THE ESTIMATION OF PRODUCTION FRONTIERS: THE AUSTRALIAN LIVESTOCK/CEREALS COMPLEX*

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An attempt is made to combine, empirically, the estimation of product transformation surfaces with the more conventional methods of linear supply analysis. This synthetic approach is used to fit simultaneously a system of six aggregate supply functions. The products covered account for more than 70 per cent of the gross value of Australian rural production.

1. *Introduction*

Production possibility schedules, like consumers' indifference surfaces, are not directly measurable; however (at least in the case of competitive industries) the observed supply structure of an industry can be used to make inferences about production transformation frontiers in much the same way as an observed demand structure can be used to infer something about the shape of consumers' indifference curves. Clearly, though, one's attempts to measure production frontiers will be greatly enhanced in industries where (a) the level of resource commitment is subject to some periodic inflexibility, and where (b) the individual firms within the industries are themselves mainly of a multiple-enterprise nature. Fortunately, both of these conditions are met, broadly speaking, by the Australian crop/livestock complex.

It is, of course, true that estimates of the responsiveness of the production mix to changing relative prices can only be attempted under an appropriate set of restricting assumptions. The problem is to select a set of assumptions which is both *useful* and *flexible*—i.e. a set which incorporates sufficient prior information to materially facilitate analysis, but which nevertheless retains enough freedom within the model to cope with the complications likely to arise in practice.

In section 2 below we shall attempt no more than a sketch of the methodology adopted—this is a subject we have discussed fully in another paper.[1] The generation of our basic data is a lengthy procedure which has also been detailed in another paper.[2] However, a brief outline

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[1] A. A. Powell and F. H. Gruen. "The Constant Elasticity of Transformation Production Frontier and Linear Supply Systems". *International Economic Review* (forthcoming).

[2] A. A. Powell and F. H. Gruen. "Problems in Aggregate Agricultural Suppply Analysis, I: The Construction of Time Series for Analysis". *Review of Marketing and Agricultural Economics,* Vol. 34, No. 3 (September 1966), pp. 112-135.

is given in section 3 after the description of our methodology. This is followed in section 4 by a discussion of the model of price expectations we have used. In the cases of beef and wool the normal Koyck-Nerlove distributed lag model is unsatisfactory; this has prompted our search for an alternative estimation of price expectations for these products. In section 5, we discuss and justify the shift factors used in conjunction with the price variables in our supply equations. Before summarizing the principal results of the model, there is further discussion of beef supply relationships, which attempts to distinguish between the short and the long responses to changes in relative beef prices. This is done in section 6. Finally, in section 7, we summarize the principal results of the simultaneously fitted core of six supply functions.

2. *A Linear Supply System and its CET Transform*

We assume the existence of discrete, well-defined gestation periods in the case of every product.[3] These production periods are taken as equal to one year for all the products concerned—i.e. for *wheat, coarse grains, wool, lamb, beef* (including veal), and *dairy products*. This assumption approximates reality closely only in the cases of the cereals. However, annual series constitute the available data, and our model has been built with operational feasibility paramount among our objectives.

Let the output of product i ($i = 1, \ldots, 6$) planned for year t be y_{it}; notice that the decisions affecting the size of y_{it} are taken during $(t - 1)$. Clearly y_{it} depends, among other things, on the product prices expected from the viewpoint of year $(t - 1)$ to prevail during year t. Denote this set of expected prices by $\{\pi_{jt}\}$ ($j = 1, \ldots, 6$). For the time being, let us put all other relevant determinants of planned output of i into a function Γ_{it}; that is to say, Γ_{it} is meant to reflect the influences of everything bar current price expectations upon the size of y_{it}. A linear system of supply equations would be written, in this notation, as

$$(1) \qquad y_{it} = \Gamma_{it} + \sum_{j=1}^{6} a_{ij}\, \pi_{jt} + \epsilon_{it} \qquad (i = 1, \ldots, 6),$$

where the a_{ij} are parameters determining the slopes of the supply curves and the extent to which the supply curve of any one product shifts in response to a change in the expected price of a competing product. On the assumption that all products concerned compete for the same resource base, we have

$$(2.1) \qquad a_{ii} > 0$$

and

$$(2.2) \qquad a_{ij} < 0 \qquad (i \neq j).$$

The final term of (1), ϵ_{it}, is an equation misclosure which we assume to be stochastic with zero mean.

Assuming that resource commitments on farms are fixed at the beginning of each production period, the basic decision confronting an entrepreneurial unit is the determination of its product mix. Movements around production possibility frontiers, like movements around indifference curves, involve symmetrical price derivatives. Thus, on this view, $a_{ij} \equiv a_{ji}$ for all $i \neq j$. Consequently, any method of fitting the system (1) which aims to make economical use of the data should incorporate

[3] A partial relaxation of this assumption will be made below in the case of beef.

this symmetry.[4] In fact, one can facilitate interpretation by rewriting the system (1) as

$$(3) \qquad y_{it} = \Gamma_{it} + \sum_{j \neq i} \tau_{ij}\, x_{ijt} + \epsilon_{it}$$

in which

$$(4) \qquad x_{ijt} = y_i^{**}[\pi_j^{**} y_j^{**} / (\pi_j^{**} y_j^{**} + \pi_i^{**} y_i^{**})][(\pi_{jt}/\pi_j^{**}) - (\pi_{it}/\pi_i^{**})].$$

Here, double asterisks indicate mean values over the sample period (1947-48 to 1964-65), whilst τ_{ij} may be interpreted as the partial transformation elasticity between products i and j; i.e. as the elasticity of the product mix ratio with respect to the marginal rate of transformation.[5] Equation (1) has N^2 price parameters $\{a_{ij}\}$; the exploitation of symmetry reduces the number of price parameters to $N(N-1)/2$ partial transformation elasticities $\{\tau_{ij}\}$. The transformation from τ_{ij}'s back to a_{ij}'s may be made by using

$$(5.1) \qquad a_{ij} = y_i^{**} \tau_{ij} \theta_{ij} / \pi_j^{**} \qquad (i \neq j),$$

and

$$(5.2) \qquad a_{ii} = -y_i^{**} \sum_{j \neq i} \tau_{ij} \theta_{ij} / \pi_i^{**},$$

in which θ_{ij} is the first expression within square brackets on the right-hand side of (4).[6]

How does one interpret the variables x_{ijt} in equation (4)? Obviously these have replaced raw price expectations $\{\pi_{it}\}$ with variables which reflect pair-specific relative prices. In equation (4), the last term in square brackets measures the price of product j relative to product i by taking the difference of price relatives for these products.[7]

The first term within square brackets on the right-hand side of (4) gives an average measure of the share of product j in the total value of output of products i and j. Such a correction for value share results in this model from the fact that transformation elasticities are, in themselves, scale-free. By this we mean that the relative scales on which two products are produced does not, in itself, affect the extent of their technological substitutability as measured by a partial transformation elasticity. But of course, the relative scales of production are relevant for assessing the impact of a change in the price of one product upon the output of another. Our formulation ensures that if the price of a product whose relative share in income is large should rise by a given amount, this will not require an unduly high τ value in order to "explain" the response in the output of another product: for example, whilst we would expect a change in the price of wheat to have quite an impact on the output of coarse grains, part of this expected response must be attributed to the sheer size itself of the wheat industry (relative to coarse grains), rather than to the basic technological possibilities for transformation of potential wheat output into production of coarse

[4] The above presupposes a well-behaved production possibility surface such that any cross-section through it will exhibit the usual desirable property of strict concavity when viewed from the origin.

[5] $\tau_{ij} \equiv \tau_{ji} = d(y_i/y_j)(\partial y_i/\partial y_j)/d[(\partial y_i/\partial y_j)\,(y_i/y_j)]$.

[6] Powell and Gruen. "The Constant Elasticity . . .", *op. cit.*

[7] The price relatives each have the mean over the sample period of expected prices as base.

grains.[8] The final element of the right-hand side of (4)—namely y_i^{**}—merely ensures that what would otherwise have been a pure index number is converted back into the units in which the output of product i is measured, thus preserving dimensionality. The expected signs of all τ_{ij}'s are negative. Otherwise a rise in the price of a competing product j would not reduce output of product i.

The fact that the class of transformation surfaces characterized by constant (partial) transformation elasticities (the CET class) is not analytically compatible with the linear supply system (1) presents us with a choice of compromises so far as interpreting estimated τ_{ij}'s. First, one could regard (1) as a convenient linearization of a CET supply system, treating the estimated transformation elasticities as a set of parameters coming from such a system. We have shown elsewhere that the CET system is an exceptionally flexible one, in that all of the obvious features of investment and technological change are easily accommodated within it.[9] A second interpretation merely accepts the computed τ_{ij}'s as estimates of partial transformation elasticities in the neighbourhood of sample mean planned outputs and expected prices, no inferences whatever being drawn about the constancy or otherwise of these elasticities.[10] We incline towards an intermediate position in which we regard the operationally interesting zone of the production possibility map—i.e. the set of observed co-ordinates on output mixes and its immediate environs—as if it belonged to the CET class. Our justification here is Occam's razor: the CET postulate provides a minimal parameter space for describing plausible output configurations, whilst we regard it as extremely unlikely that any devisable test would discriminate empirically between the CET class and any other hypothetical alternative class.

3. *The Data*[11]

A detailed statement of our sources of data, and of the preliminary transformations performed on these data, has been made in another paper.[12] Annual time series on six output indicators for the eighteen-year period 1947-48 to 1964-65 were compiled from official statistics. The units in which output was measured were, with the exception of the meats, of a kind which might reasonably reflect intended production. Thus for cereals, intended acreages were used as output indicators, whilst for wool, the number of adult sheep shorn was used. For dairy products, the index of planned output was taken to be the number of dairy cows kept. Unfortunately, no fruitful approach towards measuring intended meat production could be found, so that for beef and lamb

[8] The "scale" and "pure transformation" effects we have distinguished here are both components of a substitution effect; i.e. of a movement *around* a production frontier—they should not be confused with substitution and expansion effects of textbook fame.

[9] Powell and Gruen. "The Constant Elasticity . . .", *op. cit.*

[10] The CET class is such that, for any partial production frontier contained within it, the partial transformation elasticity is constant along the entire length of the frontier; moreover for any given pair of products, *all* partial frontiers have the same elasticity of transformation.

[11] The historical price and output indexes upon which this study is based are tabulated in Powell and Gruen, "Problems in Aggregate Agricultural Supply Analysis, I", *op. cit.*

[12] *ibid.*

actual slaughterings (tons carcass weight) were used as the output variables. In the case of wheat, the availability of official statistics on planned acreages enabled us to remove some of the (unwanted) climatic variability which might otherwise have remained in the series, even after the major source of climatic variability in production—variations in yields per acre—had been removed.[13]

Secular improvements in grain yields per acre, in wool cuts per head, and in milk gallonage per cow, have been regarded as autonomous technological improvements: our historical price series, when converted to revenues per acre, per sheep and per cow, have taken into account these productivity gains. In the case of wheat, the institutional features affecting growers' returns have been recognized in our attempts to construct a hypothetical price series. For the coarse grains aggregate, a variable weight index was constructed from price indicators for the individual aggregands; namely, barley, oats and maize.

4. *Models of Price Expectations*

For the most part, we have adhered to orthodox models of price expectations. The model most frequently used here has been the Nerlove distributed lag approach in which

$$\pi_{it} = \sum_{\nu=1}^{\infty} \beta_i(1 - \beta_i)^{\nu-1} p_{i,\, t-\nu} \tag{6}$$

where π_{it} is the price expected to prevail for product i in year t from the viewpoint of year $(t - 1)$; β_i is the coefficient of expectations for product i, and $p_{i,\,t-\nu}$ is the actual price pertaining ν years ago from the viewpoint of year t.[14] This adaptive expectations approach has been used as it stands for lamb, coarse grains and dairy products; and with some further modification, for wool and for beef. The approach to price expectations for wheat was dictated by the institutional features surrounding the administration of the wheat stabilization machinery, as detailed elsewhere.[15]

At this juncture we must foreshadow the special treatment of the supply relations for beef, discussed below in section 6. The Nerlovian approach to supply postulates that farmers respond to only one notional product price; namely, their long-term expectation. Whilst we have found this approach appropriate enough for five of the products covered here, it is unsatisfactory in the case of beef. Beef cattle can be converted from potential future output into current output with only a small time lag; viz. the time required to fatten and despatch for slaughter. Thus supply behaviour depends at minimum on two sets of price expectations; one short, the other long. Rises in the long price (with the short

[13] Work by Watson had suggested that planned acreage (though differing only by very small percentages from realized acreage) behaved considerably better as a regressand in single equation supply systems than did actual acreage. See A. S. Watson, *Wheat Stabilization Policy: A Supply Study*, unpublished M.Ag.Ec. thesis, University of New England, Armidale, 1965.

[14] M. Nerlove. *The Dynamics of Supply: Estimation of Farmers' Response to Price* (Baltimore: Johns Hopkins Press, 1958). In using the above formula (6) and variants of it introduced below, we have truncated lags after seven years, adjusting the coefficients of lagged prices upwards so that they sum to unity.

[15] Powell and Gruen. "Problems in Aggregate Agricultural Supply Analysis, I", *op. cit.*

price constant) would *suppress* current output. This is a fundamental difference between beef and all other products in our model. In so far as they concern beef, our discussions at this point focus upon *long*-term price expectations.

Both wool and beef have experienced phases of exceptionally rapid rises in price during the post-war period; the former during the Korean commodity boom of 1950-51 and the latter during the U.S. export boom of 1959-60 and 1960-61. Under these circumstances, producers can be expected to have little faith in the permanence of prevailing high prices. In an attempt to filter the influences of such abnormal prices from our long-term expected price series, we have devised and used a compromise—albeit arbitrary—between adaptive and extrapolative models of price expectations.

A Compromise between Adaptive and Extrapolative Models of Price Expectations

We accept in principle the idea that expectations are formed by taking a weighted average of past experience, and, in pragmatic spirit, we will adhere to the single-parameter formulation of the Koyck-Nerlove treatment. However, we wish to modify this treatment somewhat to allow for the fact that in situations in which prices have risen much faster than "usual", farmers may "disbelieve" a high price pertaining in the immediate past. By this we mean that in such situations farmers may discount heavily the probability that such a high price will be sustained in future periods.

Suppose, then, that p_{it-1} is last year's price for product i, and that $(p_{it-1} - p_{it-2})/p_{it-2}$—the proportional rise between last year's price and the price pertaining the year before—is "unusually" large. In such a situation we hypothesize that farmers' expectations will not be based on a weighted sum of the form (6) with p_{it-1} as the first term in a weighted average. Instead, farmers' conservatism will substitute a credible, or in Friedman's terminology, "permanent", component of p_{it-1}. In general, this will be so of all the previous years, so that price expectations $\{\pi_{it}\}$ will be generated by the formula

$$(7) \qquad \pi_{it} = \sum_{\nu=1}^{\infty} \beta_i (1 - \beta_i)^{\nu-1} \psi_{it-\nu} ,$$

where $\psi_{it-\nu}$ is the "credible" component of $p_{it-\nu}$ from the viewpoint of year t. Before attempting an operational definition of $\psi_{it-\nu}$, let us postulate that, for all t,

$$(8) \qquad \psi_{it-\nu} \rightarrow p_{it-\nu} \qquad \text{as } \nu \rightarrow \infty.$$

That is to say, if we go back far enough into the past, all actual prices become fully credible from the viewpoint of year t.

How might one estimate the $\psi_{it-\nu}$? Suppose that farmers disbelieve any price which exceeds an m-year sliding trend projection. Suppose that such a sliding trend projection in year t' has value $\hat{p}_{it'}$. Then, if the least-squares sliding trend projection were used, we would have

$$(9) \qquad \hat{p}_{it'} = \sum_{h=1}^{m} \{[2(2m + 1) - 6h]/m(m - 1)\} p_{it'-h},$$

where m, as before, is the number of years used for the sliding trend projection. Defining

$$(10) \qquad p^*_{it-\nu} = \min \{p_{it-\nu}, \hat{p}_{it-\nu}\},$$

our previous postulate about initial disbelief in the permanence of prices which rise "too fast" may be expressed as

$$(11) \qquad \psi_{it-1} = p^*_{it-1}.$$

Thus, the credible or "permanent" component of last year's price is taken as the recorded value, or a sliding trend projection utilizing data prior to last year's, whichever is less. Although designed for a situation in which prices rise "too fast", this equation may nonetheless be suitable for a situation in which prices fall at abnormal rates. Thus, by accepting the minimum of trend projection and observed value, we make explicit recognition of the renowned pessimism of farmers.

We have already intimated that what initially seems "too high" may, in perspective, seem credible. Thus we wish to build a learning process into the system whereby the credibility of actual prices increases as one moves backward in time from the date at which expectations are being formulated. Suppose that, for all practical purposes, historic prices become fully credible after n years; thus $\psi_{it-\mu}$ equals $p_{it-\mu}$ for all $\mu > n$. That is, from the viewpoint of t, all prices occurring n or more years ago are fully credible. Since last year's "credible" component is p^*_{it-1}, and since the credible component after a lag of n years is just the actual price, p_{it-n}, it seems reasonable to approximate the permanent components $\psi_{it-\nu}$ ($\nu = 2, \ldots, n-1$) during the intervening years by a weighted average of $p^*_{it-\nu}$ and $p_{it-\nu}$, where their respective weights shift linearly from one through zero. Thus for example, with $n = 3$,

$$(12.1) \qquad \psi_{it-1} = p^*_{it-1}$$

$$(12.2) \qquad \psi_{it-2} = (2/3)p^*_{it-2} + (1/3)p_{it-2}$$

$$(12.3) \qquad \psi_{it-3} = (1/3)p^*_{it-3} + (2/3)p_{it-3}$$

$$(12.4) \qquad \psi_{it-4} = p_{it-4}$$

$$(12.5) \qquad \psi_{it-\xi} = p_{it-\xi} \quad \text{for all } \xi > 4.$$

On entirely pragmatic grounds, we have postulated that $m = n = 3$. This equality assumption, as well as the specific numerical value chosen, is arbitrary: the procedure was, however, successful *ex post* in the sense that its use facilitated an economic interpretation of the roles of the long-term expected prices for beef and wool in determining beef slaughterings—where all our previous, more conventional, attempts had failed.

Coefficients of Expectations

Our estimation machinery is not geared towards simultaneously estimating the coefficients of expectations $\{\beta_i\}$; instead, all results are computed as conditional estimates for given, arbitrary, prior values of the β_i. It is clear that a sensitivity analysis based on unrestricted joint variation in all coefficients of expectations would generate a mountain of computer output. To narrow the range of possibilities to manageable limits, we have made the assumption that coefficients of expectations vary inversely with the coefficient of variation of the associated actual price series. The rationale here is that the reliability of last year's price as an estimate of the longer run price level will be high for stable series, but less so for erratic ones. Whilst ideally one would have wished to use for this purpose price series at the farm level, we have been forced to work with average annual price series; i.e. the series underlying our empirical analysis. The coefficients of variation for the five products concerned are given in Table 1.

TABLE 1

Coefficients of Variation of Price Series for Five Products, 1940-41 to 1961-62

	Wool	Lamb	Beef	Coarse grains	Dairy products
Coefficient of variation	0·63	0·43	0·51	0·44	0·45
Ratio (relative to Wool = 1)	1	0·68	0·81	0·70	0·71
Inverse ratio	1	1·47	1·23	1·43	1·40

Source: Based on Tables 9, 11, 13, 14 and 15 in Powell and Gruen, "Problems in Aggregate Agricultural Supply Analysis, I", *op. cit.*

5. *Shift Functions and Short-run Capacity*

The shift functions Γ_{it} of equation (1) are free to vary as investment and technology shift the production frontier over time. Thus, to the extent that the Γ_{it} reflect investment, they incorporate the entire history of price expectations up to, but not including, expectations held for t from the viewpoint of $(t - 1)$. Apart from their role in locating supply curves under shifts induced by investment and technology, the Γ_{it} have been specified to include drought mortality indexes in the cases of wool, lamb, and beef. In the latter two instances, this expedient reflected our inability to prefilter climatic influences from the output indicators used; a drought mortality index was added to the supply equation for wool because we believed that numbers of adult sheep shorn would, to a lesser degree, incorporate unintentional variation of climatic origin. For the sheep products—lamb and wool—the same drought index was used; this time-series has been tabulated elsewhere.[16] A similar series on beef cattle mortality is given here in Table 2.

Also included in Table 2 is a series on the estimated opening inventory of non-breeding beef cattle. This series has been used as a proxy variable to indicate the capacity of the beef industry to expand output in the short run; i.e. to locate the short-run supply curve for beef. In this, as in several other respects, our treatment of the beef supply equation departs radically from that adopted for the other five products. In the cases of all other products, lagged output was used as a proxy measure of short-run capacity. Apart from the advantage of obviating

[16] Powell and Gruen. "Problems in Aggregate Agricultural Supply Analysis, II: Preliminary Results for Cereals and Wool." *Review of Marketing and Agricultural Economics,* Vol. 34, No. 4 (December 1966), Table 17, p. 191.

TABLE 2

Ancillary Data for Beef Supply Equations

Fiscal year	Opening inventory of other beef cattle[a]	Drought mortality index[b]	Number of beef cattle breeders[c]
	(millions)	(per cent)	(millions)
1947-48	4·98	−0·65	3·856
1948-49	5·05	−0·35	3·987
1949-50	5·09	−1·24	4·171
1950-51	5·28	−1·20	4·469
1951-52	5·59	+3·67	4·834
1952-53	5·45	+0·01	4·827
1953-54	5·51	−0·07	4·943
1954-55	5·85	−0·86	4·830
1955-56	5·97	−1·16	4·889
1956-57	6·30	−1·18	5·102
1957-58	6·54	+1·89	5·597
1958-59	6·29	+1·37	5·619
1959-60	6·00	+0·03	5·413
1960-61	5·98	+0·27	5·642
1961-62	6·33	−0·05	6·104
1962-63	6·55	−0·44	6·439
1963-64	6·70	−0·58	6·775
1964-65	6·73[d]		7·021[d]
1965-66	6·53[d]		7·036[d]
1966-67	5·92[e]		

(*a*) Number of beef cattle (other than breeders) on hand at March 31 of first year shown. Obtained from Commonwealth Bureau of Census and Statistics, *Livestock Numbers, Statistical Bulletin*, No. 23 (March, 1965) and earlier bulletins.
(*b*) Deviation of crude percentage mortality rate about a "normal" value of 4·27 per cent. Source data were supplied privately.
(*c*) Number of cows and heifers on hand at March 31 of first year shown. Source as for (*a*).
(*d*) Official figures were prepared on a new basis; recorded values are our estimates.
(*e*) Preliminary; our estimate.

measurement problems, the latter approach to locating short-run production frontiers has the virtue of formal equivalence with Nerlove's distributed-lag model of supply adjustment.[17] This may be verified by writing

$$(13) \qquad y_{it} - y_{it-1} = \gamma_i \, (y_{it}^* - y_{it-1}),$$

where y_{it}^* is the desired long-run equilibrium output from the viewpoint of year t, and γ_i is the "coefficient of adjustment" of product i (reflecting technological stickiness in the adjustment of output towards its desired long-run level, and numerically equal to the proportion achieved in the first year of the eventual total increment in output). We make the further assumption that the shift functions Γ_{it} may be decomposed linearly; i.e. that

[17] Nerlove, *op. cit.*

$$\Gamma_{it} = b_i + c_{oi} y_{it-1} + \sum_{k=1}^{r_i} c_{ik} z_{ikt} \tag{14}$$

where b_i, c_{oi} and the $\{c_{ik}\}$ are constants specific to the i-th equation, with r_i equal to the number of arguments (other than lagged output) in the shift function Γ_{it}, and z_{ikt} the value at time t of the k-th variable (other than lagged output) included in the i-th shift function. Combining (1), (13) and (14) we have

$$\gamma_i \; y_{it}^* = \{b_i + \sum_{k=1}^{r_i} c_{ik} z_{ikt}\} + \sum_{j=1}^{6} a_{ij} \; \pi_{jt} + \epsilon_{it}, \tag{15}$$

in which γ_i equals $(1 - c_{oi})$. Equation (15) would be the long-run supply function if the term γ_i did not appear on the left-hand side; it follows that long-run elasticities in this model can be estimated by scaling up short-run (one year) estimates by the factor $(1 - \hat{c}_{oi})^{-1}$, where $\hat{c}_{oi}$ is the estimated partial autoregressive coefficient within the i-th supply equation. However, infinite period elasticities are operationally obscure, and much more sensitive to arbitrary assumptions about the structure of price expectations than are finite period (say 5-year) elasticities. At sample mean outputs and expected prices, short-run (one-year) elasticities of supply of product i with respect to the expected price of j are

$$\eta_{ij} = a_{ij} \; \pi_j^{**}/x_i^{**}; \tag{16}$$

the m-year elasticities are

$$\eta_{ijm} = \{1 + c_{oi}[1 - c_{oi}^{(m-1)}]/(1 - c_{oi})\} \; \eta_{ij}. \tag{17}$$

We conclude this section by listing in Table 3 the variables included as arguments of the shift function for each short-run supply equation.

TABLE 3

Structure of Shift Functions Used in Supply Analysis

Product	Units in which output is measured	Variables appearing in shift functions, Γ_{it}
Wool	m. adult sheep shorn	lagged output; drought mortality index for sheep[(a)]
Beef	m. tons carcass weight slaughtered	opening inventory of non-breeding beef cattle (millions); drought mortality index for cattle; ratio of expected long-term beef price to expected long-term wool price
Lamb	m. tons carcass weight slaughtered	lagged output; drought mortality index for sheep[(a)]
Dairy products	m. dairy cows kept	lagged output
Wheat	m. acres	lagged output
Coarse grains[(b)]	m. acres	lagged output

[(a)] Deviation of crude annual mortality rate about a "normal" value of 6·3 per cent.

[(b)] Aggregate of barley, oats and maize.

6. *Special Treatment of the Supply of Beef*

The distinction between short- and long-run price expectations has already been made. So far as the six-sector supply system is concerned, slaughterings (tons, carcass weight) were used in the beef equation as an output indicator. (To avoid confusion, we shall refer to this as the "short-run" equation.) However, in order to evaluate the impact over periods in excess of one year of changes in price expectations, an index of intended production suitable to a longer run context was needed. For this purpose, we have used a series on the number of breeding beef cattle (Table 2), terming the equation involving this response variable the "long-run" equation. Finally, an attempt was made to document the feedback mechanism by which increased current slaughterings could be expected to suppress future output.

Short-run Equation

Exactly what is meant by the "short (expected) price"? By virtue of the data available, the operational choice confronting us was between spot price and price lagged six months (annual series in each case). We chose the latter in the belief that a half-year was a better approximation than zero to the lag that might reasonably be expected in the response of slaughterings to current price conditions. We were hopeful of avoiding identification problems in the "short-run" equation, but not so much because we had specified a six-month lag in the response of current output, but because of the dominance of export price in setting local price. Hopefully, our share of the international market would be small enough to justify treating export prices as exogenously determined.

The "short-run" equation was fitted simultaneously with the supply equations for the other five products and tabular results relating to this equation will be found in the next section. The explanatory variables, all having estimated coefficients of the expected sign and *apparent* statistical significance,[18] were as follows: the opening inventory of non-breeding beef cattle, the beef drought mortality index, the price of beef lagged six months (relative to the long-term price expected for dairy products),[19] and the ratio of the expected long-run beef price to the expected long-run wool price. These variables accounted for 90 per cent of the variance in the recorded series on slaughterings.

Long-run Equation

The difficulty with using the number of beef breeders as an index of intended long-term production is that the effectiveness of a beef cow in generating output has been increasing over time. As a result of

[18] It is not proper to speak of statistical "significance" in this study as we have not used an independent set of data for each hypothesis tested. Moreover, we have based our estimates of standard errors of regression coefficients on classical least-squares formulae, which are not strictly applicable for the iteratively fitted generalized least-squares method used in the case of the six sector model. However, we conform to the common usage by using the word "significance" synonymously with the result of a Student's $|t|$-test based on the ratio of parameter estimate to its apparent sampling standard deviation.

[19] I.e. the explanatory series used was a weighted difference of price relatives of the form specified in equation (4), with i = beef and j = dairying. The regression coefficient, interpreted as the partial transformation elasticity between beef and dairy products, had an associated Student's $|t|$-value in excess of 5.

increasing productivity in the beef industry, there has been a secular decline in the proportion of breeding stock to all beef cattle. We were able to cope with an analogous situation in the case of wool by making allowance within our price series for secular productivity gains—this simply involved allowing for an increase in the weight of the adult fleece.[20] However, no direct adjustments could be made to the price series in the case of beef, and we had to fall back on indirect techniques. The equation actually fitted was

$$
\begin{aligned}
(18)\quad ln\,(\text{beef breeders})_t = &\underset{(0\cdot3549)}{-0\cdot0754} + \underset{(0\cdot3679)}{0\cdot8421}\, ln\,(\text{beef breeders})_{t-1} \\
&- \underset{(0\cdot8266)}{1\cdot3221}\, ln\,(1 + \text{drought index})_{t-1} \\
&+ \underset{(0\cdot0441)}{0\cdot0092}\, ln\,(\text{expected long beef price} \times 10^{-2}/\text{expected long wool price})_{t-1} \\
&- \underset{(0\cdot1192)}{0\cdot0468}\, ln\,(\text{beef slaughterings})_{t-1} + \underset{(0\cdot0147)}{0\cdot0063}\, t \quad (R^2 = 0\cdot97).
\end{aligned}
$$

The technique used was ordinary least squares, with the coefficient of expectations for wool and beef respectively set equal to 0·6 and 0·738 (see Table 1).[21] The logarithms in the above equation are natural, and the time scale is such that $t = 48$ at 31st March 1948; the sample data consisted of 17 annual observations corresponding to values of the regressand spanning 1948 through 1964. The residuals showed incipient evidence of autocorrelation, the Durbin-Watson statistic at 1·38 standing mid-way in the indeterminate zone between critical limits of 0·67 and 2·10.

A number of comments need to be made about this equation. First, we would have wished to fit it simultaneously with the core of six equations reported in the next section. The capacity of our computer would not permit this. Second, although the regressand lagged is the only variable in the fitted equation whose estimated coefficient differs with statistical significance from zero, all estimated coefficients have the expected signs. Their failure to attain statistical significance could well be bound up with the collinear nature of a number of regressors; thus the simple correlation between the logarithm of lagged beef breeders and time was 0·97; between the logarithm of lagged slaughterings and time, 0·83; and between the logarithms (lagged, in each case) of slaughterings and beef breeders, 0·81. Third, whilst the estimated one-year elasticity of the long beef output index with respect to its long price is 0·009, this figure by itself provides very little information. This criticism applies (with less force, perhaps) to the estimate that an increase of ten per cent in slaughterings will decrease the number of beef breeders by 4·7 per cent in the year following. In order to solve for interesting elasticities—for example (say) the five-year elasticity of slaughterings with respect to a sustained rise in the average price of beef—the short-run equation, the long-run equation (18) above, and the feedback equation (20) below, would have to be solved dynamically.

[20] Powell and Gruen. "Problems in Aggregate Agricultural Suppply Analysis, I", *op. cit.*

[21] Here, as below, we have stuck to a single arbitrary value of the coefficient of expectations for wool; namely, 0·6. Some comments on potential sensitivity are made in section 7 below.

In particular, the time pattern via which the rise in average price is achieved would influence the result, and a simulation approach would be needed before useful conclusions would emerge.

Feedback Equation

The long-run equation has provided an estimate of the impact of current slaughterings upon the number of breeding beef cattle; this, of course, must feed back into slaughtering rates in the future. The short-run equation used the opening inventory of non-breeding beef cattle as a proxy measurement of short-run capacity to slaughter.[22] The cycle is completed by specifying the relationship between the opening inventory of non-breeding cattle and the previous size of the breeding herd. We postulate that, as a result of productivity gains, the ratio of non-breeders to breeders is an increasing function of time, but that the rate of increase itself must be diminishing over time. Thus if the proportion of non-breeders is denoted by λ_t, then we might write

$$\lambda_t = a + bt + ct^2, \tag{19}$$

expecting the first derivative with respect to time, $(b + 2ct)$, to be positive for all relevant t; and the second derivative $(2c)$ to be negative. Combining this idea with the postulate that the non-breeding herd would depend mainly on the number of breeders, but would be adversely affected by droughts and/or high slaughtering rates in the preceding year, we obtained the equation

$$\begin{aligned}
(20)\quad & (\text{number of non-breeding beef cattle, end of year } t) \\
& = \underset{(1{\cdot}5561)}{3{\cdot}4082} + \underset{(0{\cdot}3585)}{0{\cdot}5343}\ (\text{beef breeders, beginning of year } t) \\
& + \underset{(0{\cdot}0095)}{0{\cdot}0059}\ (t \times [\text{beef breeders, beginning year } t]) \\
& - \underset{(0{\cdot}000451)}{0{\cdot}001427}\ (t^2 \times [\text{beef breeders, beginning year } t]) \\
& - \underset{(0{\cdot}8346)}{0{\cdot}0720}\ (\text{slaughterings, year } t) \\
& - \underset{(0{\cdot}0527)}{0{\cdot}1153}\ (\text{beef drought index, year } t) \qquad (R^2 = 0{\cdot}92).
\end{aligned}$$

The data employed consisted of annual series for values of the regressand spanning 1947-48 through 1953-54; the "end" of 1947-48, for example, meaning the value recorded in the Agricultural and Pastoral Census for 31st March, 1948. The time scale was chosen such that $t = 0$ in 1955-56. The Durbin-Watson test for positive serial correlation $(d = 1{\cdot}47)$ was inconclusive.

It will be noted that, in terms of the coefficients of equation (19), we have obtained estimates

$$\hat{b} = 0{\cdot}0059 \tag{21.1}$$

$$\hat{c} = -0{\cdot}001427. \tag{21.2}$$

[22] The decision to use this variable was arrived at after experiments with other indicators such as the total size of the beef herd; a weighted index number of breeders and non-breeders with the weights allotted to breeders a diminishing function of the long/short beef price ratio; as well as others. The inventory of non-breeding cattle provided the most easily interpretable results, as well as the highest Student's $|t|$ statistic.

With the maximum value of t in the sample period equal to $+8$, it is seen that the postulates made above concerning the behaviour of the ratio of non-breeding to breeding stock are, in fact, verified empirically only for part of the sample period; the first derivative of λ_t with respect to time is estimated to become negative in 1958-59. However, the (constant) second derivative has the correct sign.

7. *Principal Results*

The core of six supply equations was fitted using a transform of the system (1) which allowed Aitken estimators to be obtained in such a way that the symmetry of partial transformation elasticities was preserved.[23] However, certain partial transformation elasticities were constrained arbitrarily to zero.[24] This was done partly because we expected certain cross supply elasticities (e.g. between wool and dairying) to be extremely low; and, given this prior belief, a method which estimates all transformation elasticities makes inefficient use of a strictly limited quantity of data. In other cases (e.g. between pastoral products and coarse grains), elasticities were constrained to zero because some "outputs" are in fact also intermediate products within the system: witness the winter grazing in southern Australia of oats which are subsequently harvested for grain. In the case of the short-run supply equation for beef contained within the six-equation core, it made little sense to postulate potential transformation of products such as wool into beef slaughterings, given that the beef price series involved was simply actual price lagged six months: there would scarcely be time enough for the other product to adjust. Patently, this does not apply to dairying where the opportunities for marketing cattle for slaughter at short notice are at least as good as in the beef industry proper. Although the explanations tendered above do, in fact, cover all cases where transformation elasticities have been constrained to zero, estimates of partial transformation elasticities with perverse signs would involve exceptionally serious consequences for this particular model since, under the specification which enforces homogeneity in prices, cross plus own price elasticities for any given product must add identically to zero. In other words, a statistical accident resulting in a large (though statistically non-significant) coefficient of the "wrong" sign on the price of a competing product, could lead to a totally unreliable picture of a product's own price elasticity. Finally we note that, for aggregate analysis, in certain cases the very concept of partial transformation production frontiers could break down, since, in the very short run, it might be impossible to visualize moving out of product 1 (e.g. wool) into product 2 (e.g. wheat) without varying the output level of a third product (e.g. mutton).

For the purposes of tabular presentation we have split the system, as previously, into a segment unresponsive to current price expectations (the Γ_{it}) and its logical complement. Again beef is exceptional; this time in that the short-run transformation schedule between beef and dairying is conditional on the ratio of the long-term expected prices for

[23] Powell and Gruen. "The Constant Elasticity . . .", *op. cit.*

[24] Formally, this constraint has two possible interpretations: (i) that the production frontier is right-angled; (ii) that the relative prices prevailing between the two products in question do not enter into the farmers' decision process. In so far as an interpretation is forced on us, we favour the latter.

beef and wool (i.e. the latter ratio appears in the shift function). In this paper we restrict our tabulations to results based upon an arbitrary coefficient of expectations of 0·6 for wool, with the other β_i scaled as in Table 1. Apart from brevity, we are justified in using this approach on the grounds that sensitivity analysis with smaller systems had indicated that varying the set of coefficients of expectations within plausible limits did not greatly affect the estimates of one- and five-year elasticities obtained (although the sensitivity of the operationally obscure infinite-period elasticities was considerable).

TABLE 4

Coefficients of Multiple Determination, and Estimated Constants and Coefficients of Shift Variables for Six-Equation Supply Model[(a)]

Equation	Regression constant	Variables shifting supply curve		Estimated coefficient of adjustment (γ)	R^2
Wool[(b)]	6·1435 (1·319)	drought mortality index[(c)]	—0·3675 (0·641)	0·0211	0·98
		lagged output[(b)]	0·9789 (24·806)		
Lamb[(d)]	0·0153 (0·950)	drought mortality index[(c)]	—0·0099 (3·909)	0·0671	0·89
		lagged output[(d)]	0·9329 (9·502)		
Wheat[(e)]	0·5958 (0·425)	lagged output[(e)]	0·9717 (8·470)	0·0283	0·87
Coarse grains[(e)]	0·8241 (1·750)	lagged output[(e)]	0·8558 (8·611)	0·1442	0·84
Beef and veal [(d)]	—0·5794 (4·565)	drought mortality index[(f)]	—0·0325 (3·192)	not estimated	0·90
		(long-run beef/wool price) $\times$ 10^{-2}[(g)]	—0·1627 (2·900)		
		opening inventory of non-breeding beef cattle[(h)]	0·2418 (10·758)		
Dairy[(i)]	1·3970 (4·255)	lagged output[(i)]	0·5721 (5·640)	0·4279	0·83

(a) All results are conditional on an assumed coefficient of expectations (β) of 0·6 for wool. With the exception of wheat, and beef and veal, other β-values are in the ratio shown in the third line of Table 1. The wheat price series was specially derived from institutional considerations (see Powell and Gruen, "Problems in Aggregate Agricultural Supply Analysis: I", *op. cit.*). Beef price is "short price" as measured by spot price, lagged six months. Student's $|t|$-values are shown in brackets (approximate test only).

(b) Millions of adult sheep shorn.

(c) Deviation of crude annual percentage mortality rate about a "normal" value of 6·3 per cent.

(d) Millions of tons carcass weight.

(e) Millions of acres.

(f) Deviation of crude annual percentage mortality rate about a "normal" value of 4·3 per cent.

(g) "Long-run" expected prices measured by modified distributed lag model.

(h) Millions of animals.

(i) Millions of dairy cows.

TABLE 5

Estimated Partial Transformation Elasticities

Partial frontiers between [a]:	Estimated elasticity[b]
Wool/wheat	—0·1592 (1·928)
Wheat/coarse grains	—0·2905 (1·474)
Beef/dairy[c]	—0·2918 (5·139)
Lamb/dairy	—0·2602 (2·726)
Lamb/wool	—0·1348 (1·114)

[a] Partial transformation elasticities for all other pairs constrained to zero (for reasons discussed in the text).
[b] Student's $|t|$-values in brackets.
[c] Beef price is spot price, lagged six months.

TABLE 6

Estimated Coefficients of Expected Prices in Supply Equations[a]

Product[b]	Coefficient with respect to the expected price[c] of:					
	Wool	Beef[d]	Wheat	Coarse grains	Lamb	Dairy
Wool	3·7797	0	—0·5898	0	—0·0087	0
Beef	0	0·0008	0	0	0	—0·0029
Wheat	—0·5898	0	0·2082	—0·0965	0	0
Coarse grains	0	0	—0·0965	0·1096	0	0
Lamb	—0·0087	0	0	0	0·0002	—0·0008
Dairy	0	—0·0029	0	0	—0·0008	0·0152

[a] Zero entries indicate coefficient constrained as such.
[b] For units, see footnotes to Table 4.
[c] Units for expected prices are:
Wool: £ per fleece; Beef: £ per ton of carcass weight; Wheat: £ per acre per annum; Coarse grains: £ per acre per annum; Lamb: £ per ton of carcass weight; Dairy: £ per cow per annum.
[d] Short-run price. For coefficient of long-run price, see Table 4.

TABLE 7

Estimated One-Year Own and Cross Price Elasticities of Supply[a]

Product	Coefficient with respect to the expected price of:						Sum
	Wool	Lamb	Wheat	Coarse grains	Beef	Dairy	
Wool	0·0698	—0·0177	—0·0521	0	0	0	0
Lamb	—0·1171	0·3167	0	0	0	—0·1996	0
Wheat	—0·1071	0	0·1808	—0·0737	0	0	0
Coarse grains	0	0	—0·2167	0·2167	0	0	0
Beef	0	0	0	0	0·1600	—0·1600	0
Dairy	0	—0·0606	0	0	—0·1319	0·1925	0

[a] See footnotes to Table 6.

TABLE 8

One- and Five-Year Own Price Elasticities Contrasted[a]

Product	One-year elasticity	Five-year elasticity[b]
Wool	0·070	0·3347
Beef	0·160	not estimated
Wheat	0·181	0·8541
Coarse grains	0·217	0·8131
Lamb	0·317	1·3849
Dairy	0·193	0·4223

[a] See footnotes to Table 4; elasticities evaluated at sample mean outputs and expected prices.
[b] Based on formula (17).

Estimated shift functions are given in Table 4; estimated partial transformation elasticities in Table 5; estimated coefficients a_{ij} with respect to expected prices in Table 6; estimated one-year own and cross price elasticities in Table 7; and finally, estimated own one-year and five-year price elasticities of supply are compared in Table 8. Some idea of the power of the technique used here may be gleaned from the fact that five partial transformation elasticities (Table 5) transform to a set of six own plus ten cross elasticities of supply (Table 7).

The general fit of the supply equations is reasonably good, R^2's ranging from a low of 0·83 for dairy products and coarse grains to an intermediate 0·87 to 0·90 for wheat, lamb and beef, and a high of 0·98 for wool. The Durbin-Watson statistics for the six equations indicated freedom of residuals from positive serial correlation in three cases (wool, coarse grains and dairy), but were inconclusive for the remaining three (lamb, wheat, and short-run beef). All arguments of the shift functions, other than the drought index in the case of wool, had estimated coefficients which differed significantly from zero under the usual tests. Only two of the six regression constants differed significantly from zero: those for beef and dairy. Disappointingly, of the five partial transformation elasticities estimated, only two were clearly significant, one marginally significant, whilst the remaining two had Student's $|t|$-ratios well below critical significance limits. However, all variables in the six-sector system have economically "sensible" signs, whilst in the short-run equation for beef, the ratio of long expected beef price to long expected wool price clearly has a significant effect in depressing the current rate of cattle slaughterings.

As is to be expected, the one-year own price supply elasticities tend to be rather low, ranging from 0·07 in the case of wool to between 0·16 and 0·22 for beef, wheat, dairying and coarse grains with a high of 0·32 for lamb. In all cases, output in year t depends very largely on those shift factors which are used as a proxy of the short-run capacity of the supply structure; for five of the six products (with beef again the exception) this is lagged output. In fact, price variables contribute very little to the descriptive power of the system (except in the case of dairy products). To illustrate this point, in Table 9 are set out the R^2 values obtained by single equation methods for each supply equation when price variables are eliminated; these are contrasted with the values of

R^2 obtained for each equation of the simultaneously fitted system including prices.

TABLE 9

Coefficients of Multiple Determination Contrasted for Supply Equations Including and Excluding Prices

Product	Proportion of variation in regressand explained in the case of:	
	Six-equation system including prices	Single equation estimates excluding prices
Wool	0·979	0·977
Beef	0·886	0·851
Wheat	0·871	0·844
Coarse grains	0·838	0·815
Lamb	0·899	0·897
Dairy products	0·826	0·477

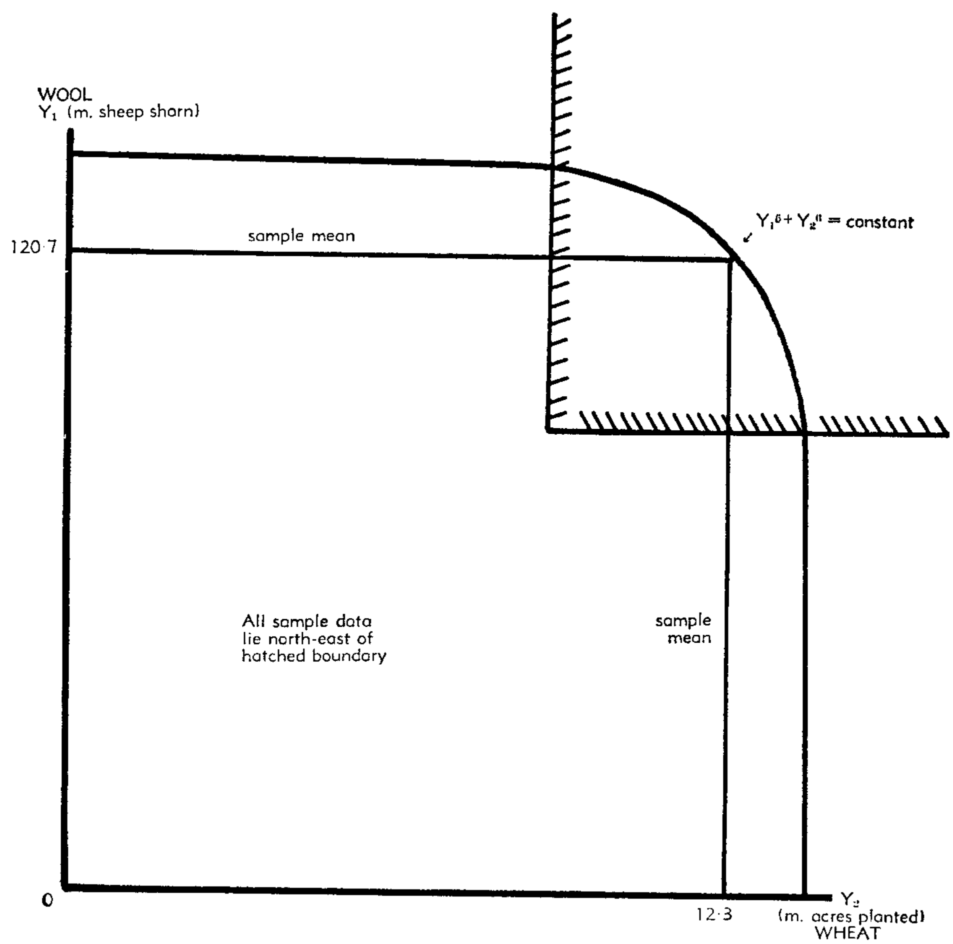

FIG. 1—Hypothetical constant elasticity of transformation production frontier belonging to the family whose partial transformation elasticity is —0·2.

The estimated partial transformation elasticities in Table 5 range from a low of $-0{\cdot}13$ for wool/lamb to a high of $-0{\cdot}29$ for wheat/coarse grains and beef/dairy. To provide readers with an illustration of a hypothetical production transformation curve based on a transformation elasticity in the above range, Figure 1 shows a hypothetical wool/wheat transformation curve with a partial transformation elasticity of $-0{\cdot}2$.

In the long run, changes in relative prices can be expected to have a considerably greater impact on the product mix. To illustrate this point we have tabulated both one-year and five-year own price supply elasticities in Table 8.[25] For coarse grains, wheat and lamb, our estimates suggest five-year supply elasticities in the neighbourhood of unity. Wool and dairy products are estimated to be in much less elastic supply, their five-year elasticities being put at 0·3 and 0·4 respectively.

[25] Here again beef is an exceptional product which does not lend itself to this type of treatment.