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AN ECONOMIC RESPONSE MODEL OF HERBICIDE APPLICATION FOR WEED CONTROL

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A biologically realistic model of crop yield response to herbicide application is presented. It includes functions for weed mortality from herbicide application and yield loss due to surviving weeds. The optimal herbicide rate and two types of decision thresholds are derived theoretically and illustrated with empirical examples. Responses of the various decision criteria to changes in parameters are also examined theoretically and empirically. A multidimensional threshold for weeds based on weed density and weed-free yield is presented. The issue of farmers using other than officially recommended herbicide rates is discussed.

There is a large and diverse literature on the economics of controlling diseases and insect pests in agriculture (McCarl 1981; Osteen *et al.* 1981; Mumford and Norton 1984). By contrast, published economic analyses of weed control have been far less common and many of those published have been narrowly focused (Pannell 1988*a*). Analyses at the farm level have almost exclusively used a simple economic threshold framework in which the herbicide dosage to be used is taken as given and the problem is to determine what weed density is sufficient to justify application of the recommended herbicide dose. There has recently been an increase in the number of publications on weed economics (e.g. Cousens *et al.* 1986; Auld *et al.* 1987; Lybecker *et al.* 1988; Zanin and Sattin 1988; Marra *et al.* 1989; Pandey 1989) but the dominance of the economic threshold paradigm for farm-level analyses has continued. There has been no theoretical analysis of the economic properties of a realistic response model for herbicide application nor of the determinants of optimal herbicide dosages.

It is clear that herbicide dosages recommended on product labels are not economically efficient in many circumstances. Due to the status of these labels as legal contracts, recommended dosages must be set at levels sufficient to kill almost all weeds almost all of the time. They are not determined on the basis of an economic comparison of alternative rates. Pannell (1989) presents several arguments in favour of facilitating, or even encouraging, downward flexibility in herbicide dosages. These include:

(a) The economically optimal herbicide dose depends on herbicide efficacy which can vary widely under different environmental conditions (Jensen and Kudsk 1988).
(b) The optimal dose depends on factors such as the crop's yield potential, the price of output and the rotation practised (Abadi Ghadim and Pannell 1990).

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(c) Lower dosages tend to reduce the rate of development of resistance to the herbicide (Gressel and Segel 1982).
(d) Farmers differ in their attitudes to risk, so if herbicides have an influence on risk, different farmers may prefer different rates.

It is obviously true that 'the very simple if-then-else decision rule common in pesticide treatment recommendations cannot be more profitable than the marginal decision rule' (Moffitt 1988, p. 630).

This paper is concerned with the determinants, other than resistance and risk, of optimal herbicide usage. In the next section, a theoretical response model on biological relationships in the literature is presented. This is then used to derive equations for the optimal herbicide rate, the threshold weed density for herbicide application and the threshold crop yield. Following this are analyses of the effects on optimal rates and thresholds of changes in a range of biological and economic parameters. Finally the derived theoretical results are illustrated with empirical results for a particular herbicide problem: post-emergence application of Hoegrass® to control annual ryegrass (*Lolium rigidum*) in wheat in Western Australia.

The Model

Lichtenberg and Zilberman (1986*a*) showed that yield response to pesticides must be represented as an indirect response; pesticides kill pests and it is the removal of pests which increases yield. Use of a response model which does not recognise this two stage process leads to biased predictions of response and erroneous conclusions about the optimal pesticide strategy.

In this paper, crop yield (Y) is represented using the following general form.

(1) $Y = Y_0[1 - D(W)]$

where Y_0 is yield with no weeds present and D is the damage function representing the proportion of yield lost at weed density W.

Cousens (1985) conducted tests of a wide range of functional forms for the damage function. He found that the following hyperbolic form best fitted published data on weed competition:

(2) $D(W) = a/[1 + a/(bW)]$

The parameter a can be interpreted as the asymptotic yield loss as $W \rightarrow \infty$. Crops typically give some positive yield even at very high weed densities, so a is normally less than one. The parameter b is the proportional yield loss per weed as $W \rightarrow 0$.

W is a function of W_0, pre-treatment weed density, and $K(H)$, the proportion of weeds killed at herbicide rate H.

(3) $W = W_0[1 - K(H)]$

The kill function must be bounded by zero and one. It is often represented in the literature by the following exponential function (e.g. Feder 1979; Moffitt *et al.* 1984; Auld *et al.* 1987):

(4) $K(H) = 1 - \exp(-kH)$

Substituting (2), (3) and (4) into (1) gives the response function:

(5) $Y = Y_0(1 - a/\{1 + a/[bW_0\exp(-kH)]\})$

Profit (π) is given by

(6) $\pi = P_y Y - P_h H - A - F$

where P_y is output price, P_h is herbicide unit cost, A is herbicide application cost and F represents costs from all other inputs which are assumed to be fixed. A consists of costs of labour and machinery use which are incurred only if herbicide is applied but which are assumed to be independent of the application rate, H.

The Optimal Herbicide Rate

The first order condition for profit maximisation on herbicide rate is

(7) $$\frac{\partial \pi}{\partial H} = P_y \frac{\partial Y}{\partial H} - P_h = 0$$

$\frac{\partial Y}{\partial H}$ can be derived as follows. By the chain rule, $\frac{\partial Y}{\partial H} = \frac{\partial Y}{\partial W} \cdot \frac{\partial W}{\partial H}$.

Assuming that $\frac{\partial Y}{\partial W}$ is unaffected by H (as implied in the model above)

(8) $$\frac{\partial Y}{\partial W} = -Y_0 \cdot \frac{\partial D}{\partial W}$$

Now

(9) $$\frac{\partial W}{\partial H} = -k \cdot W_0 \cdot \exp(-k \cdot H) = -k \cdot W$$

Combining (8) and (9), substituting into (7) and rearranging gives

(10) $$H^* = \frac{1}{k}\left[\ln(P_y) + \ln(Y_0) + \ln\left(\frac{\partial D}{\partial W}\right) + \ln(k) + \ln(W_0) - \ln(P_h)\right]$$

It should be noted that this is not in closed form as $\frac{\partial D}{\partial W}$ depends on H^*. $\frac{\partial D}{\partial W}$ in (10) is the marginal yield loss per weed at the optimal herbicide rate, H^*. Although there is no closed form solution for H^* in this model, (10) can still be used to derive a number of useful results (see below). Also note that it is necessary to check for an interior solution. Equation (10) only gives the global optimal herbicide rate if profit at H^* is greater than profit with no herbicide applied. Costs of labour, fuel, oil, grease and machinery repairs and maintenance associated with the spraying operation [A in (6)] do not vary with the herbicide dosage. Therefore zero herbicide use will be more profitable than any non-zero rate at very low weed densities or in very low yielding crops.

The Threshold Weed Density

The derivation of the threshold density is quite different from the derivation of the optimal dosage. The latter requires marginal analysis, as demonstrated in the previous section, whereas the threshold is determined by comparing profits from two discrete input levels. The result of the marginal analysis is an input level, whereas the result of the threshold analysis is a weed density above which a fixed input level should be used. The threshold is derived as follows.

If the recommended herbicide rate (H_r) is used, profit is given by

$$(11) \quad \pi(H_r) = P_y \cdot Y_0 \cdot [1 - D(W_r)] - P_h \cdot H_r - A - F$$

where W_r is weed density surviving application of the recommended herbicide rate. If no herbicide is applied, profit is

$$(12) \quad \pi(0) = P_y \cdot Y_0 \cdot [1 - D(W_0)] - F$$

Without herbicide application, weed density is higher ($W_0 > W_r$) so the level of yield loss is greater. On the other hand, savings are made on herbicide costs ($P_h \cdot H_r$) and application costs (A). For treatment at the recommended rate to be at least as profitable as non-treatment, we need $\pi(H_r) \geq \pi(0)$. i.e.

$$(13) \quad P_y \cdot Y_0 \cdot [1 - D(W_r)] - P_h \cdot H_r - A - F \geq P_y \cdot Y_0 \cdot [1 - D(W_0)] - F$$

Rearranging gives

$$(14) \quad P_y \cdot Y_0 \cdot L \geq P_h \cdot H_r + A$$

where $L = D(W_0) - D(W_r)$.

In words, the value of yield loss avoided by applying the herbicide must equal or exceed the total cost of the application. The threshold density (W_0^T) is the lowest value of W_0 at which (14) is satisfied. That is, W_0^T is the value of W_0 which makes (14) an equality.

$$(15) \quad W_0^T = \frac{P_h \cdot H_r + A}{P_y \cdot Y_0 \cdot L / W_0}$$

This is the approach taken, either explicitly or implicitly in the threshold papers cited earlier. Although thresholds have sometimes been quoted in the biological literature as if they are fixed values for particular weeds, it is clear from (15) that this is not the case [as noted, for example, by Cousens (1987)]. Any of the variables or parameters on the right hand side of (15) can affect the threshold. Their influence will be further analysed below.

The Threshold Weed-Free Yield

In applications of the economic threshold concept, the usual assumption is that all parameters and variables in the system other than W_0 are fixed and known with certainty. Once information about the value of W_0 has been obtained, the decision on whether to spray can be made depending on whether W_0 is greater than W_0^T. However W_0 is not the only variable for which information can help improve the treatment decision. For example, estimates of P_y could be improved by obtaining latest forecasts, or expectations about Y_0 could be revised in the light of observed weather patterns. Thus it seems equally valid to assume that all variables other than, say, Y_0 are fixed and known with certainty. One could then derive Y_0^T, the threshold value of Y_0 above which treatment was justified. From equation (14),

$$(16) \quad Y_0^T = \frac{P_h \cdot H_r + A}{P_y \cdot L}$$

This value would be valid for a given W_0 in the same way as W_0^T is valid for a given Y_0.

Considering the decision problem in a Bayesian framework may help clarify the use of a yield threshold of this type. Consider that before the commencement of a growing season, a farmer is uncertain about the weed-free yield for the coming season and the weed density which will be encountered. The farmer can, however, estimate subjective probability distributions for each variable. In the case of Y_0, the subjective distribution would usually be strongly influenced by the distribution of yields from previous seasons. The subjective distribution of W_0 would depend on previous weed densities, previous control measures, intended cultural practices and other variables. At the time of the treatment decision, the farmer could choose not to obtain further information and to make the decision on the basis of the initial (or 'prior') subjective distributions. In the Bayesian terminology, the decision made in this way would be the 'prior optimal act'. Alternatively the farmer could use information from weed counts to update his or her subjective distribution for weed density and base the decision on that 'posterior' distribution together with the prior distribution for Y_0. This is the procedure usually assumed in the economic threshold approach.

A third alternative would be to update subjective probabilities of Y_0 on the basis of weather data observed up to the time of the treatment decision. Although it may still not be possible to predict Y_0 with great accuracy, some improvement in prediction would be possible. Theoretically it would be possible to adjust the expected weed-free yield without also counting the weed density. (The weed-free yield is independent of the actual weed density.) This revised or posterior distribution of Y_0 could be combined with the prior weed density distribution to make the decision. This corresponds to the type of threshold defined above in (16).

Finally, the farmer could obtain information on both Y_0 and W_0 and use posterior distributions for both variables in the decision. This points to a 'multidimensional' threshold approach where the decision to spray depends on a number of variables. An approach of this type has been used for insect pests in dynamic programming studies by Shoemaker (1979) and Shoemaker and Onstad (1983). However there has been no study in which multidimensional thresholds have been estimated for weeds and there has been no study for any type of pest in which expected yield has been used in a threshold type criterion.

There is an important difference between the two types of information discussed above. In the case of weed density, the farmer can obtain good information at relatively low cost by counting weeds in the crop. Theoretically, with sufficient effort, perfect information could be obtained. On the other hand, the best that can be achieved for weed-free yield is a moderate reduction in uncertainty. Climatic conditions between the time of the decision and the end of the growing season are highly uncertain, and further information on them can only be obtained by delaying the spray application, probably at very high cost. Furthermore Y_0 cannot be observed directly in the same way as W_0, but must be inferred from climatic data. This results in further uncertainty; even with perfect knowledge about the relevant climatic variables, Y_0 cannot be predicted with certainty. Nevertheless even partial information about weed-free yields is likely to be valuable, especially in environments where yields are highly variable.

Responses to Parameter Changes

It was noted previously that both optimal treatment rates and threshold values are sensitive to the values of a range of variables and parameters. It may be useful for policy or planning purposes to predict the direction of change resulting from changed parameter values. For example it may be useful to know what types of changes are likely to result in reduced chemical usage, reducing externalities from chemical toxicity. Alternatively it may be desired to provide incentives to increase the level of control to avoid external costs from damage agent spread. In this subsection, the responses of the previously outlined decision criteria to changes in variables or parameter values are derived.

Initial weed density

In this section, the responses of the optimal herbicide rate and the threshold yield to changes in weed density are derived.

Firstly consider the optimal herbicide rate. The derivation of $\mathrm{d}H^*/\mathrm{d}W_0$ can be greatly simplified by adopting a realistic approximation for $\partial D/\partial W$. Given the hyperbolic damage function of equation (2), $\partial D/\partial W$ tends toward b as W approaches zero. In practice, application of the optimal herbicide rate reduces weed densities to levels at which $\partial D/\partial W$ is only slightly less than b. Changing W_0 does change the optimal post-treatment weed density so $\partial D/\partial W$ is also changed. However numerical examples show that for very wide ranges of W_0, $\partial D/\partial W$ at the optimal W is almost unchanged. Therefore $\partial D/\partial W$ is approximated by b in the following derivation. A derivation without this approximation is available from the author. The simplified derivations are presented here to facilitate intuitive understanding of the results.

If $\partial D/\partial W$ is approximated by b then, from (10),

$$(17) \quad \mathrm{d}H^*/\mathrm{d}W_0 = 1/(kW_0) > 0$$

and

$$(18) \quad \mathrm{d}^2H^*/\mathrm{d}W_0^2 = -1/(kW_0^2) < 0$$

That is, the optimal herbicide rate increases at a decreasing rate with increases in W_0. This is illustrated in Figure 1.

Intuitively, increasing W_0 increases weed survival for a given herbicide rate. The higher level of weed survival reduces yield and causes the dosage response curve to move downward. At high herbicide rates the downward movement is much less because there is a smaller increase in weed survival. As a result the point where the response curve is tangent to the price line (P_h/P_y) moves to the right, increasing H^*.

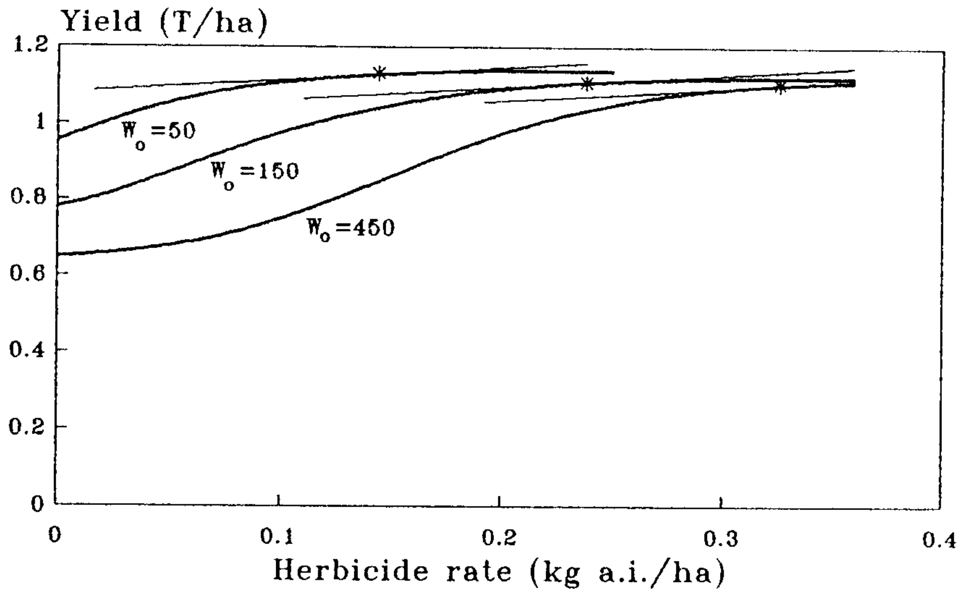

FIGURE 1—Derivation of Optimal Rates of Herbicide for Three Weed Densities

Now consider the yield threshold. From equation (16):

$$(19)\quad \frac{dY_0^T}{dW_0} = -(P_h \cdot H_r + A)/(P_y \cdot L^2)\frac{\partial L}{\partial W_0}$$

which is negative since $\frac{\partial L}{\partial W_0}$ is positive (yield loss increases with increasing initial weed density). As weed density is increased, a larger proportion of the yield is threatened and a lower expected yield is needed to justify treatment with the recommended dose, increasing the probability that herbicide will be used. This is consistent with the finding for H^* which was that a higher W_0 increases herbicide usage.

Weed-free yield

Changes in weed-free yield will affect the optimal herbicide rate and the traditional economic threshold. From (10) the effect on H^* of a change in Y_0 is

$$(20)\quad \frac{\partial H^*}{\partial Y_0} = \frac{1}{kY_0}$$

which is positive. That is, the higher the expected weed-free yield, the higher will be the optimal herbicide rate. This occurs because a higher Y_0 results in greater yield loss at a given weed density, increasing the marginal productivity of herbicide. Equation (21) shows that $\partial^2 H^*/\partial Y_0^2$ is negative so that H^* increases at a decreasing rate.

$$(21)\quad \frac{\partial^2 H^*}{\partial Y_0^2} = \frac{-1}{kY_0^2} < 0$$

The effect of Y_0 on W_0^T is as follows. It follows from (15) that

$$(22) \quad \mathrm{d}W_0^T/\mathrm{d}Y_0 = -\frac{P_h H_r + A}{P_y}\left[(Y_0 \cdot \bar{L}^2)^{-1}\frac{\partial \bar{L}}{\partial Y_0} + (Y_0^2 \cdot \bar{L})^{-1}\right]$$

where $\bar{L}$ is average yield loss per weed ($= L/W_0$). It can be seen from (22) that $\mathrm{d}W_0^T/\mathrm{d}Y_0$ is negative since $\partial\bar{L}/\partial Y_0 > 0$ (higher weed-free yield leads to higher loss per weed). In other words, if weed-free yield is higher, absolute yield loss at a given weed density is greater so a smaller weed density is sufficient to warrant treatment.

Output price and herbicide cost

The effect of output price on H^* is quite predictable. Herbicide use increases at a decreasing rate with rises in P_y.

$$(23) \quad \frac{\partial H^*}{\partial P_y} = \frac{1}{kP_y} > 0$$

$$(24) \quad \frac{\partial^2 H^*}{\partial P_y^2} = \frac{-1}{kP_y^2} < 0$$

Also, not suprisingly, higher output prices reduce both the traditional economic threshold and the yield threshold. Equation (15) gives

$$(25) \quad \frac{\partial W_0^T}{\partial P_y} = -\frac{P_h H_r + A}{P_y^2 \cdot Y_0 \bar{L}}$$

From (16):

$$(26) \quad \frac{\partial Y_0^T}{\partial P_y} = -\frac{P_h \cdot H_r + A}{P_y^2 \cdot L}$$

Both derivatives are always negative.

Conversely to these results, it can be shown that increases in herbicide chemical cost will reduce the optimal rate and increase both thresholds. These results are consistent with the argument of Lichtenberg and Zilberman (1986*b*) that government programs which inflate the price received for output or subsidise damage control input costs will tend to lead to overuse of chemicals.

The above analyses are for exogenous variables in the models. However there are also three parameters of the biological relationships which deserve attention: maximum yield suppression (*a* in the damage function, 2), weed competitiveness (*b* in equation 2) and herbicide effectiveness (*k* in equation 4). These will vary for different weeds (*a* and *b*) and different herbicides (*k*) and they may even change over time (e.g. development of resistance would reduce *k*).

Maximum yield suppression

Parameter *a* gives the maximum proportion of yield lost as $W \rightarrow \infty$. Here we consider the effect of an increase in *a* (e.g. through the introduction of a more competitive strain of the weed) on H^*, W_0^T and Y_0^T.

It is shown in the Appendix that

$$(27)\quad \frac{dH^*}{da}=\frac{2}{k}\left[\frac{1}{a}-\frac{1}{a+bW}\right]$$

which is always positive. So the greater the level of maximum yield suppression, the greater will be the optimal herbicide rate. This is consistent with previous results showing that if the level of yield loss increases, it is optimal to increase herbicide treatment to avoid that loss. H^* increases at a decreasing rate since the second derivative is negative.

$$(28)\quad \frac{d^2H^*}{da^2}=\frac{2}{k}\left[\frac{1}{(a+bW)^2}-\frac{1}{a^2}\right]<0$$

Now consider the traditional economic threshold. From (15),

$$(29)\quad \frac{dW_0^T}{da}=-\frac{P_hH_r+A}{P_yY_0\bar{L}^2}\cdot\frac{\partial\bar{L}}{\partial a}$$

which is negative since $\partial\bar{L}/\partial a$ is positive (an increase in maximum yield suppression increases the yield loss per weed). So an increase in the parameter a results in a decrease in the threshold density.

The equation for the yield threshold, (16), gives

$$(30)\quad \frac{dY_0^T}{da}=-\frac{P_h\cdot H_r+A}{P_y\cdot L^2}\cdot\frac{\partial L}{\partial a}$$

which is also negative. Thus if maximum yield suppression is increased, the yield threshold decreases. These threshold results are consistent with those for H^* and all are consistent with the intuition that if the yield loss is increased, the incentive for herbicide application increases.

Weed competitiveness

Parameter b in the damage function indicates the rate at which yield suppression approaches a, the maximum level of suppression. Biologically, a and b would depend on many of the same factors. Both indicate the competitive ability of weeds, so one would be unlikely to change without the other. Nevertheless, for illustrative purposes we consider here an assumed change in b while a is unchanged. In the Appendix it is shown that

$$(31)\quad \frac{dH^*}{db}=\frac{1}{bk}\left[\frac{2a^2}{bW+a}-1\right]$$

which is greater than zero iff:

$$W<a(2a-1)/b$$

Thus the effect of a change in b on H^* is ambiguous. If the initial weed density is sufficiently low for the post-treatment density to be less than $a(2a-1)/b$, an increase in b will increase H^*, otherwise H^* will be decreased. A necessary condition for $\frac{dH^*}{db}>0$ is $a>0\cdot 5$. Beyond this, no generalisations are possible.

The threshold calculations are relatively straightforward for b.

$$(32)\quad \frac{dW_0^T}{db} = -\frac{P_h H_r + A}{P_y Y_0 \bar{L}^2} \cdot \frac{\partial \bar{L}}{\partial b}$$

which is negative since $\partial \bar{L}/\partial b$ is positive (losses are higher at higher values of b). That is, an increase in weed competitiveness increases the avoidable yield loss at each weed density and reduces the threshold density at which treatment is justified.

The effect of b on the yield threshold is given by

$$(33)\quad \frac{dY_0^T}{db} = -\frac{P_h \cdot H_r + A}{P_y \cdot L^2} \cdot \frac{\partial L}{\partial b}$$

which is also negative. As with the other analyses, the direction of response of the yield threshold is the same as that for the density threshold.

Herbicide effectiveness

The final parameter considered here is herbicide effectiveness (k in the kill function, 4). This parameter may decrease, for example, if weeds develop genetic resistance, or it may increase by the use of improved spraying technology. From (10)

$$(34)\quad \frac{dH^*}{dk} = \frac{1}{k^2}\left[1 - ln(P_y) - ln(Y_0) - ln\left(\frac{\partial D}{\partial W}\right) - ln(k) - ln(W_0) + ln(P_h)\right]$$

Thus the effect of a change in k on H^* is ambiguous, depending on all other parameters and variables in the model. The sign of (34) may vary in different circumstances; values would need to be assigned to each of the variables before the sign of $\partial H^*/\partial k$ could be determined.

Now consider the effect of k on W_0^T.

$$(35)\quad \frac{dW_0^T}{dk} = -\frac{P_h \cdot H_r + A}{P_y Y_0 \bar{L}^2} \cdot \frac{\partial \bar{L}}{\partial k}$$

This is negative since $\dfrac{\partial \bar{L}}{\partial k}$ is positive (higher herbicide effectiveness kills more weeds and so increases the difference between yield with treatment and yield without treatment). Again, an increase in the avoidable yield loss encourages herbicide use and reduces the threshold density.

Finally consider the yield threshold

$$(36)\quad \frac{dY_0^T}{dk} = -\frac{P_h \cdot H_r + A}{P_y \cdot L} \cdot \frac{\partial L}{\partial k}$$

which again is negative since $\partial L/\partial k$ is positive. A lowering of herbicide effectiveness through development of resistance would increase the threshold yield for herbicide use.

It is notable that in all cases analysed above, the direction of response of the weed density threshold to changes in a parameter or variable is identical to that of the yield threshold. Furthermore, where unambiguous results were obtained for the optimal herbicide rate, the direction of response was opposite to that for the thresholds.

Empirical Results

There are several reasons for considering empirical examples of the principles derived in the previous sections. Firstly, the theoretical results indicate only the directions of response to the changes considered. There may be considerable variation in the magnitudes of the responses for different parameters. An empirical analysis will suggest the variables to which H^*, W_0^T and Y_0^T are most sensitive. Secondly, two of the results were ambiguous. Empirical results may indicate which direction of response tends to dominate in practice. Thirdly, the response model was simplified to make theoretical analyses more tractable. Empirical results can help by indicating whether these simplifications have affected results.

Method

The problem selected for analysis was control of ryegrass in wheat by application of Hoegrass (active ingredient diclofop-methyl). This problem was selected because of its economic importance in Western Australia where farmers consider ryegrass to be one of their most important crop weeds (Roberts *et al.* 1988). The basic biological relationships were taken from Pannell (1990). Weed survival is given by

$$(37) \quad W = W_0/[1+\exp(F)]$$

where

$$(38) \quad F = -2{\cdot}85 - 0{\cdot}995\, ln(H) - 0{\cdot}00559\, W_0 - 0{\cdot}00366\, ln(H)\, W_0$$

This function differs from the kill function in equation (4) in that the functional form is logistic and the proportion of weeds killed at a given herbicide dose is not independent of the weed density. The simplified version in (4) was adopted to facilitate theoretical analysis. Pannell (1990) estimated the following yield function:

$$(34) \quad Y = Y_0(1 - 0{\cdot}149H)\left[1 - \frac{0{\cdot}544}{1 + 0{\cdot}544/(bW)}\right]$$

where

$$(35) \quad b = 0{\cdot}0172{\cdot}\exp(-0{\cdot}801\,Y_0){\cdot}\exp(-5{\cdot}70H)$$

This differs from the damage function given in equation (2) in two respects. Firstly, the parameter b is not fixed but depends on the weed-free yield and herbicide rate and, secondly, there is an additional term representing phytotoxicity (direct damage to the crop by herbicide).

Templates for a microcomputer spreadsheet program were developed for deriving optimal herbicide rates and thresholds. The empirical analyses were conducted using values for costs, prices, weed densities and yields considered reasonable for the shire of Merredin in Western Australia's eastern wheatbelt: wheat price \$144 tonne^{-1}, Hoegrass cost \$48 per kg a.i., weed-free yield 1·2 tonnes ha^{-1}, initial weed density 200 m^{-2} and recommended herbicide rate 0·375 kg a.i. ha^{-1}.

Derivation of optimal herbicide rate and thresholds

The relatively simple analyses presented here are intended to provide an intuitive understanding of the more formal proofs given above. Derivation of the optimal herbicide rate for three different initial weed densities (50, 150 and 450 m^{-2}) has already been illustrated in Figure 1. Other parameters and variables underlying the diagram are fixed at values given above. The higher the initial weed density, the higher is the yield response attainable by applying herbicide. It can be seen from Figure 1 that as initial weed density increases, the optimal herbicide rate increases at a decreasing rate.

Now consider the traditional economic threshold, the initial weed density above which application of the fixed recommended herbicide rate (H_r) is justified. Figure 2 shows one derivation of the traditional economic threshold. It shows the benefits and costs of applying the recommended herbicide rate as functions of initial weed density. As weed density increases, the benefits of treatment increase since the yield loss avoided is greater. However treatment costs, which consist of application costs and chemical costs, are fixed since treatment rate is limited to H_r. At low weed densities, treatment costs exceed the benefits but above the threshold (W_0^T) the benefits of treating exceed the costs. At very low weed densities, benefits of treatment are negative due to herbicide directly damaging the crop.

Thus we have two quite different decision rules. In the threshold density approach, herbicide dosage is either zero or the recommended rate depending on the weed density. In the optimal rate approach, the dosage changes continuously with changes in the weed density. Levels

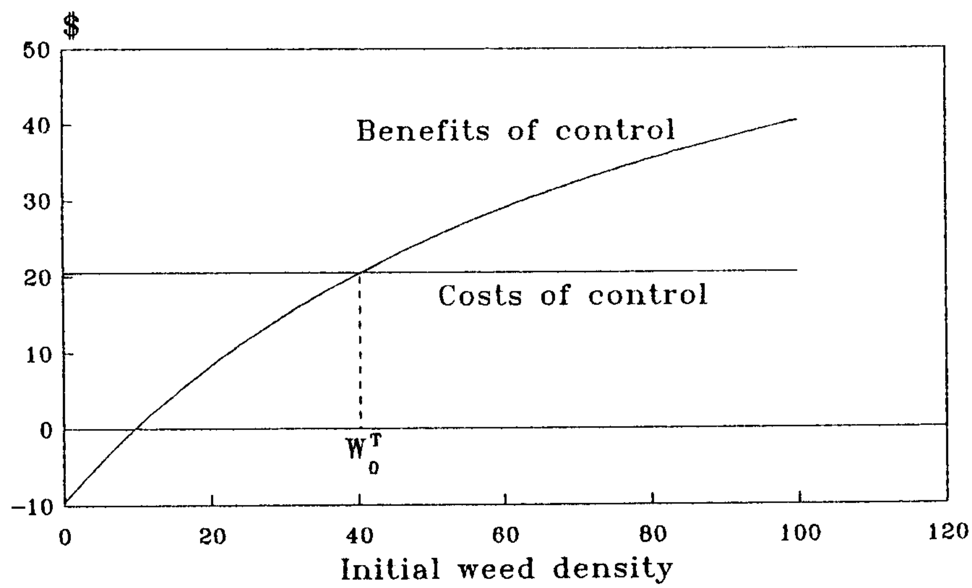

FIGURE 2—Derivation of the Threshold Density

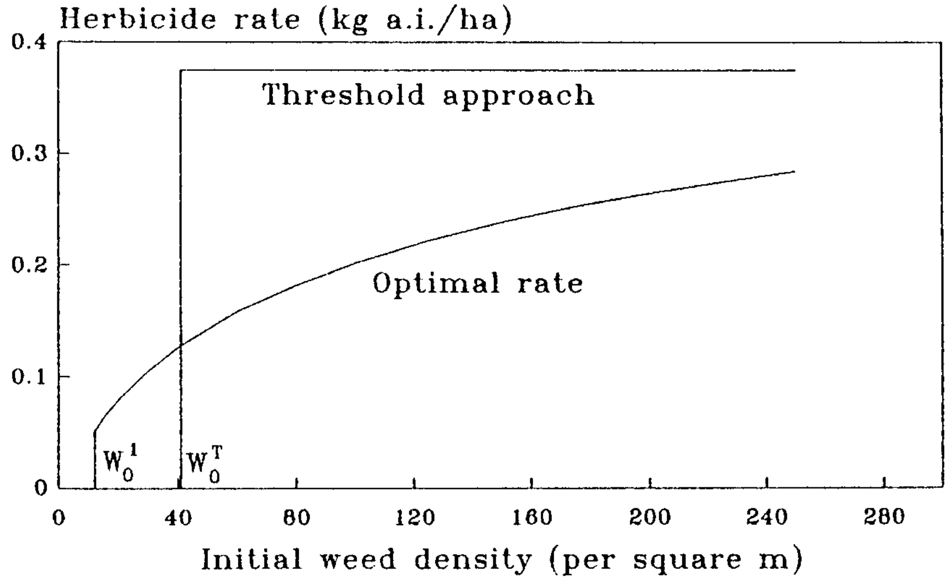

FIGURE 3—Herbicide Use Under Optimal Rate and Threshold Density Approaches

of herbicide use under the two decision rules are compared in Figure 3. Because of fixed application costs which do not vary with the rate of herbicide applied, there is a weed density below which the rate implied as optimal by marginal analysis is less profitable than no application of herbicide. Below this point (labelled W_0^1) the optimal rate is zero. This has been described by Pannell (1988*b*) as the 'optimal economic threshold'. Note that at the optimal economic threshold, the optimal herbicide rate is greater than zero.

The traditional economic threshold is greater than the optimal economic threshold. This is because the relatively high cost of the recommended dosage requires a high weed density (and, thus, the potential to avoid a high yield loss) before it is worth applying. At weed densities between W_0^1 and W_0^T, adoption of the optimal rate approach results in higher application of herbicide than the threshold approach. Above W_0^T the threshold approach leads to higher herbicide use.

Finally profits from the two approaches are compared in Figure 4. Below W_0^1 the two approaches are equally profitable since they both imply zero herbicide use. Above W_0^1 profit falls less rapidly with increasing weed density under the optimal rate approach. Above W_0^T the threshold criterion implies application of the recommended rate H_r and the rate of profit decline is reduced. In this illustration, profit from the optimal rate approach remains above that from the threshold approach. If the recommended rate had been lower the two profit curves would have converged at the point where the optimal rate equalled the recommended rate. It is obvious that expected profits from the threshold approach could never exceed those from the optimal rate approach.

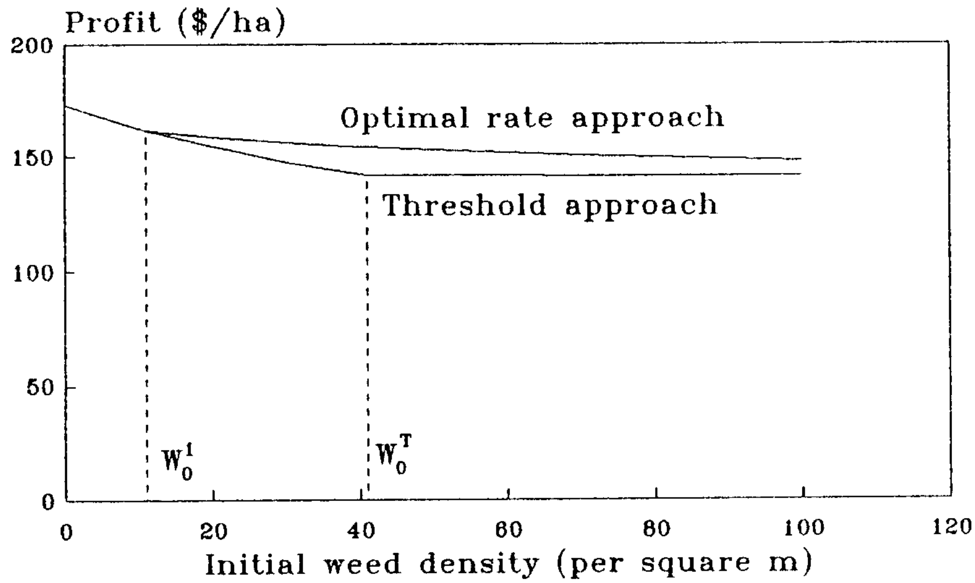

FIGURE 4—Profit Under Optimal Rate and Threshold Density Approaches

Diagrams for the yield threshold are not presented due to their similarity to those for the density threshold.

In the paper so far, thresholds have been derived based on the assumption that only one variable is unknown. This is the most common approach in the literature, with weed or pest density being the unknown variable. In reality information can be obtained about many variables in the system. Figure 5 is an illustration of a multi-dimensional threshold as advocated by Shoemaker (1976) and implemented by Shoemaker (1979) and Shoemaker and Onstad (1983).

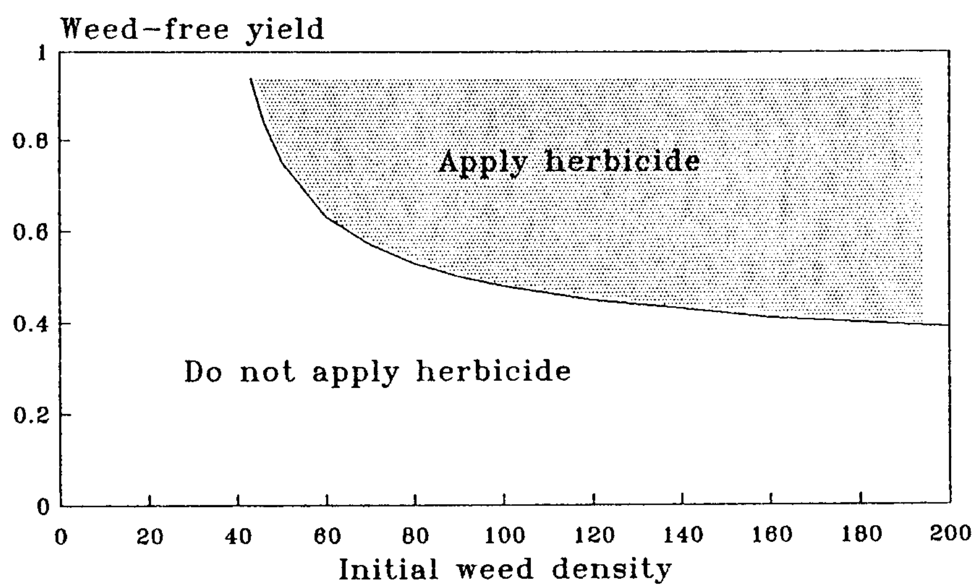

FIGURE 5—Multidimensional Threshold for Weed Density and Weed-Free Yield

The diagram shows combinations of initial weed density and weed-free yield above which application of the recommended herbicide rate is justified.

There is a negative relationship between the threshold yield and the threshold density. For any yield/density combination above the line, benefits from applying the recommended herbicide rate exceed the costs. This diagram highlights the potential for error in assuming that the weed-free yield is fixed and known with certainty. Variations in Y_0 can substantially alter the weed density above which application of the recommended herbicide rate is profitable.

Responses to parameter changes

In this section we consider the responses of the optimal rate, the threshold weed density and the threshold yield to changes in the parameters and exogenous variables of the model. Responses of the empirically estimated model are compared to the theoretical responses derived previously.

Given the standard parameter values indicated earlier, the optimal herbicide rate is 0·26 kg active ingredient (a.i.) per hectare, the threshold weed density is 41 plants per square m and the threshold yield is 0·39 tonnes per hectare. Table 1 shows the effects on these values of varying a range of parameters up or down by 20%. In each case only one change at a time is considered. All parameters other than the one being changed are assumed to be at the values given above.

TABLE 1

Effects of Parameter Changes on Optimal Herbicide Rate, Threshold Weed Density and Threshold Yield

	Optimal rate		Threshold density		Threshold yield	
	kg a.i./ha		plants m^{-2}		T ha^{-1}	
Parameter change	−20%	+20%	−20%	+20%	−20%	+20%
Parameter						
Maximum yield loss	0·26	0·27	47	38	0·50	0·33
Weed competitiveness	0·25	0·28	51	34	0·41	0·38
Herbicide efficacy	0·30	0·23	42	40	0·41	0·38
Phytotoxicity	0·27	0·26	37	45	0·38	0·41
Output price	0·25	0·28	52	34	0·50	0·33
Herbicide cost	0·28	0·25	34	48	0·32	0·47
Initial weed density	0·24	0·28	41[a]	41[a]	0·41	0·38
Weed-free yield	0·27	0·26	43	41	0·39[a]	0·39[a]
Recommended dose	0·26[a]	0·26[a]	31	52	0·34	0·47

[a]Value cannot be affected by changes in this parameter.

In general the empirical results are consistent with the earlier theoretical results. The only exception is the effect of weed-free yield on the optimal herbicide rate. With the more complex yield function of equations (34) and (35), the optimal rate rises with yield at low yields but then levels off and falls very slowly. The result in Table 1 is from the falling part of the curve.

Two of the theoretical results for the optimal rate were ambiguous. It was found that the relationship between weed competitiveness (b) and the optimal rate is positive only if post-treatment weed density is sufficiently large. However in investigations with the more complex empirical model it appears that dH^*/db is always positive, even for very low post-treatment weed densities. The other ambiguous theoretical result was for dH^*/dk. In the empirical results, the optimal rate is negatively related to herbicide efficacy. In fact herbicide efficacy is the variable to which the optimal rate is most sensitive. It seems that particular care is needed in the estimation of parameters of the kill function.

Phytotoxicity refers to the direct suppressing effect herbicides can have on the crop. It was not included in the simplified model used for theoretical analysis. However, since phytotoxicity is positively related to herbicide rate, it seems clear that a higher rate of yield loss per unit of herbicide will reduce the optimal herbicide rate and increase the thresholds. This is confirmed in Table 1.

There was also no theoretical analysis of responses to changes in the recommended herbicide rate. Table 1 shows that both thresholds are positively related to the recommended rate; the higher the recommended rate, the higher the initial weed density and/or weed-free yield need to be to justify its application.

The optimal rate and the thresholds differ in their sensitivity to parameter changes generally and they differ in the particular parameters to which they are most sensitive. Apart from herbicide efficacy, changes in parameter values had surprisingly little effect on the optimal herbicide rate. The threshold density was much more sensitive to parameter changes, being particularly responsive to weed competitiveness, output price and recommended dose. An interesting contrast between the two thresholds is that whereas the threshold density is very sensitive to weed competitiveness, the yield threshold is very insensitive to it. Maximum yield loss is the biological parameter to which the yield threshold is most sensitive. It is also responsive to output price, herbicide cost and the recommended dose. In contrast to the optimal herbicide rate, neither threshold is greatly affected by herbicide efficacy.

Caution is needed in generalising from the results in Table 1. It is shown in Figure 5 that the sensitivity of a decision criterion to parameter changes can vary widely for different values of that parameter. In Figure 5 that sensitivity of the threshold density to the weed-free yield is much greater at low weed-free yields.

In all results, the direction of response is the same for the threshold density as it is for the yield threshold. Apart from herbicide efficacy and weed-free yield, all parameter changes cause thresholds to respond in the opposite direction to the optimal rate.

Concluding Comments

Despite the neglect of economically optimal herbicide rates in the agricultural economics and weed science literatures, many farmers do use herbicide rates other than those recommended on chemical labels (Dolin *et al.* 1988). In the Western Australian wheat belt, farmers routinely cut rates, with halving of recommended dosages not uncommon. At present they are determining the optimal dosages on a trial-and-error basis. It is hoped that this study can form the basis for future research to determine optimal herbicide rates for a range of weed control problems. To this end, some field trial work in Western Australia has already been conducted and more is planned. There is also scope to extend the study by including risk, weed population dynamics and build up of resistance.

In New South Wales and Victoria there are legal impediments to downward flexibility in chemical dosages (Pannell 1989). It is to be hoped that with current concerns about chemical residuals in food, chemical build up in the environment and unnecessary use of resources, these impediments can be removed. However while they persist, the results presented here for the threshold approaches will continue to be relevant in these states.

Appendix

In this Appendix, proofs of equations (27) and (31) are presented. First consider equation (27). The effect of a on H^* in equation (10) occurs in the $\frac{\partial D}{\partial W}$ term. Thus

$$\text{(A1)} \quad \frac{\mathrm{d}H^*}{\mathrm{d}a}=\frac{1}{k}\frac{1}{\partial D/\partial W}\frac{\partial^2 D}{\partial W\partial a}$$

But

$$\text{(A2)} \quad \frac{\partial D}{\partial W}=\frac{a^2}{bW^2(1+a/(bW))^2}>0$$

so we need only solve for $\frac{\partial^2 D}{\partial W\partial a}$ to solve for $\frac{\partial H^*}{\partial a}$.

$$\frac{\partial^2 D}{\partial W\partial a}=\frac{1}{bW^2}\left[2a\frac{1}{(1+a/bW))^2}-\frac{2a^2}{bW}\frac{1}{(1+a/(bW))^3}\right]$$

$$=\frac{a^2}{bW^2(1+a/(bW))^2}\frac{2}{a}\left[1-\frac{a}{1+a/(bW)}\frac{1}{bW}\right]$$

$$\text{(A3)} \quad =\frac{\partial D}{\partial W}\frac{2}{a}[1-D/(bW)]$$

Substituting (A3) into (A1) gives:

$$\text{(A4)} \quad \frac{\mathrm{d}H^*}{\mathrm{d}a}=\frac{2}{ak}[1-D/(bW)]$$

Substituting $bW/(1+bW/a)$ for D we have:

$$\text{(A5)} \quad \frac{\mathrm{d}H^*}{\mathrm{d}a}=\frac{2}{k}\left[\frac{1}{a}-\frac{1}{a+bW}\right]$$

which is equation (27).

Now consider the proof of equation (31). The effect of b on H^* is given by

$$\text{(A6)} \quad \frac{\mathrm{d}H^*}{\mathrm{d}b}=\frac{1}{k}\frac{1}{\partial D/\partial W}\frac{\partial^2 D}{\partial W\partial b}$$

We have the equation for $\partial D/\partial W$ in (A2) and from that it can be seen that

$$\frac{\partial^2 D}{\partial W \partial b} = \frac{a^2}{W^2}\left[\frac{-2}{b^2}\frac{1}{(1+a/(bW))^2} + \frac{2aW}{b(bW)^2}\frac{1}{(1+a/(bW))^3}\right]$$

$$= \frac{a^2}{bW^2(1+a/(bW))^2}\frac{1}{b}\left[-1+\frac{a}{1+a/(bW)}\frac{2abW}{(bW)^2}\right]$$

$$= \frac{\partial D}{\partial W}\frac{1}{b}\left[-1+\frac{a}{1+a/(bW)}\frac{2abW}{(bW)^2}\right]$$

(A7) $$= \frac{\partial D}{\partial W}\frac{1}{b}\left[\frac{2a^2}{bW+a}-1\right]$$

Substituting (A7) into (A6) and simplifying gives:

(A8) $$\frac{dH^*}{db} = \frac{1}{bk}\left[\frac{2a^2}{bW+a}-1\right]$$

which is equation (31).

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