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DISTRIBUTIONS OF INDEMNITIES FOR CROP-INSURANCE PLANS: WITH APPLICATION TO GRAIN CROPS IN NEW SOUTH WALES*

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A review of crop-insurance schemes is followed by a discussion of a guaranteed-yield, crop-insurance plan. General formulae for the distribution function and mathematical expectation of indemnities for the insurance plan are presented in terms of the distribution of crop yields. Three special cases are considered in which the original yields, the square root of yields, and the logarithm of yields are normally distributed.

The insurance plan is applied on a regional basis for wheat and sorghum production in N.S.W. Given distributional information on the crops obtained from a simulation model, expected indemnities are calculated for four different insurance plans.

Introduction

Crop insurance is a method by which a farmer can ensure that his income does not fall below a certain level in times of partial or complete crop failure. Crop insurance has been tried with various modifications in many countries. The forms, coverages and general provisions have varied widely. In some cases, general protection has been provided against any or all factors that cause low yields. In other cases, protection has been restricted to specific risks such as hail or insect damage.

The importance of crop insurance in agricultural development has been emphasized by Ray [14] and Oury [13]. The latter provides a comprehensive bibliography of crop insurance. The experience in overseas countries suggests that crop insurance is worthy of examination because of its potential usefulness in offsetting some of the instabilities inherent in agriculture (Staniforth [18]; Jones and Larson [10]; and Ray [14]).

Crop insurance against specific risks (e.g., hail) is the main type of insurance available in Australia. This insurance is usually provided by private companies on a commercial basis. The Queensland Barley Board requires compulsory hail insurance; the Wheat Board provides com-

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pensation to growers who have suffered hail damage. The Queensland rice industry provides insurance against flood and hail. All-risk crop insurance is, generally speaking, not available in Australia, with the exception of a scheme for banana growers at Carnarvon maintained by the West Australian State Government, and a short-lived insurance scheme offered by the Westralian Farmers' Co-operative Limited to wheat growers (Milne [12]). The latter was launched in 1974 and ceased operating in 1975 (Hackett [8]). The scheme was voluntary and did not receive any governmental assistance. A committee from the Australian Wheat Growers' Federation in 1968 recommended the introduction of a compulsory scheme of all-risk crop insurance, initially for wheat. However, the Australian Wheat Growers' Federation has not endorsed the report [15, Section 4.81].

All-risk crop insurance is based on a guaranteed yield, measured in tonnes per hectare, which is specified for particular districts or areas. Since yield variability differs between areas, it is important that premiums differ accordingly (Halcrow [9]). The scheme covers losses caused by adverse weather, insects, plant diseases, floods, etc., but not losses due to poor management.

To effectively implement an all-risk crop insurance scheme, adequate information on crop yields is required. The characteristics of the probability distribution of yields for a farm or an area determine the premium-indemnity schedules that are appropriate in each geographical unit where insurance is offered. The calculation of premium-indemnity schedules when the crop yields are normally distributed has been outlined by Botts and Boles [2] for a crop-insurance plan similar to the one defined in this paper. If yields are not normally distributed then the calculation of premium rates using normal theory cannot be expected to guarantee that the insurance scheme will be self-sufficient. An insurance scheme is said to be self-sufficient if premiums equal the mathematical expectation of indemnities.

It seems that literature pertaining to non-normality of crop yields has been rather scarce since the path-breaking article by Day [6]. Anderson [1] discusses a method to consider the effects of non-normal yields in fertilizer recommendations. A note by Ryan [16] concludes that there is slight evidence of non-normality of yields of wheat grown in various areas of Victoria. Yeh and Wu [19] discuss the effects of non-normality of yields on crop insurance in Canada. They conclude that yields of wheat grown in several areas in Manitoba are not normally distributed.

The remainder of this paper considers a crop insurance plan and presents the expression for the expectation of indemnities in terms of the density function for crop yields. Three particular cases are considered in which the original yields, the square root of yields, and the logarithm of yields are normally distributed. By using distributional information for yields of wheat and sorghum obtained from a regional simulation model, values of expected indemnities are presented in commodity units for four different insurance plans.

Crop-Insurance Plan

Consider an insurance plan that makes indemnity payments, denoted by I , by the rule

$$(1) \quad I = \begin{cases} k(c - Y) & \text{if } Y \leq c \\ 0 & \text{if } Y > c \end{cases}$$

where crop yield, Y , is a continuous positive random variable; k is a constant between zero and one; and c is a positive constant which would generally be a fraction of the mean of the yields.

The relationship between the indemnity payable and observed yield is depicted in Figure 1. If the yield is greater than the 'coverage yield', c , then no indemnity is made to the insured individual(s). However, if yield is less than the 'coverage yield', a positive indemnity payment is made that is a fixed proportion of the difference between the 'coverage yield' and the observed yield.

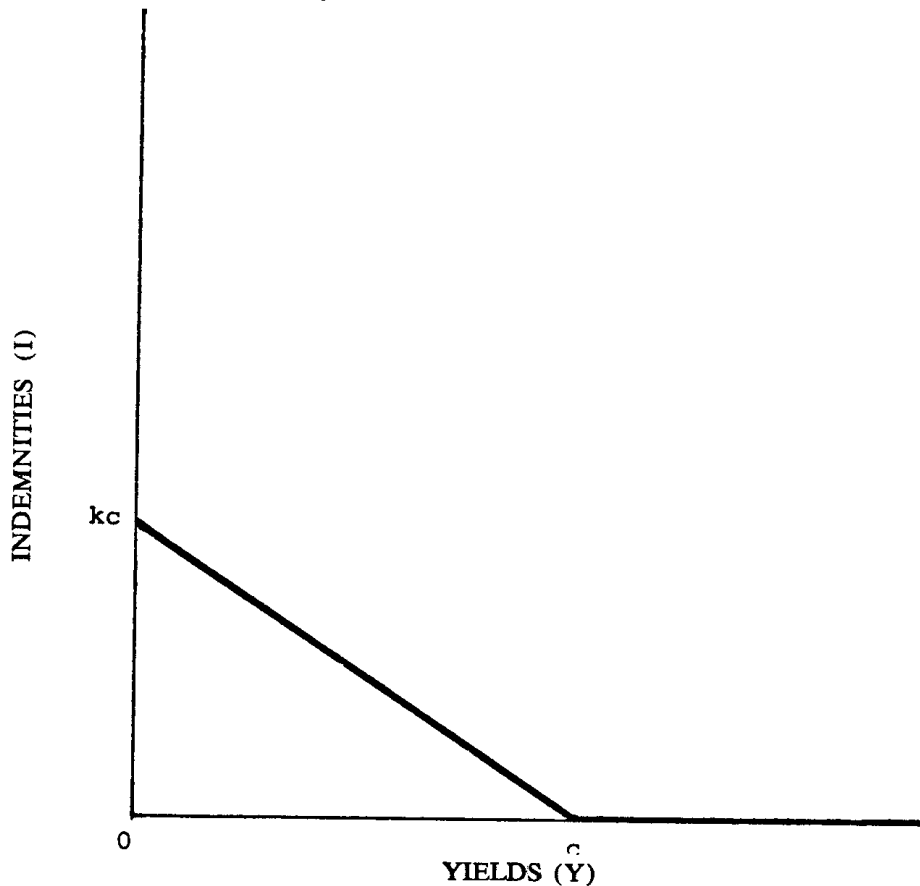


FIGURE 1—Relationship between indemnities and yields.

The relationship between the 'adjusted yield', which is the observed yield plus the indemnity payment, and the observed yields is depicted in Figure 2. The broken line through the origin is the continuation of the straight line to the right of the coverage yield, c , and it represents the case if no indemnity is paid.

The insurance plan defined by equation (1) could be implemented for a particular farm or for all farms in a region that was fairly homogeneous with respect to topography, climate, type of farming and management techniques. The lack of suitable individual farm data would generally preclude the implementation of the insurance plan for

individual farms. An area insurance scheme would be the most likely case for the implementation of the suggested insurance plan.

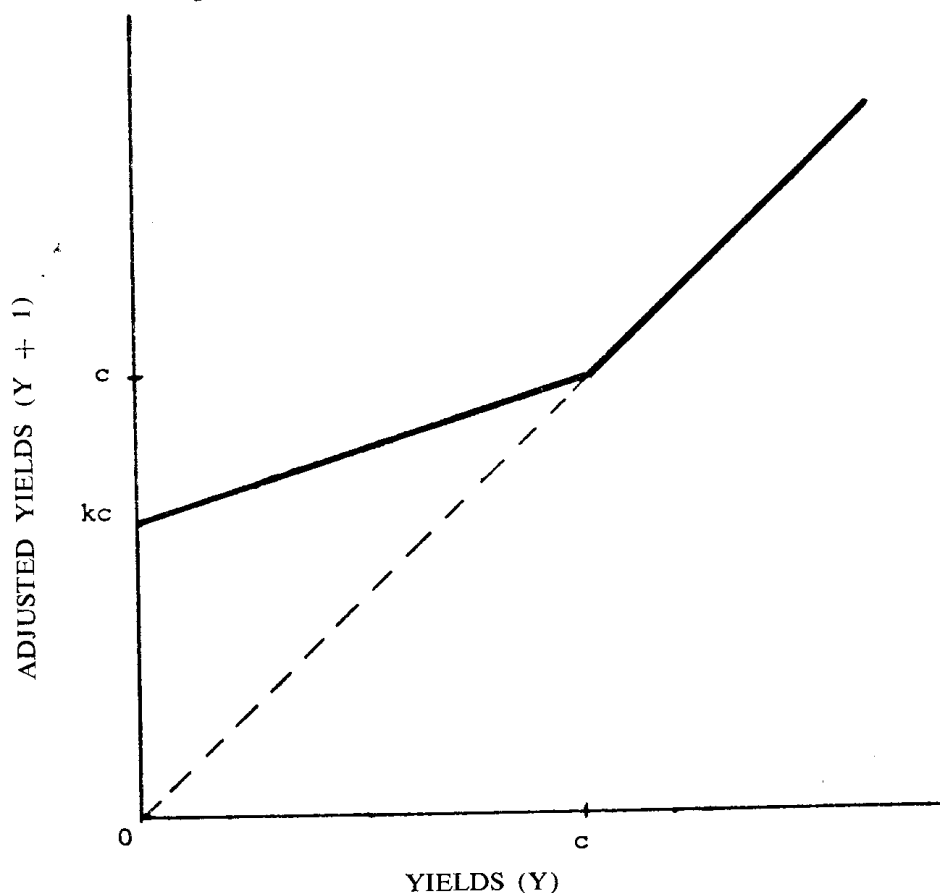


FIGURE 2—Relationship between adjusted and observed yields.

The indemnity rule (1) could be applied in a farm income stabilization plan, such as suggested by Campbell and Glau [5], Campbell [4, pp. 55-56] and Lloyd [11, pp. 23-30]. In such situations the random variable Y would denote farm income and the indemnity would be payable in monetary units. The statistical properties of the stabilization plan would be identical to those to be outlined in this paper for crop insurance.

The indemnity payment schedule associated with (1) is slightly different to that considered by Botts and Boles [2] and Francisco [7] in which indemnity payments are proportional to the difference between the mean yield and the observed yield when observed yields are sufficiently small. The latter insurance plan is such that there is a discontinuity in the indemnity function at the point of the coverage yield.

The implementation of the insurance plan (1) requires the determination of premiums to be charged. Premiums should be no smaller than the mathematical expectation of the indemnities for the particular insurance plan. The extent to which premiums should exceed the expectation of indemnities (i.e., to cover administrative costs or result

in a profit margin to the company) is not considered in this paper. The expectation of the indemnities depends on the statistical distribution of crop yields in addition to the parameters (k and c) of the crop-insurance plan.

Distributional Properties of Indemnities

In this section general expressions are given for the (probability) distribution function for indemnities and the expectation of indemnities in terms of the distribution function for crop yields. The statistical results are presented in four theorems: the first considers the general case; the last three consider the three special cases in which the crop yields are normally distributed, the square roots of yields are normally distributed, and the crop yields have lognormal distribution.

It is noted that even though the yield, Y , is assumed to be a continuous random variable, the indemnity, I , of equation (1) is not a continuous random variable, since it has a positive probability of assuming zero. This implies that the distribution function of indemnities has a discontinuity (jump) at the point zero.

The first theorem gives the distribution function and the expectation of indemnities in terms of the general notation, $F_Y(\cdot)$ and $f_Y(\cdot)$, for the distribution and density functions for crop yields. Note that $F_Y(y)$ is the probability of yields not exceeding y , and $f_Y(y)$ is the derivative of the distribution function at the point y .

Theorem 1: If crop yield, Y , has distribution and density functions, denoted by $F_Y(\cdot)$ and $f_Y(\cdot)$, respectively, then the distribution function, $F_I(\cdot)$, of the indemnities for the crop-insurance plan (1) is given by

$$(2) \quad F_I(i) = \begin{cases} 0 & \text{if } i < 0 \\ 1 - F_Y(c - i/k) & \text{if } 0 \leq i < kc \\ 1 & \text{if } i \geq kc. \end{cases}$$

Further, the expected indemnity is given by

$$(3) \quad E(I) = k[c - E(Y | Y \leq c)]F_Y(c)$$

where $E(Y | Y \leq c)$ is defined by

$$(4) \quad E(Y | Y \leq c) = \int_{-\infty}^c y f_Y(y) dy / F_Y(c).$$

The general shape of the distribution function of indemnities is shown in Figure 3. The jump in the distribution function of indemnities (2) at zero is equal to the probabilities that yields exceed the coverage yield, c . The expectation of indemnities (3) depends on the probability of yields being below the coverage yield, the parameters of the indemnity rule (k and c), and the 'conditional mean yield', $E(Y | Y \leq c)$. The conditional mean yield is the expectation of the truncated distribution of yields below the coverage yield, c . Knowledge of the distribution function for yields is clearly crucial to the determination of suitable premiums for different values of the parameters of the indemnity rule.

Although crop yields are positive it may be that their distribution is very closely approximated by a normal distribution. That is, under the assumption of normality the mean and variance are such that the probability of 'negative yields' is negligible. The approximation to normality

would be expected to be reasonable in the case of average regional yields if the number of farms (or hectares) was sufficiently large. The next two theorems deal with the normal distribution and in these theorems only the expected indemnity is given. The range for the indemnity function in these cases is the positive real line (i.e., I is not bounded by kc). The expressions for distribution functions are therefore not given.

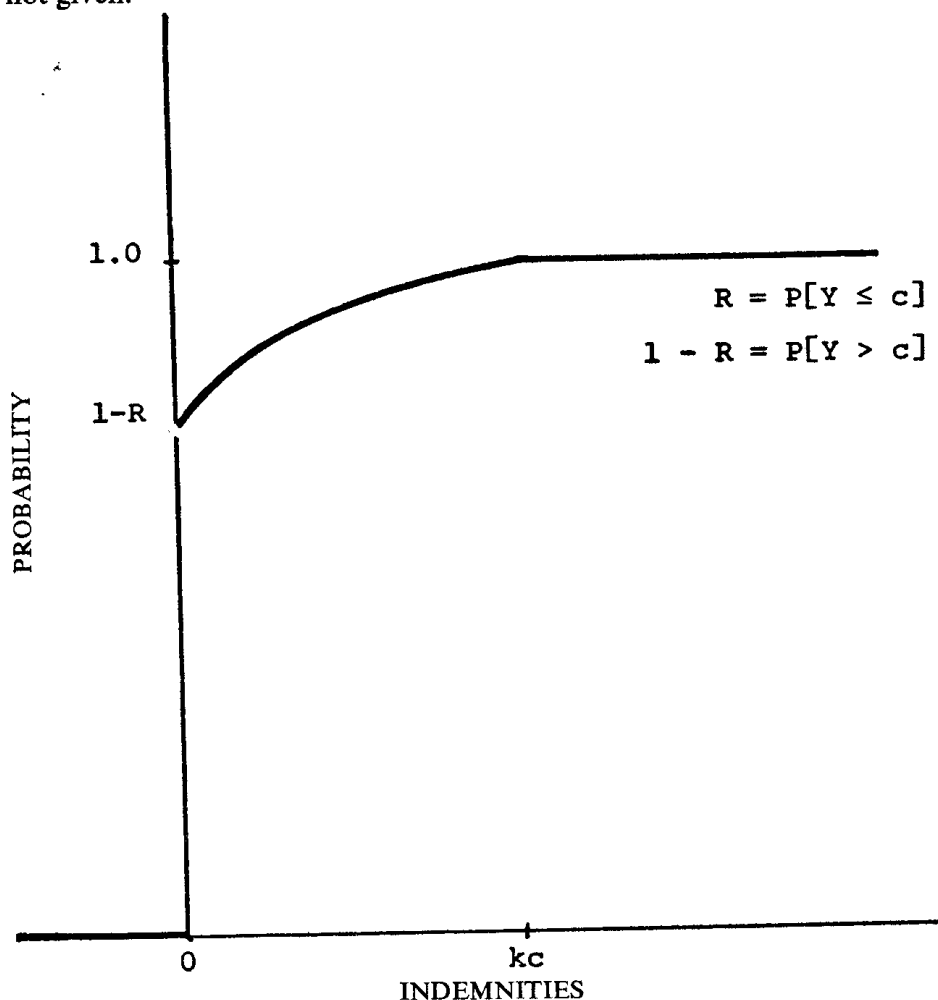


FIGURE 3—Distribution function of indemnities.

Theorem 2: If crop yield is distributed as a normal random variable with mean μ_Y and variance σ_Y^2 , then the expectation of the indemnities is given by

$$(5) \quad E(I) = k(c - \mu_Y)\Phi(z_2) + k\sigma_Y\phi(z_2)$$

where $z_2 = (c - \mu_Y)/\sigma_Y$ and $\Phi(\cdot)$ and $\phi(\cdot)$ denote the distribution and density function of the standard normal random variable,

$$\text{i.e.,} \quad \phi(z) = (2\pi)^{-1/2} \exp(-\frac{1}{2}z^2), \quad -\infty < z < \infty,$$

$$\text{and} \quad \Phi(z) = \int_{-\infty}^z \phi(t)dt.$$

When the distribution of crop yields is known to deviate significantly from normality, the result of (5) would not be reliable for approximating the expected indemnity. It may, however, be possible to obtain approximately normal distributions by choosing a suitable power transformation of crop yields (see [3]). That is, the transformed yields, Y^λ , may have approximate normal distribution for some value of $\lambda > 0$. If crop yields were negatively skewed, the value of λ would be expected to be greater than one. If crop yields were positively skewed, the value of λ would be expected to be less than one. If the value of λ is close to zero, then the distribution of crop yields is approximately lognormal.¹

In practice, if yields are known to be positively skewed,² it may be reasonable to investigate the distribution of Y^λ , where $\lambda = 1/t$, for $t = 1, 2, \dots$, and test if the transformed yields are normally distributed by using the Shapiro-Wilk W test [17]. Empirical analyses for several crops in New South Wales suggests that when regional crop yields are not normally distributed, the square-root transformation obtains approximately normal random variables (Francisco [7]). In these situations the expectation of the indemnities can be approximated with use of the result of the following theorem.

Theorem 3: If the crop yield, Y , is such that $X \equiv Y^\lambda$ has normal distribution with mean μ_X and variance σ_X^2 , then the expected indemnity is given by

$$(6) \quad E(I) = k(c - \mu_Y)\Phi(z_3) + k\sigma_X(\mu_X + c^\lambda)\phi(z_3)$$

where $z_3 = (c^\lambda - \mu_X)/\sigma_X$ and $\mu_Y = \sigma_X^2 + \mu_X^2$.

The expected indemnities for other power transformations of yields can be evaluated by procedures similar to those used to obtain the above results. We conclude this section by presenting distributional results for indemnities for the case in which the crop yields are lognormally distributed. As noted above the lognormal case can be considered the limiting case of the power transformations. The lognormal distribution is considered a more reasonable candidate for the distribution of yields in so far as the range of the random variable is the positive real line.³

Theorem 4: If the crop yield, Y , is such that $X \equiv \ln Y$ (natural logarithm) has normal distribution with mean μ_X and variance σ_X^2 , then the distribution function and expectation of the indemnity random variable (1) are given by (7) and (8), respectively

¹This follows by considering the quantity $(y^\lambda - 1)/\lambda$. By a Taylor expansion of y^λ about $\lambda = 0$ the quantity is equivalently expressed by $(\log y)y^{\lambda*}$ where λ^* is between zero and λ . Thus the limit, as λ approaches zero, of $(y^\lambda - 1)/\lambda$ is $\log y$.

²In the study by Day [6], yields of cotton and corn were found to be positively skewed, whereas oat yields were generally negatively skewed.

³The lognormal distribution has often been considered a reasonable approximation for the distribution of economic aggregates such as household expenditure on various items. The senior author is investigating this hypothesis for several categories of household expenditure using Australian data. If it was shown that farm income (or gross returns) had lognormal distribution, then the result of Theorem 4 would be particularly relevant in a study of a farm income stabilization plan with compensatory payments made according to equation (1).

$$(7) \quad F_I(i) = \begin{cases} 0 & \text{if } i < 0 \\ 1 - \Phi\{[\ln(c - i/k) - \mu_X]/\sigma_X\} & \text{if } 0 \leq i < kc \\ 1 & \text{if } i \geq kc \end{cases}$$

$$(8) \quad E(I) = k(c - \mu_Y)\Phi(z_4) + k\mu_Y[\Phi(z_4) - \Phi(z_4 - \sigma_X)]$$

where $z_4 = [\ln c - \mu_X]/\sigma_X$ and $\mu_Y = \exp(\mu_X + \frac{1}{2}\sigma_X^2)$.

Empirical Applications

We consider that the crop-insurance plan (1) is adopted on a regional basis for wheat and sorghum production in New South Wales. The random variable Y is the average regional yield for a well-defined area in a given year and μ_Y is the mean of Y . It is assumed that all farmers in the particular region receive indemnity payments when the average regional yield is less than a predetermined proportion of the (true) mean regional yield. In the notation of equation (1), the coverage yield, c , is equal to $p\mu_Y$, where p is a number between zero and one. We consider two values of p and two values of the proportion k in our illustrations, namely p equals 0.6 or 0.8 and k equals 0.60 or 0.75. That is, we consider four crop-insurance plans. Farmers are compensated if in a given year the average regional yield falls below either 60 or 80 per cent of the mean regional yield. The rate of compensation is either 60 or 75 per cent of the difference between the appropriate coverage yield and the average regional yield. Expected indemnities are calculated in kilograms per hectare for each of the four crop-insurance plans applied to the regions involved.

The distributions of crop yields within different regions of N.S.W. are closely approximated by a simulation model developed at the University of New England (Francisco [7]). The simulation model estimates the values of the parameters of the appropriate distributions of crop yields. In the cases in which regional crop yields deviated from normality, it was found that the square root of yields was satisfactorily approximated by the assumption of normality, according to the Shapiro-Wilk W test [17].

Tables 1 and 2 contain basic information on the crop-insurance plans for the several regional divisions in which wheat and sorghum are grown in New South Wales. For the two different rules for determining coverage yields, the probabilities of yields falling below the coverage yields are given, correct to the second decimal place. Expected indemnities are given in kilograms per hectare, correct to the nearest kilogram. For each region, the means and standard deviations of regional yields that are obtained from the APMAA simulation model [7] are given in the tables. For the non-normal cases, the means and standard deviations of the square root of the yields are given in the footnotes.

For example, in Table 1 the distribution of regional wheat yield in the Central Plains is satisfactorily approximated by a normal distribution with mean yield 1294.0 kg/ha with a standard deviation of 443.2 kg/ha. Insurance plans with Coverage 1 (i.e., indemnities are paid when yields fall below 60 per cent of the mean regional yield) have a probability of 0.12 that indemnities will be paid to wheat farmers in the Central Plains. When indemnities are payable for yields less than 80 per cent of the mean regional yield (plans with Coverage 2), the probability of indemnities being paid in the Central Plains is 0.28. Further, for a plan

TABLE 1
Expected Indemnities and Probabilities of Claims for a Regional Wheat Insurance Scheme in N.S.W.

Region	Yield Parameters μ_Y, σ_Y kg/ha	Coverage 1 ($c = 0.6\mu_Y$) ^a			Coverage 2 ($c = 0.8\mu_Y$) ^a		
		Probability	Premium A kg/ha	Premium B kg/ha	Probability	Premium A kg/ha	Premium B kg/ha
Central Plains	1294.0; 443.2	0.12	16	20	0.28	46	58
North Central Plains	1138.0; 378.0	0.12	22	28	0.27	38	48
North Western Slopes	1344.0; 374.2	0.07	8	10	0.24	31	39
Northern Tablelands ^b	1645.2; 492.0	0.08	7	9	0.26	39	49
Hunter-Manning	825.6; 347.9	0.17	19	24	0.32	43	54
Central Tablelands	1193.2; 417.3	0.13	16	20	0.28	44	55
Central Western Slopes	1323.5; 471.9	0.13	19	23	0.29	51	64
Southern Tablelands ^b	1320.2; 404.5	0.08	7	8	0.27	33	42
South Western Slopes ^b	1662.8; 309.2	0.01	0	1	0.14	11	14
Riverina	1691.9; 317.8	0.02	1	1	0.14	14	18

^a Coverages 1 and 2 guarantee that indemnities are payable if yields are not greater than 60 and 80 per cent of the mean yield, respectively; Premiums A and B are expected indemnities for rates of compensation of 60 and 75 per cent, respectively, of the difference between the coverage and observed yields.

^b Yields were not normally distributed. The mean and standard deviation of the square root of the yields are: Northern Tablelands (40.1; 6.1); Southern Tablelands (35.9; 5.6); South Western Slopes (40.6; 3.8).

TABLE 2
Expected Indemnities and Probabilities of Claims for a Regional Sorghum Insurance Scheme in N.S.W.

Region	Yield Parameters μ_Y, σ_Y kg/ha	Coverage 1 ($c = 0.6\mu_Y$) ^a			Coverage 2 ($c = 0.8\mu_Y$) ^a		
		Probability	Premium A kg/ha	Premium B kg/ha	Probability	Premium A kg/ha	Premium B kg/ha
Central Plains	1692.1; 591.7	0.13	22	28	0.28	63	78
North Central Plains ^b	1219.2; 326.7	0.05	3	4	0.24	23	28
North Western Slopes ^b	1494.0; 453.4	0.08	7	9	0.27	37	46
Northern Tablelands ^b	1833.3; 443.6	0.04	3	4	0.21	26	33
North Coast	2130.8; 473.3	0.04	4	5	0.18	28	36
Hunter-Manning	1754.3; 621.8	0.13	24	30	0.29	67	83
Central Tablelands	1214.7; 639.0	0.22	49	62	0.35	91	114
Central Western Slopes	1954.3; 729.0	0.14	32	40	0.29	82	102
South Coast	1065.4; 285.6	0.07	5	6	0.23	23	28
South Western Slopes ^b	1393.4; 640.6	0.03	84	104	0.37	75	94
Riverina	2491.4; 664.4	0.07	12	15	0.23	52	65

^a Coverages 1 and 2 guarantee that indemnities are payable if yields are not greater than 60 and 80 per cent of the mean yield, respectively; Premiums A and B are expected indemnities for rates of compensation of 60 and 75 per cent, respectively, of the difference between the coverage and observed yields.

^b Yields were not normally distributed. The mean and standard deviation of the square root of the yields are: North Central Plains (34.6; 4.7); North Western Slopes (38.2; 5.9); Northern Tablelands (42.5; 5.2); South Western Slopes (36.3; 8.7).

with Coverage 1 and a rate of compensation of 60 per cent of the difference between 776.4 kg/ha (0.6 times 1294.0) and the average yield for the Central Plains, the expected indemnity is 16 kg/ha. For the rate of compensation of 75 per cent, the expected indemnity is 20 kg/ha. For a plan with Coverage 2, the expected indemnities for the rates of compensation of 60 and 75 per cent are 46 kg/ha and 58 kg/ha, respectively, for the Central Plains.

Conclusions

This paper considers the statistical distribution of indemnities of a crop-insurance plan in terms of the distribution of crop yields. Matters relating to the implementation of the scheme are not discussed, although they are of obvious importance. For instance, the question of whether the scheme should be compulsory or voluntary is important. Some researchers (e.g., Halcrow [9]) prefer a voluntary scheme because farmers might be opposed to compulsory ones. There are, however, indications that compulsory schemes might be less costly to administer.

Government subsidies may be a necessary requirement for the implementation of crop-insurance schemes on a large scale. Hackett [8] notes that the lack of government assistance may have been one of the reasons for the Westralian Farmers' Co-operative abandoning its wheat insurance plan in 1975. It could be argued that a subsidy which leads to a widespread use of crop insurance might be smaller than the amount of money required for relief or disaster payments.

In spite of the obvious problems involved in the implementation of crop-insurance plans, there is little doubt that they are highly regarded as effective means of reducing the instability in crop production and farmers' receipts in overseas countries where they are operating.

Appendix

An outline of the proofs of the basic results in the theorems of the paper are given below.

Proof of Theorem 1

The distribution function of indemnities is defined by

$$F_I(i) = P[I \leq i].$$

Since the range of the indemnities is between zero and kc , then the value of the distribution function is zero if i is less than zero and one if i is greater than or equal to kc .

Thus $F_I(i) = P[I = 0] + P[0 < I \leq i]$ if $0 < i < kc$.

$$\begin{aligned} \text{But } P[I = 0] &= P[Y > c] \\ &= 1 - P[Y \leq c] \\ &= 1 - F_Y(c) \end{aligned}$$

where $F_Y(\cdot)$ is the distribution function for yields. Further, if $0 < i < kc$, then

$$\begin{aligned} P[0 < I \leq i] &= P[Y \leq c]P[k(c - Y) \leq i | Y \leq c] \\ &= P[c - i/k \leq Y \leq c] \\ &= F_Y(c) - F_Y(c - i/k). \end{aligned}$$

The result of (3) is obtained with use of conditional expectations, i.e., $E(I) = E\{E(I|Y)\}$

$$\begin{aligned} &= E\{k(c - Y | Y \leq c) \cdot P[Y \leq c] + 0 \cdot P[Y > c]\} \\ &= k[c - E(Y | Y \leq c)]F_Y(c). \end{aligned}$$

To obtain the expectation of the positive indemnities, the density function of $k(c - Y)$ over the range $Y \leq c$ is required. Alternatively, the density function of yields over the range $Y \leq c$ is required. This is simply the truncated distribution of yields with density function that is defined by

$$f_{Y|Y \leq c}(y) = \begin{cases} f_Y(y)/F_Y(c) & \text{if } y \leq c \\ 0 & \text{if } y > c. \end{cases}$$

Proof of Theorem 2

The result of (5) is obtained from (3) by evaluating the integral

$$E(Y | Y \leq c) = \int_{-\infty}^c y(2\pi\sigma_Y^2)^{-1/2} \exp[-\frac{1}{2}(y - \mu_Y)^2/\sigma_Y^2] dy / \Phi[(c - \mu_Y)/\sigma_Y].$$

This is readily shown to be

$$E(Y | Y \leq c) = \mu_Y - \sigma_Y \phi(z_2)/\Phi(z_2)$$

where $z_2 = (c - \mu_Y)/\sigma_Y$.

Proof of Theorem 3

The result of (6) is obtained after evaluating the integral of (4) in which the density function for yields, $f_Y(\cdot)$, is given by $f_Y(y) = \frac{1}{2}y^{-1}f_X(y^2)$, where $f_X(\cdot)$ denotes the normal density function with mean μ_X and variance σ_X^2 . After some tedious algebra the conditional mean yield is found to be

$$E(Y | Y \leq c) = (\mu_X^2 + \sigma_X^2) - (c^2 + \mu_X)\sigma_X \phi(z_3)/\Phi(z_3)$$

where $z_3 = (c^2 - \mu_X)/\sigma_X$.

Proof of Theorem 4

The result of (8) follows readily from (3) after evaluating the integral of (4) in which $f_Y(y)$ is defined by

$$f_Y(y) = \begin{cases} y^{-1}f_X(\ln y), & \text{if } y > 0 \\ 0, & \text{if } y \leq 0 \end{cases}$$

where $f_X(\cdot)$ is defined above. The conditional mean yield is found to be

$$E(Y | Y \leq c) = \exp(\mu_X + \frac{1}{2}\sigma_X^2)\Phi(z_4 - \sigma_X)/\Phi(z_4).$$

where $z_4 = [\ln(c) - \mu_X]/\sigma_X$.

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