



AgEcon SEARCH
RESEARCH IN AGRICULTURAL & APPLIED ECONOMICS

The World's Largest Open Access Agricultural & Applied Economics Digital Library

This document is discoverable and free to researchers across the globe due to the work of AgEcon Search.

Help ensure our sustainability.

Give to AgEcon Search

AgEcon Search

<http://ageconsearch.umn.edu>

aesearch@umn.edu

*Papers downloaded from **AgEcon Search** may be used for non-commercial purposes and personal study only. No other use, including posting to another Internet site, is permitted without permission from the copyright owner (not AgEcon Search), or as allowed under the provisions of Fair Use, U.S. Copyright Act, Title 17 U.S.C.*

COORDINATING PRODUCTION AND DISPOSAL OF COMMODITY STOCKPILES WITH APPLICATION TO AUSTRALIA'S WOOL INDUSTRY

GREG HERTZLER*

*Faculty of Agriculture, The University of Western Australia,
Nedlands, Western Australia*

Following the dismantling of a price-support program, a central bureaucracy is left with a commodity stockpile to dispose. It happened with wheat and feed grains in the U.S. in 1986 and wool in Australia in 1991. It soon may happen in Europe with grains, manufactured dairy products and other commodities which have supported prices. Obvious policies include privatising the stockpile, disposing of the stockpile by a central bureaucracy and quarantining the stockpile from the market. Each policy imposes constraints on disposal based, perhaps, on judgments of political acceptability to producers and government. In this article, optimal rules for production and disposal are derived and solved and a new policy is proposed. Then the model is applied to the disposal of Australia's wool stockpile. Results show that centralised disposal will almost always be preferred to privatisation of the stockpile. Centralised disposal is also preferred to quarantining the stockpile if interest rates are high, but quarantining is preferred if interest rates are low. Centralised disposal and quarantining are not optimal, however. Optimal production and disposal combines the efficiency of privatisation with the market power of centralised disposal. To achieve this, the property rights to the stockpile can be redefined using payment-in-kind certificates and individual transferable entitlements. The payment-in-kind certificates assign ownership of the stockpile to individual producers who then make both production and disposal decisions. The individual transferable entitlements allow the industry to produce efficiently and extend market power from the central bureaucracy to producers. For the Australian wool stockpile, optimal production and disposal would benefit the industry by an estimated \$2.7 billion.

Occasionally, governments and industry bodies wage war against commodity markets, trying to maintain prices above market-clearing levels. Excess supply results which must be purchased and stored. It is a war which cannot be won (Watson 1990; Gunasekera and Fisher 1991). Eventually the accumulated stockpile becomes too big a burden and must be surrendered at reduced prices. It happened with U.S. wheat and feed grains in 1986 and with Australian wool in 1991. It may happen in Europe

* The author would like to thank Rob Fraser, Bob Richardson and an anonymous referee for their helpful comments on this article.

with grains, manufactured dairy products and other commodities which have supported prices.

A stockpile can be one of three things. It can be a buffer stock to stabilise prices, an asset held as a risky investment within a portfolio of investments or an exhaustible resource. A stockpile accumulated by a support price program is most like an exhaustible resource. From the theory of exhaustible resources, we know the stockpile may have a scarcity value and should be rationed over time. The stockpile's scarcity value will be part of its disposal costs. In each year, a competitive industry will dispose of the scarce stockpile to the point where marginal disposal costs equal the market price. A monopolist will dispose less, where marginal disposal costs equal marginal revenue. But stockpile disposal is only one of two sources of the commodity in the market. The other is current production. Again from the theory of exhaustible resources, we know the combination of disposal and production should be least-cost. Both a competitive industry and a monopolist will produce wool to the point where marginal production costs equal marginal disposal costs. Eventually, the stockpile will be exhausted and the industry will return to normal production. But if the benefits of normal production exceed the benefits of production with disposal, the industry should resume normal production immediately by quarantining the stockpile from the market.

Unfortunately, there is a complication. Following the demise of a support-price program, the stockpile will be controlled by a central bureaucracy. The industry is half competitive, half monopolistic. Producers must compete amongst themselves and against a bureaucracy with market power. Producers will produce where marginal production costs equal the market price. The bureaucracy will dispose of the stockpile to the point where marginal disposal costs equal its marginal revenue. Because the market price exceeds marginal revenue, production is more costly on the margin than disposal, too much is produced, too little is disposed from the stockpile and the industry is inefficient. If disposal decisions were decentralised by privatising the stockpile, production and disposal decisions would be coordinated, the industry would be efficient but it would lack market power. Thus centralised disposal creates market power but causes inefficient production and disposal. Decentralised disposal is efficient but lacks market power. Ideally, a policy to maximise benefits to the industry would both create market power and coordinate production and disposal decisions.

There are three obvious policies: privatisation, centralised disposal or quarantining of the stockpile. There is a less obvious alternative, however, which has the potential to coordinate production with disposal and extend the market power of the central bureaucracy to include producers. This is a policy based on property rights. There are many details to consider in implementing such a policy but the principles are simple: first, redefine rights to the stockpile so that producers make both production and disposal decisions; and second, restrict the total quantity of production plus disposal in the marketplace to create market power. Policies based on

property rights are now widely used in natural resource management. They began 30 years ago with a seminal article by Coase (1960) and were developed further by Demsetz (1967), Dales (1968), Randall (1972), and Bromley (1991), amongst others. Now there are transferable entitlements for many resources from water to fish to emissions (Western Australia Water Resources Council 1989; Geen and Nayer 1989; Tietenberg 1985). A property rights policy was also used by the United States Department of Agriculture (Roberts *et. al.* 1989) to dispose of its wheat and feed grains stockpile beginning in 1986. From experience, producers will accept the restriction on production and disposal if they benefit from ownership of the stockpile.

The purposes of this article are to evaluate the relative merits of alternate policies for the disposal of a commodity stockpile and formulate a new policy which redefines property rights. In the following section, production and disposal decisions are derived for various policies. Next, the model is applied to the disposal of Australia's wool stockpile beginning in 1991. Benefits from the different policies are calculated and compared to optimal production and disposal. Then a new policy is proposed to achieve optimal production and disposal by redefining the rights to a stockpile. Finally, implications are drawn.

Optimal Production and Disposal over Time

Both the demand and supply for a commodity can be a challenge to model. Many commodities are exported. Export demand depends upon demand in overseas markets, supply by other producing countries and inventories held overseas. Ideally, export demand would be calculated within a world trade model that links importing and exporting countries. Supply, particularly of livestock products, may be difficult to adjust. Realistically, supply should be modelled with lags in response to price changes. Nor are many commodities uniform products. Modelling various qualities would require systems of demand and supply equations.

Putting it all together would give a complex model of world trade in differentiated qualities with dynamic supply adjustments. Constructing such a model may be important in setting policy. This study, however, constructs a parsimonious model. Demand and supply are specified for a uniform product produced without lags. Such a simple model has limitations but is more suitable than a complex model for illustrating the key features of optimal stockpile disposal.

Suppose, initially, that the industry is operated as a single enterprise by a monopolist whose objective is to maximise wealth. Wealth is the present value of net income from production and disposal. It is maximised subject to depletion by disposal of the stockpile.

$$(1) \quad J(S_0) = \text{Max} \int_0^T e^{-rt} [p(Q + D)(Q + D) - \int_0^Q c(q) dq - sS] dt \\ + e^{-rT} [N_T - sS_T] / r;$$

subject to:

$$\dot{S} = -D; \quad \text{for all } t;$$

$$Q \geq 0; \quad \text{for all } t;$$

$$S_0 = \bar{S}; \quad \text{and}$$

$$S_T \geq 0.$$

J is the net present value of income from production and disposal; Q is the quantity produced at a total variable cost of $\int cdq$, where c is the marginal production cost, and sold for a price of p ; D is the quantity disposed from stockpile S which is stored at a cost of s per unit; N is the annual net income from normal production once disposal is stopped at time T ; and r is the interest rate. Price p is a function of the total quantity marketed, $Q+D$, and marginal cost c is a function of the quantity produced, Q . After time T , annual net income from normal production and annual storage costs are assumed to be constant and received in perpetuity. Thus their net present value is calculated simply by dividing by the interest rate. The monopolist must choose the length of the disposal period and the production and disposal in each year.

Optimality conditions (Kamien and Schwartz 1981, pp. 147-148) are derived from the current-value Hamiltonian corresponding to the optimisation problem in equation 1. A Hamiltonian is a dynamic profit function. It includes net income from production and disposal for a year but also subtracts a dynamic cost for selling wool from the stockpile instead of saving it for the future.

$$(2) \quad H = [p[Q + D] - \int_0^Q cdq - sS] - \lambda D + \mu Q.$$

λ is a costate variable for the constraint on stockpile disposal and μ is a Lagrange multiplier on the inequality constraint for production. The costate variable, λ , represents marginal disposal costs which measure the effect of current disposal decisions on future profitability. Multiplying marginal disposal costs by the disposal quantity gives a dynamic measure of total disposal costs which are subtracted from net income to get dynamic profit for the year.

Assume the monopolist has power in the output market and considers the effect of its production and disposal decisions on the sale price but has no power in input markets. Optimal production and disposal decisions are found by differentiating the Hamiltonian.

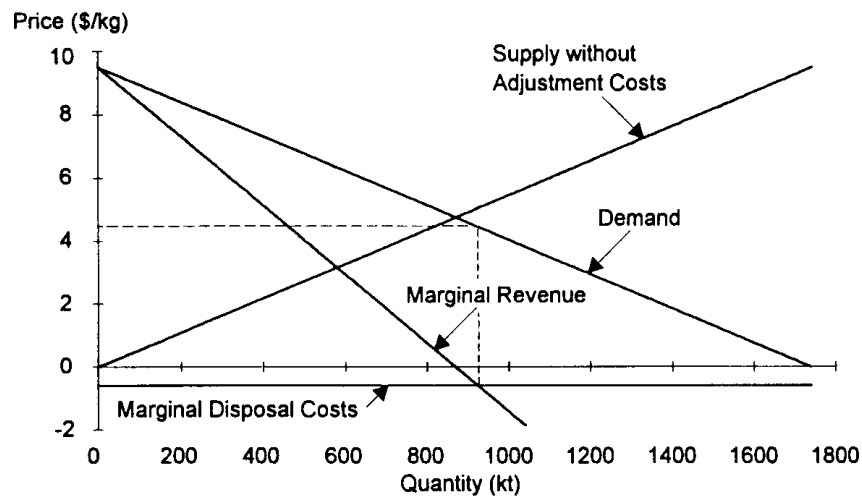
$$(3) \quad \frac{\partial H}{\partial Q} = 0 = p + p'[Q + D] - c + \mu;$$

$$(4) \quad \frac{\partial H}{\partial D} = 0 = p + p'[Q + D] - \lambda.$$

The derivative of price is p' and marginal revenue is $p + p'[Q+D]$. Thus, equation 3 equates marginal revenue plus multiplier μ to marginal production costs. Equation 4 equates marginal revenue to marginal disposal costs.

By complementary slackness, multiplier μ in equation 3 can be positive only when production Q is zero. This could occur if production is easily adjusted and disposal is inexpensive, as in Figure 1. Marginal production costs, shown as the supply curve, are always positive. The stockpile is large, has no scarcity value and marginal disposal costs equal the costs avoided by selling rather than storing the stockpile in perpetuity. Marginal disposal costs are negative. In Figure 1, the optimal disposal quantity is 925,000 tonnes where marginal revenue equals marginal disposal costs of $-\$0.60 / \text{kg}$. Marginal production costs always exceed marginal disposal costs and optimal production is zero. Unnecessary production and storage costs are avoided and the industry benefits from market power because the price of $\$4.45 / \text{kg}$ exceeds marginal disposal costs by $\$5.05 / \text{kg}$.

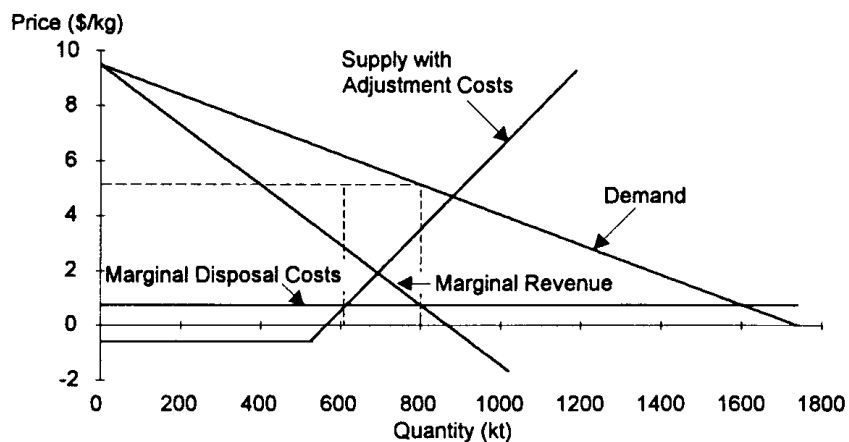
FIGURE 1
Optimal Production without Adjustment Costs and Disposal from a Large Stockpile



More realistically, production will be greater than zero. This occurs if production is not easily adjusted as the stockpile is sold and supply is inelastic. The marginal costs of production are negative at lower quantities, as shown by the supply curve in Figure 2. In addition, the stockpile may be small and have a scarcity value. If the scarcity value is large enough, marginal disposal costs can be positive. In Figure 2, optimal production of 613,000 tonnes and disposal of 189,000 tonnes both have marginal costs of $\$0.74 / \text{kg}$. Production and disposal are efficient. Marginal revenue also equals $\$0.74 / \text{kg}$ at the total quantity marketed of 802,000 tonnes. The price of $\$5.12 / \text{kg}$ exceeds marginal revenue by $\$4.38 / \text{kg}$ and the industry benefits from market power. Thus, equations

3 and 4 are used to determine the least-cost combination of production and disposal and the total quantity to be marketed, once marginal disposal costs are determined.

FIGURE 2
Optimal Production with Adjustment Costs and Disposal from a Small Stockpile



Marginal disposal costs change over time as the stockpile changes and are anchored at time T by the transversality condition for the stockpile.

$$(5) \quad -\frac{\partial H}{\partial S} = \dot{\lambda} - r\lambda = s;$$

$$(6) \quad \lambda_T = v_T - s/r.$$

The Lagrange multiplier, v , corresponds to the constraint on the ending stockpile and measures the scarcity value. Storage costs per unit divided by the interest rate, s/r , are the costs of storing a unit of the commodity in perpetuity. Solving differential equation 5, using the transversality condition (equation 6), gives marginal disposal costs at any time prior to the end of the disposal period.¹

$$(7) \quad \lambda = v_t - s/r.$$

If the stockpile is not exhausted, v is zero by complementary slackness, and the stockpile has no scarcity value, as in Figure 1. Marginal disposal costs are negative and equal to storage costs that could be avoided by disposing of a unit of the commodity rather than storing it in perpetuity.

¹ The steps in the solution are:

$$\lambda = -s/r + e^{-r(T-t)} [\lambda_T + s/r] = -s/r + e^{-r(T-t)} v_T = -s/r + v_t$$

where $v_t = e^{-r(T-t)} v_T$.

If the stockpile is exhausted, as in Figure 2, marginal disposal costs will exceed the avoided storage costs by the scarcity value. As with other exhaustible resources, the scarcity value of the stockpile increases at the rate of interest. Marginal disposal costs increase at the rate of interest plus a factor for storage costs. Unfortunately, there is no analytical solution to the general optimisation problem and marginal disposal costs must be determined empirically.

In addition to production and disposal, the optimal length of the disposal period is chosen. Here there are two alternatives. The first alternative is to continue production and disposal. The second alternative is to switch to normal production with no disposal and pay storage costs on any remaining stockpile. The profitability of the first alternative is the Hamiltonian in equation 2; the profitability of the second alternative is net income above storage costs at the end of the disposal period, $N - sS$, from the objective function in equation 1. Production and disposal should continue so long as it is the more profitable alternative. The time to switch, T , occurs when its profitability falls to equal the profitability of normal production.

$$(8) \quad 0 = p_T[Q_t + D_T] - \int_0^{Q_t} cdq - sS_T - \lambda D_T - [N_T - sS_T].$$

Switching time, T , can be any number from zero to infinity, making this free-time optimisation problem more difficult to solve than the usual fixed-time problem.

Other market structures and policies can be modelled by imposing constraints on the problem in equation 1. A policy to quarantine the stockpile and never sell it would constrain the length of the disposal period, T , to zero. The stockpile could be destroyed rather than stored, with destruction costs replacing storage costs in the objective. A policy of privatising the stockpile by selling or giving it to private individuals who lack market power would set the derivative of price, p' , to zero in equations 3 and 4. Individual producers and owners of the stockpile would equate the price, instead of marginal revenue, to marginal production and disposal costs.

Centralised disposal of the stockpile by the government or an industry body requires an assumption about the objective of the bureaucracy. A benevolent bureaucracy will try to maximise industry benefits. Production, however, will be at quantity Q where marginal production costs, c , equal the market price, p . This constrains the decisions of the bureaucracy and equations 3 and 4 must be augmented.

$$\frac{\partial H}{\partial Q} = 0 = p + p'[Q + D] - c + \mu + \eta [p' - c'];$$

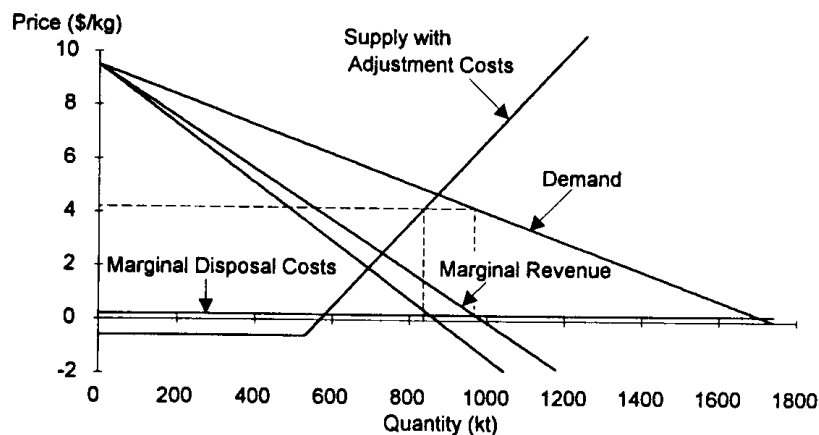
$$\frac{\partial H}{\partial D} = 0 = p + p'[Q + D] - \lambda + \eta p'.$$

Multiplier η corresponds to the constraint $p-c = 0$. Cancelling $p-c$ and assuming positive production with multiplier μ equal to zero in the first equation, solving both equations for multiplier η and equating the result gives a single optimality condition for disposal by a benevolent bureaucracy.

$$(9) \quad 0 = p + p' [Q + D] \left[\frac{c'}{c' - p'} \right] - \lambda.$$

The bureaucracy knows that an increase in disposal will lower the market price which, in turn, will cause producers to decrease their production. This decrease in production reduces the effect of disposal on the price and marginal revenue rotates, as shown in Figure 3. The total quantity marketed is 979,000 tonnes where marginal disposal costs and the new marginal revenue equal \$0.20 / kg. Marginal revenue is \$3.96 / kg below the market price of \$4.16 / kg and the industry benefits from market power. Production is 865,000 tonnes, where marginal production costs equal the price, and disposal is the remaining 147,000 tonnes. Marginal production costs exceed marginal disposal costs by \$3.96 / kg. Disposing of the marginal kilogram from the stockpile instead of producing it would save \$3.96. Although the industry benefits from the market power of the bureaucracy, it loses from inefficient production and disposal.

FIGURE 3
Uncoordinated Production and Disposal with a Central Bureaucracy Maximising Industry Benefits



If the bureaucracy is not benevolent, it will try to maximise its net income from disposal, regardless of the effect on producers. Its objective will not include net income from production but only disposal benefits and its optimality conditions will be simplified.

$$\frac{\partial H}{\partial Q} = 0 = p'[D] + \mu + \eta [p' - c'];$$

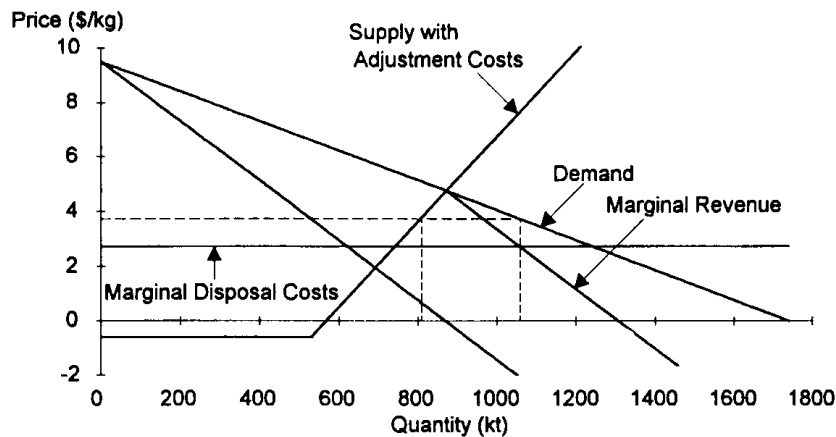
$$\frac{\partial H}{\partial D} = 0 = p + p'[D] - \lambda + \eta p'.$$

These two equations are combined into a single equation for disposal by a bureaucracy which maximises its own disposal income.

$$(10) \quad 0 = p + p'D \left[\frac{c'}{c' - p'} \right] - \lambda.$$

Because the bureaucracy does not consider the effect on woolgrowers, the new marginal revenue does not begin at the origin but, instead, where disposal is zero at the intersection of demand and supply in Figure 4. The new marginal revenue is shifted to the right. A total of 1,056,000 tonnes is marketed at the point where marginal disposal costs and marginal revenue equal \$2.72 / kg. Of this, 805,000 tonnes is produced, where price and marginal production costs equal \$3.74 / kg, and 251,000 tonnes is disposed from the stockpile. Price exceeds marginal disposal costs by \$1.02 / kg and marginal production costs exceed marginal disposal costs by the same \$1.02 / kg. Again, the industry benefits from the market power of the bureaucracy but production and disposal are inefficient.

FIGURE 4
Uncoordinated Production and Disposal with a Central Bureaucracy Maximising its Disposal Income



Application to the Australian Wool Industry

The Australian Reserve Price Scheme (ARPS) was a buffer stock which stabilised wool prices from 1970 through the 1980s. By the time it was formally abolished on 30 June 1991, the ARPS had accumulated a stockpile of 4.6 million bales of wool, about 90% of annual production, and a debt of \$2.7 billion.

Several policies were proposed for disposing of the stockpile. The original Wool Review Committee recommended that wool from the stockpile not be sold until the market indicator price exceeded 500c/kg of clean wool. Stoeckel, Borrell and Quirke (1990) proposed that sales from the stockpile be pre-announced, allowing processors and woolgrowers time to adjust. Beare, Fisher and Sutcliff (1991) derived dynamic disposal rules and did not restrict price or disposal. Less formally, several people proposed privatising the stockpile. Finally, several industry organisations advocated quarantining the stockpile, either by storing it indefinitely, denaturing it or destroying it (Wool Review Committee 1991). Initially, the Australian Wool Realisation Commission (AWRC) was formed to dispose of the stockpile using a flexible approach similar to the proposal of Beare, Fisher and Sutcliff. Soon after the AWRC began operation, a second review (Wool Industry Review Committee 1993) proposed a new entity, called Wool International (WI), which replaced the AWRC. WI is constrained by fixed disposal rules and is to be privatised by giving shares to woolgrowers.

The alternatives for disposing of the stockpile can be summarised by six scenarios: (1) optimal production and disposal, (2) competitive disposal of the stockpile by private individuals, (3) centralised disposal with the AWRC maximising industry benefits, (4) centralised disposal with the AWRC maximising disposal income, (5) the WI following fixed disposal rules and (6) quarantine of the stockpile from the market. The first two scenarios will coordinate production with disposal. The first scenario would apply if ownership the stockpile and debt obligations were immediately transferred to woolgrowers and the government could also extend market power to them. The second scenario would apply if the stockpile and debt were privatised but the government does not extend market power to woolgrowers. The last four scenarios do not coordinate production with disposal. The third scenario would require the AWRC to be benevolent and voluntarily restrict disposal while woolgrowers show no restraint. In the fourth scenario both the AWRC and woolgrowers would try to maximise their own benefits regardless of the effect on the industry. The fifth scenario restricts WI so that benefits to the industry can never exceed those with a benevolent AWRC. Finally, in the sixth scenario, the stockpile must be either stored indefinitely, denatured and sold for other uses or destroyed. The industry would return immediately to normal production without disposal.

Solving the model (equation 1) gives optimal production and disposal for the first scenario. Solving the model subject to additional constraints gives scenarios two to six. Unfortunately, the model is difficult to solve.

Beare, Fisher and Sutcliff (1991) found an analytical solution to their model by assuming marginal disposal costs converge to marginal revenue at the end of the disposal period. The more general transversality condition (equation 6) shows that marginal disposal costs need not include a scarcity value and may not converge to marginal revenue. But it is the optimality condition for the length of the disposal period (equation 8), which makes the free-time problem more difficult to solve than the typical fixed-time problem. Hertzler (1990) has developed a method to convert free-time problems to fixed-time problems and solve them by non linear programming. A special case of the method is presented in the Appendix.

The model was solved using the non linear programming software, GINO (Liebman, *et al.* 1986). Parameters of the model are listed in Table 1. Demand and supply curves, storage costs and the interest rate are similar to those of Beare, Fisher and Sutcliff (1991) except that the price is for greasy wool and is just high enough to repay the debt on the stockpile without a tax on woolgrowers. Demand and supply elasticities of 1 and 0.35 are for the medium term of 1 to 5 years (Chisholm *et al.* 1994). The interest rate of 10% has been used in other studies but is above the rate being paid on the debt which is under 7.5%. The calculation of normal producers' surplus, once disposal is finished, assumes woolgrowers will no longer be influenced by either the AWRC or WI and will lack market power. Marginal production costs can never go below marginal disposal costs for a large stockpile with no scarcity value, or $s/r = -\$0.60/\text{kg}$. Thus, normal producers' surplus will be assumed to equal the area in Figure 2 above -0.60, above the supply curve and below the intersection of supply and demand. The initial stockpile assumes a starting date of 1 July 1991 when disposal began.

TABLE 1
Model Parameters

Parameter	Equation
Demand (\$ / kg)	$p = 9.500000 - 0.005460[Q+D]$
Supply (\$ / kg)	$c = -8.821430 + 0.015599Q$
Storage costs per unit (\$ / kg)	$s = 0.06$
Interest rate	$r = 0.10$
Normal producers' surplus (\$'000,000 / year)	$N_T = 3736.96$
Initial stockpile (kt)	$S_0 = 800$

Results are summarised in Tables 2 and 3 and in Figures 5 to 8. In Table 2 is shown the net present value of benefits for each of the six scenarios. In each scenario, benefits from production far outweigh benefits to disposal. However, most benefits are from normal production following the

TABLE 2
Benefits from Five Production and Disposal Scenarios

Scenario	Production Benefits \$m.	Disposal Benefits \$m.	Industry Total \$m.	Consumer Benefits \$m.	Society Total \$m.
Optimal Production and Disposal	40,798	3,376	44,174	20,302	64,476
Competitive Stockpile Disposal	38,570	2,587	41,156	25,652	66,809
AWRC Maximising Industry Benefits	38,922	2,533	41,455	25,036	66,491
AWRC Maximising Disposal Income	38,696	2,686	41,382	25,414	66,796
WI Fixed Disposal	39,115	2,306	41,421	24,801	66,222
Quarantine of the Stockpile					
Stored	41,107	-528	40,579	22,730	63,308
Destroyed	41,107	0	41,107	22,730	63,837

disposal period. For example, a quarantine of the stockpile constrains the disposal period to zero and either stores the stockpile in perpetuity for a cost of only \$0.5 billion or denatures and destroys it for an unknown cost assumed to be zero. Production benefits are the largest for this scenario at \$41.1 billion. In other scenarios, the \$41.1 billion is received at the end of a disposal period and its present value is much smaller.

The scenario for optimal production and disposal maximises disposal benefits of \$3.4 billion and total industry benefits of \$44.2 billion. Disposal benefits could retire the \$2.7 billion debt on the stockpile and leave a \$0.7 billion surplus. The five remaining scenarios: competitive stockpile disposal, the AWRC maximising industry benefits,² the AWRC maximising disposal income,³ WI disposing of the stockpile on a fixed schedule and a quarantine of the stockpile, have similar industry benefits in the range of \$41.0 billion to \$41.5 billion. The efficiency of competitive disposal almost matches the market power of centralised disposal by the AWRC. Whether the AWRC is benevolent or not makes some, but not a lot, of difference. A benevolent AWRC trying to maximise industry benefits will prolong the disposal period, allowing more time to exercise market power. An AWRC trying to maximise its own disposal income will shorten the disposal period and let woolgrowers quickly resume normal production. The WI disposes of 91.7 kt a year with the same length of disposal period as the benevolent AWRC. Because the length of the disposal period is chosen correctly, disposal by the WI is only slightly less beneficial to the industry. For the parameters in Table 1, a quarantine of the stockpile is the least preferred option and woolgrowers would have to be taxed to repay the debt. In all other scenarios, disposal benefits will nearly repay the debt.

Also listed in Table 2 are consumer benefits calculated as consumers' surplus. Even though wool is almost entirely exported and benefits to consumers leave Australia, it may be important to anticipate how wool buyers might react to different policies. Consumer benefits are around \$25 billion for four scenarios: competitive stockpile disposal, the AWRC maximising industry benefits, the AWRC maximising disposal income and WI disposing on a fixed schedule. These offer large quantities of wool for sale. Consumer benefits are much smaller for optimal production and disposal and for quarantining the stockpile because less wool is sold.

The last five scenarios are compared with optimal production and disposal in Table 3. In total, the industry loses at least \$2.7 billion. Capturing this \$2.7 billion requires a policy which can coordinate pro-

² Beare, Fisher and Sutcliffe (1991) analyse a similar policy but also restrict marginal cost, c , to be zero in the objective function. The price-based rule of the Wool Review Committee and the disposal rule of Stoeckel, Borrell and Quirke (1990) place additional constraints on the price and on disposal. These additional constraints can only reduce benefits to the industry.

³ Beare, Fisher and Sutcliffe (1991) also analyse centralised disposal with the AWRC maximising disposal income.

TABLE 3
Gains and Losses Compared to Optimal Production and Disposal

Scenario	Production Loss (\$m.)	Disposal Loss (\$m.)	Industry Loss (\$m.)	Consumer Gain (\$m.)	Society Gain (\$m.)
Optimal Production and Disposal	0	0	0	0	0
Competitive Stockpile Disposal	-2,228	-789	-3,017	5,350	2,333
AWRC Maximising Industry Benefits	-1,876	-842	-2,719	4,734	2,015
AWRC Maximising Disposal Income	-2,102	-690	-2,792	5,112	2,320
WI Fixed Disposal	-1,683	-1,070	-2,753	4,499	1,746
Quarantine of the Stockpile					
Stored	308	-3,904	-3,595	2,428	-1,168
Destroyed	308	-3,376	-3,068	2,428	-640

duction with disposal and extend market power from the government to woolgrowers. Competitive stockpile disposal coordinates production and disposal but has no market power and loses \$3.0 billion. Centralised disposal by the AWRC creates market power but causes inefficient production and disposal and loses at least \$2.7 billion. Disposal by an AWRC which maximises only disposal income loses an additional \$0.1 billion for a total of \$2.8 billion. Fixed disposal by WI loses \$2.8 billion. A quarantine of the stockpile loses either \$3.1 billion or \$3.6 billion, depending upon whether the stockpile is costlessly destroyed or stored in perpetuity. The industry's losses are consumers' gains, however. In every scenario but the quarantine option, consumers gain more than the industry loses. In theory, side-payments from consumers to the industry in exchange for competitive disposal by woolgrowers could benefit society by a total of \$2.3 billion. In practice, this won't happen and the industry can maximise its own benefits at the expense of consumers.

Another important consideration is the timing of benefits. Woolgrowers must maintain an adequate cash-flow and the debt on the stockpile must be repaid. Figures 5 and 6 show annual benefits, undiscounted, for optimal production and disposal and for WI's fixed disposal. At the end of their respective disposal periods, both scenarios revert to normal production with annual benefits of \$3.7 billion a year, the same as a quarantine of the stockpile with costless destruction. Optimal production and disposal in Figure 5 exhaust the stockpile in 6.8 years. In each of these years, industry benefits are above the annual benefits of normal production. Disposal benefits more than repay the debt and improve the industry's cash-flow. In Figure 6, fixed disposal by WI lasts 8.8 years. Production and industry benefits are always below those for optimal production and disposal. Disposal benefits for the AWRC are similar to those for the WI but with a smoother transition to normal production. Competitive disposal is more rapid and would cause cash-flow problems for producers in early years.

Optimal production and disposal give benefits and a net present value well above those of any other scenario. The advantage over competitive disposal is easily explained. Competitive disposal lacks market power and collects no monopoly rents. The advantage over centralised disposal by either the AWRC or WI is explained by Figures 7 and 8. Production is not coordinated with disposal and the industry is inefficient. Woolgrowers, trying to maximise their individual benefits, produce too much in Figure 7 and force the price too low in Figure 8. Marginal production costs exceed marginal disposal costs by about \$4 / kg in all years. Selling one more kilogram from the stockpile and producing one less would save the industry \$4. In total, the industry could save \$2.7 billion.

FIGURE 5
Annual Benefits from Optimal Production and Disposal

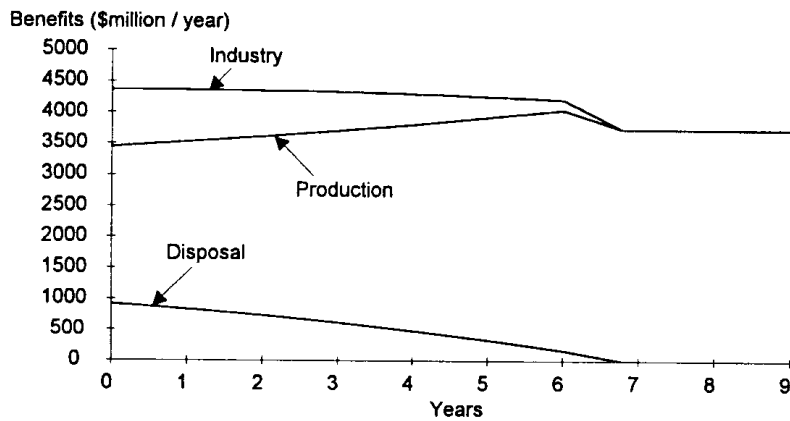


FIGURE 6
Annual Benefits from Wool International's Fixed Disposal Schedule

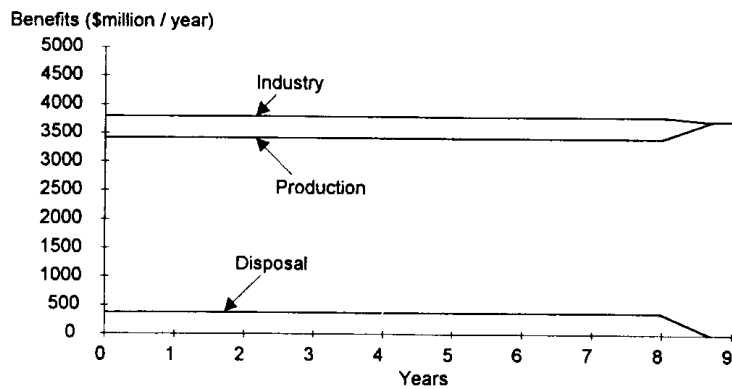


FIGURE 7
Quantities for Optimal Production and Disposal and for the AWRC Maximising Industry Benefits

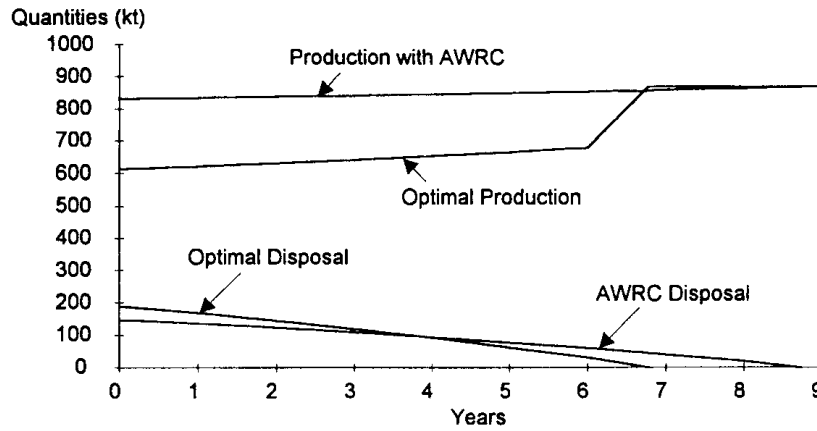
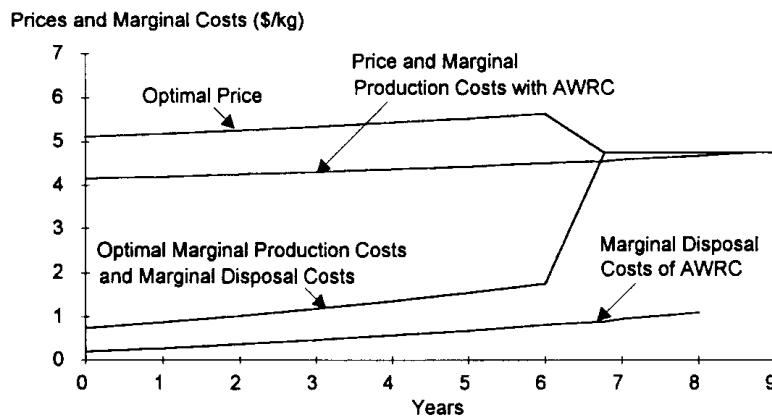


FIGURE 8
Prices and Marginal Costs for Optimal Production and Disposal and for the AWRC Maximising Industry Benefits



Many parameters of the model are difficult to measure. In Table 4 is given the relative benefits of the six scenarios for the base case of Table 2 and four parameter changes. With one exception, centralised disposal by the AWRC is the second-best policy. For lower interest rates, quarantining the stockpile is second-best. The fixed disposal schedule by WI performs relatively poorly with parameter changes because the length of the disposal period does not change. If advertising for woollen products shifts demand to intersect supply at 970,000 tonnes instead of 870,000 tonnes, the industry would lose \$0.1 to \$0.4 billion more from all scenarios except fixed disposal by WI which loses \$2.4 billion more. If production can be easily adjusted with a supply elasticity of 0.89 at a quantity of 870,000 tonnes instead of 0.35, the relative merits of the different scenarios change little except that quarantining the stockpile loses an extra \$1.1 billion. If demand has an elasticity of 2.5 instead of 1 at a quantity of 870,000 tonnes, there is no market power to exploit and the method of disposal becomes unimportant. In general, elastic supply and demand allow rapid disposal of the stockpile without depressing the price but do not affect the ranking of different scenarios. Finally, if the interest rate is 7.5% instead of 10%, benefits in early years are sacrificed to dispose of the stockpile and resume normal production in less than 4 years. In fact, immediately destroying the stockpile becomes the second-best alternative, ahead of disposal. An interest rate of 7.5% is somewhat above the rate actually paid on the debt and quarantining the stockpile should be considered as a serious option, particularly if the wool can be denatured and sold into other markets for alternative uses.

Policy Implementation

What kind of policy could achieve optimal production and disposal? A successful tax or subsidy policy is unlikely.⁴ For the base case, the required marginal tax is the difference between the optimal price and the optimal marginal costs in Figure 8 and is greater than \$4 / kg in most years. Such a tax would leave woolgrowers impoverished. Alternatively, both woolgrowers and the old AWRC or the new WI could accept quotas at the optimal production and disposal levels in Figure 7. However, woolgrowers have not accepted quotas in the past.

Perhaps a new policy is needed. Managing the wool stockpile is similar to managing a fishery, allocating water in a catchment, or controlling air pollution. Without an effective policy to manage a natural resource, producers use the resource to maximise their own short-term benefits and

⁴ Table 2 indicates the value of the stockpile is greater than the debt. The studies by Beare, Fisher and Sutcliff (1991) and by the Wool Review Committee (1991) calculate benefits smaller than the debt and propose a tax levied on future wool sales to help repay it. Initially, woolgrowers were heavily taxed, although the tax has now been lowered. But a tax to repay the debt is much smaller than an optimal tax policy to coordinate production with disposal.

TABLE 4
*Industry Losses (\$'000,000) Compared to Optimal Production and Disposal
 for Different Model Parameters*

Scenario	Base Case ^a	Strong Demand ^b	Elastic Supply ^c	Elastic Demand ^d	Lower Interest ^e
Optimal Production and Disposal	0	0	0	0	0
Competitive Stockpile Disposal	-3,017	-3,165	-3,126	-530	-3,255
AWRC Maximising Industry Benefits	-2,719	-2,921	-3,000	-429	-3,009
AWRC Maximising Disposal Income	-2,792	-2,994	-3,020	-455	-3,086
WI Fixed Disposal	-2,753	-5,120	-3,169	-790	-3,062
Quarantine of the Stockpile					
Stored	-3,595	-4,006	-4,686	-2,782	-3,386
Destroyed	-3,068	-3,478	-4,158	-2,254	-2,858

^a From Table 3.

^b For demand $p=11.6058-0.005460[Q+D]$.

^c For supply $c=-0.60+0.006149Q$.

^d For demand $p=6.65-0.002184[Q+D]$.

^e For interest rate $r=0.075$.

ignore the effects of their decisions on others. With no coordination between production and disposal, woolgrowers will maximise their own short-term benefits and market too much wool, to the detriment of the industry as a whole. The policy lessons learned in natural resource management can be applied to managing the wool stockpile.

The first lesson is that quotas administered by a central authority are inefficient (Tietenberg 1985). The least-cost producers should hold the quotas but the central authority has no way of knowing who they are. The solution is to allocate individual entitlements to producers and establish a market to make any transfers as easy as possible. These individual transferable entitlements (ITEs) are not quotas administered by a central authority but property owned by individual producers. Rights to an ITE can be bought and sold just like any other property. The second lesson is that ITEs, like other property, are wealth and must be safeguarded as such. Although ITEs will be quickly traded to the least-cost producers, the initial allocation has large distributional consequences and is crucial for fairness and acceptability of the policy. The final lesson is that producers will readily accept ITEs if the benefits to them are obvious.

By themselves, ITEs are not sufficient to coordinate the disposal of a stockpile. Some years ago, the U.S. government used a property-rights policy to dispose of its wheat and feed grains stockpile (Roberts *et. al.* 1989). Prior to 1986, the prices of wheat and feed grains were regulated by a complex policy of non-recourse loans which supported the price to farmers. For a decade, this price was set too high and the government accumulated a stockpile equivalent to just less than 90% of annual production. With the Food Security Act of 1985, the Payment-in-Kind (PIK) program was implemented. Ownership of the stockpile was transferred to farmers through a complex mechanism. Farmers were paid a subsidy, called a deficiency payment, based on past production and on the difference between an arbitrary target price and the prevailing market price. Payments were not in cash, however, but in kind, with PIK certificates.⁵ These could be bought and sold or redeemed for grain from the stockpile. In return for the certificates, farmers agreed to reduce the area planted to crops by up to 27.5%. The PIK program was voluntary but almost 90% of farmers participated and, with the help of a drought, the stockpile was reduced to 20% of annual production within 3 years.

A policy to dispose of the wool stockpile could combine PIK certificates with ITEs. The PIK certificates would transfer ownership of the stockpile without physically relocating it. The ITEs would extend market power from the government to woolgrowers. Together, PIK certificates and ITEs would ensure efficient production and disposal. Implementing such a policy would take several steps.

⁵ PIK certificates are similar to convertible wool bonds proposed as a way to transfer ownership of the wool stockpile to woolgrowers.

Initially:

1. Determine a formula for allocating the PIK certificates and ITEs to woolgrowers.
2. According to the formula, issue 800 million PIK certificates, one for each kilogram in the stockpile, to woolgrowers who agree to participate. Each certificate transfers ownership of one kilogram of wool and may be redeemed at any time. In return, woolgrowers agree to repay their proportion of the debt and pay storage costs for unredeemed certificates.

Each year:

3. According to the formula, issue 189 million ITEs, one for each kilogram of optimal disposal in year 0. Issue 613 million ITEs, one for each kilogram of optimal production in year 0. In subsequent years, issue fewer ITEs for disposal and more for production according to the schedule in Table 5.
4. Conduct frequent auctions for PIK certificates and ITEs.
5. Conduct wool sales. Woolgrowers could deliver wool or redeem PIK certificates. Wool or redeemed certificates covered by an ITE would not be taxed. Wool or certificates not covered by an ITE would be taxed according to the schedule in Table 5. These sales must be compulsory, with no direct sales to customers.
6. Invoice the initial recipients of PIK certificates for their share of debt repayments. Invoice the current owners of unredeemed PIK certificates for storage costs.

The policy has stick for woolgrowers who do not participate and a carrot for those who do. The stick is the optimal tax listed in Table 5 as the difference between the price of wool and marginal disposal costs. In practice, woolgrowers would not pay the tax. It is only a threat to prevent free-riding. For example, the initial year is graphed in Figure 2. Woolgrowers who try to free-ride will, instead, add the tax of \$4.38 / kg to their marginal production costs, equate the sum to the price and produce the optimal amount, on average. The carrot of the policy is the transfer of wealth as PIK certificates and ITEs. The market price of an ITE should equal the tax per kilogram of wool. For example, during the initial year of the policy shown in Figure 2, a woolgrower who has a PIK certificate giving him ownership of wool in the stockpile and who holds an ITE to cover the PIK certificate would require at least the wool price minus marginal disposal costs before he would sell the ITE. A grower needing an ITE to cover newly produced wool would pay the wool price minus marginal production costs. At the equilibrium, marginal production costs equal marginal disposal costs and the market price for the ITE is the wool price minus marginal costs or, in other words, the optimal tax. In addition, adjustments to production may be easier on wheat and sheep farms than on specialist farms or stations. Transferring ITEs to specialist farms and stations will also equilibrate marginal production costs across woolgrowers. Further, the market price of PIK certificates should equal marginal

TABLE 5
*Prices, Marginal Costs, Tax, Quantities and Debt Repayments
 for a Policy of Optimal Production and Disposal*

Year	Price (\$/kg)	Marginal Costs (\$/kg)	Optimal Tax (\$/kg)	Production (kt/yr)	Disposal (kt/yr)	Production + Disposal (kt/yr)	Stockpile (kt)
0	5.12	0.74	4.38	613	189	802	800
1	5.19	0.87	4.31	622	168	790	611
2	5.26	1.02	4.24	631	145	776	443
3	5.34	1.18	4.16	641	120	761	297
4	5.43	1.36	4.07	653	92	745	177
5	5.53	1.56	3.97	665	62	727	85
6	5.63	1.76	3.87	678	30	709	23
6.775	4.75	4.75	0	870	0	870	0

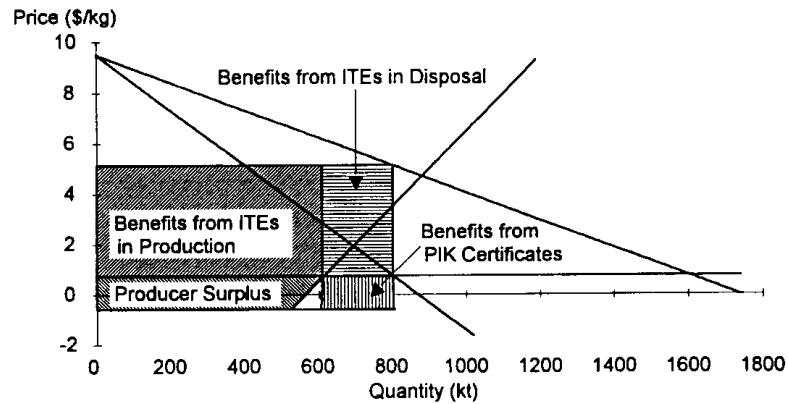
disposal costs. Suppose a grower has sold the ITE which covered a PIK certificate. He can redeem the PIK certificate, receive the wool price, pay the optimal tax and net an amount equal to marginal disposal costs. Alternately, he can sell the PIK certificate at a market price equal to marginal disposal costs.⁶ He will no longer pay storage costs but, as the original recipient of the certificate, must continue to meet debt repayments.

PIK certificates clearly define property rights to the stockpile and ITEs define marketing rights. In Figure 9, the allocation of ITEs gives wool-growers monopoly rents from both production and disposal. The allocation of PIK certificates gives them an annual surplus from disposal. Added to producers' surplus, woolgrowers retain the total value of the industry, or \$44.1 billion in Table 2 before debt repayments.⁷ Overall, the industry will be \$2.7 billion richer than with any other policy.

⁶ For proof, see footnote 6.

⁷ The relationships between benefits in Figure 9 and the objective function in equation 1) are as follows. In Figure 9, annual benefits from ITEs allocated for production are $[p - \lambda]Q$. Producers' surplus is typically $\lambda Q - \int cdq$. This would be a triangle with height extending from -8.82 , where marginal production costs intercept the vertical axis, to λ . In Figure 9, however, everything below $-s/r$ has been truncated. In total, annual benefits from production equal revenue minus variable costs, $pQ - \int cdq$, with a net present value as written in the objective function. Annual benefits from ITEs allocated to disposal are $[p - \lambda]D$ and annual benefits from PIK certificates are $v_t D$. According to equation 7, $v_t = \lambda + s/r$ and total annual benefits from disposal equal revenue plus storage costs avoided, $[p + s/r']D$. The objective function, however, writes the net present value of benefits to disposal as $\int_0^\infty e^{-rt} [pD - sS] dt$. But storage costs, $\int_0^\infty e^{-rt} sS dt$, equal $\int_0^\infty e^{-rt} s [S_0 - \int_0^t D d\tau] dt$, after solving the equation of motion and substituting for the stockpile. Integration by parts gives $[s/r] [S_0 - \int_0^\infty e^{-rt} D dt]$. Therefore, benefits to disposal can be separated into ITE benefits and PIK benefits, as shown in Figure 9, as well as a fixed cost, $\int_0^\infty e^{-rt} [[p - \lambda]D + v_t D] dt - [s/r] S_0$. The fixed cost is for storing the initial stockpile in perpetuity. PIK benefits arise by avoiding this cost. Because the scarcity value grows at the rate of interest, $v_t = e^{rt} v_0$, the net present value of PIK benefits minus the fixed cost of storage can also be written as $[v_0 - s/r] S_0$, or $\lambda_0 S_0$. The marginal disposal costs are the implicit price of wool in the initial stockpile and also the market price of PIK certificates. By Bellman's principle of optimality the stockpile at any time is the initial stockpile for optimal decisions in the future. Therefore, the marginal disposal costs in any year are the market price of PIK certificates.

FIGURE 9
*Annual Benefits from a Property Rights Policy with PIK
 Certificates and ITEs*



Conclusions

At high interest rates, disposal by a centralised bureaucracy is the second-best scenario. It makes little difference whether the bureaucracy is benevolent and tries to maximise industry benefits, is selfish and tries to maximise its own disposal income or follows a fixed disposal schedule. The bureaucracy has much less discretion than an ordinary monopolist because the size of the stockpile is predetermined and the only choice is how quickly to dispose of it. As a result, elasticities of supply and demand are relatively unimportant, so long as the length of the disposal period is chosen optimally. Elastic supply or demand causes a short disposal period. Inelastic supply or demand causes a long period. A caveat is that a fixed disposal scenario which does not have flexibility to adjust the length of the disposal period may damage the industry.

At lower interest rates, quarantining the stockpile by destroying it is the second-best scenario. Alternatively, the stockpile might be denatured and sold for alternative uses to generate additional benefits. Competitive disposal by privatising the stockpile is a viable scenario but is not as beneficial as centralised disposal or a quarantine. Optimal production and disposal are always the best scenario, combining the efficiency of competitive disposal with the market power of centralised disposal. For example, the Australian wool industry could be \$2.7 billion richer. The relative advantage of optimal production and disposal is insensitive to changes in parameters of the model.

To achieve optimal production and disposal, a new type of policy is needed. Precedents are the ITEs used in natural resource management and the PIK program used in the U.S. to dispose of its wheat and feed grain

stockpile. A policy to dispose of a stockpile should combine elements of both. PIK certificates assign ownership of the stockpile to individuals. ITEs govern the marketing of the commodity. The PIK certificates ensure coordination of production and disposal by giving both decisions to individuals. ITEs extend market power from a central bureaucracy to individuals and allow production to respond flexibly to changes in market conditions. In addition, the property-rights policy is flexible enough to let the industry adjust to any errors in its formulation.

Participation in the property rights policy is voluntary. Even without the threat of a tax, almost all producers should participate and receive an allocation of PIK certificates and ITEs. This allocation is a large transfer of wealth. Any allocation formula is politically sensitive and must be perceived as fair by all producers. And, of course, any property right must be enforced. A black market could destroy the value of the ITEs. A cost-effective method of policing the market must be devised before a property rights policy can be implemented.

Finally, the property rights policy would free the centralised bureaucracy for more important tasks. One task might be to manage market risks by providing smoothly functioning forward, futures and options markets.

References

- Beare, S. C., Fisher, B. S. and Sutcliffe, A. G. (1991), *Managing the Disposal of Australia's Wool Stockpile*, Australian Bureau of Agricultural and Resource Economics, Tech. Paper 91.2, Canberra, September.
- Bromley, D. W. (1991), *Environment and Economy: Property Rights and Public Policy*, Blackwell, Cambridge Massachusetts.
- Chisholm, T., Haszler, H., Edwards, G., and Hone, P. (1994), 'The Wool Debt, the Wool Stockpile and the National Interest: Did Garnaut Get It Right?', 38th Annual Conference of the Australian Agricultural Economics Society, Victoria University, Wellington, February.
- Coase, R. H. (1960), 'The Problem of Social Cost', *J. Law and Econ.*, 3, 1-44.
- Dales, J. H. (1968), *Pollution, Property and Prices*. University of Toronto Press, Toronto.
- Demsetz, H. (1967) 'Toward a Theory of Property Rights', *Amer. Econ. Rev.*, 57, 347-359.
- Geen, G. and Nayar (1989), M. *Individual Transferable Quotas and the Southern Bluefin Tuna Fishery: Economic Impact*, Australian Bureau of Agricultural and Resource Economics, Occ. Paper 105, Canberra, April.
- Gunasekera, H. D. B. H. and Fisher, B. S. (1991), 'Buffer Stock Schemes in Australia: A Case Study of Wool', Symposium on Management in Unregulated Markets, 21st International Conference of Agricultural Economists, Tokyo, August.
- Hertzler, G (1990), 'Dynamically Optimal Adoption of Farming Practices which Degrade or Renew the Land', 34th Annual Conference of the Australian Agricultural Economics Society, University of Queensland, St. Lucia, Brisbane, February.
- Kamien, M. I., and Schwartz, N. L. (1981), *Dynamic Optimization: The Calculus of Variations and Optimal Control in Economics and Management*, North-Holland, New York.
- Liebman, J., Lasdon, L., Schrage, L. and Waren (1986), A. *Modeling and Optimization with GINO*, The Scientific Press, Palo Alto.

- Murtagh, B. A. and Saunders, M. A. (1983), *MINOS 5.0 User's Guide*, Systems Operation Laboratory, Stanford University, Tech. Rep. SOL 83-20, December.
- Randall, A. (1972), 'Market Solutions to Externality Problems: Theory and Practice', *Amer. J. Agr. Econ.*, 54, 175-183.
- Roberts, I., Love, G., Field, H., and Klijn, N. (1989), *U.S. Grain Policies and the World Market*, Australian Bureau of Agricultural and Resource Economics, Policy Monograph No. 4.
- Stoeckel, A., Borrell, B., and Quirke, D. (1990), *Wool into the 21st Century: Implications for Marketing and Profitability*, Centre for International Economics, Canberra, November.
- Tietenberg, T. *Emissions Trading: An Exercise in Reforming Pollution Policy*, Resources for the Future, Washington, 1985.
- Western Australia Water Resources Council (1989), *Transferable Water Entitlements in Western Australia*, Publication No. 8/89, Perth, March.
- Watson, A. (1990), *Unravelling Intervention in the Wool Industry*, Centre for Independent Studies, CIS Policy Monograph 17, Melbourne.
- Wool Review Committee (1991), *The Australian Wool Industry: Recommendations for the Future*, Report to the Minister for Primary Industries and Energy, Canberra, March.
- Wool Industry Review Committee (1993), *Wool: Structuring for Global Realities*, Report to the Minister for Primary Industries and Energy, Canberra, August.

Appendix: Mathematical Programming Solution

The free-time optimal control problem in 1 can be converted to a fixed-time problem by replacing the variable, t , by a series of auxiliary variables, ϑ , one for each time period (Hertzler). If each time period is one year long, as in this example, the auxiliary variables must be constrained between zero and one. Problem 1 in discrete time with auxiliary variables becomes:

$$J(S_0) = \sum_{t=0}^{T-1} \left(\frac{1}{1+r} \right)^{\sum_{s=0}^t \vartheta_s} \vartheta_t \left[p_t [Q_t + D_t] - \int_0^{Q_t} cdq - sS_t \right] + \left(\frac{1}{1+r} \right)^{\sum_{s=0}^{T-1} \vartheta_s} [N_T - sS_T] \frac{1+r}{r};$$

subject to:

$$S_{t+1} - S_t = -\vartheta_t D_t;$$

$$\vartheta_t Q_t \geq 0;$$

$$0 \leq \vartheta_t \leq 1;$$

$$S_T \geq 0;$$

$$S_0 = \bar{S}.$$

Terminal time T is now a number larger than any possible length of time needed to dispose of the stockpile. The optimal length of the disposal period is the sum of the ϑ variables from $t = 0$ to $T - 1$. During disposal, each ϑ should be at its upper bound which, in this case, is one. Following disposal, each ϑ should be at its lower bound of zero. At the optimal time of switching, ϑ can be between zero and one. In other words, each ϑ is a switch, turning time either on or off.

There are two difficulties in solving the problem using mathematical programming software. The first difficulty is that the simple linear state equation for the wool stockpile becomes a highly non linear constraint. The MINOS software (Murtagh and Saunders) uses a projected Lagrangian algorithm for non linear constraints, can converge quickly but often gets lost amongst the non linearities. The software GINO (Liebman *et al.* 1986) uses Newton's method to ensure the non linear constraints are satisfied at each iteration and rarely gets lost. The second difficulty is that the software must allow control over differentiation. A ϑ variable turns the Hamiltonian on or off in each period. Optimality conditions require that the Hamiltonian be maximised. The normal method for doing this differentiates the Hamiltonian with respect to quantities produced and disposed from the stockpile and sets the derivatives to zero. Software which takes derivatives automatically, multiplies a derivative by ϑ and sets the result to zero. If ϑ is already zero, the result is zero without maximising the Hamiltonian. Therefore, a special set of derivatives is required which ignore ϑ . MINOS allows control of the

derivatives, GINO does not. Because existing software cannot solve the free-time problem, a practical approach is to specify a length for the disposal period, fix that many ϑ variables to one, the rest to zero, and solve the problem. Then vary the length of the disposal period by fixing a new number of ϑ variables above zero and solve the problem again. Repeat the process, searching until the objective function is maximised.