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WHICH COUNTRY LOSES THE LEAST IN A TRADE WAR?

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Trade actions, which can generally be claimed as trade wars, appear to be on the rise. This is particularly true in the case of agricultural commodities. It is a common perception that large countries will be the victors in such contests and this clearly affects the trade strategies of small countries, including Australia. The relationship between free-trade and trade-war pay-offs in the context of a prisoner's dilemma is explored in this paper. It is shown why neither a favourable terms of trade movement, a flatter import demand curve nor a larger population is, on its own, a sufficient condition for a relative victory in a trade war. The implications for small country trade strategies are then discussed.

Introduction

Since the end of the Second World War, the emphasis of trade policy in most industrialized countries has been on the process of trade liberalization. The enlargement of the General Agreement on Tariffs and Trade (GATT) to encompass most of the world's trading nations as signators and its successive successful rounds of negotiations, the expansion and extension of the European Community (EC), the signing of the Australia-New Zealand Closer Economic Relations Trade Agreement and the advent of the Canada-U.S. Trade Agreement are all manifestations of the belief in benefits from trade liberalization.[1] In spite of the policy emphasis on liberalization, trade wars have received attention from trade theorists such as Johnson (1958), Rodriguez (1974), Dixit (1987), and Kennan and Riezman (1988). In the current climate under which international trade is being conducted, their work seems to be of more than casual interest.

Progress at the current Uruguay Round of GATT talks has been illusive as the major contending parties have attempted to find an acceptable compromise for trade in agricultural commodities. Of course, 'trade liberalization' has never been a generally accepted

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[1] While the formation of trade blocs typically represents a pro-liberalization response to the slow pace of multilateral negotiations, their formation does give rise to the possibility of bloc-to-bloc conflict.

policy for agricultural commodities except by a few countries which are heavily export-dependent. Instead, the general policy line has been to aggressively seek improved access for exports while at the same timtime protecting less competitive commodities from imports. Agricultural trade policy has largely served as an adjunct to the domestic policy goal of retaining or expanding the farm population. The results have been predictable — major distortions in output, farm sectors dependent on government support, continued upward pressure on budgetary allocations to agriculture and capitalization of program benefits into relatively fixed farm assets. The latter has made it particularly difficult for governments to abandon distortionary domestic policies but budgetary pressures encourage policy makers to seek 'solutions' to the agricultural problem. The result has been that trade policy directions in agriculture have been moving away from liberalization, which implies abandonment of output distorting policies, toward attempts to use trade instruments strategically to achieve specific policy ends.

While a compromise agreement on agriculture may now be possible at the Uruguay round, it or future rounds of GATT negotiations could clearly founder on the issue of agricultural trade. If an acceptable compromise on the limitation of output distorting subsidies cannot be found, it could mean the demise of the entire GATT system or an abandonment of GATT principles for trade in agricultural commodities (Kerr, 1988). If either of these results come to pass, agricultural trade policy will likely be conducted through bilateral or bloc-to-bloc negotiations with the strategic use of trade instruments as one of the main bargaining tactics.

Even if a compromise can be found which will keep trade in agricultural commodities within the GATT, it is not likely to require sufficient changes to output distorting policies to remove the pressures on governments to seek strategic solutions for individual problems. This is supported by the trends toward bilateral negotiation of 'voluntary export restraints', expanded use of non-tariff barriers and, in particular, the strategic use of contingency protection measures such as anti-dumping duties and countervail actions.

Basically, it would appear that trade policy, at least as it concerns agricultural commodities, may continue to move away from attempts at multilateral liberalization and toward trade warfare involving the strategic use of trade instruments. This suggests that policy makers should give increased attention to the opportunistic use of trade instruments when developing their negotiating strategies. One of the commonly held hypothesis is that small countries, such as Australia, New Zealand and Canada will be losers in any trade war with larger trading partners such as the EC, the U.S. or Japan. Since the acceptance of this hypothesis would affect the strategies employed in trade negotiations, the hypothesis needs to be examined in detail. Who are the victors and the vanquished in trade wars where two countries set their trade taxes optimally? In this paper an attempt is made to answer relevant questions such as: which country claims a relative victory in the sense that its per-capita welfare loss is less than that of its opponent; when does a country win an absolute victory in the sense that it is better off as a result of the trade war than it would be under free trade.

There are many interesting facets of the trade war problem. The focus in the paper is on the comparative static relationship between cooperative versus non-cooperative payoffs. Related game-theoretic issues have been discussed by Riezman (1982), Dixit (1987), Stahl and Turunen-Red (1990) and others. Of course, both in the theory and practice of international economic life, as well as in international politics, wars may be fought even when there can be no absolute winners. Trade policy strategies must often be formulated in the context of a prisoners' dilemma.[2] For this reason, the question of relative winners and losers appears to be extremely important. Moreover, as Dixit (1987) suggests, the margin of relative victory will affect the bargaining positions of the antagonists and, thereby, influence how far negotiated solutions depart from free trade. Nevertheless, the literature on trade-war pay-offs, such as Johnson (1958) and Kennan and Riezman (1988), seems to have focussed primarily on the requirements for an absolute victory.[3]

This paper breaks new ground by exploring the shortcomings of three seemingly plausible conjectures concerning the impact of trade wars on relative per-capita welfare. The first of these conjectures is that a beneficial change in world prices is necessary and sufficient for a country to achieve a relative victory. Even with equal populations and quasi-linear tastes, however, this *terms of trade hypothesis* proves to be incorrect because, with discrete optimal trade taxes, the share of the deadweight loss borne by each country also matters. The second appealing conjecture is that if a country has a flatter import demand curve than its opponent, this suffices to guarantee a relative victory.[4] Even under quasi-linear tastes, this *flat demand curve hypothesis* does not go through without restrictions on the populations of the opposing countries. The third intuitive conjecture is that countries with larger

[2] In a static trade war game, each country has an incentive to deviate from free trade and play its optimal tariff. Thus, in a Nash equilibrium both countries assess optimal tariffs and both countries will often be worse off (and at least one country must be worse off) than under free trade. In dynamic games, free trade may be a possible equilibrium (Dixit, 1987). Nonetheless, it is often very difficult for governments which face the possibility of defeat in future elections to credibly commit their countries to free trade (Stahl and Turunen-Red, 1990).

[3] Markusen and Wigle (1989) construct a computational general equilibrium model of Canada-versus-U.S. trade conflict which invites international comparisons between the countries' net 'gains' Mayer (1981) compares the inter-country effects of various tariff reduction schemes.

[4] Since the domestic prices of the two countries differ under the trade war regime, it is simpler to compare slopes than elasticities.

populations tend to suffer relatively less harm than their smaller opponents in trade wars.[5] The key limitation of this *large country hypothesis* is that, depending on the tastes and technologies, the larger country could conceivably be less able to substitute from foreign to domestic goods as a trade war causes price and income changes.

The comparative static exploration of the payoff structure of trade wars has several aspects which are of technical interest. Obviously, the onset of a trade war leads to discrete trade taxes. Consequently, changes in national utility could be calculated via consumer surplus methods if utility functions were quasi-linear, or by using equivalent (or compensating) variations in the more general case. Such measures of discrete welfare changes could be viewed as continua of infinitesimal comparative static steps since they could be calculated by: (i) integrating (compensated) import demand functions over the range of differential changes in domestic prices; and (ii) adding on the discrete change in income from trade taxes. This approach ties in very naturally with the usual comparative static procedures for infinitesimal trade tax changes and also serves to emphasize the well-known point that welfare changes depend on distortionary losses and world price changes. Nevertheless, there are difficulties in obtaining further results from such an approach. Since the optimum trade tax is not parametric, the limit of integration for the price changes, as well as the change in income from trade taxes, is endogenous. It is this difficulty of needing to know the final equilibrium values of domestic and world prices which has led previous authors, including Johnson (1958) and Kennan and Riezman (1988), to place rather severe restrictions on functional forms. Once functional forms are specified, however, the fundamental theorem of calculus may as well be applied to calculate the changes in welfare directly.

Although quasi-linear utility functions will be assumed for simplicity in this paper, the *synthetic* comparative static procedure developed to analyze the changes in payoffs does not require restrictive functional forms. This synthetic procedure involves considering a continuum of hypothetical environments in which the free-trade and trade-war regimes co-exist.[6] In such hypothetical mixed regimes, a given fraction of the free-trade volume of trade is exchanged at free-trade prices and is exempt from trade taxes but any *additional trade* takes place at going world prices and is subject to trade taxes.[7] The

[5] For a recent example see Vanzetti and Kennedy (1989).

[6] This approach to comparative statics is *synthetic* both in the sense that the continuum of regimes are hypothetical or *artificial* and in the sense that the pure free-trade and trade-war regimes are being *mixed* or synthesized.

[7] A referee has observed that there are many examples of real-world mixed trading regimes such as tariff quotas which are quite different from the hypothetical mixed regimes utilized in this paper. In these cases, tariff-exempt trade typically takes place at the going world price rather than the world price that would prevail under pure free trade.

total impact of a trade war can be viewed as if it were the result of the gradual reduction of the free-trade component over the fixed interval from one hundred to zero per cent. Not surprisingly, the ongoing changes in trade taxes and trade-tax income during this reduction typically give rise to complex income effect terms. Nevertheless, as in many areas of cost-benefit analysis, the situation where tastes are quasi-linear and income effects are absent provides an insightful base case.

In section 2, a duality trade model similar to that of Dixit and Norman (1980) is modified by the introduction of hypothetical mixed regimes in which an exogenous fraction of trade occurs under free-trade conditions while the remainder takes place under trade-war conditions. Regime dependent trade-tax reaction functions are also derived. In section 3, the Nash equilibrium effects of infinitesimal regime shifts on domestic prices are considered and then discrete changes in domestic and world prices are determined by integrating over the full interval of regime mixes (i.e., from zero to 100 per cent free trade). In section 4, the absolute and relative welfare gains of the two countries are presented as continua of infinitesimal comparative static steps and the requirements for an absolute victory and the less stringent requirements for a relative victory are analyzed. Conclusions follow in the final section.

The Model

Net Expenditure, Import Demand and the Equilibrium World Price

Suppose that two countries, Lowersia and Uppernia, engage in international trade. Good zero will be numeraire in both countries and, as a further normalization convention, good zero will always remain untaxed in both countries.[8] In Lowersia, the per-capita consumption levels of the numeraire and non-numeraire goods are c_0 and c, the production levels are x_0 and x, and the import volumes are m_0 and m. The corresponding variables for Uppernia are denoted using upper-case letters. It will be assumed that Lowersia exports the numeraire good and imports the non-numeraire good so that $m_0 < 0$ and $m > 0$.[9] In order to avoid unnecessary complications, it will be assumed that both countries always consume strictly positive quantities of the numeraire good.

The per-capita national product or revenue functions of Lowersia and Uppernia are $g(p)$ and $G(P)$, where p and P are the domestic prices of the non-numeraire good. Of course, derivatives of these per-capita

[8] See Dixit and Norman (1980, 149-150).

[9] For expositional purposes, the non-numeraire good will be treated as a scalar, but, by treating the non-numeraire *good* as a vector (with positive and negative elements), the extension to many goods could be readily accomplished.

national product functions with respect to prices are the per-capita supply functions, $x = g_p(p)$ and $X = G_P(P)$. In order to avoid the complications which would arise from income effects, it will be assumed that preferences are quasi-linear. Consequently, the per-capita gross expenditure functions of Lowersia and Uppernia take on the quasi-linear forms, $u + e(p)$ and $U + E(P)$, where u and U are per-capita utility levels.[10] Differentiation of these per-capita gross expenditure functions with respect to price yields per-capita compensated demand functions for the non-numeraire good, $c = e_p(p)$ and $C = E_P(P)$ which depend only on prices.

The quasi-linear per-capita net expenditure functions, $u + f(p)$ and $U + F(P)$, are calculated by deducting per-capita production revenue from per-capita gross expenditure.

(1) $$u + f(p) \equiv u + e(p) - g(p)$$

where: $f_p(p) = c - x = m, f_{pp} < 0$

(2) $$U + F(P) \equiv U + E(P) - G(P)$$

where: $F_p(P) = C - X = M, F_{PP} < 0.$

Differentiation of these per-capita net expenditure functions with respect to domestic prices yields per-capita import demand functions which are independent of utility (and income) levels. Of course, Lowersia's and Uppernia's per-capita import substitution effects, f_{pp} and F_{PP}, must be negative and, thus, the slopes of the import demand curves are always negative.

In this paper, trade wars are assumed to be fought in a deterministic setting[11] with specific, or per-unit, trade taxes,[12] but the method and the qualitative conclusions appear to be more general. Lowersia's

[10] In the case of many-person economies, it could be assumed that governments pursue programs of lump-sum income redistribution that maximize the Bergson-Samuelson social welfare functions (Boadway and Bruce, 1984, 16-19) of the form, $w = nu$ and $W = NU$, where n and N are the populations of the two countries. Alternatively, the implicit representative-consumer simplification could be defended by assuming that all individuals within a country have the same tastes, the same endowments and equal claims to any tariff revenue.

[11] For simplicity, we will abstract from uncertainty. For recent treatments of the conduct of trade policy under uncertainty, see Feenstra (1987) and Bagwell and Staiger (1990). Cooper and Riezman (1989) examine how uncertainty affects the optimal choice of trade policy instruments under oligopoly. A referee has pointed out that, in practice, a country entering a trade war may deploy a wide variety of instruments because the effectiveness of each possible instrument of protection is typically uncertain.

[12] It is well known, both in the theory of industrial organization and in international trade, that when agents interact strategically, the type of instruments or weapons which are used have a bearing on the outcome of the game. In the international realm, Rodriguez (1974) has shown the non-equivalence of tariff-war equilibria, where some trade may persist, and quota-war equilibria, where all trade is eliminated. Chan (1988) attempts to endogenize the choice of instruments in a two-stage game, and Copeland (1989) examines tariff-versus-quota issues in negotiations to restrict the scope of trade wars.

domestic price is equal to the world price, π, plus the Lowersian per-unit import tax, t, and Uppernia's domestic price is equal to the world price minus the Uppernian export tax, T. Let the populations of Lowersia and Uppernia be denoted by n and N respectively. World equilibrium requires that the aggregate import demands of the two countries sum to zero.

$$(3) \qquad nf_p(\pi + t) + NF_P(\pi - T) = 0.$$

Since tastes are quasi-linear, there is a unique world equilibrium price determined by any given values of the two trade taxes.

$$(4) \qquad \pi = \Pi(t, T)$$

$$\text{where:} \qquad 0 \geq \Pi_t = \frac{-nf_{pp}}{nf_{pp} + NF_{PP}} = -1, \;\; 0 \leq \Pi_T = 1 + \Pi_t \leq 1.$$

An increase in the Lowersian import tax must reduce the world price of the non-numeraire good and an increase in the Uppernian export tax must raise the world price, but in both cases the induced change in the world price is of a smaller magnitude than the tax change.

Income and Utility in Mixed Trading Regimes

The synthetic comparative static analysis of the impact of a trade war will make use of hypothetical mixed regimes in which some trade takes place at free-trade prices and some trade is subject to tax. To begin with, let $v = nm = -NM$ denote the aggregate trade volume for the non-numeraire good. In a particular mixed regime, fraction $\theta \in [0,1]$ of the free-trade volume of trade, $\bar{v}$, is tax exempt and must exchange at the free trade world price, $\bar{\pi}$. This infra-marginal free-trade component of the total trade volume can be dubbed *administered* trade. Any additional *autonomous* or market-determined trade, $\mu \equiv v - \theta\bar{v}$, flows across the border at the going world price, π, and is subject to the optimal trade taxes of both countries.

It could be the case, for example, that the Uppernian government buys $\theta\bar{v}$ units of the non-numeraire from Uppernian suppliers at the Uppernian domestic price, P. The Uppernian government could then fulfil a contract to sell these goods to the Lowersian government at the administered price, $\bar{\pi}$. Finally, the goods obtained under contract could be resold to Lowersian buyers at the Lowersian domestic price, p. Any government deficits or surpluses would, of course, be offset by lump-sum taxes or subsidies. Regardless of the institutional details, it is obvious that a pure trade-war regime will exist when $\theta = 0$. The equally intuitive proposition that a pure free-trade regime will exist when $\theta = 1$ must be formally justified later, after optimal trade taxes have been derived.

Suppose that y and Y represent the per-capita net (non-production) incomes of the two countries. For any mix of regimes (i.e., where $0 \leq \theta \leq 1$), Lowersia's per-capita net (non-production) income consists of the trade-tax revenue it earns on any additional trade, plus the imputed income from infra-marginal trade of $\theta\bar{v}$ units of the non-numeraire good which are acquired at the free-trade price, $\bar{\pi}$, rather than Lowersia's marginal valuation, p.

$$(5.0) \qquad y = t\,[m - \frac{\theta\bar{v}}{n}] + [p - \bar{\pi}\,]\,\frac{\theta\bar{v}}{n}$$

$$(5.1) \qquad = tm + \frac{[\pi - \bar{\pi}\,]\theta\bar{v}}{n}.$$

Lowersia's per-capita net income can also be viewed as if it comprised tax revenue on the total, as opposed to taxable, trade volume adjusted by $[\pi-\bar{\pi}]\theta\bar{v}$ which, in effect, is a unilateral transfer from Uppernia to Lowersia. Since Lowersia is constrained by the trading behaviour of Uppernia and by the response of the world price to its trade tax specified in equation (4), it is useful to rewrite equation (5.1) in the following form

$$(6) \qquad y = \frac{-tNF_p(\Pi(t,T)-T) + [\Pi(t,T)-\bar{\pi}\,]\theta\bar{v}}{n}.$$

The expression for Uppernia's net income is derived through an equivalent procedure.

$$(7) \qquad Y = \frac{Tnf_p(\Pi(t,T) + t) - [\Pi(t,T)-\bar{\pi}\,]\theta\bar{v}}{N}.$$

Since each country's national budget constraint requires that its net expenditure be equal to its net income, $y = u + f(\Pi(t,T) + t)$, and $Y = U + F(\Pi(t,T) - T)$. Inversion of these relationships yields the quasi-linear indirect utility functions, $u = y - f(\Pi(t,T) + t)$ and $U = Y = F(\Pi(t,T) - T)$. Equations (6) and (7) allow the following re-statements of the indirect utility functions.

$$(8) \qquad u = \frac{-tNF_p(\Pi(t,T) - T) + [\Pi(t,T) - \bar{\pi}\,]\theta\bar{v}}{n} - f(\Pi(t,T) + t)$$
$$\equiv h(t;T, n, N, \theta)$$

$$(9) \qquad U = \frac{Tnf_p(\Pi(t,T) + t) - [\Pi(t,T) - \bar{\pi}\,]\theta\bar{v}}{N} - F(\Pi(t,T) - T)$$
$$\equiv H(T;\, t, N, n, \theta).$$

The utility levels, u and U, can be interpreted as measures of per-capita real income since tastes have been assumed to be quasi-linear and marginal utilities of income must remain constant.

Optimum Trade Taxes, Reaction Functions and the Nash Equilibrium

Each country exerts monopoly power over the autonomous flow of trade. In other words, for any exogenous value of the regime mix parameter, each country maximizes its utility by choosing its optimum trade tax conditional on its opponent's trade tax.[13,14]

$$(10.0) \qquad h_t(t; T, n, N, \theta) = \{NF_P + \theta\,\bar{v} - t\,NF_{PP}\}\frac{\Pi_t}{n} = 0,$$

$$\text{and} \qquad h_{tt} = h_{tT} - \frac{N\Pi_t F_{PP}}{n} < 0$$

$$\text{where:} \qquad h_{tT} = \{F_{PP} - t\,F_{PPP}\}\frac{N\Pi^2_t}{n}, \; h_{t\theta} = \frac{\Pi_t\bar{v}}{n} < 0$$

$$(11.0) \qquad H_T(T; t, N, n, \theta) = \{nf_p - \theta\bar{v} + Tnf_{pp}\}\frac{\Pi_T}{N} = 0,$$

$$\text{and} \qquad H_{TT} = H_{Tt} + \frac{n\Pi_T f_{pp}}{N} < 0$$

$$\text{where:} \qquad H_{Tt} = \{f_{pp} + Tf_{ppp}\}\frac{n\,\Pi^2_T}{N}, \; H_{T\theta} = -\frac{\Pi_T\bar{v}}{N} < 0\,.$$

The equations and inequalities in (10.0) and (11.0) are the first- and second-order conditions for the optimum trade taxes for Uppernia and Lowersia. The second-order conditions imply that when welfare is maximized, a country's optimal tax must become larger if its terms of trade become more favourable. The first-order conditions can be rearranged in order to determine optimal trade tax formulae.[15]

[13] It is important to note that if the first-order conditions are re-interpreted in a many-good context, intersectoral linkages become vital. In particular, the optimum trade taxes would be influenced by cross-substitution effects on import demand (i.e., f_{pp} and F_{PP} would be matrices in which the off-diagonal elements were cross-substitution effects).

[14] These national welfare optimization exercises abstract from any income distributional constraints or tax revenue constraints which face governments. Clearly such constraints are pervasive in reality but incorporating them into the model would result in unnecessary complexity. For an alternative, endogenous approach to trade policy, in which an attempt is made to model the political process, see Mayer (1984), Magee, Brock and Young (1989), and Stahl and Turunen-Red (1990).

[15] These optimum tax formulae could easily be converted to the more orthodox, *ad valorem* tariff rate versus inverse elasticity form. For example, Lowersia's optimal *ad valorem* export tax would be:

$$\frac{t}{P} = \frac{v}{\mu}\frac{1}{1-\varepsilon},$$

where $\varepsilon \equiv 1 + \frac{v}{PNF_{PP}}$ is Uppernia's elasticity of demand for imports of the numeraire good. Of course, the ratio of total trade, v, to autonomous trade, μ, would be equal to one if all trade were autonomous. In his analysis of welfare changes arising from retaliation, Johnson (1958) assumes that the countries choose optimal *ad valorem* trade taxes and then drastically simplifies matters by assuming that these elasticities of demand for imports are constant.

(10.1) $t = -\mu(NF_{PP})^{-1}$

(11.1) $T = -\mu(nf_{pp})^{-1}$.

Given that the autonomous trade volume, $\mu \equiv v - \theta\bar{v}$, is positive, both countries' optimal trade taxes are non-negative.

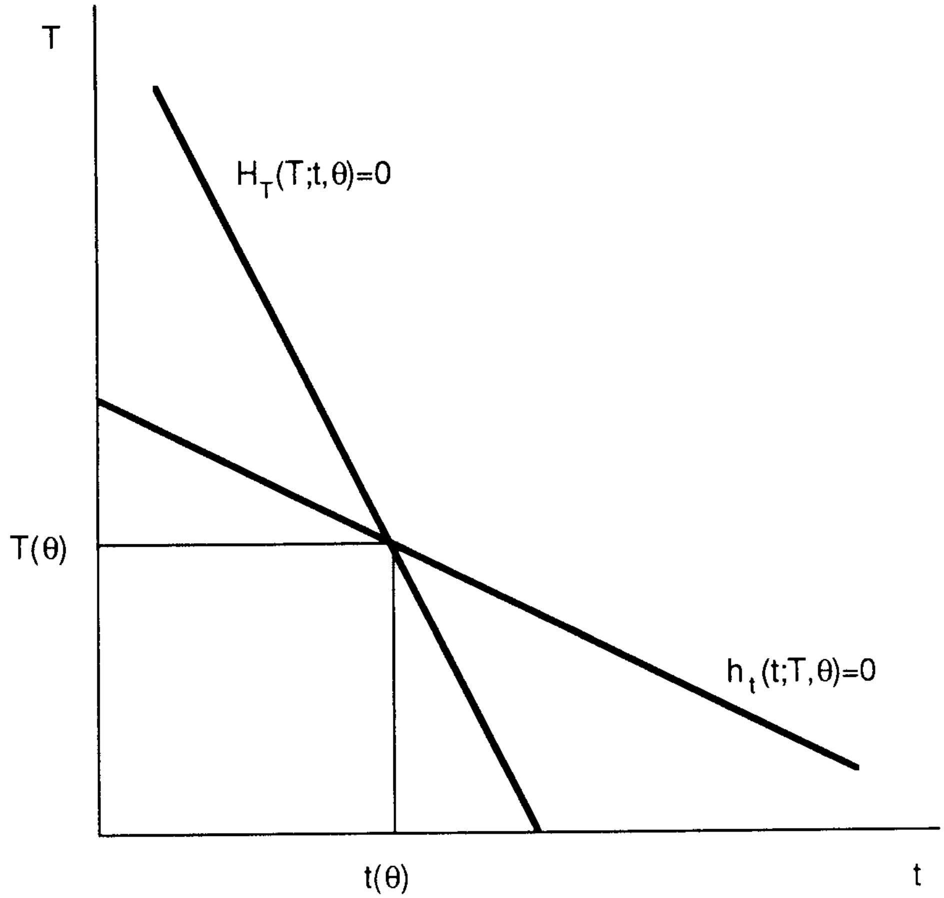
The fraction, θ, has been introduced in order to determine the mix between free-trade and trade-war regimes. On the one hand, it has been clear from the outset that a pure trade-war regime exists if and only if there is no administered trade (i.e., *iff* $\theta = 0$). On the other hand, in a pure free-trade regime both countries' trade taxes are equal to zero (i.e., $t = T = 0$) and the trade volume takes on its free-trade value (i.e., $\bar{v}$). These optimal trade taxes, however, are equal to zero if and only if there is no autonomous trade and 100% of the trade volume is administered (i.e., *iff* $\mu = 0$, $v = \theta\bar{v}$ and, therefore, θ=1).

The first-order conditions in equations (10.0) and (11.0) are implicit trade-tax reaction functions. It will be assumed that $F_{pp} - tF_{ppp} < 0$ and $f_{pp} + Tf_{ppp} < 0$ on a global basis which in turn implies that the trade taxes of the two countries must be strategic substitutes in the sense of Bulow, Geanakopolos and Klemperer (1985) (i.e., $h_{tT} < 0$ and $H_{Tt} < 0$). Although these assumptions are generally reasonable, they are more restrictive than the second-order conditions. Inspection of either country's reaction function reveals that an exogenous increase in its opponent's tax must reduce its own optimum tax and reduce it by a smaller magnitude (i.e., $-1 < -\frac{h_{tT}}{h_{tt}} < 0$ and $-1 < -\frac{H_{Tt}}{H_{TT}} < 0$). Possible configurations of the two countries' reaction functions are shown in Figure 1. Moves in the direction of pure free trade caused by an exogenous increase in the regime-mix parameter always shifts either country's reaction function such that its optimum trade tax falls for every possible value of its opponent's trade tax (i.e., $-\frac{h_{t\theta}}{h_{tt}} < 0$ and $-\frac{H_{T\theta}}{H_{TT}} < 0$). In other words, both countries' reaction functions would shift toward the origin in Figure 1.

In the Nash equilibrium corresponding to any particular value of the regime-mix parameter, θ, neither country has an incentive to change the height of its trade tax given the trade tax being assessed by its opponent. The Nash equilibrium values of each country's trade tax, $t = t(\theta)$ and $T = T(\theta)$, can be determined by simultaneously solving the two reaction functions, (10.0) and (11.0), as shown in Figure 1. The Nash equilibrium corresponding to any given value of the regime-mix parameter must be unique and stable because of the restrictions on the slopes of the two countries' reaction functions. The equilibrium trade taxes can be substituted into equation (4) in order to find the equilibrium world price, $\pi = \Pi(t(\theta), T(\theta)) - \pi(\theta)$. It is then straightforward to determine the Nash equilibrium values of domestic prices,

$p = \pi(\theta) + t(\theta) = p(\theta)$ and $P = \pi(\theta) - T(\theta) = P(\theta)$, and the total trade volume, $v = nf_p(p(\theta)) = -NF_P(P(\theta)) = v(\theta)$.

FIGURE 1
Trade-Tax Reaction Functions



The Price Changes Caused by Trade Warfare

The Impact of Infinitesimal Changes in the Mix of Regimes

The total trade tax wedge between the Uppernian and Lowersian domestic prices, $p(\theta) - P(\theta) = t(\theta) + T(\theta)$, falls monotonically as the regime mix parameter is increased and the regime moves in the direction of pure free trade.

$$(12) \qquad t_\theta(\theta) + T_\theta(\theta) = \frac{\bar{v}\delta(\theta)(nf_{pp} + NF_{PP})}{nf_{pp}NF_{PP}} < 0$$

where: $0 < \delta <(\theta) \equiv$

$$\frac{(f_{pp}F_{PP})(nf_{pp}+NF_{PP})}{(f_{pp}F_{PP})(nf_{pp}+NF_{PP})+n(f_{pp})^2\,(F_{PP}-t\,F_{PPP})+N(F_{PP})^2\,(f_{pp}+Tf_{ppp})} < 1\,.$$

Equation (12) is obtained by totally differentiating the first-order conditions in (10.0) and (11.0) and solving simultaneously. For stability $\delta(\theta)$ must be positive. This requirement is always met given the assumptions that $F_{PP} - tF_{PPP} < 0$ and $f_{pp} + Tf_{ppp} < 0$ for all values of θ. The fact that these assumptions also imply that $\delta(\theta)$ is a proper fraction serves to simplify the analysis at a later stage.

Given the relationship between the partial derivatives in equation (4), it can now be shown that the Lowersian domestic price must fall monotonically as the regime moves in the direction of pure free trade.

(13.0) $$p_\theta(\theta) = \left\{\Pi_t(t,T)+1\right\} t_\theta(\theta) + \Pi_T(t,T)T_\theta(\theta)$$

(13.1) $$= \Pi_T(t,T)\left\{t_\theta(\theta)+T_\theta(\theta)\right\}$$

(13.2) $$= \frac{\bar{v}\delta\,(\theta)}{nf_{pp}} < 0\,.$$

Analogous reasoning would establish that Uppernian domestic price must rise monotonically as the regime moves in the direction of pure free trade.

(14) $$P_\theta\,(\theta) = -\frac{\bar{v}\theta\,(\theta)}{NF_{PP}} > 0\,.$$

It is also possible to determine that the total trade volume rises monotonically as the regime moves in the direction of pure free trade.

(15) $$v_\theta(\theta) = \bar{v}\delta(\theta) > 0\,.$$

The Impact of Discrete Changes in the Mix of Regimes

The overall changes in domestic prices caused by the outbreak of a trade war can be calculated by integrating equations (13.2) and (14) over the interval from one to zero.

(16) $$\tilde{p} - \bar{\pi} \equiv p(0) - p(1) = -\frac{\bar{v}}{n}\int_0^1 \frac{\delta\,(\theta)}{f_{pp}}\,d\theta$$

(17) $$\tilde{P} - \bar{\pi} \equiv P(0) - P(1) = \frac{\bar{v}}{N}\int_0^1 \frac{\delta\,(\theta)}{F_{PP}}\,d\theta\,.$$

Here, the tilde symbol denotes the pure trade-war (i.e., $\theta = 0$) value of the indicated variables and the bar symbol continues to denote the pure free-trade (i.e., $\theta = 1$) value. It is possible to show (Gaisford, 1989) that if tastes are not quasi-linear, then complex income effect terms would also enter into equations (16) and (17).

It is well known that the onset of a trade war between optimizing countries must reduce the volume of trade but a trade war cannot reverse the direction of trade (see Johnson, 1958).

$$\tilde{v} - \bar{v} \equiv v(0) - v(1) = -\bar{v} \int_0^1 \delta(\theta)\, d\theta < 0 . \tag{18}$$

Since δ is always a positive, proper fraction and since the exchange mix parameter is integrated from zero to one, the trade volume cannot fall by more than the free trade volume of trade, $\bar{v}$.

The world price is equal to Lowersia's domestic price minus its import tax, and to Uppernia's domestic price plus its export tax. Moreover, the overall change in either country's trade tax is equal to the height of its pure trade-war optimum trade tax since trade taxes are absent under the pure free-trade regime (i.e., $\tilde{t} - 0 = -\tilde{v}(N\tilde{F}_{PP})^{-1}$ and $\tilde{T} - 0 = -\tilde{v}(n\tilde{f}_{pp})^{-1}$). Hence, the overall change in the world price is equal to the overall change in Lowersia's domestic price minus its optimum trade tax and to the overall change in Uppernia's domestic price plus its optimum trade tax (i.e., $\tilde{\pi} - \bar{\pi} = \tilde{p} - \bar{\pi} + \tilde{v}(N\tilde{F}_{PP})^{-1}$ and $\tilde{\pi} - \bar{\pi} = \tilde{P} - \bar{\pi} - \tilde{v}(n\tilde{f}_{pp})^{-1}$). Of course, any weighted average of these two methods of calculation is also acceptable.

$$\tilde{\pi} - \bar{\pi} = \tfrac{1}{2}\,[\,\tilde{v}\,[(N\tilde{F}_{PP})^{-1} - (n\tilde{f}^{-1}_{pp})] + [\,\tilde{p} - \bar{\pi}] + [\,\tilde{P} - \bar{\pi}\,]\,] \tag{19.0}$$

$$= \frac{(n\tilde{f}_{pp} - N\tilde{F}_{PP})\,\tilde{v}}{2nN\tilde{f}_{pp}\tilde{F}_{PP}} + \frac{\bar{v}}{2nN} \int_0^1 \frac{\delta(\theta)\,(nf_{pp} - NF_{PP})}{f_{pp}F_{PP}}\, d\theta . \tag{19.1}$$

Equation (19.1) was obtained by substituting equations (16) and (17) into (19.0).

It is useful to introduce simple definitions of relative size and strength.

Definition 1: A country is at least as large as its opponent if its population is at least as large.

Definition 2: A country is universally at least as strong as its opponent if the negative of its per-capita import substitution effect is of at least as large a magnitude for all values of the regime-mix parameter.

The simple notion of relative strength embodied in Definition 2 is based on the presumption that, *ceteris paribus*, being able to substitute easily between goods in order to escape foreign trade taxes should be advantageous in a trade war context.

An interesting result emerges from these simple formulations of relative size and strength.

Proposition 1: When both countries have quasi-linear utility functions, the world price cannot move against a country that is at least as large and universally at least as strong as its opponent.

Suppose that Lowersia is at least as large and universally at least as strong as Uppernia (i.e., $n \geq N$ and $-f_{pp} \geq -F_{pp}\ \forall\ \theta \in [\,0, 1\,]$). In such a situation, Lowersia's negative aggregate substitution effect must be of at least as large a magnitude as that of Uppernia for all values of the regime-mix parameter including a pure trade war (i.e., $nf_{pp} - NF_{pp} \leq 0 \; \forall \; \theta \in [0, 1]$). Consequently, the world price cannot rise because neither the initial relative-tax term nor any of the successive terms in the integral expression in equation (19.1) can be positive.

There is a simple corollary of Proposition 1.

Corollary 1.1: When both countries have quasi-linear utility functions, the world price cannot move against a country that has an aggregate import demand curve that is at least as flat as that of its opponent over the entire range of trade volumes from pure free trade to pure trade war.

If for all relevant trade volumes, Lowersia's aggregate import demand curve is at least as flat as that of Uppernia (i.e., $0 \leq (nf_{pp})^{-1} \leq -(NF_{pp})^{-1} \; \forall \; \theta \in [0,1]$) then, once again, Lowersia's aggregate substitution effect must never be of a smaller magnitude than that of Uppernia and the world price cannot rise.

Proposition 1 is important mainly in a negative sense. Even if a country has a population or size advantage over its opponent, it is by no means assured a favourable terms of trade change because relative strength is also important. Moreover, if tastes were not quasi-linear and complex income effect terms appeared in the analog to equation (19.1), the combination of simple size and strength advantages would be insufficient to guarantee a favourable world price movement.

To conclude this section, a word of warning is in order concerning the world price change equation (19.1). This equation does not indicate a monotonic relationship between increases in the fraction of the world population within a given country and more advantageous changes in the world price unless both countries happen to have linear per-capita import demand functions. In general, the values for the substitution effects of Lowersia and Uppernia arising in the Nash equilibrium associated with any particular value of the regime-mix parameter will depend upon the populations of the two countries. It is possible, however, to reconfirm a well-known asymptotic result. In the Appendix, the current methodology is used to show that a small price-taking country must experience a terms of trade deterioration in a trade war with an invulnerable opponent.

Victory and Defeat in Trade Wars

Preliminary Calculations of Real Income Changes

Since tastes have been assumed to be quasi-linear, a simple consumer surplus analysis will furnish exact (rather than approximate) measures of the changes in per-capita real income caused by a trade war.[16] For example, Lowersia's change in per-capita real income is equal to the change in net per-capita income from the advent of trade-tax revenue plus the change in per-capita net (or importer) surplus (i.e., the change in consumer's surplus plus the change in producer's surplus) from the change in the domestic price.[17]

$$(20.0) \qquad \tilde{u} - \bar{u} \equiv u(0) - u(1) = \frac{\tilde{t}\,\tilde{v}}{n} - \int_{\bar{\pi}}^{\tilde{p}} f_p(p)\,dp$$

$$(20.1) \qquad = -[\tilde{\pi} - \bar{\pi}]\frac{\tilde{v}}{n} - \int_{\bar{\pi}}^{\tilde{p}} \left(f_p(p) - \frac{\tilde{v}}{n}\right) dp\,.$$

Equation (20.1), which is obtained by adding and subtracting

$$\frac{\tilde{v}}{n}\int_{\bar{\pi}}^{\tilde{p}} dp = [\tilde{\pi} + \tilde{t} - \bar{\pi}]\frac{\tilde{v}}{n},$$

establishes a well-known intuitive result. The trade-war-induced changes in Lowersia's per-capita real income can be decomposed into the gain or loss of a *terms-of-trade rectangle* plus the loss of a *distortionary triangle*. In Figure 2, the terms-of-trade rectangle is gained by Lowersia (and lost by Uppernia) because the world price has fallen. Since this terms-of-trade gain is smaller than Lowersia's distortionary loss, Lowersian per-capita real income declines as a result of the trade war (i.e., given that $n = 1, \tilde{u} - \bar{u} = \tau - \lambda < 0$).

Analogous calculations can be made in order to determine how a trade war affects Uppernia's per-capita real income.

$$(21.0) \qquad \tilde{U} - \bar{U} = [\tilde{\pi} - \tilde{P}]\frac{\tilde{v}}{N} + \int_{\tilde{P}}^{\bar{\pi}} F_P(P)\,dP$$

$$(21.1) \qquad = [\tilde{\pi} - \bar{\pi}]\frac{\tilde{v}}{N} - \int_{\tilde{P}}^{\bar{\pi}} \left[-F_P(P) - \frac{\tilde{v}}{N}\right] dP\,.$$

In Figure 2, Uppernia experiences both a terms-of-trade loss and a distortionary loss as a result of the trade war, (i.e., given that $N = 1, \tilde{U} - \bar{U} = -\tau - \Lambda < 0$).

The Lowersia-versus-Uppernia difference in per-capita welfare, $\gamma \equiv [\tilde{u} - \bar{u}] - [\tilde{U} - \bar{U}]$, can also be compiled. Essentially, γ measures the change in the per-capita real-income gap between Lowersia and Uppernia.

[16] Recall that changes in utility levels can be interpreted as changes in per-capita real income because the quasi-linear utility functions have marginal utilities of income that are always equal to one.

[17] It is also noteworthy that in equation (20.0) the overall change in per-capita net income is equal to the per-capita trade-tax revenue collected in the pure trade-war regime. There is no additional imputed income because the entire trade volume exchanges at the going domestic price under either pure regime.

$$\gamma = \tilde{v}\left[\frac{\tilde{t}}{n} - \frac{\tilde{T}}{N}\right] - \int_{\bar{\pi}}^{\tilde{p}} f_p(p)\, dp - \int_{\tilde{P}}^{\bar{\pi}} F_P(P)\, dP \tag{22.0}$$

$$= -\frac{(n+N)\,\tilde{v}}{nN}\left[\tilde{\pi} - \bar{\pi}\right] - \int_{\bar{\pi}}^{\tilde{p}} \left[f_p(p) - \frac{\tilde{v}}{n}\right] dp - \int_{\tilde{P}}^{\bar{\pi}} \left[F_P(P) + \frac{\tilde{v}}{N}\right] dP . \tag{22.1}$$

Equation (22.0) compares the per-capita importer-surplus losses and the per-capita trade tax revenue gains of the two countries by using equations (20.0) and (21.0). Equation (22.1) compares the per-capita effects of the terms-of-trade changes and the per-capita distortionary losses of the two countries by utilizing equations (20.1) and (21.1).

It will become important to distinguish between absolute victory and relative victory in trade wars.

Definition 3: As a country experiences a per-capita real-income gain which is strictly greater than, exactly equal to, or strictly less than zero, it is said to have achieved an absolute victory, par, or an absolute defeat respectively.

Definition 4: As a country experiences a per-capita real-income gain which is strictly greater than, exactly equal to, or strictly less than that of its opponent, it is said to have achieved a relative victory, a tie, or a relative defeat respectively.

For example, Lowersia would obtain an absolute victory if $\tilde{u} - \bar{u} > 0$ and a relative victory if $\gamma > 0$. In Figure 2, both countries are absolute losers but Lowersia is the relative victor because it loses less per-capita real income than Uppernia (i.e., given that $n = N = 1, \gamma = 2\tau - \lambda + \Lambda > 0$).

There are three highly intuitive, well-known results that can be quickly restated in terms of these victory criteria for reference purposes (see Johnson, 1958 and Dixit, 1987).

Proposition 2: In a trade war, both countries cannot simultaneously achieve absolute victory, but they can both experience absolute defeat.

Corollary 2.1: A relative victory is a necessary, but not sufficient, condition for a country to achieve an absolute victory in a trade war.

Corollary 2.2: A favourable change in the terms of trade is a necessary, but not sufficient, condition for a country to achieve an absolute victory in a trade war.

These basic results follow immediately from an examination of equations (20.1), (21.1) and (22.1) as well as Figure 2.[18] Both countries

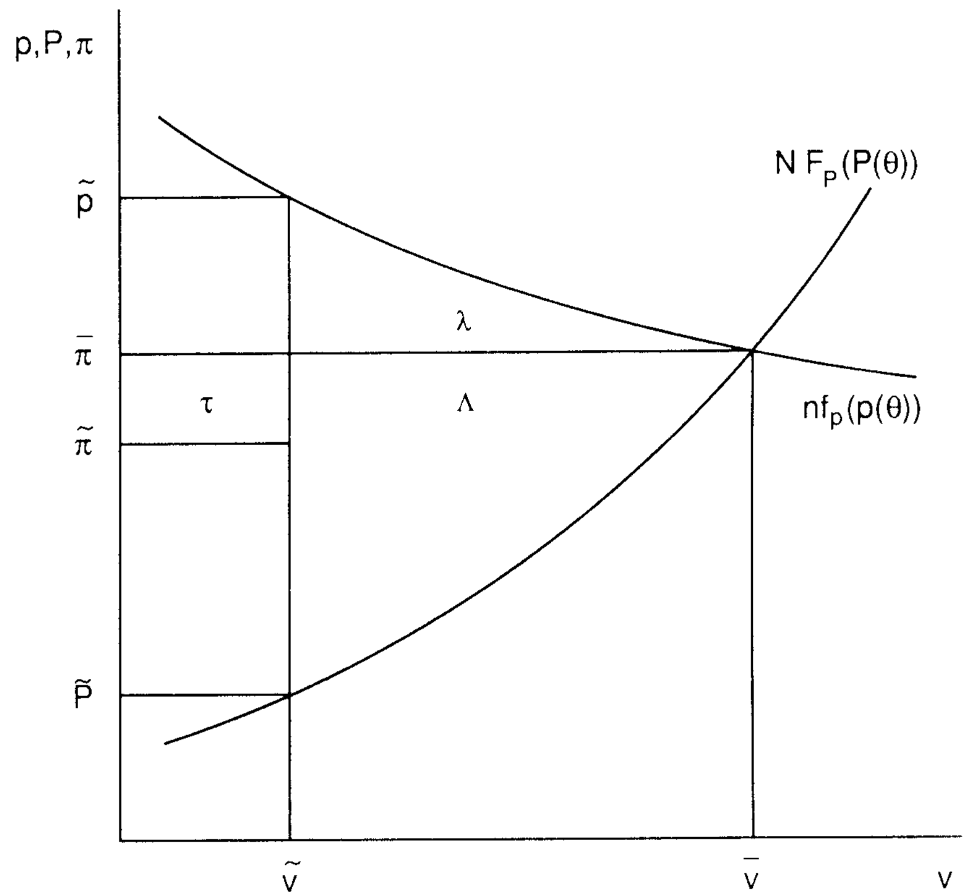
[18] Uppernia cannot win an absolute victory since world prices have moved in favour of Lowersia. In spite of the world price change and in spite of the fact that Lowersia's import demand curve is at least as flat as that of Uppernia for all relevant trade volumes, Lowersia achieves only a relative victory and not an absolute victory.

experience distortionary losses, and the only source of possible gain is from the change in the terms of trade.

Hence, trade wars may be Prisoners'-Dilemma games (see Riezman, 1982 and Dixit, 1987). For a one-shot game, each country's dominant strategy is to impose its optimal trade tax given any particular trade tax which is being levied by its opponent. Thus, a state of trade war is a Nash equilibrium even though the cooperative solution of free trade could be more desirable for both countries. Since such mutually detrimental trade wars continue to be possible Nash equilibria even if the game is repeated, it is extremely important to discover which country loses the least and emerges as the relative victor. It is now possible to make use of the dependence of prices on the regime-mix parameter, θ, in order to make more systematic observations concerning the impact of a trade war on first the absolute and then the relative levels of welfare of the two countries.

FIGURE 2

The Incidence of a Trade Ware with Equal Populations ($n = N = 1$)



Absolute Victory and Defeat in Trade Wars

The real-income changes of Lowersia and Uppernia can be reformulated in terms of changes in the regime mix parameter.[19] Equations (13.2) and (14) can be used in equations (20.1) and (21.1) respectively, and equation (19.1), which determines the overall change in world prices, can be used in both equations (20.1) and (21.1).

$$\tilde{u} - \bar{u} = -\frac{(n\tilde{f}_{pp} - N\tilde{F}_{PP})\tilde{v}^2}{2n^2 N\tilde{f}_{pp}\tilde{F}_{PP}} + \frac{\bar{v}}{2nN}\int_0^1 \left(\frac{\delta(\theta) f_p}{f_{pp}F_{PP}}\right)\left(\frac{2NF_{PP}}{nf_{pp} + NF_{PP}} - \frac{\tilde{v}}{nf_p}\right) d\theta \tag{23}$$

$$\tilde{U} - \bar{U} = \frac{(n\tilde{f}_{pp} - N\tilde{F}_{PP})\tilde{v}^2}{2n^2 N\tilde{f}_{pp}\tilde{F}_{PP}} - \frac{\bar{v}}{2nN}\int_0^1 \left(\frac{\delta(\theta) f_p}{f_{pp}F_{PP}}\right)\left(\frac{2nf_{pp}}{nf_{pp} + NF_{PP}} + \frac{\tilde{v}}{NF_P}\right) d\theta . \tag{24}$$

These equations demonstrate that it is possible to view the absolute changes in per-capita welfare in both countries as continua of infinitesimal comparative-static steps.

Equations (23) and (24) certainly do not indicate general monotonic relationships between larger populations and more favourable welfare changes. For one thing, a larger population serves to disipate any aggregate gains as well as to spread any aggregate losses. Furthermore, the equilibrium pairing of Lowersia's and Uppernia's substitution effects depends upon the populations of the two countries unless both countries have linear per-capita import demand functions. Nevertheless, well-known asymptotic results can be re-confirmed using the current approach. In the Appendix, it is shown that a trade war must be welfare reducing for a small price-taking country but must not be welfare reducing for its invulnerable opponent.

In less lopsided trade wars the welfare changes are less obvious. Nevertheless, sufficient conditions for absolute victory and defeat can be formulated based on equations (23) and (24).[20]

[19] The infinitesimal domestic price changes for Uppernia and Lowersia are $dp = p_\theta(\theta)d\theta$ and $dP = P_\theta(\theta)d\theta$. Turning to the limits of integration, the free trade price is $\bar{\pi} = \pi(1)$, which is in turn equal to $\bar{p} = p(1) = \bar{P} = P(1)$, and the trade war prices are $\tilde{p} = p(0)$ and $\tilde{P} = P(0)$.

[20] Recall that the two countries' aggregate import demands add up to zero for all values of the regime-mix parameter (i.e., $nf_p(p(\theta)) + NF_P(P(\theta)) = 0 \ \forall \ \theta \in [0,1]$).

$$\tilde{u} - \bar{u} > 0 \text{ if } \frac{\tilde{v}}{n f_p} > \frac{2}{1 + \frac{n f_{pp}}{N F_{PP}}} \quad \forall\ \theta \in [0,1] \tag{25}$$

$$\tilde{U} - \bar{U} > 0 \text{ if } \frac{\tilde{v}}{n f_p} > \frac{2}{1 + \frac{N F_{PP}}{n f_{pp}}} \quad \forall\ \theta \in [0,1]\,. \tag{26}$$

The ratio of pure trade-war to current Lowersian imports, $\frac{\tilde{v}}{n f_p}$, on the left-hand side of these inequalities is strictly less than one for all values of the regime-mix parameter greater than zero and exactly equal to one under pure trade-war conditions, when the regime-mix parameter is equal to zero. Consequently, the magnitude of Lowersia's aggregate substitution effect must exceed that of Uppernia for all regimes if sufficiency condition (25) is to be met. Similarly, the magnitude of Uppernia's aggregate substitution effect must exceed that of Lowersia's for all regimes if sufficiency condition (26) is to be met. Since both conditions clearly cannot be met simultaneously, these absolute victory criteria are in conformity with Proposition 2. If the directions of inequality in conditions (25) and (26) are reversed, this is sufficient to ensure that Lowersia on the one hand, and Uppernia on the other, suffer a welfare loss or absolute defeat.[21] Of course, such sufficiency conditions for absolute losses are not mutually exclusive.

The following results, which are based upon inequalities (25) and (26), are intuitive extentions to our understanding that trade wars may be prisoners'-dilemmas.

Proposition 3: When both countries have quasi-linear utility functions, a country which is at least as large and universally at least as strong as its opponent need not be an absolute victor but its opponent must be an absolute loser.

Corollary 3.1: When both countries have quasi-linear utility functions, a country that has an aggregate import demand curve that is at least as flat as that of its opponent over the entire range of trade volumes from pure free trade to pure trade war need not be an absolute victor but its opponent must be an absolute loser.

If Lowersia happens to be larger and stronger than Uppernia, then Lowersia's aggregate substitution effect must be of a larger magnitude than Uppernia's for all values of the regime-mix parameter including a pure trade war (i.e., if $n \geq N$ and $-f_{pp} \geq -F_{PP}$ $\forall$ $\theta \in [0,1]$, then

[21] Since $\tilde{v}/f_p = 1$ in the final pure-trade-war state where $\theta = 0$, the sign on the relative tax term as well as the sign on the integral expression in equation (23) (or equation (24)) are reversed by changing the direction of inequality (25) (or inequality (26)).

$\frac{NF_{PP}}{nf_{pp}} \geq 1 \;\; \forall \; \theta \in \; [0,1]$). Similarly, if Lowersia's quasi-linear import demand curve is flatter than that of Uppernia (i.e., $-(nf_{pp})^{-1} \leq -(NF_{PP})^{-1} \;\; \forall \; \theta \in \; [0,1]$), then Lowersia's aggregate substitution effect must be of a larger magnitude than that of Uppernia for all regimes. In either case, the right-hand side of inequality (25) will not necessarily be less than $\frac{\tilde{v}}{nf_p}$ even though it will be less than one for all regimes. Neither being large and strong nor having a flatter import demand curve is a sufficient condition for Lowersia to secure an absolute victory even under the restrictive assumption of quasi-linear tastes. Uppernia, on the other hand, must experience an absolute loss if tastes are quasi-linear because the right-hand side of condition (26) will exceed one for all regimes. If tastes were not quasi-linear, the presence of income effects would require more complex sufficient conditions for absolute victory and defeat.

Relative Victory in Trade Wars

Since trade war games may have the form of a Prisoners' Dilemma where both countries suffer absolute losses, it is pertinent to determine which country loses the least and obtains a relative victory. Such a relative victory depends on the direction of the trade-war-induced change in the per-capita real-income gap between Lowersia and Uppernia. In order to analyze relative victory, equations (13.2) and (14) can be utilized to reformulate equation (22.0) in terms of the regime-mix parameter.

$$(27) \qquad \gamma = \frac{(\tilde{F}_{PP} - \tilde{f}_{pp})\tilde{v}^2}{nN\tilde{f}_{pp}\tilde{F}_{PP}} + \frac{\bar{v}}{(nN)^2}\int_0^1 \frac{nf_p(p)\delta(\theta)(N^2F_{PP} - n^2 f_{pp})}{f_{pp}F_{PP}}\, d\theta .$$

Here, the first term is a relative per-capita tax revenue term which favours the country with the larger per-capita optimum tax and the integral expression is a relative per-capita importer surplus term which is dependent on comparisons of the slopes of the per-capita import demand functions over the full interval of mixed regimes.

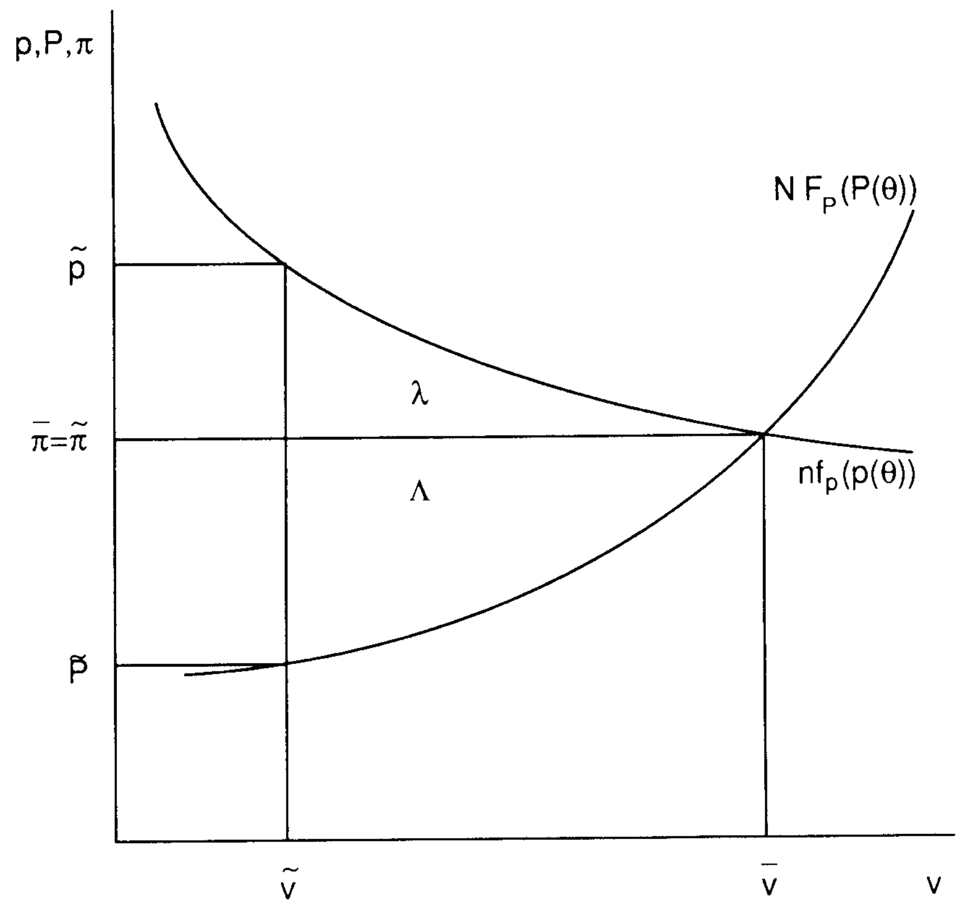
While it has been shown (by Corollary 1.2) that a favourable change in world prices is a necessary but not sufficient condition for an absolute victory in a trade war, there is no such connection between terms of trade changes and relative victory even under highly restrictive conditions.

Proposition 4: Even if tastes are quasi-linear and the two countries have equal populations, a favourable change in the terms of trade is neither a necessary nor sufficient condition for a country to achieve a relative victory in a trade war.

For convenience, assume that the population of each country is equal to unity. Comparison of the integral expression in relative victory, equation (27), with that in world-price change, equation (19.1), shows that each infinitesimal term in the former equation is weighted by the (negative of) the corresponding aggregate import demand. Since such import volumes are non-uniform across mixed regimes, the two integral expressions need not be of the opposite sign. For example, it is completely possible for the per-capita real-income gap to move in Lowersia's favour (i.e., $\gamma > 0$) at the same time that world prices move against Lowersia (i.e., $\tilde{\pi} - \bar{\pi} > 0$). The inescapable conclusion is that the *terms of trade hypothesis* is incorrect. An improvement in the terms of trade is not synonymous with a relative victory unless the distortionary triangles lost by each country happen to be of equal area. This result is reinforced in Figure 3, where Lowersia wins a relative victory in the trade war on the basis of losing a smaller distortionary triangle

FIGURE 3

The Incidence of the Trade-War Distortion with Equal Populations ($n = N = 1$ but Unchanged Terms of Trade)



than Uppernia even though the world price remains unchanged (i.e., $\gamma = \Lambda - \lambda > 0$ but $\tilde{\pi} - \bar{\pi} = 0$).[22]

It is possible to obtain a positive, but highly restrictive, result which relates relative victory in trade wars to the intuitive notions of relative size and strength.

Proposition 5: Given that both countries have quasi-linear utility functions, if one country is at least as large and universally at least as strong as its opponent, then it cannot sustain a relative defeat in a trade war. If either the size or strength relationship is strict, then the large and strong country wins a relative victory.

If Lowersia happens to be larger and stronger than Uppernia, then neither the relative per-capita tax revenue term nor the importer surplus term in equation (27) can be negative and Lowersia cannot experience a relative defeat (i.e., if $n \geq N$ and $-f_{pp} \geq -F_{PP} \forall\ \theta \in [0,1]$, then $\gamma > 0$).

The stringent conditions underlying Proposition 5 are not easily relaxed.

Corollary 5.1: Even when both countries have quasi-linear utility functions, a country that has an aggregate import demand curve that is at least as flat as that of its opponent over the entire range of trade volumes from pure free trade to pure trade war could sustain a relative defeat.[23]

Suppose that Lowersia's aggregate import demand curve were flatter than that of Uppernia and, therefore, that Lowersia's aggregate substitution effect were larger in magnitude than that of Uppernia (i.e. $(nf_{pp})^{-1} \leq (NF_{PP})^{-1} \leq 0\ \theta \in [0,1]$ which implies that $nf_{pp} - NF_{PP} \leq 0\ \forall \theta \in [0,1]$) . This condition certainly does not imply that the relative *per-capita* tax revenue term in equation (27) is positive. Moreover, because of the non-linear population effects, even the relative per-capita importer surplus term need not be positive. A flatter aggregate import demand would assure that the aggregate, but not the per-capita, real-income loss of Lowersia was less than that of Uppernia. Hence, the *flat demand curve hypothesis* is also faulty. Even with quasi-linear

[22] Notice that at the trade-war trade volume, $\tilde{v}$, Lowersia's import demand curve is flatter than Uppernia's export supply curve and, therefore, the height of Uppernia's optimal tax is larger than that of Lowersia's (i.e., $\tilde{T} \geq \tilde{t}$. Furthermore, because the Lowersian import demand curve is flatter than the Uppernian export supply curve at the free-trade volume of trade, $\bar{v}$, it is possible for the change in the Uppernian domestic price to be of a larger magnitude than the change in the Lowersian domestic price. Such a construction, by equation (19.1), opens the possibility that there is no overall change in world prices. Despite the fact that the world price has remained unchanged, Lowersia has won a relative victory in the trade war on the basis of losing a smaller distortionary triangle than Uppernia.

[23] When income effects are significant, the changes in trade taxes, which are the hallmark of trade warfare, cause shifts in compensated import demand curves which would be utilized to analyze welfare changes. This would render the flat demand curve hypothesis entirely inoperable.

tastes, a country with a flatter aggregate import demand curve does not necessarily lose less per-capita real income than its opponent unless the populations of the two countries are at least equal.

A flatter per-capita, as opposed to aggregate, import demand curve is also insufficient to ensure a relative victory because the population parameters enter into the relative per-capita importer surplus term in equation (27).

Corollary 5.2: Even when both countries have quasi-linear utility functions, the relative results of a trade war are inconclusive when one country is at least as large as its opponent but no stronger.

Suppose that Lowersia is universally stronger than Uppernia, but Uppernia is larger than Lowersia (i.e., $-f_{pp} \geq -F_{PP} \forall\ \theta \in [0,1]$ but $N \geq n$). Even though the per-capita relative tax revenue term in equation (27) favours Lowersia under these conditions, the change in the per-capita real-income gap is ambiguous because the sign of the relative per-capita importer surplus term is ambiguous. While the per-capita import demand curve is central to the assessment of welfare changes, the aggregate import demand curves are the key determinants of the price changes that generate those welfare changes.

Clearly, Corollary 5.2 implies that the *large country hypothesis* is also incorrect. Even with quasi-linear tastes, the more populous country is not guaranteed a relative victory. The structure of tastes and technologies could certainly imply that the larger country is less able to substitute from foreign to domestic goods as trade warfare restricts the trade volume. It should also be emphasized that equation (27) does not even indicate a general monotonic relationship between a country's size and its margin of relative victory. Unless both countries happen to have linear per-capita import demand functions, the equilibrium pairing of substitution effects for each mixed regime depends upon the two populations. Bigger is not necessarily better.

Conclusions

The synthetic comparative static procedure which makes use of a continuum of hypothetical mixed regimes could potentially be useful in other areas of economics, such as industrial organization, where agents interact strategically. Indeed, a two-country trade war can be viewed as an example of bilateral monopoly. In the international vein, it would appear that the addition of by-stander countries could produce further interesting results in a many-good world. For example, it seems to be conceivable that both combatants could gain absolutely from a trade war if they were to shift away from by-stander exports to a sufficient degree. Foreign investment trade wars could also be systematically examined using the synthetic comparative static approach.

The synthetic comparative static approach has been used in this paper to study discrete optimum trade taxes in much the same way as infinitesimal trade taxes. When discrete, as opposed to infinitesimal, taxes are introduced to an undistorted environment, however, trade restriction is no longer a zero-sum game. Thus, the incidence of the distortions is important in addition to the induced changes in prices. Moreover, if tastes were not assumed to be quasi-linear, money-metric measurements would be required to analyze the discrete changes in welfare (Gaisford, 1989).

A number of intuitive results have been confirmed concerning the effect of trade wars on the terms of trade and levels of absolute welfare. The question of relative welfare changes is also crucial because trade policy is often conducted in a Prisoners'-Dilemma context and because the relative payoffs are likely to influence how closely trade treaties resemble free trade. It has been shown that overly hasty 'theoretical' pronouncements concerning relative victory are extremely dangerous. Neither a favourable movement in the terms of trade, a flatter import demand curve, nor a larger population is, on its own, a sufficient condition for even a relative victory in a trade war.

It is clear then that an alternative policy to avoidance may be available to small country negotiators when faced with the possibility of threats of strategic trade war by large country negotiators. Certainly, only the possibility of an alternative outcome to the normal large country hypothesis has been presented. Providing quantitative estimates of changes to trade regimes is seldom easy but it should be no more difficult in the trade war case than in the case of liberalization. Of course, great faith has been put in the benefits of liberalization by policy makers, no matter how poor the empirical evidence as to the benefits. The theoretical results of this paper suggest that, in the face of a possible further shift away from liberalization in the trade in agricultural commodities, there should be a similar shift in research by applied economists. Without the relevant information, strategic opportunities may be missed.

Appendix

Trade Warfare Between a Small Price-Taking Country and an Invulnerable Opponent

The methodology of the paper is capable of generating well-known results for the extreme case where one combatant is an archetypical small price-taking country and the other is *invulnerable*. Let a country be defined to be: (i) strictly invulnerable if its population is finite and its per-capita substitution effect approaches infinity for all values of the regime-mix parameter, and (ii) weakly invulnerable if its aggregate substitution effect approaches infinity for all values of the regime-mix parameter. For example, Lowersia is strictly invulnerable if $-f_{pp} \rightarrow \infty$ $\forall\ \theta \in [0,1]$ and Lowersia is weakly invulnerable if $-nf_{pp} \rightarrow \infty\ \forall\ \theta \in [0,1]$. Lowersia would be strictly invulnerable if it possesses either perfect substitutes in production (e.g., a Ricardian technology) which is diversified under free trade, or perfect substitutes in consumption which is diversified under free trade. Furthermore, Lowersia would be weakly, but not strictly, invulnerable if its population were to approach infinity.

It is shown by equations (16) and (17) that if a country is at least weakly invulnerable, then its domestic price remains unaltered in response to a trade war. If Lowersia is invulnerable, then Lowersia's domestic price is independent of the mix of regimes according to equation (13.2) (i.e., if $-nf_{pp} \rightarrow \infty\ \ \forall\ \theta \in [0,1]$ then $p_\theta(\theta) = 0$) and, therefore, it is shown in equation (16) that a trade war would not change Lowersia's domestic price. Since Uppernia is a small price-taking country, it would have a lower domestic price, which is equivalent to a lower marginal benefit of its export, according to equation (17). From equation (19.1) it can be concluded that the trade-war induced change in the world price must favour invulnerable Lowersia (i.e., $\tilde{\pi} - \bar{\pi} < 0$) . In this situation, Uppernia is a small price-taking country having an optimal trade tax which is equal to zero, but Lowersia assesses a positive tax in order to exploit its monopoly power.

The welfare changes that emerge from equations (23) and (24) are equally intuitive. A weakly invulnerable country cannot suffer an absolute defeat in a trade war and its opponent must be an absolute loser. Furthermore, a strictly invulnerable country must obtain an absolute victory in a trade war. If Lowersia is weakly invulnerable, then there must not be a reduction in Lowersian welfare and there must be a reduction in Uppernian welfare (i.e., if $-nf_{pp} \rightarrow \infty\ \forall\ \ \theta \in [0,1]$, then $\tilde{u} - \bar{u} \geq 0$ and $\tilde{U} - \bar{U} < 0$). If Lowersia is strictly invulnerable, then Lowersia must receive a welfare gain (i.e., if $-f_{pp} \rightarrow \infty,\ \forall\ \ \theta \in [0,1]$, the $\tilde{u} - \bar{u} > 0$). In either case, Lowersia would experience no distortionary loss and a terms of trade gain. If Lowersia's invulnerability were attributable to a population which approached infinity, however, Lowersia would only succeed in maintaining par because its finite terms of trade gain would be dissipated over its infinite population. In any case, small price-taking Uppernia would clearly be an absolute loser since it would experience a terms trade loss and a distortionary loss.

References

Bagwell, K. and Staiger, R. (1990), 'A theory of managed trade,' *American Economic Review*, 80 (4), 779-795.

Boadway, R. W. and Bruce, N. (1984), *Welfare Economics*, Blackwell, Oxford.

Bulow, J., Geanakopolos, J. and Klemperer, P. (1985), 'Multimarket oligopoly: strategic substitutes and complements,' *Journal of Political Economy*, 93 (3), 488-511.

Chan, K.S. (1988), 'Optimum trade policies and retaliation,' *Canadian Journal of Economics*, 21 (2), 277-288.

Cooper, R. and Riezman, R. (1989), 'Uncertainty and the choice of trade policy under international oligopoly,' *Review of Economic Studies*, 56 (1), 129-140.

Copeland, B.R. (1989), 'Tariffs and quotas with two instruments of protection,' *Journal of International Economics*, 26 (1/2), 179-186.

Dixit, A.K. (1987), 'Strategic aspects of trade policy,' in T. Bewley ed., *Advances in Economic Theory, Proceedings of the 5th World Congress of the Econometrics Society*, Cambridge University Press, Cambridge UK.

Dixit, A.K. and Norman, V. (1980), *Theory of International Trade*, Cambridge University Press, Welwyn UK.

Feenstra, R. (1987), 'Incentive compatible trade policies,' *Scandinavian Journal of Economics*, 89 (3), 373-387.

Gaisford, J. (1989), 'Which country loses the least in a trade war?' Department of Economics, University of Calgary, Discussion Paper 122.

Johnson, H. G. (1958), 'Optimum tariffs and retaliation,' in H.G. Johnson, *International Trade and Economic Growth*, Harvard University Press, Cambridge.

Kennan, J. and Riezman, R. (1988), 'Do big countries win tariff wars?,' *International Economic Review*, 29 (1), 81-85.

Kerr, W. A. (1988), 'International trade in beef — technical issues for the current GATT negotiations,' *Journal of Agricultural Taxation and Law*, 10 (1), 55-66.

Magee, S., Brock, W. and Young, L. (1989), *Black Hole Tariffs and Endogenous Trade Theory*, Cambridge University Press, Cambridge UK.

Markusen, J. R. and Wigle, R. M. (1989), 'Nash equilibrium tariffs for Canada and the United States: the roles of country size, scale economies and capital mobility,' *Journal of Political Economy*, 97 (2), 368-386.

Mayer, W. (1981), 'Theoretical considerations on negotiated tariff adjustments,' *Oxford Economic Papers*, 33 (1), 135-153.

Mayer, W. (1984), 'Endogenous tariff formation,' *American Economic Review*, 74 (5), 970-985.

Riezman, R. (1982), 'Tariff retaliation from a strategic point of view,' *Southern Economic Journal*, 48 (3), 583-593.

Rodriguez, C. (1974), 'The non-equivalence of tariffs and quotas under retaliation,' *Journal of International Economics*, 4 (3), 295-298.

Stahl, D. and Turunen-Red, A. (1990), 'Tariff games: sustainability of free trade with variations in policy regimes,' University of Texas at Austin, Center for Economic Research, Discussion Paper No. 90-03.

Vanzetti, D. and Kennedy, J. (1989), 'Optimal retaliation in international commodity markets,' *Review of Marketing and Agricultural Economics*, 57 (1,2,3), 93-117.