THE EFFECT OF FERTILISER ON RISK:
A HETEROSCEDASTIC PRODUCTION
FUNCTION WITH MEASURABLE
STOCHASTIC INPUTS

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The sources of production risk are many and diverse in nature. Estimating risk as a black box, without explicit recognition of its sources, can lead to inferior estimates of optimal inputs under risk aversion. In this paper, a method is presented for estimating production functions with measurable stochastic inputs and for generating the parameters of the probability distributions of yield for various environments and input levels. Based on this method, it appears that moderate risk aversion can account for a 6.7 per cent to 16.7 per cent reduction in nitrogen use (relative to the risk-neutral solution) for selected rice producing areas of the Philippines. Estimating optimal inputs without environment-specific information about the sources of risk leads to large errors. This underscores the value of collecting information about the sources of risk and of exercising caution when such information is not available.

Estimating the probability distributions of yield is a major part of both positive and normative analyses of decision making under risk (Roumasset 1976, 1979; Anderson, Dillon, and Hardaker 1977). Anderson et al. (1977) have classified methods of estimating production risk as 'gross' or 'analytical'. In the gross approach, production risk is treated as a black box, without any attempt to identify the stochastic sources of risk. One such approach is to estimate the functional relationship between the moments of the yield distribution and managed input levels (Day 1965; Doll 1972; Anderson 1973; Antle 1983; Antle and Goodger 1984). Alternatively, heteroscedastic production functions have been specified, with the variance of the stochastic error term allowed to vary with the level of managed inputs (Just and Pope 1978, 1979; Anderson and Griffiths 1981). A problem with the gross approach is that the exclusion of the measurable stochastic variables which contribute to production risk inhibits understanding of the causal factors underlying yield variability and limits generalisation of the results to different environments.

In the analytical approach, there is an attempt to identify and estimate the independent or joint effects of the major stochastic sources of production variability and to incorporate these in the production function (for example, Byerlee and Anderson 1969; de Janvry 1972; Ryan and Perrin 1973; Roumasset 1976; Rosegrant and Herdt 1981). The primary disadvantage of the analytical approach to date has been the restrictions placed on the interaction between controlled inputs and the stochastic elements of production. In particular, unmeasured

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stochastic inputs have been represented in the production functions by a homoscedastic error term. In this paper, a heteroscedastic response function with measurable stochastic variables is presented. The model makes maximum use of available information, allows asymptotically efficient estimation, permits disaggregation of the general production function by agro-climatic environment through estimation of the joint probability distributions of the measurable stochastic variables, and permits unrestricted estimation of the impact of managed inputs on yield variability.

In the first section of the paper, the heteroscedastic production function with measurable stochastic inputs is specified and applied to experimental data for modern rice varieties from farmers’ fields in the Philippines. In the second section, the methodology is described for estimating the yield distributions based on the joint probability distribution of the stochastic variables. Estimated parameters of the yield distributions for different crop seasons and water regimes are presented. In the third section, an expected utility maximisation framework is used together with the yield distributions to assess optimal fertiliser use under different attitudes to risk. Next, results derived from the heteroscedastic production function are compared with simpler alternative specifications of the production function. The final section contains general observations about the treatment of stochastic production relationships.

Estimation of the Production Function

Specification

A general production function can be represented as:

\[ y = f(x, z, v, u) \]

where \( y \) is yield per hectare, \( x \) is a vector of managed inputs, \( z \) is a vector of fixed factors and environmental inputs, \( v \) is a vector of measurable stochastic inputs, and \( u \) is a vector of non-measurable stochastic inputs (see also Anderson et al. 1977). A number of alternative specifications of the production function were examined. The alternatives incorporated different degrees of generality in the treatment of expected yield response to nitrogen fertiliser and yield variance due to non-measurable stochastic variables.

In the preferred heteroscedastic production function, the expected yield is first estimated as a function of the independent variables (managed inputs, fixed factors, and measurable stochastic inputs). Measurable stochastic variables are included in direct and interactive terms with managed inputs. The variance of yield from the non-measurable inputs (that is, the variance of the stochastic disturbance term from the expected yield function) is then estimated as a function of the independent variables. This specification is preferred because it provides the most general specification of interaction between nitrogen and measurable stochastic variables and the most general relationship between the variance of the non-measurable stochastic variables and measurable variables; because it permits consistent and asymptotically efficient estimation of parameters; and because it provides a slightly superior statistical fit to alternative specifications. Results for the two best alternatives are discussed below.
The general form of the model is:

\[
Y_t = X_t'\beta + \varepsilon_t, \quad t = 1, 2, \ldots, T
\]

\[
E(\varepsilon_t) = 0, \quad E(\varepsilon_t) = \sigma_t = h(X_t, \alpha), \quad E(\varepsilon_t \varepsilon_s) = 0, \quad t \neq s
\]

where \( Y_t \) is the \( t \)th observation on yield and \( X_t \) is a \((k \times 1)\) vector containing the \( t \)th observation on the \( k \) independent variables, \( \varepsilon_t \) is the \( t \)th normally distributed stochastic disturbance term and \( \alpha \) and \( \beta \) are \((k \times 1)\) vectors of unknown parameters. A number of alternative specifications of the variance term function have been suggested (Just and Pope 1978, 1979; Harvey 1976; Judge, Griffiths, Hill, and Lee 1980). The specification \( \sigma_t^2 = X_t'\alpha \) is used here because it provides the best statistical fit to the data.

**Estimation results**

Consistent and asymptotically efficient generalised least squares estimators for \( \alpha \) and \( \beta \) were obtained using the estimation procedure given by Amemiya (1977), and described in detail by Judge et al. (1980, pp. 136–7). The estimation technique was applied to data compiled from a series of yield response experiments for modern rice varieties on farmers’ fields in Central Luzon, the Philippines, by the International Rice Research Institute beginning with the wet season of 1973 and ending with the dry season of 1977, for a total of four crop years (eight seasons). The experimental design was based on fertiliser, weed control, and insect control, with other inputs and cultivation techniques left to the farmers’ discretion. The levels of use of non-test inputs and the values of agro-climatic variables were measured through close monitoring of the plots (IRRI 1977; Herdt and Mandac 1981). The pooled data set consists of 3617 observations.

The estimated expected yield and variance functions are summarised in Table 1. The Breusch-Pagan test was utilised to test for homoscedasticity of the error term. The Breusch-Pagan test statistic is distributed asymptotically as chi-square with \( k-1 \) degrees of freedom under the null hypothesis of homoscedasticity (Breusch and Pagan 1979; Judge et al. 1980). The computed value of the test statistic is 455.7, indicating that the null hypothesis of homoscedasticity should be rejected.

The parameter estimates are agronomically reasonable and statistically highly significant.\(^1\) The managed inputs (nitrogen, phosphorus, insect control cost and weed control) all have a positive impact on yield. The marginal product of nitrogen declines as the level of nitrogen increases. Pest damage, disease, moisture stress, typhoon, age of seedling and clay content reduce yields, while solar radiation and organic matter increase yields.\(^2\) Yields on farmer-controlled plots are

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\(^1\) Several variables (nitrogen, pest damage index, stress days, and farmer/researcher dummy) are included in interaction terms but not without interaction. Parameter estimates for these variables in non-interactive form were statistically insignificant when included with interaction terms. The non-interactive terms were therefore deleted. The impact of these variables on yield variance is captured by the interaction terms.

\(^2\) The impact of clay content on yield has ranged from highly positive to highly negative in previous production function analyses (Rosegrant and Herdt 1981, Herdt and Mandac 1981). However, with both organic matter and stress included in the function, the negative relationship is plausible, as Wickham (1971) points out that at any given level of water shortage, heavy soil increases the stress on the root system of the plant.


**TABLE 1**

*Generalised Least Square Estimates of the Parameters of the Expected Yield and Variance Functions for Modern Rice Varieties, Philippines*

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated parameters&lt;sup&gt;a&lt;/sup&gt;</th>
<th>Variance function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expected yield function</td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>163.08 (194.14)</td>
<td>701 890.70 (332 649.62)</td>
</tr>
<tr>
<td>Nitrogen (kg/ha) × solar radiation&lt;sup&gt;b&lt;/sup&gt;</td>
<td>1.06 (0.06)</td>
<td>-242.68 (103.27)</td>
</tr>
<tr>
<td>Nitrogen squared</td>
<td>-0.055 (0.01)</td>
<td>62.42 (11.11)</td>
</tr>
<tr>
<td>Phosphorus (kg/ha)</td>
<td>7.03 (1.19)</td>
<td>-3 134.96 (2 035.69)</td>
</tr>
<tr>
<td>Insect control cost (peso/ha)</td>
<td>0.45 (0.05)</td>
<td>232.62 (90.51)</td>
</tr>
<tr>
<td>Weed control index (scale: 1–8)</td>
<td>52.86 (6.04)</td>
<td>10 387.42 (10 284.57)</td>
</tr>
<tr>
<td>Seedling age (days)</td>
<td>-15.36 (2.20)</td>
<td>-11 440.00 (3 763.16)</td>
</tr>
<tr>
<td>Nitrogen × pest damage index (70)</td>
<td>-0.11 (0.01)</td>
<td>-64.88 (23.94)</td>
</tr>
<tr>
<td>Disease incidence index (per cent)</td>
<td>-25.58 (1.77)</td>
<td>5 230.97 (3 763.16)</td>
</tr>
<tr>
<td>Organic matter in soil (per cent)</td>
<td>256.13 (32.75)</td>
<td>67 540.45 (55 818.55)</td>
</tr>
<tr>
<td>Clay content of soil (per cent)</td>
<td>-18.90 (1.55)</td>
<td>130.67 (2 613.40)</td>
</tr>
<tr>
<td>Solar radiation</td>
<td>68.56 (7.12)</td>
<td>523 296.67 (118 661.38)</td>
</tr>
<tr>
<td>Solar radiation × stress days&lt;sup&gt;c&lt;/sup&gt;</td>
<td>-1.22 (0.32)</td>
<td>-1 397.30 (539.58)</td>
</tr>
<tr>
<td>(Nitrogen + phosphorus) × stress days squared</td>
<td>-0.024 (0.003)</td>
<td>15.31 (5.51)</td>
</tr>
<tr>
<td>Typhoon dummy</td>
<td>-213.79 (74.75)</td>
<td>-113 583.00 (127 621.35)</td>
</tr>
<tr>
<td>Typhoon dummy × (nitrogen + phosphorus)</td>
<td>-3.27 (0.61)</td>
<td>1 590.93 (1 026.41)</td>
</tr>
<tr>
<td>Farmer/researcher dummy&lt;sup&gt;d&lt;/sup&gt; × (nitrogen + phosphorus)</td>
<td>-1.60 (0.34)</td>
<td>82.75 (591.07)</td>
</tr>
<tr>
<td>Year (trend)</td>
<td>383.79 (20.56)</td>
<td>-95 292.30 (35 293.44)</td>
</tr>
</tbody>
</table>

<sup>a</sup> The numbers in parentheses are estimated standard errors.

<sup>b</sup> Solar radiation in kcal/cm², from 45 days before harvest to harvest.

<sup>c</sup> Stress days from 60 days after transplanting to 20 days before harvest.

<sup>d</sup> Dummy variable for farmer (1) or researcher (0) supervised plot.
slightly lower than those supervised by researchers, and yields increased substantially over time. Nitrogen productivity is positively related to solar radiation and negatively related to moisture stress, pest damage, typhoon and farmer supervision. Increases in solar radiation intensify the negative impact of stress.

Of particular interest in this analysis is the effect of nitrogen on the error term component of yield variance. At mean levels of the other variables, the error term variance first decreases, then increases as the nitrogen level increases. However, the impact of nitrogen on yield variability must take into account also the variability of the measurable stochastic variables which interact with nitrogen. To assess this impact, and the impact of other variables on yields variance, the parameters of the probability distributions of yield are estimated, taking into consideration measurable and non-measurable stochastic variables.

Estimation of the Parameters of the Probability Distributions of Yield

A major advantage of the inclusion of measurable stochastic variables is the ability to distinguish the yield distributions resulting from different combinations of agro-climatic variables. Here, parameters of the yield distributions (expected yield, variance, and coefficients of skewness and kurtosis) are estimated for four combinations of water regime and season. The latter effect the yield distributions through the relevant probability distributions of solar radiation, moisture stress and typhoon occurrence.

The moments of the probability distribution of yield for any set of managed inputs can be estimated from the joint probability distribution of the observable and non-observable stochastic variables. It is assumed that the measurable stochastic variables are independently distributed. Information is unavailable on the long-term conditional probability distributions of the stochastic variables, but correlation coefficients among the variables estimated from the eight seasons in the data set are low (in the range 0.03 to 0.14), so the assumption of independence appears reasonable.

With the independence assumption, the moments of the yield distribution for any managed input level can be estimated by repetitively sampling from the probability distributions of the stochastic variables and the stochastic disturbance term and computing point estimates of yield. For the analysis here, 1000 iterations of the sampling process produce stable estimates of the moments of the yield distribution.

To undertake this method, probability distributions for pest damage and disease damage were estimated from the response function data set. The probability distribution of typhoon occurrence was computed from...

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1 The direct time effect on yield is handled through a trend variable. Estimation of the joint probability distribution of the stochastic variables permits assessment of the impact on production variability of both variation over time and of cross-sectional variation of the stochastic variables. An alternative would be to use an error components approach.

4 In many environments, stochastic variables such as solar radiation and moisture stress may not be independently distributed. Under such conditions, the sampling procedure used here can be extended to a stochastic process model which preserves the covariance among the stochastic variables. See, for example, Larsen and Pense (1981).
farmer estimates over five crop years in Central Luzon. Wet and dry season solar radiation distributions were estimated from research station records in Central Luzon, 1966–75. Stress day distributions were estimated for four water regimes: dry season, good irrigation, low seepage and percolation (the rate of loss of water through the soil); dry season, average irrigation, medium seepage and percolation; wet season, average irrigation, medium seepage and percolation; and wet season, rainfed, medium seepage and percolation. The estimation procedure uses a stochastic water balance model which simulates weekly rainfall and irrigation, based on sampling from distributions of irrigation flows for Central Luzon sites and from the weekly rainfall distribution at Cabanatuan City, Nueva Ecija, 1949–74. Stress days are computed as a function of weekly water additions (irrigation plus rainfall) and losses (seepage and percolation plus evapotranspiration and drainage) from the site. Iteration of the model produces a stable estimate of the distribution of stress days for each water regime (Rosegrant 1978; Rosegrant and Herdt 1981). For purposes of the Monte Carlo sampling procedure, the stochastic disturbance term is assumed to be normally distributed, independent of the observable stochastic variables, and to have a mean of zero. The variance of the disturbance term is computed separately for each combination of managed inputs and sampled values from the measurable stochastic variables.

To assess the impact of nitrogen on the moments of the yield distribution, other non-stochastic inputs were fixed at mean values from the data set. Parameters of the yield distributions were estimated for nitrogen levels from 0 kg/ha to 200 kg/ha in increments of 4 kg/ha. Expected yield, standard deviation, and coefficients of variation, skewness and kurtosis are given in Table 2 at various levels of nitrogen for two of the season/water regimes modelled.

**TABLE 2**

*Estimated Parameters of the Yield Distributions at Different Nitrogen Levels for Dry Season, Good Irrigation, and Wet Season, Rainfed Conditions*

<table>
<thead>
<tr>
<th>Nitrogen level (kg/ha)</th>
<th>0</th>
<th>40</th>
<th>80</th>
<th>120</th>
<th>160</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Dry season, good irrigation</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected yield (t/ha)</td>
<td>2.68</td>
<td>3.42</td>
<td>3.99</td>
<td>4.38</td>
<td>4.59</td>
<td>4.63</td>
</tr>
<tr>
<td>Standard deviation (t/ha)</td>
<td>1.16</td>
<td>1.11</td>
<td>1.16</td>
<td>1.30</td>
<td>1.51</td>
<td>1.75</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.43</td>
<td>0.32</td>
<td>0.29</td>
<td>0.30</td>
<td>0.33</td>
<td>0.38</td>
</tr>
<tr>
<td>Coefficient of skewness</td>
<td>0.14</td>
<td>0.10</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
<td>0.11</td>
</tr>
<tr>
<td>Coefficient of kurtosis</td>
<td>2.90</td>
<td>3.01</td>
<td>2.98</td>
<td>2.95</td>
<td>2.93</td>
<td>2.90</td>
</tr>
<tr>
<td><strong>Wet season, rainfed</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected yield (t/ha)</td>
<td>2.04</td>
<td>2.43</td>
<td>2.65</td>
<td>2.70</td>
<td>2.59</td>
<td>2.35</td>
</tr>
<tr>
<td>Standard deviation (t/ha)</td>
<td>1.01</td>
<td>1.02</td>
<td>1.13</td>
<td>1.32</td>
<td>1.53</td>
<td>1.70</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.50</td>
<td>0.42</td>
<td>0.43</td>
<td>0.49</td>
<td>0.59</td>
<td>0.72</td>
</tr>
<tr>
<td>Coefficient of skewness</td>
<td>0.18</td>
<td>0.11</td>
<td>0.11</td>
<td>0.14</td>
<td>0.26</td>
<td>0.46</td>
</tr>
<tr>
<td>Coefficient of kurtosis</td>
<td>2.78</td>
<td>2.86</td>
<td>2.84</td>
<td>2.78</td>
<td>2.69</td>
<td>2.71</td>
</tr>
</tbody>
</table>

* With low seepage and percolation (heavy soils).
* With medium seepage and percolation (medium soils).

* Insect control cost, which had a very high mean of ₱393/ha due to its use in treatments for the response function experiments, was set at a more representative ₱100/ha.
The production function specification permits clear distinctions to be made among production relations in different environments. Expected yield under dry season irrigated conditions is higher than wet season rainfed expected yields, and the difference increases as nitrogen level increases, due to the more rapidly declining marginal product of nitrogen under rainfed conditions. The standard deviations of yield are quite similar, so the coefficient of variation for wet season rainfed rice is substantially higher than for dry season irrigated rice.

The yield distributions in both seasons are positively skewed for all levels of nitrogen though the degree of skewness is small. The coefficient of skewness increases rapidly at high levels of nitrogen in the wet season but is virtually constant in the dry season. The yield distributions are also somewhat flatter than the normal distribution, except in the dry season, from 30 kg/ha to 50 kg/ha of nitrogen, where the distribution has virtually the same coefficient value as the normal distribution (see Kendall and Stuart 1977, pp. 87–8, and Day 1965 for computation and interpretation of the coefficients of skewness and kurtosis).

**Optimal Fertiliser Use Under Different Risk Preferences**

The extent to which production risk constrains farmer input use has been the subject of considerable debate. Some research (for example, Herath, Hardaker and Anderson 1982; Moscardi and de Janvry 1977) has found risk to be important in explaining shortfalls from risk-neutral optima while other work (for example, Roumasset 1976; Rosegrant and Herdt 1981) has found input use to be relatively insensitive to differences in risk preference. The production function specification described here, in conjunction with an expected utility maximisation model of farmer behaviour is a highly flexible method for estimating optimal fertiliser use under different risk preferences in different seasons/water regimes.

Farmers are assumed to choose the level of nitrogen fertiliser so as to maximise expected utility. Utility is represented as:

\[ U = (1-r) \cdot M^{1+r} \]

where \( r \) is the partial risk-aversion coefficient and \( M \) is the stochastic income or gains from nitrogen fertiliser. The discrete probability distributions of \( M \) are computed from the yield distributions by nitrogen level at the specified price of rice, nitrogen, and interest rate, assuming farmers borrow to purchase nitrogen. For risk neutrality, the rule reduces to maximisation of expected net gains. Increasing degrees of risk aversion are represented by increasing values of \( r \). The values of \( r \) used here, 0.812 for moderate risk aversion and 4.625 for severe risk aversion, are drawn from Binswanger (1980, 1981).\(^6\) To maintain computational feasibility, alternative nitrogen levels are assessed in steps of 4 kg/ha.

The price of rough rice was set at P1.21/kg, the average Philippine farm price for the six primary harvest months in 1981 (March, April,

\(^6\) The constant partial risk-aversion utility function is employed because of the availability of consistent measurements of farmer risk preferences using this function. The coefficient for moderate risk aversion is the common endpoint of the moderate and intermediate risk-aversion intervals from Binswanger (1980, 1981). The severe risk-aversion coefficient is the midpoint of the severe interval.
and May for the dry season and October, November, and December for the wet season). The nitrogen price of $P7.44/kg was based on the average retail price (August 1981 ex-warehouse price plus marketing and transport costs) for nitrogen from urea and ammonium sulphate, weighted by the tonnage use of these fertilisers in 1981. The interest rate was set at 24 per cent per annum, reflecting direct interest costs plus transaction costs for institutional loans.

Optimal nitrogen levels for alternative environments and risk preferences are shown in Table 3. Reductions in nitrogen use from risk-neutral optima due to moderate risk aversion range from 6.7 per cent under wet season average irrigation to 16.7 per cent for wet season rainfed conditions. With severe risk aversion, input use is reduced by 13.3 per cent (wet season average irrigation) to 30.8 per cent (dry season average irrigation) from the risk neutral optima.

**TABLE 3**

*Optimal Nitrogen Use under Different Risk Preferences by Season and Quality of Irrigation*

<table>
<thead>
<tr>
<th>Cropping season</th>
<th>Irrigation quality</th>
<th>Risk neutral</th>
<th>Moderate risk aversion</th>
<th>Severe risk aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dry</td>
<td>Good[a]</td>
<td>124</td>
<td>112 (9.7)[b]</td>
<td>96 (22.6)</td>
</tr>
<tr>
<td>Dry</td>
<td>Average</td>
<td>104</td>
<td>88 (15.4)</td>
<td>72 (30.8)</td>
</tr>
<tr>
<td>Wet</td>
<td>Average</td>
<td>60</td>
<td>56 (6.7)</td>
<td>52 (13.3)</td>
</tr>
<tr>
<td>Wet</td>
<td>Rainfed</td>
<td>48</td>
<td>40 (16.7)</td>
<td>36 (25.0)</td>
</tr>
</tbody>
</table>

[a] With low seepage and percolation (heavy soils). Other cases are with medium seepage and percolation (medium soils).

[b] Figures in parentheses are percentage reductions from risk-neutral optimum.

The results can be used to derive rough estimates of the extent that risk aversion reduces optimal nitrogen use in different environments. According to Binswanger (1980) and Binswanger and Sillers (1981), most farmers are moderately risk averse, with more than three-quarters of farmers tested exhibiting moderate to intermediate risk aversion (partial risk-aversion coefficients of 0.316 to 0.812 and 0.812 to 1.74 respectively). This implies that in environments with relatively low probability of moisture stress, optimal fertiliser use should be adjusted downward from the risk-neutral optimum, by roughly 8 per cent (9 per cent for dry season and irrigation and 6.7 per cent in the wet season with average irrigation) to account for risk preferences of the majority of farmers. Under conditions of greater probability of moisture stress, moderate risk aversion calls for reducing optimal nitrogen use by roughly 16 per cent (15.4 per cent under dry season, average irrigation, and 16.7 per cent for rainfed, wet season conditions). Moderate risk
aversion does not greatly reduce optimal nitrogen use from risk neutral levels.\(^7\)

**Sensitivity of Optimal Nitrogen Estimates to Alternative Production Function Specifications**

Estimation procedures for the heteroscedastic production function with measurable stochastic inputs are relatively complex, and intensive and comparatively costly data collection efforts are required. It is therefore worthwhile to consider whether simpler specifications may also allow consistent disaggregation of the production function across agro-climatic environments. The estimated parameters for two alternative production function specifications are presented in Table 4. The first alternative includes measurable stochastic inputs in interaction terms with managed inputs (as in the general specification), but the disturbance term is homoscedastic, and the function is estimated using ordinary least squares. In the second alternative, measurable stochastic inputs are included separately from the managed inputs, and the error term is heteroscedastic, with the error term variance a function of the independent variables. The model is comparable to the Just-Pope (1978, 1979) specification, in that the impact of managed inputs on yield variance is limited to the error term variance function, excluding interaction effects with measurable stochastic variables.

Both alternatives exhibit a good statistical fit, with appropriate signs and highly significant parameter estimates (see Table 4). Many of the parameter estimates from the first alternative are similar to those of the general model but estimates of important parameters, such as nitrogen x solar radiation and nitrogen squared, differ by 15 per cent or more.

Differences among the specifications can be traced through to the estimated parameters of the yield distributions and to the implied optimal nitrogen use across environments and risk preferences. The estimated parameters of the yield distributions under wet season, rainfed conditions for the alternative production function specifications are given in Table 5. The coefficients of variation and skewness for the general model and the two alternatives are shown in Figure 1. The broad differences in parameter estimates among models for the other environments are comparable to the differences outlined here for the wet season.

For the homoscedastic specification with measurable stochastic inputs which interact with managed inputs, expected yield response is similar to that of the general model, but the marginal product of nitrogen diminishes less rapidly. The increase in standard deviation as nitrogen increases is much slower, so the coefficient of variation is

\(^7\) This result is not dependent on the form of the utility function. Results are consistent using the function \( V = 1 - e^{-CM} \), for which partial risk aversion increases with the scale of the random gains from nitrogen. Although there are no empirical estimates of the relative risk-aversion coefficient, \( c \), to represent farmer risk preferences, the range 0.0025 to 0.005 appears appropriate for moderate risk aversion (Antle and Goodger 1984 use 0.001 for low risk aversion and 0.01 for high risk aversion). With the increasing partial risk-aversion function, reductions from risk-neutral optima due to moderate risk aversion average 9.5 per cent across environments for the lower \( c \) value, and 15.6 per cent for the higher value, compared to 12.5 per cent for the constant partial risk-aversion function.
TABLE 4

Parameter Estimates for Expected Yield and Variance Functions for Modern Rice Varieties, Philippines, with Alternative Production Function Specifications

<table>
<thead>
<tr>
<th>Variable</th>
<th>Estimated parameters&lt;sup&gt;d&lt;/sup&gt;</th>
<th>Alternative 1&lt;sup&gt;b&lt;/sup&gt;</th>
<th>Alternative 2&lt;sup&gt;c&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Expected yield function</td>
<td>Expected yield function</td>
<td>Variance function</td>
</tr>
<tr>
<td>Intercept</td>
<td>157.66 (210.21)</td>
<td>-521.93 (212.17)</td>
<td>1 401 974.00 (347 023.27)</td>
</tr>
<tr>
<td>Nitrogen × solar radiation</td>
<td>0.90 (0.05)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen</td>
<td>9.70 (1.20)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nitrogen squared</td>
<td>-0.034 (0.005)</td>
<td>-0.01 (0.01)</td>
<td>36.36 (9.67)</td>
</tr>
<tr>
<td>Phosphorus</td>
<td>5.37 (1.22)</td>
<td>0.46 (1.12)</td>
<td>-944.72 (1 816.77)</td>
</tr>
<tr>
<td>Insect control cost</td>
<td>0.49 (0.05)</td>
<td>0.54 (0.05)</td>
<td>303.02 (89.92)</td>
</tr>
<tr>
<td>Weed control index</td>
<td>54.23 (6.47)</td>
<td>52.66 (6.34)</td>
<td>13 480.25 (10 614.37)</td>
</tr>
<tr>
<td>Seedling age</td>
<td>-14.48 (2.42)</td>
<td>-14.41 (2.35)</td>
<td>-11 463.00 (3 872.64)</td>
</tr>
<tr>
<td>Nitrogen × pest damage index</td>
<td>-0.11 (0.01)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pest damage index</td>
<td>-8.65 (1.25)</td>
<td>-4 574.75 (1 610.83)</td>
<td></td>
</tr>
<tr>
<td>Disease incidence index</td>
<td>-25.05 (1.89)</td>
<td>-28.42 (1.84)</td>
<td>3 727.52 (3 055.34)</td>
</tr>
<tr>
<td>Organic matter in soil</td>
<td>254.91 (34.78)</td>
<td>277.37 (34.89)</td>
<td>107 779.50 (58 895.90)</td>
</tr>
<tr>
<td>Clay content of soil</td>
<td>-20.53 (1.62)</td>
<td>-18.88 (1.61)</td>
<td>-3 519.27 (2 707.13)</td>
</tr>
<tr>
<td>Solar radiation</td>
<td>77.14 (7.36)</td>
<td>136.42 (5.74)</td>
<td>34 589.44 (9 581.56)</td>
</tr>
<tr>
<td>Solar radiation × stress days</td>
<td>-1.38 (0.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Stress days</td>
<td>-59.21 (3.15)</td>
<td>5 285.90 (5 339.29)</td>
<td></td>
</tr>
<tr>
<td>(Nitrogen + phosphorus) × stress days squared</td>
<td>-0.025 (-0.003)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Typhoon dummy</td>
<td>-238.66 (82.01)</td>
<td>-651.71 (39.93)</td>
<td>-276 540.00 (68 281.48)</td>
</tr>
<tr>
<td>Typhoon dummy × (nitrogen + phosphorus)</td>
<td>-2.84 (0.64)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Farmer/researcher dummy × (nitrogen + phosphorus)</td>
<td>-1.66 (0.34)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Farmer/researcher dummy</td>
<td>-67.59 (39.53)</td>
<td>5 627.53 (62 528.11)</td>
<td></td>
</tr>
<tr>
<td>Year</td>
<td>383.03 (20.74)</td>
<td>397.05 (21.68)</td>
<td>141 515.00 (37 143.04)</td>
</tr>
<tr>
<td>R&lt;sup&gt;2&lt;/sup&gt;</td>
<td>0.68</td>
<td></td>
<td></td>
</tr>
<tr>
<td>F ratio</td>
<td>458.42</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<sup>a</sup> The numbers in parentheses are standard errors.

<sup>b</sup> Measurable stochastic inputs are included in interaction terms with managed inputs. Error term is homoscedastic. Estimation is by ordinary least squares.

<sup>c</sup> Measurable stochastic inputs are included separately from managed inputs. Error term is heteroscedastic. Estimation is by generalised least squares.

<sup>d</sup> See Table 1 for definition of units of variables.
Table 5

Estimated Parameters of the Yield Distributions at Different Nitrogen Levels for Wet Season, Rained Conditions with Alternative Production Function Specifications

<table>
<thead>
<tr>
<th>Nitrogen level (kg/ha)</th>
<th>0</th>
<th>40</th>
<th>80</th>
<th>120</th>
<th>160</th>
<th>200</th>
</tr>
</thead>
<tbody>
<tr>
<td>Alternative 1&lt;sup&gt;a&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected yield (mt/ha)</td>
<td>2.07</td>
<td>2.39</td>
<td>2.60</td>
<td>2.71</td>
<td>2.71</td>
<td>2.61</td>
</tr>
<tr>
<td>Standard deviation (mt/ha)</td>
<td>1.05</td>
<td>1.09</td>
<td>1.14</td>
<td>1.21</td>
<td>1.28</td>
<td>1.35</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.51</td>
<td>0.46</td>
<td>0.44</td>
<td>0.45</td>
<td>0.47</td>
<td>0.52</td>
</tr>
<tr>
<td>Coefficient of skewness</td>
<td>0.18</td>
<td>0.12</td>
<td>0.08</td>
<td>0.05</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Coefficient of kurtosis</td>
<td>2.72</td>
<td>2.76</td>
<td>2.76</td>
<td>2.74</td>
<td>2.72</td>
<td>2.64</td>
</tr>
<tr>
<td>Alternative 2&lt;sup&gt;b&lt;/sup&gt;</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Expected yield (mt/ha)</td>
<td>1.45</td>
<td>2.31</td>
<td>2.64</td>
<td>2.94</td>
<td>3.21</td>
<td>3.44</td>
</tr>
<tr>
<td>Standard deviation (mt/ha)</td>
<td>1.15</td>
<td>1.16</td>
<td>1.19</td>
<td>1.27</td>
<td>1.39</td>
<td>1.53</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>0.51</td>
<td>0.46</td>
<td>0.44</td>
<td>0.45</td>
<td>0.47</td>
<td>0.52</td>
</tr>
<tr>
<td>Coefficient of skewness</td>
<td>0.18</td>
<td>0.12</td>
<td>0.08</td>
<td>0.05</td>
<td>0.03</td>
<td>0.06</td>
</tr>
<tr>
<td>Coefficient of kurtosis</td>
<td>2.72</td>
<td>2.76</td>
<td>2.76</td>
<td>2.74</td>
<td>2.72</td>
<td>2.64</td>
</tr>
</tbody>
</table>

<sup>a</sup> Measurable stochastic inputs are included in interaction terms with managed inputs. Error term is homoscedastic.

<sup>b</sup> Measurable stochastic inputs are included separately from managed inputs. Error term is heteroscedastic.

Relatively flat across nitrogen levels, compared to the U-shaped relationship for the general model. The largest difference between the models is in the coefficient of skewness, which declines as nitrogen increases in the homoscedastic model. The difference in kurtosis is small.

In the heteroscedastic model with non-interactive measurable stochastic variables, expected yield is lower at low nitrogen levels, but the marginal product of nitrogen is relatively high, with slowly diminishing returns. Variability increases with nitrogen application, but at a slower rate than for the general model and the coefficient of variation declines with increasing nitrogen. Skewness is substantially higher at low levels of nitrogen, but then declines. There is again little difference in kurtosis.

Optimal nitrogen use for the alternative production function specifications is shown in Table 6. For environments of intermediate production variability (dry and wet seasons with average irrigation), the optimal input choices implied by the homoscedastic specification with interactive measurable inputs are fairly close (within 0 per cent to 14 per cent) to the optimal choices for the general model (see Table 3). However, for the dry season, good irrigation environment, with relatively little variation in the measurable stochastic variables, the expected marginal productivity of nitrogen is higher and the positive impact of nitrogen on variability is understated, resulting in much larger optimal nitrogen levels, particularly for risk-averse preferences. For unfavourable environments, such as wet season, rained conditions, yield variability is higher and expected yield lower in the economically relevant range, resulting in lower optimal nitrogen levels than for the general model.
The model with a heteroscedastic error term but no interaction between measurable stochastic inputs and managed inputs does not permit appropriate adjustment of the production function according to different environments. With only one exception, optimal input choice is constant across environments. The sensitivity of optimal nitrogen use to risk preferences is greatly overstated under this specification. Averaging across environments, the effect of switching from the risk-neutral to the moderately risk-averse preferences is to reduce optimal nitrogen use by 29 per cent, compared to an average of 12.5 per cent for the general model.

**Concluding Remarks**

A heteroscedastic production function with measurable stochastic variables was applied to experimental yield response data for modern rice varieties in farmers' fields in the Philippines. By incorporating both fixed and stochastic environmental variables, the specification permits inferences to be made about environments for which there is no experimental data. After estimating the appropriate frequency distribution for the expected time of planting and entering the fixed environmental variables, the joint probability distribution of the
TABLE 6

Optimal Nitrogen Use under Different Risk Preferences by Season and Type of Irrigation for Alternative Production Function Specifications

<table>
<thead>
<tr>
<th>Cropping season</th>
<th>Irrigation quality</th>
<th>Risk preference</th>
<th>Moderate risk aversion</th>
<th>Severe risk aversion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Risk neutral</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Alternative 1</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dry</td>
<td>Good(^c)</td>
<td>148</td>
<td>144</td>
<td>(2.7)(^d)</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>112</td>
<td>100</td>
<td>(10.7)</td>
</tr>
<tr>
<td>Wet</td>
<td>Average</td>
<td>60</td>
<td>56</td>
<td>(6.7)</td>
</tr>
<tr>
<td></td>
<td>Rainfed</td>
<td>36</td>
<td>28</td>
<td>(22.2)</td>
</tr>
<tr>
<td><strong>Alternative 2</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dry</td>
<td>Good(^c)</td>
<td>132</td>
<td>100</td>
<td>(24.2)</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>132</td>
<td>100</td>
<td>(24.2)</td>
</tr>
<tr>
<td>Wet</td>
<td>Average</td>
<td>132</td>
<td>100</td>
<td>(24.2)</td>
</tr>
<tr>
<td></td>
<td>Rainfed</td>
<td>132</td>
<td>84</td>
<td>(36.3)</td>
</tr>
</tbody>
</table>

\(^a\) Measurable stochastic inputs are included in interaction terms with managed inputs. Error term is homoscedastic.

\(^b\) Measurable stochastic inputs are included separately from managed inputs. Error term is heteroscedastic.

\(^c\) With low seepage and percolation (heavy soils). Other cases are with medium seepage and percolation (medium soils).

\(^d\) Figures in parentheses are percentage reductions from risk-neutral optimum.

Stochastic variables can be estimated in order to derive site and season specific production functions with stochastic properties that are appropriate for estimating optimal inputs under risk aversion. Relatively simple simulation techniques were used to characterise the stochastic environmental variables. However, the model can be generalised to accommodate covariance of stochastic variables.

Parameters of the yield distributions based on the production function were estimated for four environments ranging from wet season, rainfed to dry season, good irrigation. These estimates were used together with an expected utility maximising model of farmer behaviour to assess optimal nitrogen levels under various risk preferences. Unless farmers are severely risk averse, production risk does not cause large reductions in nitrogen use from risk-neutral levels. For farmers who are moderately risk averse, the reduction in optimal nitrogen from risk-neutral levels is from 6.7 per cent to 16.7 per cent, depending on environment.

The insensitivity of optimal nitrogen levels to differences in risk preferences underscores the robustness of earlier conclusions along the same lines. Using a more primitive analytical method for specifying probabilities of various damage states-of-the-world, Roumasset (1976) found that nitrogenous fertiliser has only a small impact on risk and that models that account for risk aversion fail to explain actual nitrogen use better than a carefully specified risk-neutral model. Using an approach with measurable stochastic variables but a homoscedastic production function, Rosegrant and Herdt (1981) found that risk preferences had only a minor effect on the optimal level of fertiliser.

Since the aforementioned studies were for irrigated rice or rainfed rice with relatively favourable rainfall distributions in the Philippines, the
possibility remains that risk aversion is an important determinant of
fertiliser demand in other environments. For example, risk aversion
may be important in areas prone to severe drought if fertiliser increases
risk and prospects for diversification and risk-sharing are poor. The
methodology detailed above is well-suited to test this hypothesis.

The work reported here also supports the persistent result reported
elsewhere that yield distributions are not normally distributed and their
second and third moments are significantly affected by the level of
fertiliser (Day 1965; Roumasset 1976; Mendoza 1980; Antle and
Googar 1984). For example, the results in Table 2 suggest that nitrogen
fertiliser has a U-shaped effect on the coefficient of variation.

The results on skewness are also consistent with a hypothesis which
can be derived from previous studies. Day (1965), Roumasset (1976)
and Mendoza (1980) found that nitrogen decreases the coefficient of
skewness in well-controlled environments. Anderson (1973) and
Mendoza (1980) found that nitrogen increases the coefficient of
skewness in less controlled environments with higher probabilities of
moisture stress. The results in Table 2 are consistent with these findings.
One possible explanation is that removing most of the constraints to
high yields, including moisture stress and lack of nutrients, tends to
increase the mode of the distribution, leaving a negatively skewed tail
representing low probability damage states. However, in a relatively
uncontrolled situation, for example, without irrigation, higher nitrogen
use raises yields in varying degrees, depending on the other sources of
damage that are present. Such a situation is reflected by a positively
skewed distribution. While this conjecture requires further statistical
modelling and testing, it serves to highlight the heterogeneity of
distributional forms across different environments.

The comparison of results from the heteroscedastic production
function with interactive measurable stochastic inputs to alternative
specifications demonstrates the importance of appropriate specification
of the production function. Although the statistical fit of the alternative
production functions was good, the functions did not allow for
consistent discrimination among alternative agro-climatic
environments and appropriate separation of the impact of managed
inputs and stochastic variables on yield variability. As a result,
estimated optimal nitrogen levels using these specifications were
inaccurate.

Experimental or survey design often precludes utilisation of the best
specification by excluding important variables from data collection.
The results here indicate that more intensive collection of
environmental data and collection of consistent data over time for a
given location are worth the additional costs involved. Failure to collect
the appropriate data can lead to incorrect inferences regarding farmer
behaviour.

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