PRODUCTION AND INCOME UNCERTAINTY IN THE WOOL INDUSTRY: AN AGGREGATIVE APPROACH

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Production instability has been rightly regarded as one of the key problems of Australian agriculture. Campbell has emphasised that little attention has been focussed on reducing fluctuations in output as a means of cushioning income variations, although a great deal of interest has been taken in various measures designed to stabilize price.[1] An attempt is made in this paper to split up total variability of aggregate wool income into two components : the first corresponding to production uncertainty, and the second to price uncertainty. It is only by this sort of approach that we are in a position to gauge the possible effectiveness of any scheme designed to stabilize aggregate income, whether such a scheme act through price or output. This analysis suggests that over the last thirty years production instability has been of very much less significance than shifts in overseas demand in accounting for fluctuations in *aggregate* wool income,[2] confirming the views taken by Blau,[3] McMillan[4] and Lewis.[5] It also highlights the critical dependence of Australian export income on overseas demand conditions.

Method of Analysis[6]

Gross income is the product of two factors, price and quantity. However, these two components are not independent. When considering the output of a major producer of wool, such as Australia, due allowance must be made for the interaction of Australian supply upon price. In years of short wool clips price will be bolstered to some extent, thus providing a measure of " built-in " stabilization for aggregate wool

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[1] Keith O. Campbell, " The Challenge of Production Instability in Australian Agriculture ", *Australian Journal of Agricultural Economics*, Vol. 2, No. 1 (July 1958,) pp. 3-23.

[2] This paper has nothing to say about the variability of individual farmers' incomes.

[3] Gerda Blau, " Wool in the World Economy ", *Journal of the Royal Statistical Society*, Vol. CIX, Part III, (1946), pp. 179-235. See especially p. 216.

[4] R. B. McMillan, " Organized Marketing of Wool ", *Economic Record*, Vol. XXV Supplement (August 1949), pp. 59-72 ; especially p. 65.

[5] J. N. Lewis, " The Present Position of Wool in Australia ", paper read before the Winter Forum of the Australian Institute of Political Science, Sydney, July 1959 (mimeo.). See especially p. 10.

[6] A mathematical treatment is given in the Appendix. The components of variance model used is not amenable to precise exposition in plain language, and readers desiring a rigorous presentation must consult the Appendix.

revenue. Ideally to allow for this we should need to know with certainty the form of the manufacturers' demand schedule for Australian wool. So long as this demand curve remained stable all variations in gross Australian wool income would be directly attributable to shifts in Australian output caused by seasonal conditions—droughts and the like; and to a very much lesser extent to secular shifts in supply.[7]

Of course the demand schedule for raw Australian wool is not stable. Changes in consumer incomes and tastes overseas, as well as seasonal shifts in supply in other wool producing countries,[8] contribute to its instability. We are in fact faced with a volatile demand relationship whose underlying form we cannot do much more than guess at. However, only certain forms of a price-quality demand curve are plausible. In what follows a constant elasticity curve[9] is assumed since this has a number of methodological advantages which will become clear.

I have stated above that total aggregate income variability could be accounted for by variations in production alone if the demand curve did not shift. This immediately suggests a way of analysing income variability into its components. If we consider a time-series of outputs all we need do is assume that the demand curve does not shift, and calculate what the income variability would have been under these circumstances. This is possible since once the form of the demand curve is specified, for any given output there corresponds but one price. The product of this theoretical price and the output itself would indicate the level of revenue to be expected if the demand curve did not shift. I term that amount of variation in this hypothetical series which can serve to explain observed variations in aggregate wool income, the *supply effect*.[10] For the given time-series we can then compare the *supply effect* with the total amount of income variability actually observed, or *total effect*.[11] The ratio *supply effect/total effect*, which I have termed r, will then give an index of the proportional contribution made to aggregate income variability by production instability. In subsequent discussion this index is expressed as percentage.

These ideas are illustrated in Figure 1. q_0 represents the size of the Australian wool clip in one year, and q_1 the (lower) clip in the year following. The observed movement in price is from p_0 to p_1. However, only the change from π_0 to π_1 can be explained in terms of the stable demand curve D-D'. The difference between this supply induced price change and the total change must be due to shifts in demand. The statistical analysis upon which this paper is based examines variation in the revenue series generated by these components of price change.

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[7] Trend in Australian supply accounts for less than 7 per cent of total income variance under a wide range of assumptions.

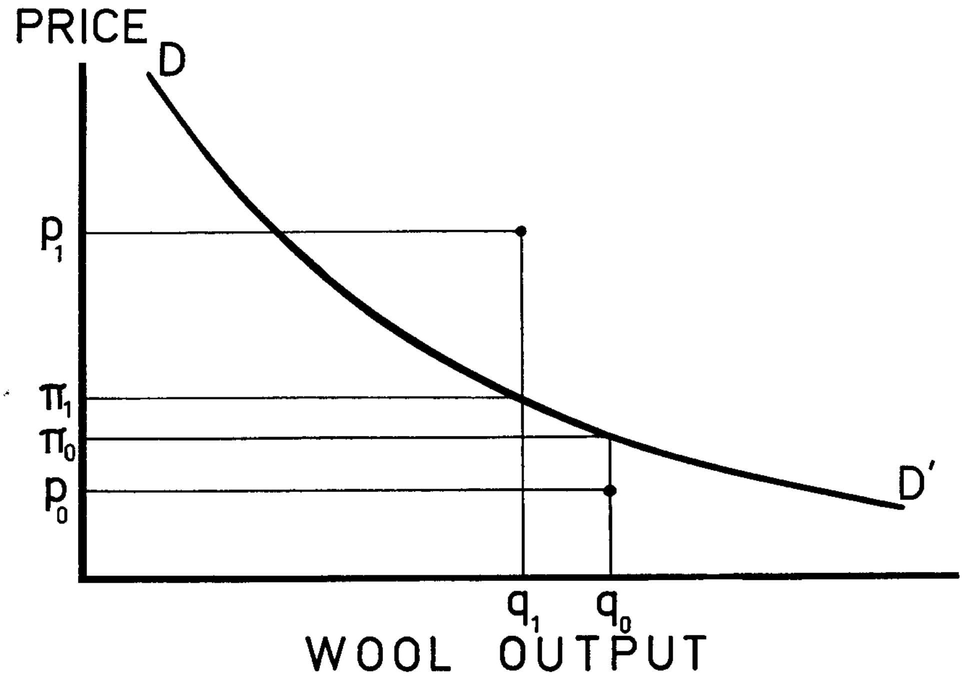
[8] Keeping in mind that shifts in overseas supply will affect manufacturers' demand for *Australian* wool.

[9] Viz., a curve of the form $q = k\pi^{\eta}$, where q is the quantity of Australian wool demanded at a price π, η is the price elasticity of demand for Australian wool, and k is a constant.

[10] For a precise definition of the supply effect see the Appendix.

[11] Variance of the observed income series is used as a measure of variability.

Figure 1



The Data

The data used to illustrate this method were a time-series of Australian outputs and average wool prices from 1928-29 to 1957-58 inclusive.[12] This period was chosen mainly because a suitable price index was available only for these thirty years.[13] Since this period contains a number of war years during which prices were stable at negotiated rates, r was re-calculated eliminating these years.[14]

Choosing a Demand Curve

Although I do not believe the results would be greatly different for any other plausible form of demand curve, the type of curve used to compute r was a curve of constant elasticity. The great advantage of this procedure lies in the fact that we are able to plot the index r as a function of η, the price elasticity of demand. This is important since we are not in a position to know *precisely* what elasticity value it is

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[12] Bureau of Agricultural Economics, *Statistical Handbook of the Sheep and Wool Industry*, Canberra, 1956. Recent figures were obtained from the National Council of Wool Selling Brokers summary charts.

[13] The index used was the index of import prices prepared by the Commonwealth Bank of Australia. This index has the twin advantages of being correlated with factor costs in wool buying countries, as well as being an appropriate index to judge the severity of fluctuations in export earnings at home. This is discussed more fully in the Appendix.

[14] Eliminating these years does not make a great deal of difference to the results, however. (See Table II.)

reasonable to assume. However, the estimates of Horner[15] and Philpott[16] do give us broad guidance here. As luck would have it, r is not too sensitive to the value of η we choose within a plausible range.

Besides η, a demand curve of the type discussed above has a second constant associated with it, k. In tabulating the index r, a least squares estimate has been used for k, treating the selected value of η as a sure number, and assuming a random disturbance in demand.[17] So much for methodology.

Results

In Figure 2, r is plotted against η, the short-run price elasticity of demand for Australian raw wool. These results are repeated in numerical form in Table I. In Table II the corresponding values of r are shown when the wartime period is eliminated from the data. In all cases an

Figure 2

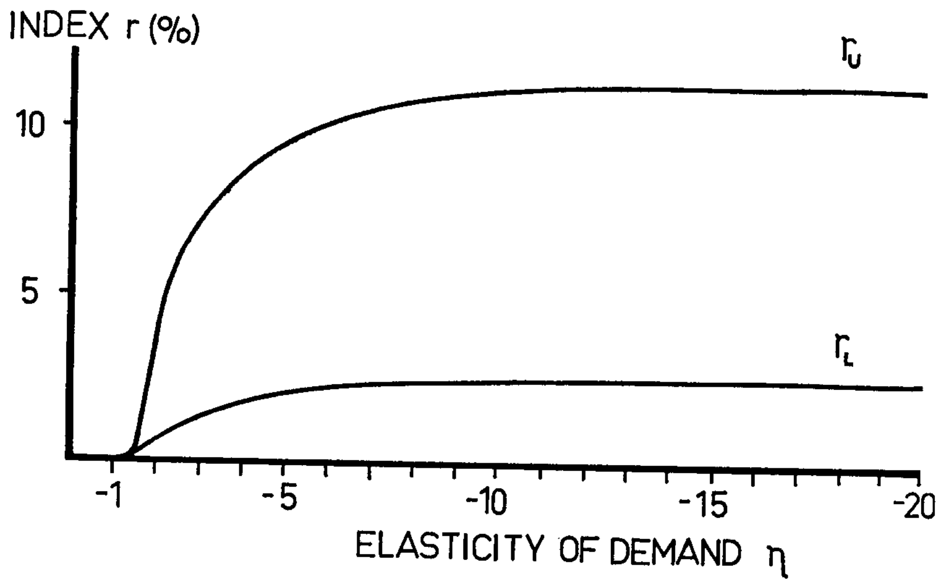

Figure 2. Upper and lower bounds for estimated percentage contribution made to variance of aggregate wool income by the supply effect : based on thirty year time series, 1928-29 to 1957-58 inclusive.

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[15] F. B. Horner, " Elasticity of Demand for the Exports of a Single Country ", *Review of Economics and Statistics*, Vol. 34, No. 4 (November 1952), pp. 326-342. The work of both Horner and Philpott (cited below) was geared to estimate *long run* rather than short run elasticities. However of necessity these estimates do give us an idea of the average level of the short run elasticities which prevailed over the period. This is particularly true since the work of both investigators pre-dated the development of distributed lag estimation of long run elasticities.

[16] B. P. Philpott, " Wool in the New Zealand Economy ", *Economic Record*, Vol. 33, No. 65 (August 1957), pp. 216-233.

[17] This is put in more rigorous terms in the Appendix. I am indebted to Dr. H. S. Konijn of Sydney University for suggesting that I adopt the above procedure to evaluate k. Earlier estimates used by the author were far less efficient and resulted in much wider limits for r.

upper and a lower bound for r are given : r_U and r_L respectively. There are strong arguments for taking r_U as an extreme upper limit for the contribution made to aggregate wool income variance by fluctuations in supply.[18]

The first thing to notice about the above results is that they are at all events consistent with theory in giving zero values of r_U when the elasticity of demand is assumed unity. Fluctuations in production have no effect on income when this condition holds.

More important, an appropriate value of η to consider will probably lie somewhere between -1.0 and -2.2. Horner in estimating the price elasticity of export demand for Australian wool quoted a range of -1.59 to -2.15 for the pre-war period.[19] The corresponding range of values for r_U is 4.5 to 8.4 per cent. Taking a more cautious interpretation, any values of η between -1.0 and -3.0 would result in values of r_L less than 1 per cent., and values of r_U less than 11.1 per cent. I suggest that this latter figure can be taken as an extreme upper limit for the relative importance of production instability in generating aggregate income fluctuations over the thirty year period studied.

TABLE I

Estimated percentage contribution made to variance of aggregate wool income by production instability 1928-29 *to* 1957-58 *inclusive*

Elasticity of demand assumed η	Lower bound r_L (%)	Upper bound r_U (%)
−20.0	2.5	11.2
−10.0	2.3	11.1
−5.0	1.8	9.6
−3.0	1.2	7.4
−2.5	1.0	6.1
−2.0	0.7	4.1
−1.5	0.3	0.4
−1.3	−2.1*	0.1
−1.1	−5.7*	0.0
−1.0	−8.2*	0.0
−0.9	−11.1*	0.0

*Negative values indicate that with the given demand elasticity, shifts in the demand curve alone would account for more variation than is actually observed.

[18] The bounds given do not constitute a statistical confidence interval. The upper limit is arrived at by attributing to production instability all income variance which is not explainable in terms of the first order effects of the model. r_U will thus considerably overestimate r.

[19] -1.59 is Horner's " shortest run " estimate. It seems fairly safe to take -1.0 as a lower limit. Similarly it seems safe to take -2.2 as an upper limit, this corresponding to his upper (long run) estimate. See Horner, *op. cit.*, p. 335.

TABLE II

Estimated percentage contribution made by production instability to aggregate wool income variance, 1928-29 *to* 1957-58, *excluding years of the U.K. Wool Purchase marketing arrangements*, 1939-40 *to* 1945-46.

Elasticity of demand assumed η	Lower bound r_L (%)	Upper bound r_U (%)
−20.0	2.4	15.6
−10.0	2.2	15.0
−5.0	1.7	13.5
−3.0	1.1	11.1
−2.5	1.0	9.7
−2.0	0.6	7.4
−1.5	0.3	2.9
−1.3	−0.2*	0.1
−1.1	−5.0*	0.0
−1.0	−8.2*	0.0
−0.9	−12.5*	0.0

*See the footnote to Table I.

Implications for Policy ?

If one is bold enough to assume that future shifts in overseas demand for Australian wool, as well as our future experience of droughts, will follow a similar distribution to the pattern for the last thirty years, certain policy implications follow.

Firstly, even though production instability may be a major component in the instability of individual wool growers' incomes, it does not follow that this will be so in the aggregate. Thus any measures designed to stabilise income by eliminating or curbing production uncertainty will have their greatest impact on individual wool producing firms. A reduction in income uncertainty for individual firms may lead to a reduction in capital rationing within the industry and consequent gains in efficiency.[20]

Secondly, complete stabilization of wool production would not stabilize our wool export earnings by more than eleven per cent., and probably by considerably less than this amount. Moreover, since few policy measures could hope to stabilize output by more than a very limited amount, it seems very unlikely that export earnings will be stabilized *to any great extent* by attempts to reduce production variability in the wool industry.

[20] See Campbell on this point, *op. cit.*, p. 17.

However, it would be quite wrong to infer from the above that any particular scheme designed to stabilize wool prices could be expected to add more to the stability of export income than a scheme based on stabilizing output. Each scheme would have to be evaluated on its own merits.

Some Objections

I prefaced the last section by a necessary *caveat.* Many may not be willing to assume that future fluctuations in overseas demand will be as violent as in the past. There are some good reasons for inferring this. The historical period considered contained the great depression as well as a period of rapid inflation. Counter-cyclical policies of governments overseas can be expected to curb such violent fluctuations in the future. As demand becomes more and more stable the relative importance of production uncertainty as a source of income variability at home will increase. Thus the r index as computed from historical data will tend to underestimate the importance of production instability.

Whilst the above objection doubtless contains a good deal of truth, I cannot see any reasonable way to allow for it in the model. However, I wonder if any of us can conjure up a picture of an international economy in which commodity prices do not fluctuate, and sometimes violently so, despite the existence of a multitude of stabilizing policies. I may be wrong, but I do not expect the relative importance of production and demand instability to change *drastically* in the near future.

A second objection might be raised on methodological grounds. There is no allowance made in the model for secular shifts in demand. This I feel is more worthy of notice. However, even though allowance can be made for the secular component of shifts in supply, it is not feasible to make a corresponding adjustment to allow for secular shifts in demand. Variability arising out of this source will form part of the residual variance, all of which has been attributed to production instability when computing r_U. The bias introduced will tend to overestimate r.

Conclusion

An attempt has been made to analyse into components aggregate income variability for the wool industry. If past experience is any indication, production instability is not a major source of variation in wool export earnings, accounting for less (perhaps much less) than eleven per cent of variability over the thirty-year period ending in 1957-58. It is possible that demand will be more stable in the future than in the past, and this could lead to an increase in the relative importance of production uncertainty. However, the author does not anticipate that the position will change a great deal in the near future.

Appendix

Basic Postulates

We have a series of N observations of outputs and prices for Australian raw wool ; say q_i and P_i $(i = 1, \ldots N)$ respectively. The first problem is to deflate the series P_i in a meaningful way. This problem could well be indeterminate except for the lucky fact that we can expect an index of import prices to be related both to the real value of money overseas and in an important sense also to its value at home.

Special assumption (I) : Suppose that an index of Australian import prices adequately reflects both the real value of money to overseas manufacturers, and also the real value of exports to the Australian economy.

The justification for the first part of the above lies in the presumption that factor prices other than wool[21] in textile manufacturing countries will be fairly highly correlated with the price of Australian imports. The second part of the assumption contains its own justification.

Let A_i be an index of Australian import prices. Denote the series of deflated prices P_i/A_i by p_i. Then gross real wool income in any year i is given by

$$R_i = p_i q_i \quad \ldots\ldots(1)$$

For the purpose of this paper total variability V or the *total effect*, is defined as the variance of R : viz.,

$$V = \mathrm{var}(R) = \sum_{i=1}^{N} (R_i - \bar{R})^2/N \quad \ldots\ldots(2)$$

where $\bar{R} = \sum_{i=1}^{N} R_i/N$ is the mean of the R_i.

Special assumption (II) : Suppose that the short run elasticity of supply for Australian wool is approximately zero.

Special assumption (III) : Let the underlying short-run price-quality demand relationship for Australian wool have a constant elasticity.

In Special assumption (II) above, by " short-run " is meant " from year to year ". This assumption is necessary to ensure that in what follows income variability generated (or eliminated) by short-run changes in production plans, and/or storage operations, will not be attributed to (nor subtracted from) variability generated by random[22] fluctuations in supply.

Special assumption (III) raises difficult (but fundamental) problems of methodology, and cannot be discussed in detail here. Suffice it to say that as a working assumption this probably does not introduce errors of any appreciable magnitude. Rejection of this assumption leaves one groping blindly for a simple (but plausible) alternative.

Suppose that the demand curve for Australian raw wool is stable and of the form

$$q = k\pi^{\eta}, \quad \ldots\ldots(3)$$

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[21] But including woollen substitutes.

[22] I.e., environmentally induced fluctuations.

where q is the quantity demanded at a real price π,

η is the price elasticity of demand for raw Australian wool,

and k is a constant.

Then under the above hypothesis the gross real income associated with an Australian output q_i is

$$H_i = q_i \pi_i \quad \ldots\ldots\ldots\ldots (4)$$

where $$\pi_i = (q_i/k)^{1/\eta} \quad \ldots\ldots\ldots\ldots (5)$$

We are free to assume any value of η we like. It is desired for any chosen value of η to select a value of k in such a way that the observed series of prices p_i and the hypothetical series π_i are comparable. It is also desired to select a value of k in a statistically efficient manner. The method used is to evaluate a least squares estimate of k from the model :

$$q = kp^{\eta}u, \quad \ldots\ldots\ldots\ldots (6)$$

where u is a random variable which when transformed into the logarithms has zero mean and constant variance.[23]

Rewriting (6) in the logarithms we obtain :

$$\ln.q = \ln.k + \eta \ln.p + \ln.u \quad \ldots\ldots\ldots\ldots (7)$$

Treating the assumed value of η as a sure number, we have as observable the variables $(\ln.q_i)$ and $(\eta \ln.p_i)$. Minimising the sum of squares $\sum_{i=1}^{N} (\ln.q_i - \ln.k - \eta \ln.p_i)^2$ with respect to k, a least squares estimate of k is given by the equation :[24]

$$k^* = \exp(\Sigma \ln.q_i/N - \eta\, \Sigma \ln.p_i/N) \quad \ldots\ldots\ldots\ldots (8)$$

Hence for any given value of η the series π_i is calculated by putting $k = k^*$ in equation (5) above.[25] The series H_i is calculated by substitution in (4).

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[23] This is directly equivalent, so far as the estimation of k is concerned, to assuming a multiplicative random " error " v in price, such that $p = \pi v$; where v has unit geometric mean and $ln.v$ has constant variance.

[24] The limits of summation, which are from $i = 1$ to N in all cases, are dropped from the notation in what follows.

[25] Several different ways of estimating k were considered, including a least squares estimate based on an additive, rather than a multiplicative, random " error ". Estimates based on an additive disturbance are not symmetrical as in the case above. (See footnote 23.) A least squares estimate assuming an additive disturbance in q made little sense, giving negative values of r over the range of η considered. A least squares estimate which is more consistent with the model is $k^{**} = \{ \Sigma q^{2/\eta} / \Sigma p q^{1/\eta} \}^{\eta}$, which is based on an additive disturbance in p. This yielded values of r_U a little higher than when k^* was used. However this was due to an increase in the residual (unexplained) variance ; the estimate of r_L actually decreased.

Finally, fitting straight line demand curves gave results little different from constant elasticity curves so far as the estimation of r was concerned. In order to make the results for the straight line fit comparable to the curve fit, the elasticity of the straight line was taken at the mean of the q_i, and a family of conditional estimates of the remaining parameter evaluated from methods directly analogous to those discussed above for the estimation of k.

Components of Variance

Consider an element of variation, $(R_i - \bar{R})$. This can be rewritten

$$(R_i - \bar{R}) = (R_i - H_i) + (H_i - \hat{R}_i) + (\hat{R}_i - \bar{R}) \quad \ldots\ldots(9)$$

where $\hat{R}_i$ is the revenue which would be expected under the stable demand hypothesis in year i, if supply were stable apart from trend.

$\hat{R}_i$ is further defined by

$$\hat{R}_i = \hat{q}_i \,.\, (\hat{q}_i / k^*)^{1/\eta} \quad \ldots\ldots\ldots\ldots\ldots\ldots(10)$$

where $\hat{q}_i$ is obtained for each year from the regression of Australian output on time.

Squaring (8), summing over all i and dividing by N we obtain :

$$V = \text{var}(R) = \Sigma(R_i - H_i)^2/N + \Sigma(H_i - \hat{R}_i)^2/N + \Sigma(\hat{R}_i - \bar{R})^2/N$$
$$+ 2\{\Sigma H_i(R_i + \hat{R}_i - H_i) + \bar{R}\,\Sigma(\hat{R}_i - R_i) - \Sigma\hat{R}_i^2\}/N \quad \ldots\ldots(11)$$

On the right hand side of (11) above, the first term is a mean sum of squares (MSS) due to variations in demand ; the second term is an MSS due to non-secular fluctuations in supply ; the third term is an MSS due to trend in output ; and the final term is a residual or error term representing the amount of variation which cannot be explained in terms of the first order effects of the model. Denote these by D, S, T and Z respectively. D is defined as the *demand effect* and S as the *supply effect*. Maxima and minima for the ratios D/V, S/V, T/V and Z/V are given in Table III for values of η in the range -0.9 to -20.0.

TABLE III

Maxima and minima of estimated percentage contributions made to total variance of aggregate wool income by various components, 1928-29 *to* 1957-58 *inclusive* : η *in the range* -0.9 *to* -20.0

Source of Variation

	Demand	Supply	Trend	Error
	D/V (%)	S/V (%)	T/V (%)	Z/V (%)
Maximum	107.2 ($\eta = -0.9$)	2.5 ($\eta = -20$)	6.9 ($\eta = -20$)	9.3 ($\eta = -20$)
Minimum	81.3 ($\eta = -20$)	0.0 ($\eta = -1.0$)	4.1 ($\eta = -1.0$)	−15.8 ($\eta = -0.9$)

Interpretation

Under the terms of this model, demand instability is by far the greatest source of variation in aggregate wool income. Non-secular changes in supply, on the other hand, seem to account for very little of this variation. However, because of various inadequacies in the model a fairly large amount of variation is left unexplained when certain values of η are assumed. This reflects in part the implausibility of these values. Other effects not accounted for by the model, such as trend in demand, will also contribute to the residual variance. To be on the safe side all the unexplained variation has been attributed to production instability when quoting an upper limit for r, the percentage contribution made from this source to total variance.

Thus we define lower and upper bounds for this ratio by[26]

$$r_L = S/V \quad \ldots\ldots (12)$$

and

$$r_U = (S + Z)/V \quad \ldots\ldots (13)$$

These are the ratios tabulated in the body of this article for various values of η.

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[26] Except when $Z < 0$, in which case the expressions for r_U and r_L are interchanged.