be expected if the courses of action recommended for the time-of-market decision by the several criteria were put into practice. It will be readily seen that the higher the average payoff, the higher its degree of instability.

Figure II, on which the " risk-income point " corresponding to each criterion is plotted, should in theory have superimposed on it a series of indifference curves characterizing a given farmer's rate of exchange of size for stability of payoff. Although data on the latter are lacking, some observations can be made.

The highest-return/most-risky course of action is recommended by criteria B and G (notation from Table IV) ,whilst A, C, E, and J lead to the least-return/most-stable outcome. H and F offer some compromise. It will be observed that moving from the B, G point to the A, C, E, J point on Figure II involves a small sacrifice of size of payoff, but a considerably larger reduction in variance. Thus it would appear that,

Fig. 2. " Risk Income Points " resulting from the Application of Various Criteria to the Time-of-Market Problem.



other things being equal, only a gambler would prefer the maximum outcome or Hurwicz optimistic criteria over the others, whereas the risk-income indifference patterns of more cautious farmers contemplating this decision would lead them to prefer criteria such as the Laplace, Wald or Savage.

Such conclusions as these are in no way general, but it is suggested that (a) they may be broadly applicable to a class of game problems, where larger profits are associated with greater variability, and *vice versa* ; (b) they indicate one possible way of approaching the problem of choice between decision criteria.

*Conclusion*

In this paper several decision criteria have been applied to a game between a particular farmer and nature. We have suggested that the problem of choice between criteria can be to a certain extent resolved in this and similar games by considerations of the risk-income preference patterns of the player.

It is interesting to note finally that the farmer at Tenterfield who first mentioned this decision problem to the author, has been playing strategy 7 for the last five years with some success. In his case, however, he has sufficient financial reserves to enable him to seek the highest average return without being unduly perturbed by its variability.