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AN OPTIMAL MANAGEMENT MODEL FOR INTENSIVE AQUACULTURE — AN APPLICATION IN ATLANTIC SALMON*

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In this paper the optimal management strategy for intensive aquaculture is viewed in terms of a combined strategy of releasing the optimal number of recruits and harvesting those recruits at the optimal harvesting time. A model which can be used to determine the optimal management strategy is developed. In the model the optimal harvesting model documented by Bjørndal (1988, 1990) in which harvesting and feed costs are considered, is extended by including release costs and how they influence the optimal number of recruits. The model forms the basis for an empirical analysis in which the optimal management strategy for a yearclass of Atlantic salmon farmed in Australia during 1989-91 is considered.

Introduction

Aquaculture (fish farming, fish culture, mariculture, sea ranching) can be defined as the human cultivation of organisms in water (fresh, brackish or marine). Aquaculture is distinguished from other aquatic production by the degree of human intervention and control that is possible. It is in principle more similar to forestry and animal husbandry than to traditional capture fisheries. In other words, aquaculture is stock raising rather than hunting (Bjørndal, 1990).

In this paper, intensive aquaculture, in which fish are farmed in floating sea cages and the farm manager controls aspects of production such as stocking, feeding, and harvesting of fish, is considered. Attention is given to one of the most commonly cultured fish, the Atlantic salmon (*Salmo salar*), and of particular interest is their optimal management. The optimal management model presented is also applicable to other fish species. Prior to outlining the model, the production process in the farming of Atlantic salmon is briefly described.

Atlantic salmon are anadromous fish. In the wild, mature fish leave the sea and return to fresh water to spawn. Eggs hatch in autumn and winter.

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The fry remain in fresh water until spring when they undergo a complex physiological and behavioural change known as the smoltification process (Laird and Needham, 1988). During this process they adapt to salt water, and when adaption is complete smolts migrate to the sea. After spending up to five winters at sea, Atlantic salmon return to their natal river to spawn. Some smolts, termed grilse, mature early and return to spawn after only one winter at sea (Shaw and Muir, 1987).

Based on the lifecycle of salmon in the wild, Bjørndal (1990) divides the biological production process in salmon aquaculture into four stages: (1) production of broodstock and roe; (2) production of fry; (3) production of smolts; and (4) production of farmed fish. The production of farmed fish can be considered independently of the other three stages of the process — fish farmers purchase smolts from salmon hatcheries and release them into sea cages in which they are farmed to marketable size. In the case of Atlantic salmon, fish are usually farmed for one to two years prior to harvesting (Shaw and Muir, 1987).

Optimal management of aquaculture involves determining the optimal levels of the production variables over which the farm manager has control, such as the initial stocking level, feeding schedule and harvesting time. Optimal management issues have been analysed by Arnason (1989), Bjørndal (1988, 1990), Hochman *et al.* (1990), Karp *et al.* (1986), Lillestøl (1986), and Talpaz and Tsur (1982), using both stochastic and deterministic models solved by optimal control and dynamic programming techniques. In this paper, the analysis of optimal harvesting time documented by Bjørndal (1988, 1990) is extended to also analyse the optimal stocking level. This is achieved by including release costs in Bjørndal's (1988, 1990) optimal harvesting model, in which harvesting and feed costs are also considered.

In the model, the optimal management strategy for the fish farmer is viewed in terms of two simultaneous strategies, the optimal stocking strategy and the optimal harvesting strategy. In order to achieve optimal management, the fish farmer must determine a combined strategy of releasing the optimal number of recruits — that is, the optimal number of salmon smolts with which to stock the farm — and harvesting those recruits at the optimal harvesting time. It is assumed that other factors influencing the production process are fixed. Thus, for a given feeding schedule, for example, the fish farmer's objective is to determine the optimal stocking level and the optimal harvesting time.

In order to simplify the analysis, the primary strategy of the farm manager, which is to determine the optimal harvesting time for a given stock of fish, is initially modelled independently of the optimal stocking decision. The optimal harvesting strategy is considered in terms of investment theory. From an economic point of view, the stock of farmed fish is a form of growing capital. The farm manager's strategy is therefore to determine the harvesting time which maximises the net present value of the capital investment in fish (Bjørndal, 1988, 1990). Underlying this analysis of the optimal harvesting strategy is the assumption that the number of recruits is given exogenously. Since the initial stocking level can be controlled by the farm manager, this assumption is considered unrealistic. It is relaxed and consideration is also given to the optimal stocking decision.

Bioeconomic Analysis

The basis of the bioeconomic analysis undertaken in this paper is a simple biological model for a yearclass of fish. (When fish are released to enclosures in a fish farm, the stock is referred to as a yearclass, as all fish are of the same age.) Throughout the analysis it is assumed that the conditions in the culture environment are stationary.

Biological Model

During its lifetime a yearclass of farmed fish is affected by the biological processes of growth and natural mortality. This can be represented by an adapted Beverton-Holt model:

$$N(0) = R, \tag{1}$$

$$\frac{dN(t)}{dt} \equiv N'(t) = -M(t)N(t), \quad 0 \leq t \leq T, \tag{2}$$

$$N(t) = Re^{-\int_0^t M(\tau)d\tau}. \tag{3}$$

In this model, t represents the time from the release of the yearclass (i.e. from $t = 0$), and T is some upper time limit for keeping the fish (i.e. the time of death or sexual maturation). $N(t)$ is the number of fish which survive to time t, and $dN(t)/dt$ denotes its change over time. R is the number of recruits in the yearclass, and $M(t)$ represents the instantaneous natural mortality rate.

The natural mortality rate of the yearclass can vary over time due to disease and environmental factors. In this analysis, however, $M(t)$ is assumed to be constant. That is:

$$M(t) = M.$$

Equations (2) and (3) are thereby simplified as follows:

$$\frac{dN(t)}{dt} \equiv N'(t) = -MN(t), \quad 0 \leq t \leq T, \tag{2'}$$

$$N(t) = Re^{-Mt}. \tag{3'}$$

The growth of the fish in the yearclass is also influenced by many factors. It can be described by an implicit function of the following form:

$$w'(t) \equiv \frac{dw(t)}{dt} = g(t, w(t), N(t), F(t)), \tag{4}$$

in which $w(t)$ represents the weight per fish and $F(t)$ is the quantity of feed consumed per fish. Growth is also affected by environmental factors, the main one being seawater temperature which influences feed consumption (Lillestøl, 1986). For simplicity, however, growth is specified as a function of time only:

$$w'(t) = g(t).$$

(The relationship between growth, fish density, and feed consumption is considered later in the analysis.) The weight per fish at any time t is therefore given by:

$$(5) \qquad w(t) = w(0) + \int_0^t w'(\tau)d\tau\,,$$

in which $w(0)$ is the initial weight of the fish at the time of release (i.e. at $t = 0$), and the second term in the expression is the growth from $t = 0$ until t. Each fish grows towards a maximum weight which is reached when its growth rate, $w'(t)$, diminishes to zero.

Assuming all fish in the yearclass are of equal weight throughout their lifetime, the biomass of the yearclass, represented by $B(t)$, can be calculated as follows:

$$(6) \qquad B(t) = N(t)w(t) = Re^{-Mt}w(t).$$

As time increases, the weight of each fish increases in accordance with equation (5). Hence, the biomass of the yearclass also increases, even though the number of fish decreases due to natural mortality. Changes in the biomass over time are given by:

$$\begin{aligned} B'(t) &= w'(t)N(t) + w(t)N'(t) \\ (7) \qquad &= w'(t)N(t) - Mw(t)N(t) \\ &= [\frac{w'(t)}{w(t)} - M\,]B(t), \end{aligned}$$

in which $w'(t)/w(t)$ is the relative growth rate of the fish, which is a decreasing function of time, at least within the time interval relevant for harvesting. (This is related to the sigmoid shape of the weight function, showing that in the range where harvesting is relevant, the growth rate of the fish is diminishing.)

The Optimal Management Model

The purpose of the bioeconomic analysis is to consider the optimal management strategy in Atlantic salmon farming within the constraints of the biological model just outlined. The analysis is initially focused on the optimal harvesting strategy for a single yearclass investment. (Optimal rotation for an infinite series of yearclass investments is not considered, since such an analysis requires the assumption that when one yearclass is harvested, the next one is immediately released. This implies that recruits are available throughout the year, which is not the case with Atlantic salmon, since smolts become available at only one time of the year, in spring.) The analysis is then expanded and consideration is also given to the optimal stocking strategy.

In the analysis, which draws heavily on Bjørndal (1988, 1990), the influence of different variable costs of production on optimal stocking level and optimal harvesting time is examined. Fixed costs are disregarded since they do not influence the harvesting time or the number of recruits which maximise the net revenue that an investment in fish generates.

Throughout the analysis, it is assumed that the farm manager's objective profit function is evaluated at the time of initial stocking (i.e. at $t = 0$). (An alternative approach would be to evaluate the objective function later in the production cycle when the fish farmer is deciding between harvesting and holding onto the stock of fish. With this approach a new origin would be assigned to the production cycle and $t = 0$ would be later than the time of initial stocking. This approach would be well suited to solving the model for the optimal harvesting time, but would be less well suited to solving for the optimal number of recruits, since R^* would be the optimal number of smolts at $t = 0$ rather than the optimal number of recruits at the time of release.)

The Optimal Harvesting Model

In the analysis of the optimal harvesting strategy, a hypothetical situation with zero variable costs of production is initially considered. This situation serves as a means of comparison for further analysis. The influence of harvesting and feed costs on optimal harvesting time is then considered. Release costs are not examined since they are fixed, given the assumption that the number of recruits is determined exogenously. Release costs are examined when this assumption is relaxed and the optimal number of recruits is also analysed, since release costs become variable and relevant to the optimal stocking decision.

Zero costs: In this hypothetical case of zero variable costs of production, the farm manager's strategy is to determine the harvesting time which maximises the present value of the gross biomass value of the yearclass.

The gross biomass value, represented by $V(t)$, is calculated as follows:

$$(8) \qquad V(t) = p(w)B(t) = p(w)Re^{-Mt}w(t),$$

in which $p(w)$ is the (gross) price per kilogram of fish, which is assumed to be fixed.

The farm manager's strategy is to determine the harvesting time which maximises an objective profit function of the following form:

$$(9) \qquad \pi(t) = V(t)e^{-rt},$$

in which $\pi(t)$ is the present value of harvesting at time t, and r is the (continuous-time) interest rate. The solution, found by manipulating the first-order condition for profit maximisation (i.e. $\pi'(t) = 0$), is t^*, at which time:

$$(10) \quad V'(t^*) = rV(t^*).$$

(The second-order condition for profit maximisation is:

$$V''(t^*) < -r^2V(t^*).$$

As is the case throughout this analysis of the optimal harvesting strategy, it is assumed that the profit function is concave in t, so that the second-order condition is satisfied.)

The optimal harvesting time therefore occurs when the return on the investment in fish (given by the change in the biomass value over time, $V'(t)$) equals the opportunity cost of the investment (which is the return that could be earned from an alternative investment, $rV(t)$).

This condition can be manipulated to yield the Fisher rule familiar in forestry literature in which the proportional increase in the biomass value equals the interest rate:

$$(11) \quad \frac{V'(t^*)}{V(t^*)} = r.$$

Equation (10) can also be expressed more explicitly, in terms of the specific components of the biomass value:

$$(12) \quad V'(t^*) = [\frac{p'(w)}{p(w)} w'(t^*) - M + \frac{w'(t^*)}{w(t^*)}]V(t^*) = rV(t^*).$$

This can be rewritten to give the following optimality condition:

$$(13) \quad \frac{p'(w)}{p(w)} w'(t^*) + \frac{w'(t^*)}{w(t^*)} = r + M.$$

The left hand side of equation (13) expresses the marginal revenue from refraining from harvesting the yearclass (i.e. the price appreciation due to growth, assuming $p'(w) > 0$, plus the relative growth rate of the fish) which is declining over time, and the right hand side expresses the marginal cost (i.e. the interest rate plus the natural mortality rate) which is constant. It is optimal to harvest the yearclass when the marginal revenue from refraining from harvesting equals the marginal cost. Notably, an increase in the price appreciation due to growth or the relative growth rate of the fish would increase the optimal harvesting time, while an increase in the interest rate or the natural mortality rate would reduce the optimal harvesting time.

Harvesting costs: Harvesting costs may be of two types: (i) a cost that depends on the biomass harvested, and (ii) a cost that depends on the number of fish harvested. These costs are not mutually exclusive and total harvesting costs may comprise both types. This will depend on the nature of the harvesting process. For example, if the biomass is harvested as a whole, harvesting costs will be of a per-kilogram type, while they will be of a per-fish type if the fish are harvested individually. In this analysis, each type is considered independently.

The farm manager's strategy is to determine the harvesting time which maximises the present value of the net biomass value (i.e. the gross biomass value less the harvesting costs) of the yearclass.

(i) Harvesting costs dependent on the biomass harvested

Assuming a fixed harvesting cost per kilogram of fish of C_K, harvesting the entire yearclass biomass at time t incurs total harvesting costs of $C_K B(t)$. The farm manager's strategy is therefore to choose the harvesting time which maximises the following objective profit function:

$$(14) \quad \pi(t) = [p(w)B(t) - C_K B(t)]e^{-rt} = [p(w) - C_K]B(t)e^{-rt},$$

in which $[\, p(w) - C_K \,]$ is the net price per kilogram of fish. The optimal harvesting time, t^*, must therefore satisfy the condition:

$$(15) \quad \frac{p'(w)}{p(w) - C_K} w'(t^*) + \frac{w'(t^*)}{w(t^*)} = r + M.$$

Price appreciation is now on the basis of the net price per kilogram of fish rather than the gross price, as was the case in equation (13) for the hypothetical situation of zero costs. Marginal revenue is therefore increased for every value of t, and hence the optimal harvesting time is later. If price is not dependent on weight (i.e. if $p'(w) = 0$), however, the price appreciation term is zero, and the optimal harvesting time is independent of the per-kilogram harvesting cost.

(ii) Harvesting costs dependent on the number of fish harvested

Assuming a fixed harvesting cost per fish of C_S, harvesting all the fish in the yearclass at time t incurs total harvesting costs of $C_S N(t)$. In this case the fish farmer's objective profit function is:

$$(16) \quad \pi(t) = [V(t) - C_S N(t)]e^{-rt} = [p(w)w(t) - C_S]Re^{-(M+r)t}.$$

The optimal harvesting time must now satisfy the following rule:

$$(17) \quad \frac{p'(w)}{p(w)} w'(t^*) + \frac{w'(t^*)}{w(t^*)} = (r + M)\,[\frac{p(w)w(t^*) - C_S}{p(w)w(t^*)}].$$

Compared with the hypothetical case of zero costs (cf. equation (13)), the marginal revenue term is unchanged. The marginal cost, however, is reduced. It is therefore optimal to wait longer before harvesting the yearclass (as was also the case with per-kilogram harvesting costs). By not harvesting until later, the discounted value of the per-fish harvesting costs is reduced due to natural mortality. Discounting itself also reduces the present value of the harvesting costs. (Compared with equation (15) for per-kilogram harvesting costs, both marginal revenue and marginal cost in equation (17) are reduced. It is therefore an empirical question whether the optimal harvesting time with per-fish harvesting costs is earlier or later than is the case with per-kilogram harvesting costs.)

Feed costs: In order to analyse the influence of feed costs on the optimal harvesting time, the following relationship between feeding and growth is recognised:

$$(18) \quad f_t = \frac{F(t)}{w'(t)},$$

in which f_t is the feed conversion ratio and $F(t)$ is the quantity of feed consumed per fish. The feed conversion ratio is defined as the number of kilograms of feed required for a kilogram's growth in weight. As a simplifying assumption f_t is assumed to be constant, and hence the quantity of feed consumed per fish varies over time according to its growth, as follows:

$$(19) \quad F(t) = f_t w'(t).$$

(Equation (19) implies that there is no feed consumption if the growth rate of the fish is zero. It is likely however, that feed consumption is a function of both the growth rate, $w'(t)$, and the weight, $w(t)$, of the fish, since fish need food not only to grow but also to maintain weight (Bjørndal, 1988, 1990). Hence, equation (19) represents a very simple relationship between feed consumption and growth of the fish.) Feed consumption for the yearclass is therefore described by:

$$(20) \quad F(t)N(t) = F(t)Re^{-Mt} = f_t w'(t)Re^{-Mt}.$$

Assuming the yearclass is fed continuously from the time of release (i.e. from $t = 0$), the total (cumulative) feed quantity at any time t, denoted by SF_t, is calculated as follows:

$$(21) \quad SF_t = \int_0^t F(\tau)Re^{-M\tau}d\tau.$$

Total feed costs discounted back to $t = 0$ are therefore:

$$\int_0^t C_F F(\tau)Re^{-(M+r)\tau}d\tau,$$

in which C_F is the price per kilogram of feed, which in this analysis is assumed to be constant over time.

In this case the fish farmer's strategy is to choose the harvesting time which maximises the objective profit function below:

$$(22) \quad \pi(t) = V(t)e^{-rt} - \int_0^t C_F F(\tau)Re^{-(M+r)\tau}d\tau.$$

The solution, t^*, is given by the following optimality condition:

$$(23) \quad \frac{p'(w)}{p(w)} w'(t^*) + \frac{w'(t^*)}{w(t^*)} = r + M + \frac{C_F F(t^*)}{p(w)w(t^*)}.$$

Marginal revenue is the same as in the case of zero costs (cf. equation (13)). Marginal cost however is increased. It is therefore optimal to harvest the fish earlier than is the case when feed costs are not considered. Notably, an improvement in the feed conversion ratio would increase the optimal harvesting time.

Harvesting and feed costs: When harvesting and feed costs are both considered in the same analysis, and harvesting costs are dependent on the biomass harvested, the fish farmer's objective profit function is:

$$(24) \quad \pi(t) = [p(w) - C_K]B(t)e^{-rt} - \int_0^t C_F F(\tau) R e^{-(M+r)\tau} d\tau.$$

The optimal harvesting time must now satisfy the following condition:

$$(25) \quad \frac{p'(w)}{p(w) - C_K} w'(t^*) + \frac{w'(t^*)}{w(t^*)} = r + M + \frac{C_F F(t^*)}{[p(w) - C_K]w(t^*)}.$$

When harvesting costs are dependent on the number of fish harvested, the objective function above is reformulated as follows:

$$(26) \quad \pi(t) = [V(t) - C_S N(t)]e^{-rt} - \int_0^t C_F F(\tau) R e^{-(M+r)\tau} d\tau,$$

and the optimality condition is given by:

$$(27) \quad \frac{p'(w)}{p(w)} w'(t^*) + \frac{w'(t^*)}{w(t^*)} = (r + M)\left[\frac{p(w)w(t^*) - C_s}{p(w)w(t^*)}\right] + \frac{C_F F(t^*)}{p(w)w(t^*)}.$$

Harvesting costs imply that it is optimal to harvest the fish later than otherwise, while feed costs imply that it is optimal to harvest the fish earlier than otherwise. The net effect on the optimal harvesting time of both harvesting and feed costs is therefore ambiguous and can only be evaluated empirically.

The Optimal Stocking And Harvesting Model

Throughout the analysis of the optimal harvesting strategy, the number of fish in the yearclass was assumed to be given exogenously. This assumption is now relaxed and consideration is also given to the optimal stocking strategy. In order to solve the optimal management model for both the optimal stocking level and the optimal harvesting time it is assumed however, that the biomass-value function is density dependent and a function of the number of recruits in the yearclass (i.e. $V = V(R,t)$). This assumption holds if the weight function is density dependent, as follows:

$$w = w(N,t) = w(Re^{-Mt},t) = w(R,t).$$

If the weight function is density independent (i.e. $w = w(t)$), the number of recruits with which the farm is stocked is irrelevant to the optimal management decision. In this case, optimal management only requires that the yearclass is harvested at the optimal harvesting time.

In the analysis, the influence of release costs on the optimal stocking level and the optimal harvesting time is initially examined. Harvesting and feed costs are then included for consideration.

Release costs: The farm manager's strategy is now to determine simultaneously the harvesting time and the number of recruits which maximise the present value of the net biomass value (i.e. the gross biomass value less release costs) of the yearclass.

Assuming a release cost per recruit of C_R, total release costs are $C_R R$. The farm manager's strategy is therefore to maximise the following objective profit function:

$$(28) \quad \pi(R,t) = V(R,t)e^{-rt} - C_R R.$$

The solution is the optimal number of recruits, R^*, and the optimal harvesting time, t^*, which together satisfy the following first-order conditions for profit maximisation:

$$(29) \quad \frac{\partial V}{\partial R}e^{-rt} = C_R,$$

$$(30) \quad \frac{\partial V}{\partial t} = rV(R,t).$$

(The second-order conditions for profit maximisation are:

$$\frac{\partial^2 V}{\partial R^2}e^{-rt} < 0,$$

$$\frac{\partial^2 V}{\partial t^2} < -r^2 V(R,t).$$

It is now assumed that the profit function is concave in both R and t, so that the second-order conditions are satisfied.)

According to equation (29) the discounted value of the marginal revenue with respect to recruits must equal the release cost per recruit. Hence this condition determines the optimal number of recruits dependent on the harvesting time. Assuming growth is a negative function of fish density, $V(R,t)$ is decreasing in the number of fish released and it is optimal to release fewer recruits than would otherwise be the case. Equation (30) determines the optimal harvesting time dependent on the number of recruits, in accordance with the familiar condition that the return on the investment in fish equals the opportunity cost (cf. equation (10)). Notably, the optimal harvesting time is independent of the per-recruit release cost.

Harvesting, feed and release costs: When harvesting, feed and release costs are all considered in the same analysis, and harvesting costs are exemplified by a cost per kilogram of biomass harvested, the fish farmer's objective profit function is:

$$(31)\quad \pi(R,t) = V(R,t)e^{-rt} - C_R R - C_K w(R,t)Re^{-(M+r)t} - \int_0^t C_F F(R,\tau)Re^{-(M+r)\tau}d\tau.$$

The solutions for optimal management are given by the following conditions:

$$(32)\quad \frac{\partial V}{\partial R}e^{-rt} = C_R + C_K[\frac{\partial w}{\partial R}R + w(R,t)]e^{-(M+r)t} + \int_0^t C_F[\frac{\partial F}{\partial R}R + F(R,\tau)]e^{-(M+r)\tau}d\tau,$$

$$(33)\quad \frac{\partial V}{\partial t} - C_K[\frac{\partial w}{\partial t} - (M + r)w(R,t)]Re^{-Mt} - C_F F(R,t)Re^{-Mt} = rV(R,t).$$

When harvesting costs are dependent on the number of fish harvested, the fish farmer's objective profit function becomes:

$$(34)\quad \pi(R,t) = V(R,t)e^{-rt} - C_R R - C_S Re^{-(M+r)t} - \int_0^t C_F F(R,\tau)Re^{-(M+r)\tau}d\tau,$$

and the optimality conditions are given by:

$$(35)\quad \frac{\partial V}{\partial R}e^{-rt} = C_R + C_S e^{-(M+r)t} + \int_0^t C_F[\frac{\partial F}{\partial R}R + F(R,\tau)]e^{-(M+r)\tau}d\tau,$$

$$(36)\quad \frac{\partial V}{\partial t} + (M + r)C_S Re^{-Mt} - C_F F(R,t)Re^{-Mt} = rV(R,t).$$

Assuming density dependence provides an unambiguous stocking solution to the optimal management model — if growth is a negative function of fish density, it is optimal to release fewer recruits than would otherwise be the case. The optimal harvesting solution is ambiguous however, since harvesting costs imply that it is optimal to harvest the fish later than otherwise, while feed costs imply that it is optimal to harvest the fish earlier than otherwise. The net effect on the optimal harvesting time of both harvesting and feed costs can only be evaluated empirically.

Application of the Optimal Management Model

The optimal management model developed earlier was solved empirically for a yearclass of Atlantic salmon farmed in Australia during 1989-91.

The yearclass was released in October 1989 and was harvested selectively over a seven month period prior to completion in March 1991. The yearclass was initially released to several 'smolts' cages in which the salmon smolts were farmed until the middle of the following year. During June, July and August 1990 the 'smolts' cages were graded and the salmon reallocated to 'graded' cages. Selective harvesting of the 'graded' cages commenced in September 1990.

Monthly fish farm reports traced the development of the yearclass for its lifetime. A detailed description of the data obtained from these reports and from other confidential documents provided by the farm manager is presented in Hean (1992). The data was obtained in strict confidence however, and cannot be disclosed.

The approaches adopted to the analysis of the price and growth data, and the estimation of the parameters of the model, are briefly outlined below.

The 1989 yearclass was sold primarily on two markets — the Australian market and the Japanese market — in two product modes — fresh salmon and smoked salmon. As a simplifying assumption however, the yearclass was assumed to have been sold exclusively as fresh salmon. On both markets, the price per kilogram of fresh salmon was dependent on the weight of the salmon, with price increasing with weight, over a range of weight categories (i.e. $p'(w) > 0$). According to Lillestøl's (1986) analysis of price data for Atlantic salmon, increasing step functions of weight can be reasonably well approximated by linear functions of weight. This approach was also adopted in this analysis.

Growth data (i.e. average weight and fish density data) was obtained for each cage farmed in each month, for the lifetime of the yearclass. Unfortunately, there was no record of how the salmon were reallocated from the 'smolts' cages to the 'graded' cages. Hence it was impossible to trace the development of any particular cage of salmon from the time of release until harvest. The approach taken was to aggregate the per-cage data on a monthly basis and base the analysis on time-series data for the yearclass as a whole for its lifetime.

The parameters of the model estimated for the 1989 yearclass were the natural mortality rate, the feed conversion ratio, and the harvesting, feed and release costs (measured in 1990-91 Australian dollars). The average natural mortality rate, M, and average feed conversion ratio, f_t, were estimated for the yearclass as a whole and assumed to have held throughout its lifetime. The estimates were 2.13% and 1.93 kg feed/kg growth respectively. On the advice of the farm manager the harvesting cost of the yearclass, a cost assumed to be totally attributable to labour, was estimated on a per-kilogram basis. The estimate for C_K was \$0.09. The average per-kilogram cost of feed was estimated and assumed to have held for the lifetime of the yearclass. C_F was estimated to be \$1.22. The release cost per recruit was assumed to include both the labour cost of releasing a recruit and the actual cost of the recruit. The estimate for C_R was \$2.44.

The price and growth data was used to estimate a price and weight function for the yearclass, using the method of Ordinary least squares.

The estimated price function (with t-statistics in parenthesis), which is a linear approximation to the actual price data for the 1989 yearclass, is:

$$\underset{(21.23)}{p(w) = 9.88} + \underset{(13.31)}{0.82w(t)}, \qquad r^2 = 0.8879 \tag{37}$$

in which price is measured in 1990-91 Australian dollars and weight in kilograms. The function confirms a priori expectations that the average price of the salmon in the 1989 yearclass was a positive function of the weight of the salmon (i.e. $p'(w) > 0$).

The estimated weight function for the typical farmed salmon (with t-statistics in parenthesis), that best fits the time-series growth data for the 1989 yearclass, is:

$$\begin{array}{lll} (38) & w(t) = \underset{(13.38)}{0.033t^2} - \underset{(-8.11)}{0.001t^3}, & R^2 = 0.9964 \end{array}$$

in which weight is measured in kilograms and time is measured in months from the time of release (i.e. from the beginning of October 1989, or $t = 0$). This is not an unexpected result given the sigmoid shape expected of the weight function for farmed fish. It is also consistent with the weight function estimated by Bjørndal (1988, 1990).

According to the weight function, the typical salmon in the 1989 yearclass would have reached its maximum weight (4.123 kg) 19.248 months after release (i.e. during May 1991). The yearclass, however, would have reached its maximum biomass weight only 17.849 months after release (i.e. during March 1991). As the average price of the salmon increased with the weight of the fish (i.e. $p'(w) > 0$), the biomass value would have been maximised later, 18.142 months after the yearclass was released (i.e. during April 1991).

The estimated price and weight functions, together with the parameter estimates for natural mortality, feed conversion, and harvesting, feed and release costs, were used to solve the optimal management model for the 1989 yearclass using a spreadsheet analysis. The estimated weight function is density independent however, and therefore not a function of recruits. Hence, the number of recruits with which the farm was stocked was irrelevant to the optimal management decision. All that was required for optimal management was that the yearclass was harvested at the optimal harvesting time.

The optimal harvesting time for the 1989 yearclass, predicted by the model, is presented in Table 1, for a range of discount rates, and for the different cost cases considered earlier. The empirical results confirm qualitatively the results predicted by the theoretical analysis: an increase in the discount rate reduces t^*, per-kilogram harvesting costs imply an increase in t^*, and feed costs imply a decrease in t^*. When harvesting and feed costs are both considered in the same analysis, the influence of feed costs on the optimal harvesting time dominates, and t^* is reduced compared to the situation without such costs. This result reflects the fact that feed comprises the largest share of costs on the fish farm.

There are two general conclusions which can be drawn from the results presented in Table 1. Firstly, the optimal harvesting time is relatively insensitive to changes in the discount rate, and secondly, the optimal harvesting time is not significantly influenced by the variable costs considered. These conclusions were also drawn by Bjørndal (1988, 1990) in his analysis of optimal harvesting time.

TABLE 1
The Optimal Harvesting Time (in Months) for the 1989 Yearclass of Atlantic Salmon

r Discount rate	t^* in the zero costs case	t^* in the harvesting costs case	t^* in the feed costs case	t^* in the harvesting and feed costs case
0	18.142	18.143	17.949	17.949
0.01	17.589	17.592	17.293	17.294
0.02	17.019	17.022	16.612	16.613
0.05	15.240	15.245	14.479	14.481
0.07	14.046	14.053	13.059	13.059
0.10	12.339	12.347	11.078	11.076
0.12	11.299	11.307	9.918	9.915
0.15	9.923	9.930	8.456	8.450
0.20	8.112	8.118	6.670	6.662

The efficacy of the optimal management model as a decision-making tool is demonstrated by the results of this analysis. For a discount rate of 5%, the model predicts that in the harvesting and feed costs case the optimal harvesting time for the 1989 yearclass was 14.481 months after release, during December 1990, which was the median month in which the yearclass was selectively harvested. Since stochastic variation in the growth conditions across the cages in which the yearclass was farmed would have resulted in the optimal harvesting time varying from cage to cage, and given that the model predicts the average optimal harvesting time for the yearclass as a whole, the model's predictive power is strong.

Concluding Comments

In this paper a model which can be used to determine the optimal management strategy for intensive aquaculture has been analysed. The contribution to knowledge of this analysis is threefold.

Firstly, by including release costs and how they influence the optimal number of recruits in a yearclass, in Bjørndal's (1988, 1990) model of optimal harvesting time, in which harvesting and feed costs are also considered, a realistic model of optimal management has been developed. In the model, the two main production variables over which the fish farmer has control, the number of recruits and the harvesting time, are considered. The model could be further developed to include other decision variables, such as the feeding schedule.

Secondly, the distinction between fixed and variable costs of production in the analysis of optimal management has been clarified, a distinction not made clear in Bjørndal's (1988, 1990) analysis. Optimal management is only influenced by variable costs of production — fixed costs do not influence either the harvesting time or the number of recruits which maximise the net revenue that an investment in fish generates. In the context of the optimal harvesting model, the variable costs of production are harvesting and feed costs. Release costs are fixed since the number of recruits is determined exogenously. In the context of the simultaneous optimal stocking and harvesting model, release costs become variable since the fish farmer can determine the number of recruits with which to stock the farm.

The third and most significant contribution of the analysis is to identify that the assumption of density-dependent growth imposes a major limitation on the optimal management model — the model cannot be solved for the optimal number of recruits if the growth of farmed fish is density independent. This provides an avenue for future research — it may be possible to modify the optimal management model in order to solve for both R^* and t^* regardless of density-dependent growth. In the model, the objective profit function is a linear function of the number of recruits in the yearclass. If the profit function were nonlinear in recruits however, density-dependent growth would not be required in order to solve for R^*. This would be the case, for example, if the release cost per recruit were declining in the number of recruits released. It must also be remembered that although the growth of farmed fish may be density independent at low levels of fish density, as was the case for the 1989 yearclass considered in the empirical analysis, this may not be so when fish density is high. Hence, density-dependent growth remains an important issue for the management of intensive aquaculture.

The practical use of the optimal management model is unfortunately limited by the simplifying assumption adopted throughout the analysis, that the conditions in the culture environment are stationary. Hence, the natural mortality rate for the yearclass is assumed to be constant, and all fish have the same weight and growth characteristics throughout their lifetime.

The assumption that natural mortality is constant, is considered reasonable for Atlantic salmon given the mortality data obtained for the 1989 yearclass. Natural mortality was at its highest soon after the release of the smolts when their adaption to salt water may not have been complete. After this time, natural mortality was spread more evenly over the production cycle, although it did increase in the summer periods during which the yearclass was farmed, when water temperatures were warm. These increases were considered insignificant and their impact on the optimal harvesting time would have been minor. Atlantic salmon are harvested during summer, however. The fish are also held at close to their maximum stocking rate at the end of the production cycle. Fish density can affect natural mortality, hence a high stocking rate accompanied by warm water temperatures could result in a substantial increase in natural mortality. The combined influence of fish density and water temperature on a fish farmer's harvesting decision is therefore important since a significant increase in natural mortality would reduce the optimal harvesting time. (Similarly, if the natural mortality rate were redefined to reflect risk factors other than death, such as the early onset of sexual maturity, an increase in these factors would also reduce t^*.) The impact of this assumption on the optimal harvesting time for other fish species for which natural mortality is not constant, would need to be considered in such applications of the model.

The assumption that all fish in the yearclass have the same weight and growth characteristics throughout their lifetime has a more serious implication for the model — it is optimal to harvest the whole yearclass instantaneously at the optimal harvesting time. In reality however, environmental conditions are not stationary, and due to stochastic variation in the growth conditions across the cages in which the yearclass is farmed, fish have different relative growth rates. Hence, it will be optimal to harvest the yearclass selectively over a period of time. This provides another avenue for future research — the optimal management model could be developed to include selective harvesting if some of the natural variation in the culture environment were retained in the analysis. This could be achieved, for example, by including sea water temperature in the analysis, since growth and sea water temperature are nonlinearly related in the sense that feed consumption is low at both low and high water temperatures.

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