ON-FARM FACTORS INFLUENCING INVESTMENT IN CROP SOWING MACHINERY*

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Farmers in grain industries make important decisions about investment in crop sowing machinery. This paper shows how some on-farm factors affect profit-maximising levels of investment in crop sowing machinery. The paper examines the effect on optimal investment of discontinuities in sowing opportunities, varietal portfolios and soil portfolios.

Introduction

Malcolm (1994) has observed:

The key to continuing to be a farmer is to get the big decisions on land purchase, machinery investment and resource improvement right;… (p.19)

In Australian dryland agriculture investment in crop sowing machinery usually requires the purchase of tractors, tillage and seeding equipment and spray units. In making decisions about the purchase of such equipment a panoply of advice and information is often available to farmers. Although rarely giving advice directly, agricultural economists have developed concepts and analytical tools that explain or facilitate these investment decisions.

For example, the issue that has attracted most attention in the literature on machinery selection has been the impact of timeliness costs on machinery use and investment (Van Kampen 1971, Tulu et al. 1974, Hughes and Holtman 1976, Danok et al. 1980, Edwards and Boehlje 1980, Whan and Hammer 1985, Wetzstein et al. 1990). In many cropping systems the sowing and harvesting of crops requires the timely use of machinery. Delays to sowing or harvesting operations can greatly reduce crop yields. In many grain growing regions timeli-

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ness costs arise through a combination of machinery and labour capacity, edaphic, weather and crop physiology effects. Soil conditions can deteriorate to delay or prevent crop sowing (e.g. waterlogging). Weather conditions can spoil crops not yet harvested and crops can suffer yield loss through delays to sowing, spraying or harvest. Investing in larger machinery capacity and more labour reduces timeliness costs, but machinery ownership and labour costs are greater.

Although timeliness costs have received the most attention in the literature, other influences upon investment decisions such as taxation (Reid and Bradford 1983, Smith 1990), the lumpiness of investment decisions (Danok et al. 1980), changes in technologies (Stoneham and Ockwell 1981, Epplin et al. 1982), investment allowances (Chisolm 1974, Vanzetti and Quiggin 1985) and crop sowing tactics (Bathgate 1993) have all received some attention. No model reported in the literature claims to examine all factors likely to affect machinery selection. The objective of most studies has been to identify the optimal set or level of investment in cropping machinery, usually given ceteris paribus assumptions about other factors known to affect machinery investment (e.g. McIsaac and Lovering 1976, Danok et al. 1978).


An important contribution of this paper to the literature on appropriate cropping machinery investment is an analysis that allows for varietal and soil portfolios. As commercial farms in the grain industries continue to grow in size (Longmire 1995) and grain segregation increases, it will become more common for farms to maintain a portfolio of grain varieties and to be characterised by a diversity of soils. Hence the linkage between varietal portfolios, soil type diversity and machinery investment may become increasingly relevant to many farmers. This paper describes how discontinuities in crop sowing opportunities and the availability of varietal and soil portfolios can affect optimal investment in crop sowing machinery.

The next section introduces the basic model for determining the profit maximising investment in crop sowing machinery. This model is used to illustrate in the following section how discontinuities in sowing opportunities and varietal and soil portfolios can influence

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1 Risk neutrality is assumed throughout this paper.
investment in crop sowing machinery. In final sections of the paper sensitivity analysis is conducted and conclusions are drawn.

The Basic Model

Kingwell (1995) used an algebraic model of profit from crop production to show how discontinuities in crop sowing opportunities and varietal and soil portfolios affect optimal investment in crop sowing machinery. Another approach adopted here is to use the same profit model but to use integration to express profit.

Before presenting the model, the biology of crop sowing is described briefly. In dryland agriculture typically crop yields are a function of the day of sowing as shown in Figure 1. When there are no machinery breakdowns, no logistic hold-ups and soils remain workable, then continuous sowing is possible. In this case the set of sowing days coincides with a set of consecutive calendar days and as shown in Figure 1, crop yield can be a linear function of the day of sowing. In practice a linear decline in crop yield is often observed in field trial and crop growth simulation studies of crop yield, given continuous sowing opportunities (e.g., Delane and Hamblin 1989, Western Australian Department of Agriculture 1992). This linearity assumption therefore is incorporated in the model.

**FIGURE 1**
**Dryland Wheat Yield Responses to Day of Sowing**

Crop yield as a linear function of sowing day can be represented as:

\[
Y_x = Y_s - cx
\]

where

- \(Y_x\) is the crop yield (t/ha) resulting from sowing on day \(x\) of sowing, assuming continuous sowing opportunities
- \(Y_s\) is the yield (t/ha) on the area sown to crop on the first day of sowing (\(x=0\)) and
c is the rate of yield decline per day’s (t/ha/day) delay in crop sowing.

For the example in Figure 1, \( Y_s = 1.8 \) t/ha, \( c = 0.06 \) t/ha/day and not shown in Figure 1 is \( x_y = Y_s/c = 30 \) days, where \( x_y \) is the sowing day for which yield is zero.

Consider a simple case where a farmer wishes to select investment in crop sowing machinery so as to maximize profit from crop production. The farmer sows continuously, a single variety on only one soil class. Profit from crop production is:

\[
\pi = pQ - hS - m - g
\]

where

- \( \pi \) is profit ($) from sowing \( S \) hectares of the crop,
- \( p \) is the price ($/t) of the crop,
- \( h \) is the production costs per hectare ($/ha),
- \( m \) is the fixed overhead costs ($) associated with planting \( S \) hectares of the crop,
- \( g \) is the cost of investment ($) in seeding gear, and \( Q \) is the grain production (t) from the \( S \) hectares of crop.

In equation 2, \( Q \) is a function both of the yield parameters in equation 1 and the crop sowing machinery’s work rate (R) which is the average number of hectares sown each day during the sowing of \( S \) hectares of the crop. To facilitate exposition of the model, \( g \) is assumed to be a simple linear function of \( R \): \( g = a + bR \). This assumes the marginal cost of work rate, \( b \), is constant so that the cost of acquiring an additional unit of sowing capacity is the same across different sizes of sowing gear.

For a decision-maker with a fixed area (S) to be sown, the decision problem is to determine \( R \) so that profit \( (\pi) \) is maximized. Profit in equation 2 can be re-expressed with \( Q \) a function of yield decline and work rate as:

\[
\pi = p \int_0^{s/R} R(Y_s - cx)dx - a - bR - hS - m
\]

\[
= p\left[Y_s - \frac{cS^2}{2R}\right] - a - bR - hS - m
\]

The first-order condition for maximum profit with respect to \( R \), ignoring any constraints on \( R \), is:

\[
\frac{d\pi}{dR} = p\frac{cS^2}{2R^2} - b = 0
\]
which implies

$$R_{opt} = S\sqrt{\frac{pc}{2b}}$$

Thus the optimal work rate is directly related to the area sown (S), positively related to the price of the crop (p) and the yield forgone by sowing one day later (c), and negatively related to the marginal cost of the machinery work rate (b).

This result needs to be modified to reflect the impossibility of negative yield. As equation 4 stands it is feasible for the optimal number of days of sowing, x_{opt} = S/R_{opt}, to be greater than the sowing day for which yield is zero. To avoid the possibility of negative yield, the profit-maximization problem must be subject to the constraint that x ≤ Y_s/c, or equivalently R (= S/x) ≥ Sc/Y_s. This leads to the result:

$$R_{opt} = \text{Max} \left( S, \frac{pc}{2b}, \frac{Sc}{Y_s} \right)$$

Note that for the interior solution of (4), R_{opt} is not dependent on Y_s, the yield of the variety on the earliest possible day of sowing.

### On-farm Factors affecting Investment in Crop Sowing Machinery

**Discontinuities in the Opportunity to Sow Crops**

In Mediterranean dryland environments rainfall events signal the start of the sowing period. These rainfall events provide opportunities for tillage of the soil and control of germinating weeds. Whereas the basic model assumes sowing is continuous, in practice rainfall patterns are not always sufficiently regular to provide continuous opportunities for sowing (Kerr and Abrecht 1992). Sometimes the amount and pattern of rainfall allows only a few days of crop sowing before soil profiles become so wet that paddocks cannot be trafficked by farm machinery, so a delay in crop sowing occurs. Conversely, rainfall can be inadequate, only permitting a few days of crop sowing before soil profiles become dry or hard, again prohibiting adequate seed bed preparation (Wetzstein et al. 1990).

Such discontinuities in sowing opportunities are typical of dryland agriculture and complicate a farmer’s decision about investment in crop sowing machinery. For example, investing in larger gear will enable more crop to be sown sooner, resulting often in higher yields but at cost of higher investment in machinery.

It is worth noting that discontinuities in sowing opportunities may also arise, not just from weather events, but also from machinery.
breakdown or accident or the ill-health of machinery operators. Further, agronomic considerations may introduce delays in crop sowing. For example, sowing in paddocks containing herbicide resistant weeds may be delayed because of the need to kill these weeds by cultivation.

Discontinuities in sowing opportunities alter the set of feasible days for crop sowing. For a given investment in crop sowing machinery (R) and area sown to crop (S), discontinuities cause production to decline and variance of production to increase\(^2\). Areas of crop are sown later resulting in lower yields. The effect of discontinuities can be incorporated in the model by increasing the yield decline parameter c. For example, if the land can only be accessed every other day, c is doubled. In this case, equation 5 shows that sowing capacity should be increased by a factor of \(\sqrt{2}\) or 1.4.

**Varietal Portfolio**

Thus far, the case of a farmer growing a single variety has been considered. But what if a farmer has access to two varieties? In the following analysis it is assumed it is technically feasible to sow either of two varieties at any stage in the sowing programme, that there are sufficient seed stocks of each variety, that seed costs are the same and that changeover costs are negligible.

![Graph: Yield Responses to Day of Sowing for Two Varieties](image)

**FIGURE 2**

*Yield Responses to Day of Sowing for Two Varieties*

Figure 2 illustrates the case of two varieties with yields as different linear functions of the day of sowing. The yield equations are:

\[ Y_1 = Y_{1e} - \alpha \theta \]

\[ Y_2 = Y_{2e} - \alpha \theta \]

\( Y_{1e}, Y_{2e} \) are the base yields of the two varieties.

\( \alpha \) is the yield decline parameter.

\( \theta \) is the day of sowing.

\( \alpha \theta \) is the yield decrease.

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\(^2\) The increased variance of production increases the variance of profit and would encourage a risk averse farmer to invest even more in sowing capacity in the presence of discontinuities.
\[ Y_1 = Y_{s1} - ex \quad \text{for variety 1}; \quad \text{and} \]
\[ Y_2 = Y_{s2} - fx \quad \text{for variety 2} \]

where \( Y_1 \) and \( Y_2 \) are the yields of varieties 1 and 2 when sown on day \( x \), with the rate of yield decline being \( e \) and \( f \) for each variety respectively and \( e > f \). \( Y_{s1} \) and \( Y_{s2} \) are the yields of variety 1 and 2 respectively when sown at the commencement of crop sowing and \( Y_{s1} > Y_{s2} \). The sowing day for which yields of varieties 1 and 2 are equated is \( x_{y1} = (Y_{s1} - Y_{s2})/(e-f) \). After this day variety 2 outyields variety 1. Not shown in Figure 2 is day \( x_{y2} = Y_{s2}/f \) for which yield for variety 2 is zero.

A profit-maximizing decision-maker with a fixed crop area \((S)\) and varieties of equal expense in use, will always first sow with that variety which initially is higher yielding. The decision problem in the case of two varieties as shown in Figure 2 is:

\[
\text{Max } \pi = pR \left[ \int_{x_{y1}}^{x_{y1}} (Y_1 - ex) \, dx + \int_{0}^{S/R} (Y_2 - fx) \, dx \right] - a - bR - hS - m
\]

\[
= pR \left\{ [Y_{s1} x - e x^2] \bigg|_{x_{y1}}^{x_{y1}} + [Y_{s2} x - f x^2] \bigg|_{x_{y1}}^{S/R} \right\} - a - bR - hS - m
\]

\[
= pR \left\{ Y_{s1} x_{y1} \frac{e}{2} x_{y1}^2 + Y_{s2} \frac{S}{R} - \frac{fS^2}{2R^2} - Y_{s2} x_{y1} \right\}
\]

\[
+ \frac{fx_{y1}^2}{2} - a - bR - hS - m.
\]  

The first-order condition for an interior maximum is:

\[
\frac{d\pi}{dR} = p(x_{y1}(Y_{s1} - Y_{s2}) - x_{y1}^2 \frac{(e-f)}{2} + \frac{fS^2}{2R^2}) - b = 0
\]

which implies the optimal rule:

\[
R_{opt} = S \sqrt[2b - p(Y_{s1} - Y_{s2})^2]{p(f)} \frac{p(f)}{(e-f)}
\]

Equation 7 gives the optimal work rate provided this implies non-negative crop yields, and that both varieties are sown. If it is optimal to sow both crops, then the number of sowing days for crop 1 is \( x_{y1} \) and yield for variety 1 is positive. To ensure yield for variety 2 is also positive, it is necessary that \( x_{opt} < x_{y2} \), where \( x_{opt} \) is the total number of sowing days for both varieties. Thus the optimal work rate for both varieties sown, and for positive yields, is:
\[ R_{opt} = \text{Max} \left( S \left[ \frac{pf}{2b} - \frac{p(Y_{S1} - Y_{S2})^2}{e-f} \right] \frac{Sf}{Y_{S2}} \right) \]

However, it must be checked that it is optimal to sow both varieties. This can be effected by determining whether the number of sowing days for variety 1 alone is less than \( x_{y1} \) using the single variety rule:

\[ R_{opt} = S \sqrt{\frac{pe}{2b}} \]

If it is, then in this case the single variety rule applies to the two variety problem.

**Soil Portfolio**

Commonly on larger farms cropping occurs on more than one soil class. Physical differences between soils can affect the power requirements of crop sowing machinery, and the yield response to the time of sowing can differ across soils. Hence, the nature and mix of soils to be cropped can influence the optimal investment in crop sowing machinery. This is illustrated for the case of two soils, a single variety and fixed sowing programmes on each soil, where \( S_1 \) and \( S_2 \) are the hectares sown to crop on each soil respectively. Figure 3 illustrates possible yield declines associated with the time of sowing on each soil class.

**FIGURE 3**

*Yield Responses to Day of Sowing on Two Soil Classes*

![Graph showing yield responses to day of sowing on two soil classes.](image-url)
The work rate of crop sowing machinery on soil 1 may be some proportion \( r \) of the work rate on soil 2, so that \( R_2 = r R_1 \). The size of \( r \) depends on the physical characteristics of each soil and the crop preparation techniques employed. For example, work rates on heavy clay soils are generally less than work rates on light sandy soils and crop establishment rates involving multiple machinery passes are less then those involving a single or few machinery passes (Pannell and Bathgate, 1994).

No unique sowing opportunities are assumed to exist for either soil. This means it is technically feasible to sow on either soil at any time in the sowing programme. The decision on which soil class to commence crop sowing is addressed in the Appendix. Essentially the optimal decision rule is to commenced crop sowing on whichever soil class displays the greater rate of yield decline according to time of sowing.

Let the equations for the yield schedules shown in Figure 3 be:

\[
Y_1 = Y_{11} - jx \quad \text{for sowing on soil class 1 and}
\]

\[
Y_2 = Y_{12} - kx \quad \text{for sowing on soil class 2 where}
\]

\( Y_1 \) and \( Y_2 \) are the yields of crops on soil classes 1 and 2 when sown on day \( x \), with the rate of yield decline on each soil class being \( j \) and \( k \) with \( j > k \). \( Y_{11} \) and \( Y_{12} \) are the yields on soil class 1 and 2 respectively for crop sowing on the earliest possible day and \( Y_{11} < Y_{12} \).

The decision problem is to maximize profit through selection of \( R_{opt} \) and by planting first on soil class 1 if \( j > r k \) or planting first on soil class 2 if \( r k > j \) (see Appendix). In the former case profit is:

\[
\pi = \left[ \frac{p R}{S_i/R} \right] \left\{ \left( \int_0^{S_i/R} (Y_{11} - jx) dx \right) + \left( \frac{(S_i/R + S_2/R)}{S_i/R} \right) \left( \int_0^{S_i/R} (Y_{12} - kx) dx \right) \right\}
\]

\[
- a - bR - h(S_i + S_2) - m
\]

\[
= \left[ \frac{p R}{S_i/R} \right] \left\{ \frac{Y_{11} S_i}{R} - \frac{j S_i^2}{2R^2} + \frac{Y_{12} (S_i + r^{-1} S_2)}{R} - \frac{k (S_i + r^{-1} S_2)^2}{2R^2} \right\}
\]

\[
- \frac{Y_{11} S_i}{R} + \frac{k S_i^2}{2R^2} - a - b R - h(S_i + S_2) - m
\]

The first-order condition for maximum profit with respect to \( R \), ignoring any constraints on \( R \), is:

\[
\frac{d\pi}{dR} = \frac{p}{2} \left( j S_i^2 R^{-2} + k (S_i + r^{-1} S_2)^2 R^{-2} - k S_i^2 R^{-2} \right) - b = 0
\]

which implies
\[ R_{opt} = \sqrt{\frac{p(jS_1^2 + 2r^{-1}kS_1S_2 + r^{-2}kS_2^2)}{2b}} \]

As before, \( R_{opt} \) needs to be constrained to ensure yields are non-negative. Constraints on the number of sowing days are:

\[ x_{1opt} \leq \frac{Y_{1i}}{j} \text{ and } (x_{1opt} + x_{2opt}) \leq \frac{Y_{2i}}{k} \]

where \( x_{1opt} \) and \( x_{2opt} \) are the optimal number of sowing days on soil class 1 and 2 respectively. Recognizing that \( R_{opt} = S_i/x_{1opt} = S_2/x_{2opt} \) leads to the optimal rule:

\[ R_{opt} = \text{Max} \left[ \sqrt{\frac{p(jS_1^2 + 2r^{-1}kS_1S_2 + r^{-2}kS_2^2)}{2b}}, \frac{S_{1i}}{Y_{1i}}, \frac{k(S_1 + r^{-1}S_2)}{Y_{2i}} \right] \]

for the case where sowing on soil class 1 occurs first. The optimal rule when \( rk > j \), and sowing on soil class 2 occurs first, is:

\[ R_{opt} = \text{Max} \left[ \sqrt{\frac{p(r^{-2}kS_2^2 + 2r^{-1}jS_1S_2 + jS_1^2)}{2b}}, \frac{j(S_1 + r^{-1}S_2)}{Y_{1i}}, \frac{S_{2i}k}{Y_{2i}} \right] \]

Interestingly, \( R_{opt} \) interior solutions do not depend on \( Y_{1i} \) and \( Y_{2i} \) but do depend on the relative rates of yield decline on soil classes 1 and 2 (j and k).

To illustrate the effect of soil portfolios upon optimal investment in work rate for crop sowing, consider the following parameter values: the on-farm price of the crop (p) is $125 per tonne, the marginal cost of the machinery's work rate for sowing (b) is $250 per hectare per day, the rates of yield decline (j, k) during the sowing interval are 0.030 and 0.015 tonnes per hectare per day respectively, the crop yields on the first possible day of sowing on soil class 1 (\( Y_{1i} \)) and soil class 2 (\( Y_{2i} \)) are 1.5 and 1.6 tonnes per hectare respectively, soil class 1 area (\( S_1 \)) and soil class 2 area (\( S_2 \)) both are 500 hectares and the work rates of sowing machinery on each soil class are the same (r=1). In this case \( R_{opt} \) is 68.4 hectares per day. Note that if 1000 hectares of soil class 1 only were sown then \( R_{opt} \) would be 86.6 hectares per day. Hence, in this example access to cropping on another soil class that is characterised by a lesser yield decline reduces \( R_{opt} \).

If the work rate of the cropping machinery were to differ between the soil classes then \( R_{opt} \) could increase or decrease depending on the work rate relativity. For example, if \( r \) equals 0.5 then \( R_{opt} \) increases to 96.8 hectares per day. In this case the sowing work rate on soil class 2 is half that on soil class 1, and in spite of the lesser rate of yield decline on soil class 2, investment in work rate increases.
Elasticity of Investment in Work Rate

The sensitivity of \( R_{opt} \) to parameter changes can be gauged from the elasticity of \( R_{opt} \) with respect to parameter values. For the case of one variety and one soil class, and assuming an interior solution, the elasticities are independent of parameter values. Thus from optimality condition (4), the elasticities with respect to price of the crop \( (p) \), yield forgone by sowing one day later \( (c) \), the marginal cost of work rate \( (b) \) and area sown \( (S) \) are \( \varepsilon_{R_{p}} = 0.5 \), \( \varepsilon_{R_{c}} = 0.5 \), \( \varepsilon_{R_{b}} = -0.5 \) and \( \varepsilon_{R_{S}} = 1 \). This means, for example, that if crop price increases by one per cent, the optimal work rate increases by only 0.5 per cent.

Given that prices of machinery have increased proportionately less than crop prices in recent years, this simple investment model would indicate an increase in the optimal investment in sowing capacity.

Further Considerations

The simple investment model also illustrates how plant breeding can influence investment in crop sowing machinery. Increasing the portfolio of varieties to reduce yield decline within a cropping programme can lessen the need for additional investment in work rate for crop sowing. This benefit from plant breeding usually is overlooked in most benefit-cost analyses of varietal improvement.

In practice, to apply the models already outlined would require collecting data relating to investment in cropping gear, crop areas and varietal yields according to time of sowing. For example, data for the relationships in equations (1) and (2) could be derived from crop growth simulation models (eg Robinson 1993), varietal trial data (Agriculture Western Australia 1994 and Garside 1979) and machinery survey data (e.g., Humphry 1987).

If other factors such as the effects of marginal tax rates and investment allowances (see Vanzetti and Quiggin 1985) and non-linearities in machinery cost and grain yield functions were to be considered then non-linear programming would be a preferable analytical tool. It is not surprising, given the many factors influencing machinery investment decisions, that programming models have often been employed in examining machinery use and investment (e.g., Gustafson 1993).

Conclusion

Using a simple model of profit from crop production it has been shown how various on-farm factors can influence investment in work rate for crop sowing. The effect on such investment of discontinuities in sowing opportunities and varietal and soil portfolios is examined. Results indicate that discontinuities in sowing opportunities increase the optimal work rate and therefore the level of investment in crop sowing machinery. Introducing a varietal portfolio, where a second variety is sown later in the sowing programme, reduces optimal investment
levels. Inclusion of a portfolio of soils within the sowing programme can increase or decrease the optimal investment in crop sowing machinery, depending on the characteristics of each soil and tillage technologies.

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Appendix

The Sowing Rule in the Case of Two Soil Classes

On many dryland farms the sowing of crops often involves sowing on more than one soil class. Optimal investment in the work rate of crop sowing machinery (R) involves deciding on which soil class to commence crop sowing. Consider the case of two soil classes with planted area requirements of \( S_1 \) and \( S_2 \) hectares respectively. For any particular \( R \), assuming \( R \) is the same for each soil class, this implies sowing for \( S_1/R \) days on one soil class and sowing for \( S_2/R \) days on the other. If sowing occurs first on soil class 1 then total production (\( TP_1 \)) is:

\[
TP_1 = \int_{0}^{S_1/R} (Y_{11} - jx)dx + \int_{S_1/R}^{(S_1+S_2)/R} (Y_{12} - kx)dx
\]

If sowing occurs first on soil class 2 then total production is:

\[
TP_2 = \int_{0}^{S_2/R} (Y_{22} - kx)dx + \int_{S_2/R}^{(S_1+S_2)/R} (Y_{11} - jx)dx
\]

Expanding the terms in equation A1 gives:

\[
TP_1 = Y_{11} \frac{S_1}{R} - \frac{jS_1^2}{2R^2} + Y_{12} \left( \frac{S_1}{R} + \frac{S_2}{R} \right) - k \left( \frac{S_1}{R} + \frac{S_2}{R} \right)^2 - Y_{12} \frac{S_1}{R} + \frac{kS_1^2}{2R^2}
\]

which simplifies to

\[
TP_1 = Y_{11} \frac{S_1}{R} + Y_{12} \frac{S_2}{R} - \frac{jS_1^2}{2R^2} - \frac{kS_1^2}{2R^2} - \frac{kS_1S_2}{2R^2}
\]

By similar argument if sowing occurs first on soil class 2 then total production is:

\[
TP_2 = Y_{11} \frac{S_1}{R} + Y_{22} \frac{S_2}{R} - \frac{jS_2^2}{2R^2} - \frac{kS_2^2}{2R^2} - \frac{jS_1S_2}{2R^2}
\]

The only difference between equations A3 and A4 is the last term in each equation which causes \( TP_1 > TP_2 \) if \( k < j \) and \( TP_2 > TP_1 \) if \( k > j \). Thus the decision rule for maximising total production is to sow first on whichever soil class displays the greater rate of yield decline.

In the case of two work rates, \( R_1 \) on soil class 1 and \( R_2 \) on soil class 2, with \( R_2 = rR_1 \), similar reasoning leads to the rule: sow first on soil class 1 if \( j > rk \); on soil class 2 otherwise.