A NOTE ON BREEDING FLOCK COMPOSITION IN RELATION TO ECONOMIC CRITERIA

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Decisions on whether to cull or to retain for further production are of major importance in all classes of stock farming. Although there have been several studies relating flock composition to culling, reproduction and death rates,[1] none of these have attempted to compare the *profitability* of various culling policies, their objective being only to show how flock composition and numbers of stock available for culling vary with variation in reproduction and death rates. Clearly, for given basic population parameters and a given framework of fixed farm resources, there should be a culling policy which is optimum for the particular farm. This note presents a linear programming model which can be used to determine the optimum culling policy for a Romney breeding flock.[2]

For any age class of stock, the decision on whether to cull or to retain for further production depends basically on three factors:

(i) The price that can be obtained for the cull animal.
(ii) The expected future production if the animal is not culled and the prices that can be expected for the products.
(iii) The animal's expected resource requirements over the future production period.

With a breeding flock, the expected future production comprises wool and lambs. Unless lambing percentages are abnormally low, there are more young stock coming forward each year than are required to maintain ewe flock numbers. Thus, a certain number of stock must always be sold—assuming fixed carrying capacity, labour supplies, etc. The decision is, how many animals of each particular age class should be sold?[3]

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[1] See for example: Hickey, F., "Death and reproductive rates of sheep in relation to flock culling and selection". *N.Z. Jnl. Agr. Res.* 3: 332, 1960. Grainger, W. and Walsh, J. D., "Equations relating the composition of beef cattle herds to certain basic data". *Aust. Jnl. Agr. Eco.* 3: 58, 1959.

[2] For our purposes, a "breeding flock" is defined as a flock in which ewes are the major stock class. Sufficient numbers of female progeny are retained to maintain breeding ewe numbers. Thus, a breeding flock is "self-contained" and rams are the only stock purchased. This is a common policy on hill country sheep farms in New Zealand.

[3] The terminology used for sheep classes in this paper may be summarized as follows: Lambs are born in the Spring (September) and remain lambs until February-March of the following year. From that period to September they are classed as hoggets and thereafter as 2-tooths, 4-tooths, 4 yr. old etc. as each year goes by.

This decision is not a simple one. If all farm resources could be considered to be variable the decision would be simply to choose the culling policy which equated the marginal cost of retaining an animal in the flock with the marginal return from its expected future production. In practice, however, most farm resources can be considered to be fixed in the short term. The marginal cost of retaining an animal is determined by both the direct costs (shearing, dipping, etc.) and the opportunity cost in terms of scarce resources which have alternative uses for other stocking activities.

In a breeding flock, the number of stock of any particular age class that can be wintered is limited by the number of stock of the immediately younger age class coming forward. Clearly, the number of ewe hoggets available equals the number of ewe lambs born minus losses to the hogget stage minus the number of ewe lambs sold. The number of ewe lambs available depends on the number of breeding ewes wintered, which in turn, depends on the number of ewe hoggets wintered. Thus a breeding flock is 'self-contained' and internally interdependent.

This situation is illustrated in Table 1. Table 1 gives the first simplex tableau of a linear programme designed to select the optimum culling policy for a hypothetical sheep farm, 250 acres in size.

The basic data relating to prices, wool weights, lambing percentages and sheep losses used to derive Table 1 are given in Tables 2 and 3.[4]

In Table 1, the fundamental restriction is Winter carrying capacity. For our hypothetical example we consider a single winter stocking rate of four ewe equivalents per acre. In addition we assume that ten hoggets are equivalent to six ewes.

The figure of 250 in the B column of Table 1 is land available for wintering sheep. The coefficients at the intersections of the land row and the stock wintering activities (columns), indicate the carrying capacity requirements of each stock activity at the given stocking rate. For example, the 0·25 coefficient at the intersection of the 'ewe 2-tooth winter' activity, P_{16}, and the land restriction, indicates that each ewe wintered requires 0·25 acres of winter carrying capacity. Similarly, the 0·15 coefficient at the intersection of the 'ewe hogget winter' activity, P_{14}, indicates that each hogget wintered requires 0·15 acres of winter carrying capacity.

Activities P_2 to P_{13} are stock selling activities and replace conventional disposal activities. The positive coefficients in the selling activity columns represent the per unit requirements of each stock selling activity of the corresponding stock class in the B column. For example, activity P_2 'Ewe Lambs sell (fat)' requires 1·03093 ewe lambs born for each ewe lamb sold fat. The coefficient is greater than unity to allow for losses (3 per cent) from birth to sale.

Activities P_{14} to P_{20} inclusive are stock wintering activities. The positive coefficients in the stock wintering activity columns represent the per unit requirements of the stock class (represented by the column) of the (immediately younger) stock class represented by the row. The coefficients are greater than unity to allow for losses. For example, the 'ewe 2-tooth winter' activity (P_{16}) has a positive requirement of 1·05263 units of the ewe hogget restriction per 2-tooth ewe wintered.

[4] The coefficients representing losses and lambing percentages are based on data obtained by Hickey F., *op. cit.* for the New Zealand Romney. Prices used for the programme are based on ruling (1962/63) prices for Manawatu.

TABLE 1

First Simplex Tableau for Culling Policy Programme

	$C \rightarrow$	0	0	1.50	0	1.50	3.09	1.75	3.50	2.80	2.50	2.25
Restriction \ Activity	B	P_1 Disposal	P_2 Ewe lambs sell (fat)	P_3 Ewe lambs sell (store)	P_4 Wether lambs sell (fat)	P_5 Wether lambs sell (store)	P_6 Wether hogget sell	P_7 Ewe hogget sell	P_8 Ewe 2-tooth sell	P_9 Ewe 4-tooth sell	P_{10} Ewe 6-tooth sell	P_{11} Ewe 4 year sell
Carrying capacity	250	1					0.15					
Ewe lambs (fat)	0		1.03093									
Ewe lambs (store)	0			1.03627								
Wether lambs (fat)	0				1.03093							
Wether lambs (store)	0					1.03627	1.08108					
Ewe hogget	0							1	1.03093			
Ewe 2-tooth	0									1.02302		
Ewe 4-tooth	0										1.03093	
Ewe 6-tooth	0											1.04439
Ewe 4 year	0											
Ewe 5 year	0											
Fat lambs	0		−1		−1							

	$C\rightarrow$	2.00	0.80	1.34	1.34	1.75	1.83	1.83	1.75	1.67	1.5
Activity / Restriction	B	P_{12} Ewe 5 year sell	P_{13} Ewe 6 year sell	P_{14} Ewe hogget winter	P_{15} Ewe hogget winter	P_{16} Ewe 2-tooth winter	P_{17} Ewe 4-tooth winter	P_{18} Ewe 6-tooth winter	P_{19} Ewe 4 year winter	P_{20} Ewe 5 year winter	P_{21} Fat lambs sell
Carrying capacity	250			0.15	0.15	0.25	0.25	0.25	0.25	0.25	
Ewe lambs (fat)	0			1.08108		−0.29325	−0.35400	−0.36300	−0.40950	−0.40725	
Ewe lambs (store)	0				1.08108	−0.09775	−0.11800	−0.12100	−0.13650	−0.13575	
Wether lambs (fat)	0					−0.29325	−0.35400	−0.36300	−0.40950	−0.40725	
Wether lambs (store)	0					−0.09775	−0.11800	−0.12100	−0.13650	−0.13575	
Ewe hogget	0			−1	−1	1.05263					
Ewe 2-tooth	0					−1	1.04712				
Ewe 4-tooth	0						−1	1.06383			
Ewe 6-tooth	0							−1	1.09290		
Ewe 4 year	0	1.06952							−1	1.14942	
Ewe 5 year	0		1.08108							−1	
Fat lambs	0										1

This allows for 5 per cent losses from the ewe hogget stage (September) to the 2-tooth lambing stage (September). The negative coefficients indicate that each wintering activity supplies lambs. It was assumed that 75 per cent of the total number of lambs produced would be suitable for sale fat, while 25 per cent would be of store quality. Thus, each 2-tooth ewe supplies 0·29325 fat ewe lambs, 0·09775 store ewe lambs, 0·29325 fat wether lambs and 0·09775 store wether lambs, or a total of 0·782 of a lamb for each 2-tooth ewe wintered (a lambing percentage of 78·2% for 2-tooth ewes, see Table 2).

TABLE 2

Data on Prices, Wool Weights and Lambing Percentages

Sheep Class	Prices (£'s)	Wool Weight (lbs)	Lambing Percentage
Fat lambs	Varied		
Store lambs	1·50		
Hoggets	1·75	8·0	
2-tooth ewes	3·50	10·5	78·2
4-tooth ewes	2·80	11·0	94·4
6-tooth ewes	2·50	11·0	96·8
4 year ewes	2·25	10·5	109·2
5 year ewes	2·00	10·0	108·6
6 year ewes	0·80		

Note: Wool price used was 40d per lb.

The programme, as formulated in Table 1, effectively 'ties together' the various stock activities.

The optimum culling policy would be expected to differ with variation in prices and production coefficients. For example, if the ratio of the prices of store stock/fat stock was high (as in the case of a 'grass market') the optimum policy may be to retain all ewe progeny until the 2-tooth stage to be sold as stores. On the other hand, if this ratio was lower, it may be more profitable to sell as many as possible ewe lambs (apart from those necessary to maintain ewe flock numbers) as fat

TABLE 3

Data on Stock Losses

Sheep Class	Period	Losses
Fat lambs	Birth to Dec.	3·0%
Store lambs	Birth to Dec.	3·5%
Hoggets	Birth to Sept.	7·5%
Hogget to 2-tooth	Sept. to March	3·0%
Hogget to 2-tooth	Sept. to Sept.	5·0%
2-tooth to 4-tooth	Sept. to Sept.	4·5%
4-tooth to 6-tooth	Sept. to Sept.	6·0%
6-tooth to 4 year	Sept. to Sept.	8·5%
4 year to 5 year	Sept. to Sept.	13·0%

Note: Ewe losses to sale (March) are presumed to be half those from September to September.

lambs. The 'fat lambs' row and the 'fat lambs sell' activity (P_{21}) are incorporated to allow parametric variation of fat lamb prices.[5]

Results of the Analysis

The programme was run on the New Zealand Government Treasury augmented IBM 650 computer. The results are summarized in Table 4.

TABLE 4

Programme Results

Activity	Description	Level	Revenue per unit	Revenue from Activity	Land Requirement per unit	Total land Requirement
P_4	Wether lambs sell (fat)	270·924	—	—		
P_5	Wether lambs sell (store)	89·842	1·50	134·763		
P_8	Ewe 2-tooth sell	113·674	3·50	397·859		
P_{12}	Ewe 5-year sell	165·828	2·00	331·656		
P_{14}	Ewe hogget	258·356	1·34	345·163	0·15	38·753
P_{15}	Ewe hogget	86·119	1·34	115·055	0·15	12·918
P_{16}	Ewe 2-tooth	215·921	1·75	377·861	0·25	53·980
P_{17}	Ewe 4-tooth	206·205	1·83	377·355	0·25	51·551
P_{18}	Ewe 6-tooth	193·832	1·83	354·712	0·25	48·458
P_{19}	Ewe 4-year	177·356	1·75	310·373	0·25	44·339
P_{21}	Fat lamb sell	270·924	1·50	406·386		
	TOTAL			3,151·183		249·999

From Table 4 it can be seen that, for the given prices and production coefficients, the most profitable culling policy is to sell all wether progeny as lambs. Ewe lambs are retained, and the surplus after fulfilling breeding flock replacement requirements are sold as store 2-tooth ewes. Ewes are cast for age at 5 years.

Using price-variable parametric programming, it was found that the plan given in Table 4 was optimum over the range in fat lamb prices, P_{fl}, given by:

$$1{\cdot}50 \leqslant P_{fl} \leqslant 2{\cdot}78 \qquad (1)$$

Revenue, R, from this plan is given by equation (2) below:

$$R = 2{,}744{\cdot}797 + 270{\cdot}924\, P_{fl}, \qquad (2)$$

subject to (1) above.

At a fat lamb price higher that £2·78, 'ewe lambs sell (fat)', P_2, came into the optimum plan, replacing activity P_8 (ewe 2-tooth sell).

The relationship between revenue from this sheep policy and fat lamb price, for $P_{fl} \geqslant 2{\cdot}78$, is given by:

$$R = 2{,}409{\cdot}704 + 391{\cdot}392\, P_{fl} \qquad (3)$$

$$\text{Subject to: } P_{fl} \geqslant 2{\cdot}78$$

Discussion:

It should be emphasized that the example given is a simplified one illustrating the method used rather than a detailed culling policy for any

[5] Candler, W. V.: "A modified simplex solution for linear programming with variable prices." *Jnl. Farm Eco.* 39: 409, 1957.

particular farm. Indeed the problem as formulated in Table 1 can be solved by other, and possibly more simple methods.[6]

However, several modifications can easily be made to a tableau of the type given in Table 1, making the programme more realistic for any particular farm situation. Some of these modifications are:

1. In Table 1, there is only one limiting resource, Winter carrying capacity. Although, for many New Zealand farms, carrying capacity over the Winter and early Spring is likely to be the major factor limiting the number of stock carried, it is unlikely to be the only factor. Other limiting resources such as labour, carrying capacity at other times of the year, etc., may be important in any particular case. Any further restrictions could be easily incorporated in the programme.
2. Stock buying activities could be incorporated allowing the farm to buy in stock to supplement flock numbers. For example '2-tooth buy' could be included as an activity with a positive $Z—C$ value, supplying (through a negative coefficient in the 2-tooth row) 2-tooths into the ewe flock. In the same way, 'one year' and 'two year' cast ewes could be included as stock buying activities allowing selection of the most profitable stock replacement policy. The programme could also be expanded to include cattle activities.
3. An extension to the number of prices varied would give optimum plans for a greater number of price situations. This information would be useful in selecting a stable plan.
4. The programme, as formulated in Table 1, does not allow investigation of the most profitable timing of stock operations such as culling and shearing. The addition of such activities such as 'December shearing', 'March lamb selling', etc., would allow such a refinement to be made.
5. For some farms, the optimum farm plan may include various cash cropping activities as well as stock activities. Clearly, by including the appropriate activities and restrictions, together with a 'breeding flock matrix' of the form given in Table 1, the optimum combination could be determined.
6. Stocking rate variation: We may wish to consider the effect of varying stocking rates on total revenue. Where increased stocking rates result in lower production per animal (lambing percentage and wool production), and/or higher death rates, this may be simply achieved by additional stock activities whose coefficients reflect the expected changes in productive performance and winter land requirements at the higher stocking rates considered. In our example, activity P_{16}, ewe 2-tooth winter, has a coefficient of 0·25, in the first row, indicating a carrying capacity of 4 ewe equivalents per acre.

   A second activity, P^*_{16}, for wintering 2-tooth ewes, with a coefficient of 0·125 in this row would indicate a carrying capacity of 8 ewe equivalents per acre.
7. Long term genetic effects. The result of considering the effects of genetic selection on the optimum policy given in Table 4 could be

[6] For example, a series of appropriate budgets could give a culling policy which is very likely to be more profitable than any other feasible plan. This simplified problem may also be solved using dynamic programming.

that it may be profitable to cull more animals of any particular age class now because of expected genetic improvement in the future. We are now concerned, not with maximising present returns from the flock, but maximizing the present value of the expected return over some future period. Of course, the animals culled in the policy given by Table 4 would be the worst of their age class, and hence some long term genetic improvement would be expected. However, this policy is not necessarily the most profitable in the long term. Consideration of the use of programming in providing economic criteria for selection indices is being investigated and will be discussed in a further paper.