MARKOV CHAINS: BASIC CONCEPTS AND SUGGESTED USES IN AGRICULTURAL ECONOMICS

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Agricultural economists are often interested in characterizing or summarizing how economic processes and institutions have changed through time as well as what paths they are likely to take in future time periods. Given this interest or objective, we are therefore interested in methods of analysis that will accomplish these purposes and that are simple to apply. Within this context the major purpose of this paper is to discuss the concept of a Markov chain process and to indicate its potential usefulness in analyzing problems where detailed time-ordered data exist over some time span. As a particular vehicle for the discussion, a limited example concerning the past and potential size distribution of a sample of hog-producing firms in central Illinois will be analyzed.

Although the basic concepts of Markov chains were introduced around 1907, their use by economists is of relatively recent vintage. Solow in 1951 and Champernowne in 1953 applied this probabilistic approach to the analysis of income and wage distributions.[1] Hart and Prais also employed the technique in an investigation of business concentration.[2] Prais in addition applied the technique in measuring social mobility.[3] Adelman used the same approach in analyzing the size distribution of firms within the steel industry and extended the work by Hart and Prais.[4] Sparks used the technique with data from a consumer food panel to analyze consumer food purchases of pork and beef.[5]

The Concept of a Markov Chain Process[6]

To provide a base for the analysis to follow, we will sketch in this section the basic concepts of a Markov chain process and state the assumptions, definitions, and theorems underlying this method that are necessary for our purpose.

If in any given sequence of experiments the outcome of each particular experiment depends on some chance event, then any such

1. R. Solow, "Some Long-Run Aspects of the Distribution of Wage Incomes," *Econometrica*, 19:333-34, July 1951; D. G. Champernowne, "A Model of Income Distribution," *Economic Journal*, 63:318-51, June 1953.

2. P. E. Hart and S. J. Prais, "The Analysis of Business Concentration, A Statistical Approach," *Journal of the Royal Statistical Society*, Series A, 119:150-75, October 1956.

3. S. J. Prais, "Measuring Social Mobility," *Journal of the Royal Statistical Society,* Series A, 118:56-66, July 1955.

4. I. G. Adelman, "A Stochastic Analysis of the Size Distribution of Firms," *Journal of the American Statistical Association,* 53:893-904, December 1958.

5. W. R. Sparks, "On Markov Chains in Demand Analysis," unpublished paper, Michigan State University, 1960.

6. For an excellent and relatively complete discussion of finite Markov processes, see J. G. Kemeny and J. Laurie Snell, *Finite Markov Processes*, Princeton: D. van Nostrand Company, Incorporated, 1960.

sequence is called a stochastic process. The process is finite if the set of possible outcomes is finite. There are many types of stochastic processes, and they can be classified by indicating special properties of the outcome functions.

Definition: A finite stochastic process with outcome functions f_o, $f_1, \ldots, f_n$ is a stationary Markov chain process if the starting state, given by f_o, is fixed and

$$Pr[f_o = t \mid (f_{n-1} = s) \& (f_{n-2} = r) \& \ldots \& (f_1 = a)] = Pr[f_n = t \mid f_{n-1} = s] \quad (1)$$

$$Pr[f_n = t \mid f_{n-1} = s] = Pr[f_m = t \mid f_{m-1} = s] \quad (2)$$

for all $m \geqslant 1$ and $n \geqslant 2$ and any possible sequence of outcomes $a, \ldots s, t$.[7] The prior statement can be read as the probability of $f_n = t$ given $f_{n-1} = s$ and $f_{n-2} = r$, etc.

The interpretation of this definition is that the outcome of a given experiment depends only on the outcome of the immediately preceding experiment and that this dependence is the same at all stages (stage refers to a particular place in the sequence of experiments).

An equivalent definition given by Kemeny *et al.* is as follows: "A Markov chain process is determined by specifying the following information: There is given a set of states $(s_1\ s_2, \ldots s_r)$. The process can be in one and only one of these states at a given time and it moves successively from one state to another. Each move is called a step. The probability that the process moves from s_i to s_j depends only on the state s_i that it occupied before the step. The transition probability p_{ij} which gives the probability that the process will move from s_i to s_j is given for every pair of states. Also an initial starting state is specified at which the process is assumed to begin."[8]

Given the information specified above, it would then be possible to construct a possibility tree and attach branch weights that would describe the process as it moves through any finite number of steps. Alternatively the transition probabilities p_{ij} can be represented in the form of a transition matrix P:

$$P = \begin{array}{c} \\ s_1 \\ s_2 \\ \cdot \\ s_n \end{array} \begin{array}{c} \begin{array}{cccc} s_1\ s_2 & \ldots & & s_n \end{array} \\ \begin{bmatrix} p_{11}\ p_{12} & \ldots & p_{1n} \\ p_{21}\ p_{22} & \ldots & p_{2n} \\ \cdot\quad\cdot & \ldots & \cdot \\ p_{n1}\ p_{n2} & \ldots & p_{nn} \end{bmatrix} \end{array} \quad (3)$$

$$\text{Where } \sum_j p_{ij} = 1 \quad (4)$$

$$\text{and } p_{ij} \geqslant 0 \text{ all } i \text{ and } j. \quad (5)$$

The elements of (3), the p_{ij}, denote the probability of moving from state s_i to state s_j in the next step. Since the elements of this matrix are non-negative and the sum of the elements in any row is 1, each row of the matrix is called a probability vector and the matrix [p] is a stochastic matrix. This matrix, together with an initial starting state, completely defines a Markov chain process; i.e., given this information, we could determine the outcome of, say, the nth step. In matrix language this can be developed in the following way: Let

7. J. G. Kemeny *et al., Finite Mathematical Structures,* New York; Prentice-Hall, 1959, p. 148.

8. *Loc. cit.*

w^0 represent the initial vector or starting state. Then

$$w^{(0)}P = w^{(1)} \tag{6}$$
$$w^{(1)}P = w^{(2)} \tag{7}$$
$$\vdots \qquad \vdots$$
$$w^{(n-1)}P = w^{(n)} \tag{8}$$

Alternatively $w^{(n)}$ may be written as

$$w^{(n)} = w^0 P^n \tag{9}$$

Therefore, if we start in state i, then $w^{(1)}$ is the ith row of P, $w^{(2)}$ is the ith row of P^2, and thus w^n is the ith row of P^n. Therefore, the rows of P^n give us the outcome vectors for various starting states. The p_{ij} will be the probability that the process will be in state j after n steps if it is started in state i.

If in addition we require all states to be accessible — meaning that there is a nonzero probability of moving from state i to state j in a finite number of time periods — then the chain is called irreducible.[9] For the transition matrix [P] to be irreducible, a sufficient condition is that some power of this matrix has only positive components. If this condition is met, the transition matrix [P] defines a Markov chain process that is regular.

Since we will be working with regular Markov chains, we now state two theorems relating to the existence and uniqueness of an equilibrium solution.

Theorem 1. If P is a transition matrix for a regular chain, then:

1. The powers P^n approach a matrix T.
2. Each row of T is the same probability vector w.
3. The components of w are all positive.[10]

Theorem 2. If P is a transition matrix for a regular chain and T and w are as in Theorem 1, then the unique vector w is the unique probability vector such that $wP = w$.[11]

These theorems state that if P is a transition matrix for a regular chain there exists a unique vector w that is both a fixed vector for P and a probability vector. The distribution at time n tends toward this vector irrespective of the initial distribution or starting state.

We can now use these theorems in deriving an equilibrium solution for a regular stochastic matrix. From Theorem 1 we know that

$$P \longrightarrow T \text{ as } n \longrightarrow \infty \tag{10}$$

and

$$T = e'w, \tag{11}$$

where $e = (1, 1, \ldots 1)$ and w is the equilibrium vector.

Therefore, one way of deriving the equilibrium vector is to multiply P by itself a large number of times.

Alternatively, from Theorem 2 we know that in equilibrium the distribution vector must be invariant, i.e.,

$$wP = w \tag{12}$$

Therefore $w(P\text{-}I) = 0, \tag{13}$

which is a system of n–1 linearly independent equations and n unknowns.

9. W. Feller, *An Introduction to Probability Theory and Its Applications*, New York: John Wiley and Sons, 1950, p. 340.

10. Kemeny et al., *op. cit.*, p. 392.

11. *Loc. cit.*, p. 393.

Since w is a probability vector, we have

$$\sum_j w_j = 1 \tag{14}$$

Using this information, we can now combine equations (13) and (14) and form a system of n linearly independent equations and n unknowns from which we can solve for the unique values of w.

We will end this discussion of some of the basic concepts of Markov chains with a definition and theorem regarding absorbing Markov chains.

Definition: A state in a Markov chain is an absorbing state if it is impossible to leave it; i.e., $p_{ij} = 1$ when $i = j$. A Markov chain is absorbing if (1) it has at least one absorbing state and (2) from every state it is possible to go to an absorbing state (not necessarily in one step).[12]

Theorem. In an absorbing Markov chain the probability that the process will be absorbed is 1.[12]

Given the basic definitions, assumptions, and concepts, we will apply this probabilistic procedure to the derivation of an equilibrium size structure of the hog enterprise for a sample of Illinois farms. Because of the restricted nature of the sample of data used, we will emphasize possible uses of this measurement technique.

The Data

Data from a sample of 83 hog-producing firms in central Illinois will be used as a basis for the empirical analysis to follow. The basic data are from records kept by these producers over the period 1946-58. Number of litters of hogs produced by each firm in each year is the variable selected for measurement. The descriptive statistics relating to the principal characteristics of the litter size distribution of this sample of hog producers in each of 13 years are presented in Table 1.

TABLE 1

Some Characteristics of Frequency Distribution of 83 Illinois Hog Producers Based on Number of Litters per Year, Livestock Area, 1946-58[a].

Year	Mean	Median	Standard deviation	Skewness
1946	37.6	37	23.1	0.078
1947	41.3	36	28.4	0.560
1948	41.5	37	24.3	0.555
1949	45.2	40	28.5	0.547
1950	51.3	45	30.6	0.618
1951	51.0	44	32.4	0.648
1952	46.7	38	32.9	0.793
1953	44.9	31	33.2	1.256
1954	48.3	45	35.4	0.280
1955	52.6	44	37.0	0.697
1956	46.5	43	34.1	0.308
1957	45.7	44	33.1	0.154
1958	52.1	50	40.9	0.154

[a] E. R. Swanson and D. R. Meline, "Hog Supply Response of Farm Bureau Farm Management Service Cooperators," Department of Agricultural Economics, University of Illinois, AERR-36, October 1960. To qualify for the sample, the farms must have produced hogs in three or more of the 13 years.

12. *Loc. cit.*, p. 404.

This summary description shows that over these years there has been an upward trend in the mean, median, and standard deviation. Skewness increased to about the middle of the period and then decreased. Although these descriptive statistics reflect some characteristics of the annual distribution, they fail to portray the paths taken by individual producers in making year-to-year changes in their production. Assume that the hypothetical "representative" producer is indicated by the mean. Even though there is an upward trend in this value, we cannot tell from the table whether there have been year-to-year changes in the *position* of individual firms relative to others within the array. Both the ease and the direction of such movements have important implications for analyzing problems at the firm as well as at government levels.

The Model

We now proceed to employ some of the ideas from the previously cited economic applications of Markov chain analysis in the study of this sample of hog producers. In specifying the model we assume that (1) the firms that engage in hog production can be grouped according to some criterion of size into classes or states and (2) the evolution of a hog enterprise through these states can be regarded as a stochastic process, with probabilities of transition constant in time and the probability of moving from one state to another a function only of the two states involved. In this framework the movement of the hog enterprise within the farm business depends on the size at the beginning of the period and the number of years involved and is independent of the previous history of the enterprise.

The degree of simplification of this specification is immediately apparent. For example, we ignore all changes in product and factor prices, technology, *etc.*, over time and represent the result of all these forces by one variable — number of litters of hogs. In addition, we assume that the effects of all these factors, which we summarize in terms of transition probabilities, remain invariant throughout the relevant future time period. While this is obviously a strong restriction, Adelman[13] notes that this specification is analogous to that used in many of the analyses involving long-run comparative statistics.

Within this framework, litters of hogs produced are the variable whose movement over time is to be analyzed, and the following class intervals are used in defining the admissible states (Table 2). Therefore, in any one year it is possible for a hog producer to be in any one of the seven specified positions. A more refined classification may be desirable for some purposes, but this specification should serve to illustrate the technique.

Having defined the data and the ranges for each class, we traced the year-to-year history of each hog producer in terms of his movement among the various classes. In developing the transition matrix, which reflects the behaviour of the sample of hog producers, let a_{ij} represent the number of firms moving from class i to class j through the 13 years under consideration. Then the transition probabilities become

$$p_{ij} = a_{ij} \Big/ \sum_{j=0}^{6} a_{ij} \tag{15}$$

13. Adelman, *op. cit.*, p. 894.

TABLE 2

Class	Class Limits
	(Number of litters produced)
S_0	0
S_1	1-20
S_2	21-40
S_3	41-60
S_4	61-80
S_5	81-100
S_6	> 100

Anderson and Goodman[14] have shown that the maximum likelihood estimates of the stationary transition probabilities p_{ij} are

$$\hat{p}_{ij} = a_{ij} \Big/ \sum_{j=0}^{6} a_{ij} \qquad (16)$$

Therefore, $\hat{P}$ will be used to denote the estimated transition matrix $[\hat{p}_{ij}]$.

Empirical Results

To test the hypothesis of the dependence of the firms' hog production in year $t + 1$ on year t against the possibility of being statistically independent, a chi square test was used. This test resulted in a chi square value of 1,549 and is statistically significant at the 99 percent confidence level. Given this information, we used equation (16) to estimate the transition matrix. Since each hog-producing firm moved (or had the option to move) from one state to another 12 times during the 13-year period, the transition matrix is based on 996 observations.

$$\hat{P} = \begin{array}{c} \\ S_0 \\ S_1 \\ S_2 \\ S_3 \\ S_4 \\ S_5 \\ S_6 \end{array} \begin{array}{ccccccc} S_0 & S_1 & S_2 & S_3 & S_4 & S_5 & S_6 \\ .693 & .194 & .065 & .016 & 0 & .032 & 0 \\ .126 & .412 & .403 & .042 & .008 & 0 & .008 \\ .043 & .134 & .593 & .193 & .034 & .003 & 0 \\ .009 & .017 & .201 & .515 & .205 & .048 & .004 \\ .014 & .007 & .035 & .246 & .514 & .148 & .035 \\ .016 & .016 & .016 & .063 & .266 & .359 & .266 \\ .017 & .017 & 0 & .017 & .069 & .207 & .672 \end{array} \qquad (17)$$

This transition matrix, in itself, gives some useful insights into the dynamic aspects of hog production. For example, the entries in the cells on the diagonal indicate that there was, during the period, a strong tendency for producers to remain within a given class from one year

14. For a discussion of maximum likelihood estimates of stationary transition probabilities and statistical tests applicable to Markov chains, see T. W. Anderson and L. A. Goodman, "Statistical Inference About Markov Chains," *Annals of Mathematical Statistics,* 28:89-110, March 1957.

to the next. Probabilities on the principal diagonal are all larger than any of the other elements in each row. This stability is, of course, partly due to the somewhat arbitrary definition of class intervals; a more refined classification would detect smaller year-to-year shifts in number of litters. Note that there is a strong tendency for growth in the 1- to 20-litter class; the probability of increasing to the 21- to 40-litter class (0.403) is almost as great as that of remaining in the 1- to 20-litter class (0.412). Comparisons of the likelihood of an increase of any size or a decrease of any size from a given state may also be made by simply summing the elements to the right of the diagonal in that row (for increases) and to the left (for decreases). For example, with a beginning state of class 3 the probability of an increase is only slightly greater than that of a decrease (0.257 compared with 0.227).

In addition it can be noted from the transition matrix that the most probable outcome, excluding remaining in the same class, is that the firms either move up or down one class at a time. Going from no hog production to some magnitude of production occurs predominantly in the classes included in the 1- to 40-litter range. The probability of going out of production is also greater for the smaller hog units than for the large.

Given the elements of the transition matrix, we can also construct a conditional probability matrix whose elements represent the conditional probability that the hog firm will go to state j if it leaves state i. The elements of this matrix can be computed as follows:

$$\hat{m}_{ij} = \hat{p}_{ij} / 1 - \hat{p}_{ii} \qquad (18)$$

Equilibrium vector

Given the transition matrix and the starting state or vector, we may now find it interesting to analyze the structure that the sample of hog firms would eventually reach if the trend persisted through time. As noted earlier, for regular chains the equilibrium size distribution does not depend on the initial starting state or vector. The period of time necessary to reach equilibrium does, of course, depend on the starting distribution. When equations (12), (13), (14), and the transition matrix P are used, the equilibrium size distribution vector that results is

$$w = (.098\ .099\ .234\ .207\ .170\ .093\ .099) \qquad (19)$$

Before we discuss this equilibrium vector, we should comment on the meaning of equilibrium in a Markov process. Adelman has succinctly interpreted an equilibrium structure in a firm-size model as follows: ". . . may be defined as that distribution for which the average number of corporations entering a given stratum per period equals the average number of businesses leaving it. Our concept of equilibrium is statistical in nature for the industry, and dynamic for the individual firm. In other words, equilibrium in this paper does not imply that there is no movement of enterprises between strata. On the contrary, the stochastic conception of equilibrium explicitly requires that firms move in and out of each class. But on the average forces acting to increase the number of enterprises in a given size range are exactly counterbalanced by those tending to decrease it . . .".[15]

15. Adelman, *op. cit.*, p. 895-96.

For purposes of discussion and comparison, the size distribution actually observed in 1946 and the 1946-58 averages are presented in Table 3 along with the equilibrium size distribution.

TABLE 3

Actual Size Distribution 1946 and 1946-58 Average and the Equilibrium Size Distribution

	State						
	S_0	S_1	S_2	S_3	S_4	S_5	S_6
Equilibrium	.098	.099	.234	.207	.170	.093	.099
1946	.036	.157	.410	.265	.084	.025	.012
1946-58 average	.074	.116	.306	.230	.148	.066	.060

Note that in equilibrium about 10 percent of the firms will produce no hogs (state S_0). This is an increase over both 1946 and the average for the period relative to this state. Decreases from the actual to the equilibrium situation are noted for states 1, 2, and 3 (those classes producing fewer than 61 litters). In contrast, increases from the actual to the equilibrium situation are noted in the three large size classes, 4, 5, and 6. Although these results are not so extreme as some current predictions in regard to increases in the size of hog firms, they are consistent in direction with what is currently being predicted.

The effect of concentration of production on the structure of production among large producers may be more readily seen by comparing the percent of total production in each size class (Table 4). The 1946 class means were used to estimate distribution of production in 1946-58 and in equilibrium.

TABLE 4

Concentration of Production, 1946, Average for Period and Equilibrium

	State						
	S_0	S_1	S_2	S_3	S_4	S_5	S_6
Equilibrium	—	.019	.134	.197	.227	.159	.264
1946	—	.040	.325	.344	.157	.058	.070
1946-58 average	—	.025	.019	.247	.233	.127	.181

In equilibrium, 65 percent of the litters were produced in states 4, 5, and 6, that is, sizes above 60. In contrast, only 28.5 percent of the litters were produced by these classes in 1946.

Given the estimated transition matrix P the equilibrium size distribution vector toward which the system converges, and the starting state or vector, we can investigate the time path(s) followed in reaching the fixed size distribution vector. We noted previously that p_{ij} represents the probability that the process will be in state j after n years, given that it started in state i. Therefore, we can investigate the probabilities for each state en route to equilibrium by raising P to various powers. Such a matrix for the seventh year is

$$\hat{P}^6 = $$

	S_0	S_1	S_2	S_3	S_4	S_5	S_6
S_0	.21	.16	.29	.16	.09	.05	.04
S_1	.14	.14	.31	.20	.12	.05	.04
S_2	.11	.12	.29	.22	.15	.06	.05
S_3	.07	.09	.24	.23	.19	.09	.09
S_4	.06	.07	.19	.22	.21	.12	.13
S_5	.06	.06	.15	.19	.21	.14	.19
S_6	.06	.05	.12	.16	.20	.16	.25

(20)

Note that each row gives the probabilities of being in each possible state six years after starting in the *ith* state. By comparing each row of $\hat{P}$ with (19) we see that even in a comparatively short period a significant advance has been made toward the fixed equilibrium vector. Alternatively we can premultiply the transition matrix $\hat{P}$ by a vector of the distribution of hog firms (number or percent) in each state (e.g., 1958) to derive the structure in the next period. Repeating this process will trace out the path of size distribution *en route* to equilibrium. In this example starting with the size distribution that existed in 1958, the equilibrium vector was approximately reached in 18 years.

Mean lifetime and mobility

The transition matrix $\hat{P}$ may also be used to determine the average number of years the hog firm remains in the same litter class and to construct an index of firm mobility. Adelman developed the mean lifetime of a firm in the *ith* state as follows:[16]

Let T_i represent the total time spent in the interval by all the s_i^o firms originally included therein. This is

$$T_i = s_i^o + s_i^o\hat{p}_{ii} + s_i^o\hat{p}^2_{ii} + \ldots \qquad (21)$$

$$L_i = T_i/s_i^o = 1 + \hat{p}_{ii} + \hat{p}^2_{ii} + \ldots = 1/1 - \hat{p}_{ii} \qquad (22)$$

Equation (22) can be then used to compute the mean lifetime of a hog firm in the *ith* interval of our example. Although the mean lifetime of a firm in the *ith* state gives some indication of mobility, if it is to be meaningful some basis of comparison is needed. To provide this standard of comparison, Adelman [17], following the work of Prais[18], defines a perfectly mobile industry as one for which the probability that a firm will move from Class *A* to Class *B* during a single period is independent of *A*. The transition matrix *T* of equations (10) and (11) fulfills these requirements, and this equilibrium structure is also one that will be reached by the group of firms in our example. Therefore, given the transition matrix *P* and the equilibrium transition matrix *T* Adelman[19] defines the index for industrial mobility for time *n* as

$$I^n = \left[\sum_{i=0}^{6} t_i/(1-t_i)\right] \Big/ \left[\sum_{i=0}^{6} s_i^n/(1-\hat{p}_{ii})\right] \qquad (23)$$

16. *Ibid.*, p. 897.
17. *Loc. cit.*
18. S. J. Pais, *op. cit.*, p. 59, 61.
19. Adelman, *op. cit.*, p. 897.

where t_i is the element in the *ith* column in any of the (identical) rows of T.

The results of applying equation (22) to the data for hog firms are given in Table 5.

TABLE 5

Mean Lifetime of a Hog Firm in a Given State

Mean lifetime

State	1946-58 average	(Years)	Perfectly mobile industry Ratio
S_0	3.26	1.11	2.89
S_1	1.70	1.11	1.53
S_2	2.46	1.31	1.87
S_3	2.06	1.26	1.63
S_4	2.06	1.21	1.70
S_5	1.56	1.10	1.42
S_6	3.05	1.11	2.75

The range of litters included in each state conditions the values obtained for mean lifetimes, and thus for inferential purposes only the ratios between the 1946-58 average and the perfectly mobile industry are meaningful. These ratios indicate that firms in the states 0 and 6 (no hog production and greater than 100 litters) are less mobile than the remainder of the classes of firms. Since there is no larger interval into which hog firms in state 6 may move, it is reasonable to expect them to have less mobility than the other classes. Except for states 0 and 6, the actual mean lifetimes of the hog firms do not appear to vary significantly.

The mobility indices for the distribution of the firms in 1946 and in equilibrium are 55.0 and 52.6. Therefore, the mobility of firms in 1946 was about 55 percent of that in a perfectly mobile structure. A decrease in the index from 55 to 52.6 indicates a tendency for the mobility of hog firms to decrease.

Use of absorbing chains

Having discussed the uses of regular Markov chains by means of an example, we may now find it interesting to inquire into the possible uses of absorbing Markov chains. We will also use data relating to the size distribution of hog firms for this analysis. Although the definition of absorbing states is somewhat artificial for this problem, the example will illustrate another possible use of this type of analysis. A state in a Markov chain was previously defined as an absorbing state if it is impossible to leave it. Furthermore, a Markov chain is absorbing if it has at least one absorbing state and from every state it is possible to go to an absorbing state.

Now for purposes of exposition let us assume that states 0 and 6 are absorbing. Hence, whenever a firm does not produce hogs for a year or produces more than 100 litters of hogs for a year, it stays in this class. Therefore, in canonical form the transition matrix for this situation is as follows:

	S_0	S_6	S_1	S_2	S_3	S_4	S_5
S_0	1	0	0	0	0	0	0
S_6	0	1	0	0	0	0	0
S_1	.126	.008	.412	.403	.042	.008	0
S_2	.043	0	.134	.593	.193	.034	.003
S_3	.009	.004	.017	.201	.515	.205	.048
S_4	.014	.035	.007	.035	.246	.514	.148
S_5	.016	.266	.016	.016	.063	.266	.359

(24)

In the preceding matrix there are two absorbing states, S_0 and S_6, and five nonabsorbing states, $S_1 \ldots S_5$. Let the southeast sub-matrix of (24) be designated by Q. Then the elements of $(I\text{-}Q)^{-1}$ give the mean number of years in each transient state for each possible nonabsorbing starting state. This computation results in the following matrix:

$(I\text{-}Q)^{-1} =$

	S_1	S_2	S_3	S_4	S_5
S_1	2.96	4.88	3.48	2.31	.82
S_2	1.68	6.57	4.36	2.89	1.03
S_3	1.31	4.63	6.01	3.60	1.30
S_4	1.03	3.57	4.18	4.89	1.46
S_5	.67	2.21	2.51	2.51	2.34

(25)

Thus, starting in state 1, the mean number of years in state 1 before absorption is 2.96; in state 2, 4.88, *etc.* If we obtain the sum for the elements in each row of (25), we get the mean years required for a given starting state before being absorbed. This operation results in the following vector:

S_1	14.45
S_2	16.53
S_3	16.85
S_4	15.13
S_5	10.24

(26)

Thus the mean number of years before absorption, starting in state 1, is 14.45; in state 2, 16.53, *etc.* We may now consider the probability that an absorbing chain will end up in a particular absorbing state. Let the southwest submatrix of (24) be designated as R. Then when $(I\text{-}Q)^{-1}$ is postmultiplied by R the following probabilities result:

	S_0	S_6
S_1	.66	.34
S_2	.59	.41
S_3	.49	.51
S_4	.41	.59
S_5	.27	.73

(27)

Thus, for example, starting in S_1 there is a .66 probability of absorption by S_0 (no hog production) and a .34 probability of absorption by S_6 (greater than 100 litters of hogs). For state 3 the probabilities of no hog production or more than 100 litters are approximately equal. In general, the larger the number of litters in the starting state, the smaller the probability of being absorbed by the state S_0.

Possible Areas of Application

In the first section of this paper, we noted a few ways in which Markov chains have been used in economics. We hope that the example will suggest other alternative applications to the reader. In general, any characteristic that can be quantified can be analyzed so long as the previously stated conditions are satisfied or hold approximately.

In our example one outcome of the firm operation, hog production, was analyzed. The analysis could, of course, be extended to other enterprises or combinations of enterprises. For example, each state might be composed of given ranges of both cattle and hog production. In this way information could be shed on the past and future organizational pattern of the firms.

The size distribution of agricultural producing firms or operating units is another area in which this method might be effectively employed. For example, there is currently much interest in the potential future size distribution of agricultural firms. With the appropriate historical data, this method could indicate what changes had occurred in the structure of producing firms and what the future time path might be. These firms could be partitioned by regions, resource availability and productivity, etc., to indicate present and future differences in structure and rates of adjustments. Estimating the index of mobility would make it possible to rank the firms by these subclassifications.

Interest also is centered in the area of market structure, or understanding the organization and performance of agricultural industries.[20] This method might provide a means of measuring past and future changes in structure. The impact of certain disturbances on market structure could be obtained by altering certain elements of the transition matrix. Problems relating to size and location of agricultural marketing firms might be solved by this method. In such problems, size and/or location would be used to specify the states. Livestock slaughtering plants in the United States are one example of an industry that in the past 25 years has experienced changes in both size distribution and location. This method could be used to characterize these changes and to indicate the future if past trends continue.

In the area of economic growth and development, this method might be used to get a good idea of the structure of changes for a region or country and what the long-run consequences might be if the current structure was maintained. Other viable structures might be

20. R. L. Clodius and W. F. Mueller, "Market Structure Analysis as an Orientation for Research in Agricultural Economics," *Journal of Farm Economics,* XLIII:515-553, August 1961.

reflected in terms of transition matrices, and thus the future consequences of each of these structures might be specified.

These are a few areas in which it appears that this technique might be employed. Undoubtedly the reader will think of others. It should be mentioned at this point that the method is demanding in terms of data. For example, in the firm problems one must have fairly complete information in regard to the micro units over time. Thus availability of data may sometimes cause difficulty. However, if the needs were known, data collection agencies could be contacted with a view to getting data consistent with the model requirements.

Summary

The Markov chain method has been suggested as a means of characterizing or summarizing economic data and of projecting the time path of certain economic variables. We have presented the basic concepts of Markov chains and have given an example involving hog-producing firms to show what information it would be possible to obtain. No specific inferences are warranted from this example; however, the results do seem to be reasonable and consistent with impressions in the field. Finally, we have suggested a limited number of areas in which this method might be used.