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## FACTOR SUBSTITUTION IN AUSTRALIAN AGRICULTURE

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Studies of the ease of substitution between inputs in production have generally been carried out within a production framework of an explicit functional form. In this study, a somewhat different approach is followed. A model of derived demand for primary factors of production, land, labour and capital is formulated to enable inferences to be made about the characteristics of the unspecified production function. The model is used to obtain estimates of the pairwise Allen-Uzawa substitution elasticities which are secondary parameters of the underlying production function. The reported FIML estimates from aggregate time series data for the period 1920/21 to 1969/70 indicate very low and marginally different substitution elasticities between different pairs of factors, suggesting that both the Cobb-Douglas and CES production function specifications for the Australian agricultural sector are inappropriate.

### *Introduction*

A consequence of technological change in agriculture is the changing relative prices of inputs. A knowledge of the technical prospects for substitution between inputs is essential in order to describe the manner in which such inputs combine in production. The elasticity of substitution parameter (*ES*) provides a measure of the ease with which a pair of factors substitute in production.<sup>1</sup> This parameter forms an essential component of input demand and production relationships, describing the extent to which changing factor prices influence input demand and hence optimal production techniques. At a more aggregate level, accurate knowledge of the ease of substitution between relevant factors could be used to throw light on such diverse problems as the nature of structural unemployment, the likely consequences of input specific subsidies on the pattern of input demands, and the issue of changes in income distribution between factors of production.

In the conventional two input case where factor-factor substitution isoquants can be depicted in two dimensions, the value of the *ES* will

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<sup>1</sup> In the case of a production process employing only two factors of production, the *ES* is defined (in terms of movement around an isoquant) as the ratio of the percentage change in factor proportions to the percentage change in the marginal rate of substitution between factors. In a production process of more than two factors, the concept can be considered in terms of the partial *ES* between various factor pairs. For a linear homogeneous production function, the (Allen-Uzawa) partial *ES* between a pair of factors is conveniently defined in terms of the co-factors and determinant of the Hessian matrix (bordered by first order partial derivatives) obtained by differentiating the production function and first order conditions for cost minimization. The partial *ES* ( $\sigma_{ij}$ ) between  $X_i$  and  $X_j = (Y/X_i X_j) F_{ij} / F$ , where  $Y$  is output,  $F_{ij}$  the co-factor of  $X_j$ , and  $F$  the determinant of the bordered Hessian.

lie somewhere between zero and infinity. A zero value corresponds to the Leontief fixed input-output production relationship (right-angled isoquant) with changes in factor prices leaving factor proportions unchanged. At the other extreme a very high value of the *ES* implies close to a straight line isoquant or near perfect substitution, resulting in 'corner' solutions in which only one of the two inputs (namely the cheaper one) is used.

Numerous attempts have been made to measure the elasticity of substitution between factors of production in manufacturing industries, and less frequently in agricultural industries. Such studies have yielded a great diversity of empirical results, raising doubts about the ability of current methodology to obtain useful estimates of the *ES* between factors. The analysis is generally undertaken within a neo-classical theory of production. Traditionally, the estimating framework has centred around a production function of a particular functional form which is consistent with economic theory, and the estimation of the substitution parameter from input demand functions derived under the behavioural assumption of cost minimization. The labour market side condition of the *CES* production function has been the most frequently used estimating model.<sup>2</sup>

Although a number of (unsuccessful) attempts have been made to establish an aggregate production function for Australian agriculture (Gruen *et al.* [10], Throsby and Rutledge [26]), such attempts have generally assumed a Cobb-Douglas functional form where the *ES* is constrained to unity. The only reported estimates of the *ES* in Australian agriculture are those of Bates [2], A. Powell [20] and Duncan [7]. For the period 1920 to 1970, Bates obtained an estimate of the capital-labour *ES* of 0.26 using the *CES* labour market side condition.

A. Powell fitted, simultaneously, equations (derived from the *CES* function) relating percentage changes in unit labour and unit capital requirements to the rate of technological change, bias of technological progress and the percentage change in factor prices. For the period 1948 to 1961, he obtained an estimate of the capital-labour *ES* of 0.55. The labour series used by Powell referred to hired labour only, with the owner-operator's labour assumed to be fixed, whereas the series used by Bates included both hired and owner-operator labour.

Duncan fitted a *CES* labour market side condition to production data in the New South Wales arid zone and obtained a capital-labour *ES* of 0.06. (Duncan's labour series included both hired and owner-operator labour.)

Overseas studies of factor substitution at a sector level have been reported by Johnson [15] and Hussey [14] for New Zealand, Lianos [18] and Binswanger [3] for the U.S.A. and Fishelson [9] for the Israeli agricultural sector. These studies, with the exception of that

<sup>2</sup> The *CES* function may be written  $Y = V(\sum \alpha_i X_i^{-\rho})^{-1/\rho}$  where  $Y$ : output,  $\{X_i\}$ : inputs and  $V$ ,  $\{\alpha_i\}$  and  $\rho$  are parameters representing efficiency, distribution and substitution respectively. The labour market side condition is obtained from the first order conditions for constrained minimization of the function. In a time series framework the estimating equation is generally of the form:

$$\ln(Y/N) = (1 - \sigma) \ln k + (1 - \sigma)gt + \sigma \ln W/P$$

where  $N$ : labour input,  $W/P$ : real wage,  $t$ : time,  $g$ : efficiency growth of labour,  $k$ : constant and  $\sigma = 1/1 - \rho$ : *ES* between labour and capital.

of Binswanger, have followed the *CES* side condition approach. Johnson, Hussey, and Lianos recognized only two factors (capital and labour) and obtained *OLS* estimates of the *ES* between capital and labour from an estimating equation which contained information only on output, labour inputs and wage rates. For the period 1921 to 1967 Hussey obtained an *ES* of 0.05. Johnson's estimates were also low, ranging from 0.01 to 0.21 for the period 1946 to 1967. In contrast, the Lianos estimate was 1.5 (1949 to 1968).

Although Fishelson's model was still restricted by the *CES* requirement, his estimate of the capital-labour *ES* (0.5 for the period 1952 to 1969) was obtained from a simultaneous model of both *CES* side conditions and must therefore be considered an improvement over the single side condition specification.

In contrast to the above studies, Binswanger employed a translog production function to obtain different pairwise substitution elasticities between a range of inputs. His approach allows considerably more flexibility and hence realism from that permitted by a *CES* treatment.

Using pooled cross-sectional and time series data, Binswanger obtained partial *ES* estimates of 0.20 (land-labour), 1.22 (land-machinery) and 0.85 (labour-machinery).

In most agricultural production, land constitutes an important factor input in the sense that its rental value (assuming capital and labour rental payments are made according to their marginal productivity) is a significant component of value added. Hence the estimating model should be at least flexible enough to allow for varying substitution elasticities between pairs of the three factors, land, labour and capital. The traditional capital-labour split with the *CES* function on which most empirical work to date has been based is unnecessarily restrictive since it implies that land is either strictly complementary to the use of capital and labour or is perfectly substitutable for one of capital or labour. When the *CES* function is used with more than two factors, the *ES* between different pairs of factors is constrained to equality.

#### *The Model*

There are two broad approaches which can be adopted in formulating a suitable estimating model.

- (i) Derive a set of estimating equations from a production function specification which is sufficiently flexible to allow the *ES* to vary between factor pairs. Two such specifications are the translog production function [4] and the *CRESH* (constant ratio elasticity of substitution homothetic) function discussed by Hanoach [12] (although to date no estimates of the *ES* from a *CRESH* specification have been published).
- (ii) A less conventional approach is to formulate a model in terms of derived demand equations from an underlying production function without specifying the explicit form of this function. Shephard [24], Lau [17], Jorgensen and Lau [16], Diewert [5] and others have established the duality between the cost function and the production function. Hence any given cost function with appropriate continuity and curvature implies the existence of an underlying production function with the correct economic properties. From an operational viewpoint, there is no reason at all why the

explicit functional form of the underlying production function should be stated since its relevant parameters can be estimated from the cost function or in this case the input demand function which follows as the first derivative of the cost function.<sup>3</sup>

In the approach that follows, the underlying production function is not specified and estimation of the *ES* is undertaken directly from the factor demand equations which are formulated in a manner consistent with neo-classical theory. This approach is very similar to that adopted by Thirsk [25] in a study of factor substitution in Columbian agriculture.

We begin by specifying the *n* factor demand equations associated with the production function  $Y = f(X_i)$  as:

$$X_i = YF(P_1, P_2, \dots, P_j, \dots, P_n) \quad (i = 1, \dots, n), \quad (1)$$

where  $X_i$  is the demand for the *i*th factor,  $Y$  is output (value added), and  $P_i$  are the *n* factor input prices. The system is homogeneous of degree one in output and homogeneous of degree zero in factor prices. That is, doubling all input prices at a fixed level of output will leave all inputs unchanged. If all inputs are doubled, then output is doubled. Homogeneity of degree zero in prices and degree one in output is required because output is an argument of the input demand equations.

The analysis is performed in terms of primary factors of production, land, labour and capital. Hence output is a value added concept. In depicting the production technology as being described by a value added production function we are assuming that the function is weakly separable. That is, the marginal rate of substitution between factors *i* and *j* within the primary factor subset is independent of the quantities of intermediate inputs used. While more realism could be imparted into the analysis by considering both intermediate and primary inputs, data requirements would be considerably more demanding. All inputs are regarded as endogenous and factor prices and output are determined exogenously.

Assume that over the range of variation in the sample data, the factor demand equations can be approximated by the following exponential form:

$$X_i = A_i Y (P_1^{\theta_{i1}} \cdot P_2^{\theta_{i2}}, \dots, P_j^{\theta_{ij}}, \dots, P_n^{\theta_{in}}), \quad (2)$$

<sup>3</sup> The input demand function (2) satisfies the conventional restrictions such as:  $\partial f / \partial p_i < 0$ ,  $\partial X_i / \partial Y > 0$ . Given this input demand function and hence cost function which follows directly from the demand function, then duality theory establishes the existence of an underlying production function with the required conventional properties such as concavity and positive marginal products. The cost and production functions are dual in the sense that each may be derived from the other. While the underlying production function cannot be written in explicit form, it can be written as a mathematical statement showing the amount of output that can be produced from a given set of inputs. The cost function (*C*) is derived from the input demand function (2) by multiplying by  $P_i$  and summing across *i*.

$$\text{Hence } C = \sum_i P_i X_i = \sum_i A_i Y (P_1^{\theta_{i1}} P_2^{\theta_{i2}} \dots P_i^{\theta_{ii}+1} P_j^{\theta_{ij}} \dots P_n^{\theta_{in}}) \quad (i)$$

Note that  $\frac{\partial C}{\partial P_i} = X_i$  (the input demand function)

The production function follows from (i) as;

$$f(X_i) = \max_Y \{Y \mid P_i X_i \geq C(P_i, Y) \forall P_i > 0\} \quad (ii)$$

Equation (ii) says that associated with the set of inputs ( $X_i$ ) there is some  $f(X_i)$  such that output is maximised subject to the requirements that the value of inputs ( $P_i X_i$ ) is greater than or equal to the cost of producing that output.

where  $A_i$  is a constant, and  $\theta_{i1}, \dots, \theta_{in}$  are parameters incorporating substitution effects. The economic interpretation of the  $\theta_{i1}, \dots, \theta_{in}$  can be deduced by differentiating (2) with respect to the  $j^{\text{th}}$  factor price and rearranging:

$$\partial X_i / \partial P_j = \theta_{ij} X_i / P_j$$

Hence,

$$\theta_{ij} = (\partial X_i / \partial P_j) P_j / X_i \quad (3)$$

(the output compensated cross elasticity of demand for factor  $i$  with respect to the price of factor  $j$ ).

It can be shown that:

$$\sigma_{ij} = \theta_{ij} / S_j, \quad (4)$$

where  $S_j$  is the cost share of input  $j$  in total costs and  $\sigma_{ij}$  the (Allen-Uzawa) partial  $ES$ .<sup>4</sup>

Applying (2) to our three factor system gives, after eliminating  $\sigma_{ii}$  terms by enforcing homogeneity ( $\sum_j \sigma_{ij} S_j = 0$ ) and symmetry ( $\sigma_{ij} = \sigma_{ji}$ )

three factor demand equations which can be combined into two equations, each containing the three partial  $ES$  terms. Allowance can be made for efficiency growth of the  $X_i$  of the form

$$X_{eit} = \sigma_i e^{g_i t} X_{it}$$

where  $X_{eit}$  represents inputs measured in efficiency units and the  $\{g_i\}$  are annual rates of factor augmentation of the  $X_i$ . The resultant equations are

$$\ln(X_2/X_3) = \sigma_{21} S_1 \ln(P_1/P_2) + \sigma_{31} S_1 \ln(P_3/P_1) + \sigma_{23} [S_2 \ln(P_3/P_2) + S_3 \ln(P_3/P_2)] + t(g_3 - g_2) + k_1, \quad (5)$$

$$\ln(X_1/X_3) = \sigma_{21} S_2 \ln(P_2/P_1) + \sigma_{23} S_2 \ln(P_3/P_2) + \sigma_{31} [S_3 \ln(P_3/P_1) + S_1 \ln(P_3/P_1)] + t(g_3 - g_1) + k_2, \quad (6)$$

where

$$k_1 = \ln A_2 / A_3 + \ln a_3 / a_2;$$

$$k_2 = \ln A_1 / A_3 + \ln a_3 / a_1.$$

Equations (5) and (6) constitute the simultaneous estimating model.

### *Specification of Variables and Parameters*

#### *Factor Inputs*

The data base compiled by R. Powell [21] was used.

$X_1$  : unimproved land area;

$X_2$  : hired labour inputs;

$X_3$  : depreciated capital stock (constant prices) consisting of structures and plant and machinery. The implied assumption is that capital service flow is proportional to the depreciated capital stock.<sup>5</sup>

<sup>4</sup> See for example, Alan A. Powell [19], pp. 12-13.

<sup>5</sup> Capital inputs are notoriously difficult to measure in production function studies. Following Griliches [11] and Yotopoulos [29], an alternative measure of capital service flow was formulated under a 'one-hoss shay' situation as the annuity associated with an  $n$  year moving sum ( $n$  = working life) of cumulated gross investment of capital types. This series resulted in very similar parameter estimates to those reported.

*Output (Value added)*

Although value added does not appear as an argument in the system of estimating equations, it must be calculated to ensure calculation of the residual rental payment to capital (see next section). Value added should represent the amount available to reward the three factors of production after intermediate inputs have been rewarded according to their *MVP*. The series was constructed as follows: value added = gross value of farm production + changes in livestock inventories - non-factor expenses. Component series used in the construction of the value added series were all obtained from R. Powell [21].

The value added series was adjusted for weather influences for the years 1945/46 to 1964/65 using the weather index in [10]. The task of constructing a weather index at this level of aggregation is a daunting one. Since an index was available for part of the time series studied, it was used. However, since weather effects in Australia are seldom continent-wide, the impact of the index is negligible. Because of the level of aggregation, it was considered that no further filtering of the value added series was required.

*Factor Prices*

Since these enter the estimating equations as ratios, they are calculated in money terms. It follows from the behavioural assumptions of the model (cost minimization in the context of a linearly homogeneous production function with output predetermined) that the sum of payments to factors will equal factor income. The problem is therefore to allocate payments to each factor such that each is rewarded according to its *MVP* where *MVP*'s represent market prices in the long run under perfect competition. However, market prices need not equal *MVP*'s in the short run in a dynamic environment in which expectations are imperfect (or where competition is incomplete). This task is particularly difficult in the Australian situation where the family farm firm owns rather than rents the three factors. Information on market rental prices is poor or non-existent and imputation procedures must be adopted.

Briefly the price data were as follows:

- $P_1$  : rental price of land. The capitalized rent (market price) will reflect the discounted present value of expected returns from the land. Under the assumption that land yields an inexhaustible flow of services over time,  $P_1$  is the perpetuity associated with the market value  $V_t$ , i.e.,  $P_1 = rV_t$ , where  $r$  is the discount rate (maximum bank overdraft rate).
- $P_2$  : price of hired labour. A series on average earnings of rural employees was used. This series was derived from R. Powell's series on total earnings of rural employees and the total number of rural employees.
- $P_3$  : price of capital computed as value added minus payments to land and to hired labour expressed as a proportion of the capital stock at current prices.<sup>6</sup>

<sup>6</sup> Payments to capital must be sufficient to reward the owner-operator for his management and labour inputs. Hence the combined owner-operator-capital factor bears all the risk of the income stream not borne by the land. (This seems

The parameters and remaining variables have the following interpretation:

- $\sigma_{21}$  : partial *ES* between hired labour and land.
- $\sigma_{31}$  : partial *ES* between capital and land.
- $\sigma_{23}$  : partial *ES* between hired labour and capital.
- $g_1, g_2, g_3$  : efficiency growth of land, labour and capital respectively (per cent per annum).
- $t$  : time (years).
- $S_1, S_2, S_3$  : cost shares of land, labour and capital respectively in total cost.

### Estimation and Results

Aggregate data for the period 1920/21 to 1969/70 were fitted. The model (5)–(6) is linear in the parameters but subject to restrictions across equations (due to the symmetry of *ES*'s). It is not linear in the endogenous variables  $\{X_i\}$ , due to the presence of shares  $\{S_i\}$  on the right. Consistent estimates were achieved by first obtaining instrumental variables estimates (*IVE*'s) of the  $\{S_i\}$ <sup>7</sup> and then fitting the restricted system using the full-information maximum likelihood (*FIML*) technique<sup>8</sup> on the linear system (5)–(6) with *IVE*'s of  $S_i$ 's treated as exogenous variables. A result of this is that the  $t$  ratios (which were computed from *FIML* theory) probably are somewhat overstated. The results are shown in Table 1.

TABLE 1

*Factor Substitution Elasticities: Australia, 1920/21 to 1969/70*

|                                    | $\sigma_{21}$ | $\sigma_{31}$ | $\sigma_{23}$ | $g_3 - g_1$ | $g_3 - g_2$ | $R^{2a}$ |
|------------------------------------|---------------|---------------|---------------|-------------|-------------|----------|
| Equation (5) } 0.193               | 0.124         | 0.207         | -0.021        | -0.013      | 0.897       |          |
| Equation (6) } (5.19) <sup>b</sup> | (5.44)        | (5.51)        | (9.69)        | (10.25)     | 0.784       |          |

<sup>a</sup> A quasi  $R^2$  can be computed as [(variance of endogenous variable — variance of structural residual)/variance of endogenous variable].

<sup>b</sup>  $t$  values are in brackets.

The serial properties of the simultaneous equation system were investigated using a likelihood ratio test described by Hendry [13]. The test involves a comparison of the value of the log likelihood function associated with the basic model (5)–(6), and with a respecified model

fairly typical of the actual situation in agriculture.) The different treatment of hired versus owner-operator labour is consistent with the labour fixity phenomenon frequently observed in the rural sector.

<sup>7</sup> The  $\{S_i\}$  are generally computed as  $P_i X_i / Y$  where the  $\{X_i\}$  are quantities endogenously determined. To ensure consistent estimates, the  $S_i$  were regressed against the exogenous variables  $\{P_i\}$ ,  $\{P_i P_j\}$ ,  $\{P_i^2\}$ ,  $Y$ , and the resultant  $\{\hat{S}_i\}$  used in the second stage estimation of the model. The explanatory power of the share regressions was high with  $R^2$ 's of 0.96, 0.90, and 0.94 respectively for  $S_1$ ,  $S_2$ , and  $S_3$ .

<sup>8</sup> C. R. Wymer [28].



allowing for a first order autoregressive (*AR*) structure of errors in both equations.<sup>9</sup> The estimated substitution and efficiency growth parameters, together with the first order autocorrelation coefficients  $\phi_1$  and  $\phi_2$  of the *AR* model are shown in Table 2.

TABLE 2

*Factor Substitution Elasticities: Australia, 1920/21 to 1969/70*  
(First order autoregressive transformation of equations (5) and (6))

|   | $\sigma_{z1}$        | $\sigma_{z2}$ | $\sigma_{z3}$ | $g_3 - g_1$ | $g_3 - g_2$ | $\phi_1$ | $\phi_2$ | $R^{2a}$ |
|---|----------------------|---------------|---------------|-------------|-------------|----------|----------|----------|
| First order transformation of equations (5) and (4) | 0.079                | 0.058         | 0.097         | -0.025      | -0.010      | 0.889    | —        | 0.975    |
|   | (2.160) <sup>b</sup> | (2.480)       | (2.540)       | (3.790)     | (3.080)     | (15.53)  | 0.845    |          |
|   |                      |               |               |             |             |          | (19.01)  | 0.961    |

<sup>a</sup> Quasi  $R^2$  computed as in Table 1.

<sup>b</sup>  $t$  values are in brackets.

The most notable feature of the results is that all three partial *ES*'s are much closer to zero than to one, though all are significantly different (at the 95 per cent level of significance) from zero. These results suggest that technical prospects for substitution among primary inputs in Australia are very low. The capital-labour *ES* is greater than the labour-land *ES* which is greater than the land-capital *ES* though the absolute differences in magnitude between three partial *ES*'s are small.

A somewhat higher value of the labour-capital *ES* than that obtained might have been anticipated. There is a good deal of evidence to suggest that the labour mobility of farm operators is low. The strong attachment of some farmers to their farms is shown by their willingness to suffer substantial reductions in money incomes in agriculture rather than seek an alternative occupation. Provided farm returns are adequate in a welfare sense then the tendency is for the owner to remain fixed irrespective of relative price movements against him.<sup>10</sup> Tweeten [27] has defined the reservation price for farm labour as the price below which farm labour earnings must fall before alternative employment will induce labour to leave, given potential earnings in non-farm employment, transfer costs, psychic income from farming and expectations of

<sup>9</sup> Under the null hypothesis of no serial correlation the appropriate test statistic is a likelihood ratio test given by  $2T(L_{(1)} - L_{(0)})$  which is distributed  $\chi_n^2$ .  $T$  is the number of observations,  $L_{(1)}$  the value of the log likelihood function of the *AR* model,  $L_{(0)}$  the value of the likelihood function of the original model and  $n$  the number of degrees of freedom which in this case corresponds to the number of *AR* coefficients specified. The result ( $\chi_n^2 = 293.4$ ) involves the rejection of the null hypothesis.

<sup>10</sup> Some numerical evidence of this is provided by Ryan and Duncan [22] who estimated, from an ad hoc specification, the long run cross elasticity of demand for labour with respect to the real price of capital equipment and supplies to be 0.18 for owner-operators and 0.75 for hired farm labour.

future farm earnings and living in the city. Given these sorts of considerations, it was anticipated that by treating owner-operator labour as inseparable from capital, hired labour inputs would be more likely to respond to changes in relative factor prices in a manner which is consistent with the behavioural theory of the model.

Land and labour can be thought of as behaving as substitutes in agricultural production. A given amount of output can be produced using high labour-land ratios or low labour-land ratios. However, it is difficult to postulate *a priori* how the size of the labour-land *ES* will compare with the capital-labour *ES*. The low value of the *ES* obtained implies that diminishing returns set in rapidly as labour is increased relative to land.

It is more difficult (though not impossible) to envisage a situation in which substitution between capital and land takes place. In fact, it is reasonable to postulate a degree of complementarity between capital and land. That is, higher capital-labour ratios being associated with higher land-labour ratios. This theoretical framework can accommodate complementarity between a pair of factors through a negative *ES* on that factor pair.<sup>11</sup> The result of this analysis suggests an almost zero substitution elasticity between land and capital.

Efficiency growth is 2.5 per cent per year greater for the land factor than for the capital factor and 1.3 per cent greater for the labour factor than for the capital factor. That is, efficiency growth of land exceeds efficiency growth of labour which in turn exceeds efficiency growth of capital.<sup>12</sup> The conclusion that rates of factor augmentation differ between factors has commonly been observed in studies in this area. See, for example, David and Van de Klundert [6], Sato [23], and Duncan and Binswanger [8] for manufacturing industry, and Fishelson [9] and Bates [2] for agriculture. In their two-factor capital-labour models, both Fishelson and Bates found that efficiency growth for labour exceeded that for capital.

Although the results are of limited operational interest because of the high level of aggregation of the data, they suggest that primary factor inputs into Australian agriculture over the period studied have been largely unresponsive to changes in their relative prices.

<sup>11</sup> From the homogeneity conditions of the three factor system:

$$\begin{aligned} S_2\sigma_{12} + S_3\sigma_{13} &> 0 \\ S_1\sigma_{12} + S_3\sigma_{23} &> 0 \\ S_1\sigma_{13} + S_2\sigma_{23} &> 0, \end{aligned}$$

since  $\sigma_{ii}$  terms are  $< 0$ , being proportional to the ratio of  $F_{ii}$  to  $F$ , with the requirements for cost minimization being that  $F < 0$  and  $F_{ii} > 0$  (Allen [1]). Although the values of the partial elasticities of substitution can be positive or negative, it can be seen from the above inequality conditions that the positive values must be more numerous or important than the negative values. It follows that either all three partial *ES* ( $\sigma_{12}$ ,  $\sigma_{13}$ ,  $\sigma_{23}$ ) are positive or that one of the partial elasticities is negative and the other two positive.

<sup>12</sup> While the  $\{g_i\}$  are interpreted as indicating the annual percentage rate of factor augmentation, they imply nothing about the source of efficiency growth. For example, an increase in  $g_2$  is designated as labour augmenting even though it may have resulted from better equipment.

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