EQUATIONS RELATING THE COMPPOSITION OF BEEF CATTLE HERDS TO CERTAIN BASIC DATA

W. GRANGER, B.V.Sc., AND J. D. WALSH, B.A.

Equations are derived which relate the components of beef cattle herds to certain basic data. By enabling the construction of series of models to represent real or theoretical situations the equations provide a useful tool for analysing the influence of selected factors upon herds or populations—particularly upon their components, turnoff, and practicable rates of selection. The technique should be useful in economic, animal husbandry, and genetic studies, where decisions have to be taken upon management policy, and in evaluating proposals for private or public investments in station improvements or utilities for the cattle industry.

The Equations

The mixed herd or population of beef cattle comprises, according to the pattern of husbandry adopted, some or all of the following categories of stock: breeders, castrated males of various ages; females of various ages, up to the age at which they are either classed into the breeders or culled; culled females possibly speyed and possibly of various ages; females cast from the breeding herd comprising possibly groups which have been held for varying periods after casting; and bulls.[1] If certain basic data of a given herd or population are represented by symbols it is possible to express each component in terms of these symbols, and, having regard to the particular pattern of husbandry concerned, to construct an equation which expresses the relationship of the various components to each other and to the total. By substitution of values for the symbols, the equation can be used to determine an unknown or to examine associations with independent variables.

Let T equal the number of stock in the total herd;

let B equal the number of breeders in the total herd; and

let m equal the average fertility of the breeders, expressed as a percentage ratio of calves branded to cows joined and mustered at branding;

then the average number of branded calves in the herd is Bm. ...... (1)

Let b_1, b_2, etc., equal wastage, expressed as a percentage ratio, in castrated male stock during the first, second, third, and succeeding years after branding;

[1] In this paper the term "culled" cows or heifers is applied to those rejected when selecting young cows to add to the breeders, and "cast" is applied to cows taken from the breeders: i.e., cast from the breeders for age or for any other reason.

then the average numbers of castrated male stock about 1, about 2, etc., years old are, respectively,

$B \frac{m}{2} (1 - b_1)$, $B \frac{m}{2} (1 - b_1) (1 - b_2)$ .... etc. (2)

Similarly, let h_1, h_2, etc., equal wastage, expressed as a percentage ratio, in branded female stock not added to the breeding component during their first, second, third, and succeeding years after branding.

then, as in the case of castrated male stock, the average numbers of young unclassed female stock are, depending upon the age of classing into the breeders,

$B \frac{m}{2} (1 - h_1)$, $B \frac{m}{2} (1 - h_1) (1 - h_2)$ .... etc. (3)

Let w equal the average annual wastage of breeders expressed as a percentage ratio;

then Bw breeders are lost each year, and that number of replacements must be classed into the breeders to maintain this component. (4)

If it is practice to cast a proportion of breeders either at one operation or periodically throughout the year, let c equal the average annual cast, expressed as a percentage of the number of breeders;

then the number of cast cows, C, equals Bc .... (5)

and that number of replacements also must be added to the breeders each year, if their numbers are to be maintained. If the cast cows are kept for a period the numbers are reduced in succeeding years by c_1, c_2, etc., analogous to the factors b_1, b_2, etc., and h_1, h_2, etc.

The number of females offered for classing is, from (3),

$B \frac{m}{2} (1 - h_1)$, or $B \frac{m}{2} (1 - h_1) (1 - h_{2},)$ etc.,

depending on the age at which they are classed, and the number required to maintain the numbers of breeders equals replacement of wastage and of those cast; i.e. by addition of (4) and (5):

$Bw + Bc$, or $B (w + c)$:

and the number of culls is, therefore,

$$B \frac{m}{2} (1 - h_1) - B (w + c), \text{ or}$$

$$B \frac{m}{2} (1 - h_1) (1 - h_2) - B (w + c)$$

etc., depending upon the age at which the young cows are classed .... (6)

Let p equal the percentage of bulls of all ages to breeders held in the herd;

then P, the number of bulls, equals Bp. .... (7)

Example

To determine the components of a herd when policy is to maintain a constant number of breeders under the following circumstances: the average percentage of calves branded to breeders which are joined and mustered at the subsequent branding is 80%; the average loss in male calves in the first year after branding is 10%; the average losses in succeeding years up to turnoff of bullocks at $3\frac{1}{2}$ years old are 5%; the average annual losses in branded heifer calves, in weaned heifers and in culled heifers are 5%; the heifers are classed into the breeders to calve at 3 years old and culled heifers are disposed of unspeyed at $3\frac{1}{2}$ years old; the average annual wastage in the breeders is 9%, and 14% of breeders are cast each year and disposed of within a year of casting; and 4% of bulls to breeders are kept.

The equation applying to this situation is T = B breeders + Bm branded calves + $B\frac{m}{2}(1-b_1)$ one-year-old bullocks + $B\frac{m}{2}(1-b_1)(1-b_2)$ two-year-old bullocks + $B\frac{m}{2}(1-b_1)(1-b_2)(1-b_3)$ three-year-old bullocks + $B\frac{m}{2}(1-h_1)$ one-year-old heifers + $B\frac{m}{2}(1-h_1)(1-h_2) - B(w+c)$ cull heifers two years old + $\left(B\frac{m}{2}(1-h_1)(1-h_2) - B(w+c)\right)(1-h_3)$ cull heifers three years old + Bc cast cows + Bp bulls.

The herd components per thousand breeding cows, obtained by simple substitution, are set out in column 2 of Table 1. By adding these it is found that T, total herd, the unknown in this equation, equals 3,642. The herd components are expressed per thousand head of mixed herd in column 3, the value in each case being obtained by multiplying the value in column 2 by 1000/3642.

Table 1

Herd Component (1)	*Per Thousand Breeding Cows* (2)	*Per Thousand Mixed Herd* (3)
Breeders	1,000	275
Branded calves	800	220
Bullocks, 1 year	360	99
Bullocks, 2 year	342	94
Bullocks, 3 year	325	89
Heifers, 1 year	380	104
Heifers, 2 year culls	131	36
Heifers, 3 year culls	124	34
Cast Cows	140	38
Bulls	40	11
TOTAL HERD	3,642	1,000

Discussion

The widest use of equations of this nature will lie, it is thought, not in direct application to real situations but in the examination of theoretical or model herds or populations as a means of analysing the influence of selected factors upon herd components, total population, turn-off and practicable degrees of selection.

The relationship between the components of females in the herd is an important one. It is possible to state this as an equation relating the initial cull heifer component to the annual crop of heifers offered for classing less the number classed into the breeders to replace losses by wastage and casting; i.e. (assuming heifers are classed at 2½ years):

$$\text{Cull heifers} = B\,\frac{m}{2}\,(1 - h_1)\,(1 - h_2) - B\,(w + c).$$

This equation, referred to for convenience as the "equation for females," may be used not only to determine the culled heifer component but also to examine whether natural increase is sufficient to maintain herd numbers; the relationship between wastage of breeders, culling and casting rates; and the practicable degree of selection when classing heifers into the breeders. If on solution the value for culled heifers is negative, natural increase will not provide for the maintenance of the breeding component; if the value for the culled heifer component is zero there is a "break-even" situation; where the equation gives a positive value, natural increase provides a surplus of females. As indicated in the equation such a surplus may be disposed of by culling or by casting, or partly by both. Needless to say, the value of w, the percentage annual wastage of breeders, is an important influence in determining whether or not such a surplus is available, and its magnitude. If there is a surplus and culling of young females is practised, the degree of selection which is practised or is practicable is a factor of importance in genetical studies. $B\,(w + c)$ heifers are to be selected from $B\,\frac{m}{2}\,(1 - h_1)\,(1 - h_2)$ or whatever expression is appropriate to the circumstances. The degree of selection is therefore given by the equation

$$\frac{B\,(w + c)}{B\,\frac{m}{2}\,(1 - h_1)\,(1 - h_2)} = \frac{2\,(w + c)}{m\,(1 - h_1)\,(1 - h_2)} \qquad (8)$$

Another way of deriving an expression for the degree of selection of young heifers which is practicable, which links this paper with one[2] dealing with the same topic in sheep, is to consider the number of times in her life each cow is joined and subsequently mustered at calf-branding, and call this n. Then it is self-evident that the number of

---

[2] W. Granger, "Selection of Breeding Ewes: Dependence of Practicable Degree of Selection of Young Ewes upon Vital Statistics," *Austr. Vet. Jl.*, 20 : 253 (1944).

replacements required annually is B/n, and the degree of selection practicable is given by the equation,

$$s = \frac{\frac{B}{n}}{B\frac{m}{2}(1-h_1)(1-h_2)} = \frac{2}{n\,m\,(1-h_1)(1-h_2)} \qquad (9)$$

Of course, since $\frac{B}{n} = B\,(w + c)$, and therefore $w + c = \frac{1}{n}$

it is true that (9) could have been derived directly from (8) by substitution.

The following example serves to show some uses of the equations in forming economic assessments.

A property of 330 square miles in northern Australia carried 5,400 head of branded cattle. Being somewhat remote from the rail which takes cattle to fattening areas or abattoirs, it turns off for droving to the trucking points bullocks at about 5 years of age. Animal husbandry practice is poor, and the branding percentage achieved is only 50%. Wastage from breeders is great and no heifers are turned off, but the breeding component is maintained by incorporating all heifers into the breeders between 1 and 2 years. Ten per cent. of branded calves are lost, but only 1% of 1-year-old bullocks, and the loss in bullocks 2 years old and over is negligible. Percentage of bulls held to breeders is 5. In addition, a certain number of cleanskins, say roughly 5% of the branded herd and of varying ages is known to be on the property. Under these conditions, herd components and turn-off determined as described earlier in this paper, would be as in column A of Table 2.

Suppose the property changes hands. The new owner considers that by putting down two additional waters, constructing better yards, erecting subdivision fences, and improving management, he can raise the safe carrying capacity to 7,000 head; reduce "cleanskins" to negligible numbers; reduce wastage in breeders and losses in branded calves to 10% and 7% respectively; eliminate the 1% loss in 1-year-old bullocks and raise branding to 80%. The reduction in breeder wastage will enable him to spey a proportion of heifers and turn them off as culls at 4 to 5 years; and also to cast 15% of breeders each year for age, replacing losses and cast with 2-year-old heifers. The components of the herds and turnoff determined by the appropriate equation would then be as shown in column B of Table 2 and this provides a basis for the buyer's budgeting to assess the profitability of the investment on improvements.

Suppose now that the Government is considering expenditure on roads and utilities at the railing point to encourage the operation of road-trains in lieu of droving and suppose that there is a ready market for young stores if advantage is taken of these facilities to turn them off. Column C of Table 2 shows the herd components and turnoff if steers and cull heifers were now turned off at 2 years from this property.

Table 2

Categories of Stock	*A*	*B*	*C*
Breeders	1,299	1,553	2,211
Branded calves	586	1,155	1,644
One-year-old bullocks	579	572	814
Two-year-old bullocks	573	572	814
Three-year-old bullocks	573	572	—
Four-year-old bullocks	573	572	—
Five-year-old bullocks	573	572	—
One-year-old heifers	579	572	814
Two-year-old cull heifers	—	183	260
Three-year-old cull heifers	—	183	—
Four-year-old cull heifers	—	183	—
Cast cows	—	233	332
Bulls	65	78	111
Cleanskins (various ages)	270	—	—
TOTAL HERD	5,400 (a)	7,000	7,000
Turnoff (Categories)			
Two-year-old bullocks	—	—	814
Five-year-old bullocks	573	572	—
Two-year-old cull heifers	—	—	260
Four-year-old cull heifers	—	183	—
Cast cows	—	233	332
TOTAL TURNOFF	573 (b)	988	1,406
% turnoff	10.6	14.1	20.9

(a) exclusive of cleanskins;

(b) in practice under the animal husbandry conditions applying in the example, the turnoff would probably be made up of stock varying from 2 to 8 years (modal age of turnoff 5 years) and would include a proportion of "cleanskins."

The inducement to the pastoralist to use road-trains is easily seen. By aggregating survey data of this kind in the area concerned the Government would be able to assess the potential contribution which its expenditure would make to the cattle industry of the region. Such studies would also show the relative importance and inter-dependence of property improvement and marketing opportunity in influencing herd composition and turnoff.

Acknowledgments

We wish to thank Miss Helen Newton Turner for criticisms of an early (1956) draft of this paper, Mr. J. H. Kelly and, more lately, Mr. W. Allen, who have discussed matters relating to this paper.