A NEW LOOK AT BUDGETING FROM THE STANDPOINT OF LINEAR PROGRAMMING

W. V. CANDLER[1]

University of New England

This paper will take a new look at budgeting. It will be argued that with the development of linear programming, budgeting can be seen to have several disadvantages as a research tool. The development of linear programming as an aid to farm management research will then be examined. This examination will lead to the development of a slight refinement of conventional budgeting methods. This refinement may reasonably be termed parametric budgeting.

Linear Programming as a Research Tool

The simplex method for solving linear programming problems has now been generally available for ten years. These ten years have seen its rapid acceptance by agricultural economists, and by industry, until today it is one of the most fashionable tools for the analysis and solution of production problems. At the "research" level, if not in extension, programming has gone a long way towards replacing budgeting as the production economist's most useful tool. The reason for this is not hard to find: Budgeting requires the analyst to nominate two points on the production possibility surface for comparison, whereas with linear programming if the analyst can provide the basic information which defines the production surface, the simplex method will then "automatically" find the highest attainable income.

Because the analyst nominates the plans to be compared, budgeting can be relied on to give "sensible results." Budgets are not drawn up for plans including the production of half a pig, or including 236 acres of wheat on a farm fenced in 250-acre paddocks, and budgets can be used for production situations where one or more factors exhibit increasing marginal product. The above is all to the credit of budgeting as an extension and research tool, but it can be said that *by nominating the alternatives to be examined, budgeting can beg the question.* Nomination of alternatives begs two questions which are likely to be of importance in research:

Question 1. What are the gaps in our knowledge of the problem being analysed? It may be possible to state that a particular property could carry 2,000 wethers or 1,300 breeding ewes. Given this information, and relevant prices, an appropriate budget could be constructed, but from a research point of view this is not nearly good enough. Important subsidiary questions which the budget begs are: Is the

[1] The author is indebted to Mr. E. J. Waring for his helpful criticism of an earlier draft of this paper, and to Mr. T. Harris for drawing the graphs.

marginal rate of substitution of wethers for ewes linear, or is there a kink? If there is a kink does it occur at a ratio of 1:9 or 1:1 or 9:1? What is the resource which limits production? And, how sensitive would the farm plan be to drought at different times of year? Completion of a programming matrix means that sufficient assumptions have been made to answer all of the above questions. If, as happens in practice, it proved impossible to obtain reasonable estimates of some of the required input coefficients, or resource availabilities (say, feed at different times of year) then it is an advantage of the programming approach that it immediately highlights this ignorance. Budgeting, on the other hand, would tend to gloss over the deficiency.

Question 2. Are farmers' rules of thumb really sound? Programming is often accused of producing ridiculous results.[2] This accusation can be met by incorporating the farming rules of thumb in the programming matrix. However, a better reply is that it is exactly this characteristic of violating present farming rules of thumb which is one of the most valuable research characteristics of the programming method. In a preliminary examination of the production possibilities of North-West New South Wales, Waring[3] found that, with quite reasonable assumptions, a farmer could double his income if he put 90% (instead of 20%) of his land into wheat and oats. Now obviously this is not the sort of result which should be published as a great discovery, and I am told that there are good agronomic reasons why cropping should not be pushed to this extreme. The result does suggest, however, that an expansion of the acreage under crops would be profitable, and that research workers should be asked to find just how far this expansion can safely be carried.[4] Waring has now redesigned his programming matrix to take account of the important agronomic restrictions. To repeat, from the research viewpoint "ridiculous" programming results are unexpected results and often raise useful questions about farming dogma which might otherwise be docilely accepted.

It might appear that budgeting has been too severely criticised as a research tool; "after all, budgeting is well suited to the solution of simple farm management problems." The only difficulty with this statement is that often the term "simple problem" is merely a euphemism for "a problem in which less than the full range of possibilities is to be considered."

Another question which programming is singularly well able to tackle is: What lines of agricultural research are likely to prove of greatest value?[5] If one knows that wethers are limited by mid-winter grazing

---

[2] One poor gentleman attempted to work out a minimum-cost fowl feed and discovered that they should only eat grit!

[3] E. J. Waring, *personal communication.*

[4] This may seem a mundane conclusion in view of the present wheat-wool price ratio; however, the farmer claimed, in giving Mr. Waring the original data, that he was not sure if it paid him to grow wheat at the present time! A more spectacular result is that in the short run *the opportunity cost of growing lucerne* in this area is *£3/10/- per acre.*

[5] E. O. Heady, "Economic Concepts in Directing and Designing Research for Programming Use of Range Resources," *Journal of Farm Economics,* vol. 38 (1956), p. 1604.

and the breeding flock by early spring grazing, then research which expands fodder availability in either of these periods is likely to be valuable. If, on the other hand, surplus autumn growth already goes to waste, then discoveries which increase the amount of autumn feed are unlikely to increase income. The programming approach quickly identifies the scarce resources, and thus indicates which lines of research are likely to be profitable.

For rough, general thinking about a problem budgeting is undoubtedly useful, but for a full analysis, even under Australian conditions, linear programming would usually be required. It is remarkable how many alternatives a grazier has when different husbandry practices are recognised as separate activities, and the possibilities of running cattle, conserving fodder or buying or breeding ewes or improving pasture are taken into account together with an analysis of feed and labour requirements of the programme in different months of the year.

Budgeting as an Aid to Extension

Few people would suggest that programming can compete with budgeting as an aid to extension. In particular budgeting has the two advantages that:

1. Budgets can easily be understood by farmers and hence budgeting results are more likely to be accepted. Budgeting demands no act of faith that the computations have been carried out correctly; indeed it can be done *by* farmers rather than *for* them.

2. Budgeting can be done quickly and requires little or no computing assistance. Certainly it avoids the need for, and expense of, a high-speed electronic computer.

Linear programming and budgeting are likely to be complementary in extension. The plans from programming a few typical, or "benchmark" farm situations can be used by extension personnel as a guide to the sort of plans which, when budgeted out, will increase farm profits. To an audience of agricultural economists it is unnecessary to go further into the importance of budgeting as an aid to extension—both as a check by the extension officer on the soundness of his recommendations, and as a way of illustrating to the farmer exactly how these recommendations are likely to affect his income.

Development of Programming Methods in Agriculture

It is useful in looking at budgeting from the standpoint of linear programming to consider the historical development of programming methods in agriculture. Three phases in this development can be distinguished:

1. Static Phase: In this phase one particular farming situation was described, and the corresponding optimum farm plan was computed.[6]

---

[6] G. A. Peterson, "Selection of Maximum Profit Combinations of Livestock Enterprises and Crop Rotations," *Journal of Farm Economics,* vol. 37 (1955), p. 546.

Thus a particular set of prices was assumed and particular amounts of land, capital and other resources were taken to be available.

Extension personnel were soon demanding more information ("What is the use of an optimum farm plan for $2,000 of cash inputs to a farmer with $10,000 available?"). This demand led to the development of the second or comparative-static phase.

2. Comparative-Static Phase: In this phase, a series of optimum plans were derived—one for each variation of the underlying assumptions. Thus if capital was felt to be an important variable, a series of plans might be derived for $2,000, $4,000, . . . $10,000 and unlimited supplies of capital.[7] The Comparative-Static approach has given way, in turn, to parametric programming.

3. Parametric Phase: In this phase the assumptions which extension officers think should be varied, are varied *continuously,* so that programmes are *developed* for, say, all levels of capital.[8] [9] Parametric programming has two important advantages in the author's opinion over the "comparative-statics" approach described above.

(a) Parametric programming requires little more computing effort than is required for the derivation of a single "static" linear programme.

(b) Parametric programming gives a much more complete picture of the production situation than can be obtained from a series of distinct programmes.

Development of Budgeting Methods

Budgeting methods which correspond to the *static* and *comparative-static* phases of programming have been known for a long time. Budgeting methods which correspond to parametric programming have not, to the author's knowledge, been used by agricultural economists.[10] Yet parametric methods would appear to be potentially as useful in budgeting as they have already been in programming. In most budgeting situations there are a few assumptions which are extremely vulnerable. Thus, there are few agricultural economists who would care to defend one particular figure as being "a reasonable long run price for wool." This means that *any one wool price* used for budgeting (or programming) is bound to be arbitrary in the sense that a price ten per cent.

---

[7] E. O. Heady and J. G. Gilson, *Optimum Combinations of Livestock Enterprises and Management Practices on Farms Including Supplementary Dairy and Poultry Enterprises.* Research Bulletin 437, Ag. Expt. Stat. Iowa State College, Ames, Iowa (1956).

[8] E. O. Heady and L. D. Loftsgard, *Farm Plans for Maximum Profits on the Cresco-Clyde Soils in Northeast Iowa,* Research Bulletin 450, Ag. Expt. Stat. Iowa State College, Ames, Iowa (1957), and W. Candler, *Linear Programming as an Aid to Economic Analysis,* A.N.Z.A.A.S. 1958, Section G. Mimeo.

[9] Mathematicians use the term *parameter* for a quantity which is given for any particular problem, yet can vary from problem to problem. Thus for a particular farm in a particular year, the acreage and the price received for wheat will be fixed; but between farms the acreage will vary, and between years the wheat price will vary. Hence acreage and wheat price can be termed parameters. If, in this case, optimum programmes were derived for a *range* of acreages and *all* reasonable wheat prices then these plans could be described as the results of parametric programming.

[10] Apart from W. Candler, *Wool and Wethers,* Farm Management Report No. 1, University of New England, Armidale, N.S.W.

higher or lower could equally well be defended by an economist of a slightly more optimistic or pessimistic temperament. It follows that decision making is much more likely to be intelligent if the optimum plans for a range of relevant prices are known, than if only the best plan for one (arbitrary) price is known.

Parametric Budgeting

By analogy with programming it seems reasonable to use the term parametric budget for a budget in which one or more of the assumptions is allowed to take on all values within a relevant range. The principles of parametric budgeting are extremely simple. First decide which assumptions (or parameters) should be varied, say, on a wether property, wool clip, wool price and stocking rate. Then give each of these parameters a letter, say, wool clip = w, wool price = p, and stocking rate = s. Third, compute the budget in the normal way, except that each time a parameter enters the budget, use the corresponding letter, rather than a specific numerical value. Expressing the revenue from running a wether in the New England district in pence, the above steps might lead to a revenue equation such as:—

$$R = wp - 5w - 367$$

where R is the revenue per wether (in pence) net of variable cost, w is the pounds of wool cut per wether, and p is the price of wool in pence.

The first term on the right hand side of this equation is the gross revenue from wool sold, the second term represents the selling costs, while the third term refers to running costs including the cost of flock maintenance.

To express the returns, net of variable cost, on a per acre basis it is only necessary to include stocking rate in the above equation:—

$$R' = wps - 5ws - 367s.$$

where R′ is the return per acre, net of variable cost, and s is the stocking rate per acre.

It should be obvious that for *any* relevant values of w, p and s, the above equations can be used to give a corresponding revenue. Thus once the budgeting required to derive these equations has been carried out, the results corresponding to any price-production assumptions of interest can quickly be derived. If the revenue per acre (net of variable costs) was required for a property carrying 1.5 wethers per acre, clipping 11 lbs. of wool, with wool selling for 50d. was required, then since s = 1.5, w = 11, and p = 50, the equation would read:—

$$\begin{aligned} R' &= (11)(50)(1.5) - 5(11)(1.5) - 367(1.5) \\ &= 550(1.5) - 55(1.5) - 367(1.5) \\ &= 128(1.5) \\ &= 192 \end{aligned}$$

thus the return per acre would be 192d. or 16/-.

A change of price often affects costs as well as returns. Thus an increase in the price of wool will likely increase the cost of flock maintenance. If it is possible to express the cost of flock maintenance

in terms of pounds of wool, then this allows the parameter "wool price" to affect both cost and returns, and hence to be accurately reflected in the final revenue equation.

The most direct advantage of parametric budgeting for the research or extension worker is, in the author's opinion, that it provides a concise summary of a wide range of possible budgeting results. There is, however, another advantage of parametric budgeting in that when the results are properly presented they should concentrate attention on important gaps in our knowledge.

Presentation of Results

There are essentially three ways of presenting the results of parametric budgeting. These are as graphs, as a nomogram or as a pro forma budget.

Graphical results, as in Figure 1, should be easily understood by anyone with a training in production economics. The two axes of the graph have been assigned to two appropriate parameters (wool clip per wether, and wethers per acre) and appropriate iso-revenue lines have been inserted. Since three parameters are being varied in this figure, separate graphs have been drawn for wool prices of 50, 60, 70 and 80 pence per lb.

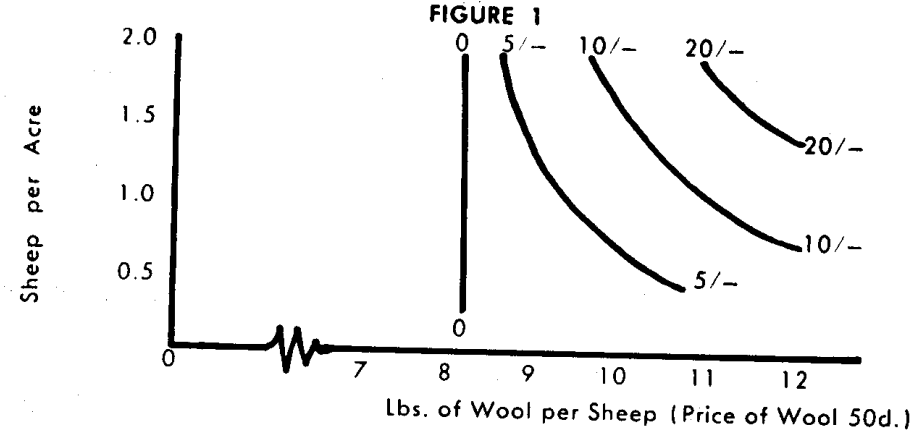
Figure 2 is a nomogram which can be used to express the results of parametric budgeting. If $p = 50$, $w = 11$ and $s = 1.5$, then point A corresponds to assumed values of p and w. Moving from A parallel to the iso-revenue lines one comes to B on the pivot line. From B and the assumed value of s we get C, and moving from C parallel to the second set of iso-revenue lines we reach D, and can see that the corresponding revenue per acre is about 16/-. This checks, of course, with the budget computed above.[11] Nomograms would probably be of most use to extension workers in saving them from running through a budget each time they want an approximate result. Farmers who have experienced nomograms in engineering handbooks would also, likely, find the nomogram approach useful. Nomograms are not likely to be very useful in highlighting technical problems about which little is known, since it is hard to visualise the relevant production economics relationships.

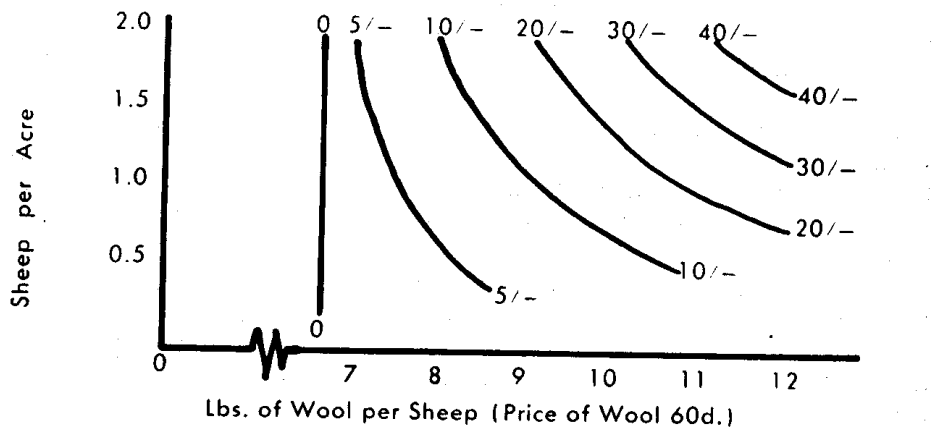
The third way of expressing the result of parametric budgeting would be as a *pro forma* budget. In this case one budget is presented, and the farmer is expected to fill in his own figures:—

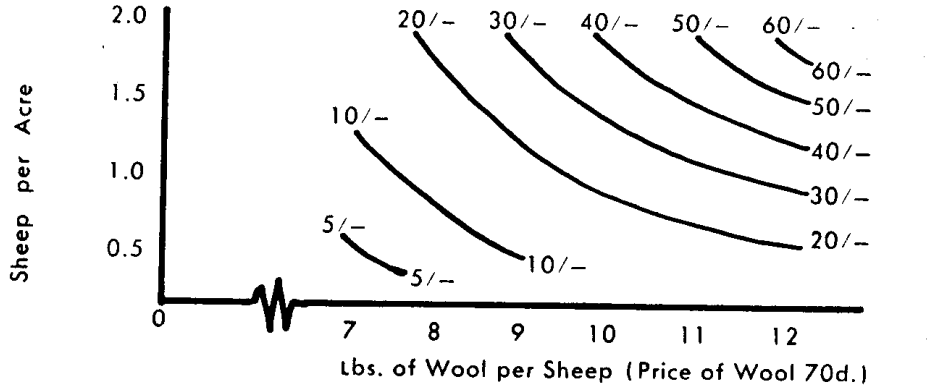
Return from running one wether cutting *11* lbs. of wool, selling for *50*d. (11)(50)	550d.
Less selling cost 5d. per lb. on *11* lbs. 5(11)	55d.
Net return on wool clip	495d.
Less cost of running one wether	367d.
Net return on one wether	128d.
with a stocking rate of *1.5* per acre, the return per acre is (128)(1.5)	192d.

[11] The author is indebted to Mr. Eric Waring for insisting that the results of parametric budgeting could be expressed in this way.

FIGURE 1


Sheep per Acre
2.0
1.5
1.0
0.5
0
0
5/-
10/-
20/-
20/-
10/-
5/-
0
7
8
9
10
11
12
Lbs. of Wool per Sheep (Price of Wool 50d.)


Sheep per Acre
2.0
1.5
1.0
0.5
0
0
5/-
10/-
20/-
30/-
40/-
40/-
30/-
20/-
10/-
5/-
0
7
8
9
10
11
12
Lbs. of Wool per Sheep (Price of Wool 60d.)


Sheep per Acre
2.0
1.5
1.0
0.5
0
20/-
30/-
40/-
50/-
60/-
60/-
50/-
40/-
30/-
20/-
10/-
5/-
10/-
5/-
7
8
9
10
11
12
Lbs. of Wool per Sheep (Price of Wool 70d.)

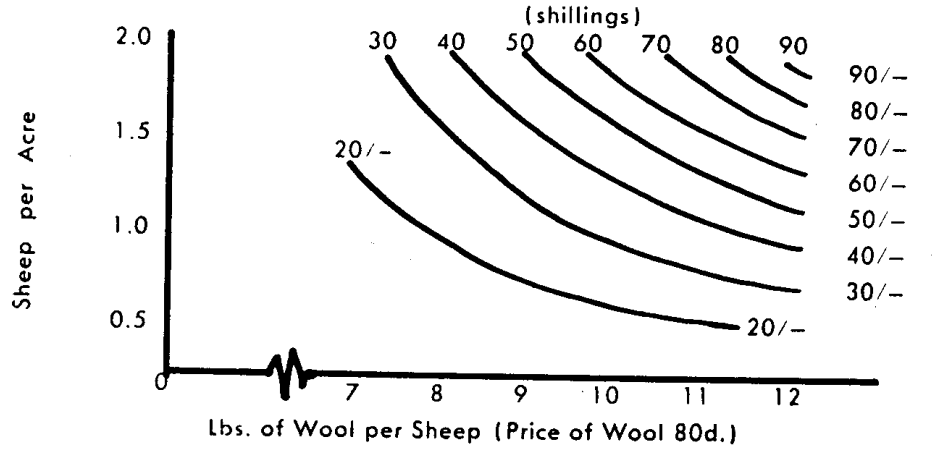
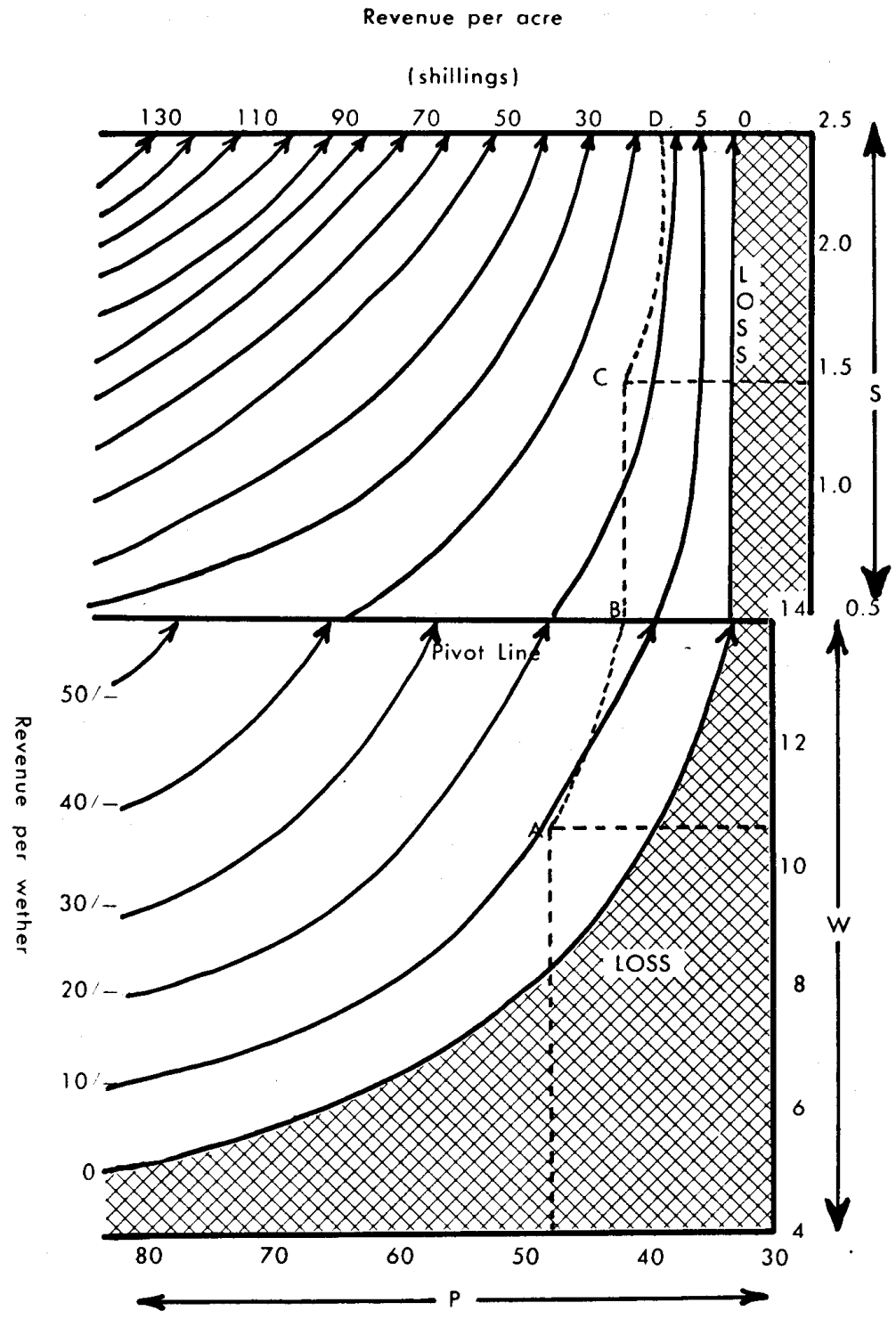

(shillings)
Sheep per Acre
2.0
1.5
1.0
0.5
0
30
40
50
60
70
80
90
90/-
80/-
70/-
60/-
50/-
40/-
30/-
20/-
20/-
7
8
9
10
11
12
Lbs. of Wool per Sheep (Price of Wool 80d.)

FIGURE 2


Revenue per acre
(shillings)
130
110
90
70
50
30
D
5
0
2.5
2.0
1.5
1.0
0.5
S
LOSS
C
B
14
Pivot Line
50/–
40/–
30/–
20/–
10/–
0
Revenue per wether
12
10
8
6
4
W
A
LOSS
80
70
60
50
40
30
P

In this case the farmer only has to repeat the budget, inserting his own values for the underlined assumptions.[12]

Conclusion

Linear programming is unlikely to supersede budgeting as an extension tool. Budgeting results, like programming results, depend for their usefulness on the accuracy of the underlying assumptions. By their very nature many of these underlying production and price relationships must vary from farm to farm, and year to year. Parametric programming methods have been developed in an attempt to assist decision making in an uncertain environment. It has been argued that attention to parametric budgeting techniques would extend still further the usefulness of the budgeting method.

DISCUSSION

F. G. Jarrett (University of Adelaide): I would like first of all to congratulate Dr. Candler on a most useful discussion on the use of both budgeting and linear programming procedures in research and extension. Any attempt to narrow the gap between those who use conventional budget procedures and those working on more powerful research tools is well worthwhile. Dr. Candler's point that both techniques have a role to play in production economics and workers in this field should not become one-eyed barrackers of either method certainly needs emphasising.

I doubt if I would go so far as to agree with Dr. Candler that at the "research" level linear programming has gone a long way towards replacing budgeting as the production economist's *most* useful tool. The techniques of marginal analysis still appear useful aids for discussing the class of problems where decreasing marginal returns seem relevant. There is also a large class of problems where cost information on alternative methods of doing a given job seems more appropriate than bringing to bear the big guns of linear programming. Such questions as the costs of various methods of fodder conservation, feeding rates for livestock, water rates in irrigated farming appear to me to be areas where simpler techniques than linear programming would yield useful results.

On the questions of limiting resources and farmers' rules of thumb I admit the usefulness of linear programming in pin-pointing both the limiting resource and in evaluating the sensibility of rule of thumb methods. However, perhaps Dr. Candler overstates the use of linear programming in these two connections. A good budgeteer is, and should be, aware of limiting resources before he draws up a budget. For example, one does not have to be a linear programmer to be aware that on most South Australian dairy farms the availability of winter feed is the limiting factor in stocking rates. On the second

---

[12] Note that the *presentation of results* is designed to show the revenue corresponding to any set of values for the parameters. Obviously it cannot give more information than this. It cannot, for instance, give the revenue if the cost of running a wether does not correspond to the assumptions in the original budget. If one wants to treat running costs as a parameter, then giving these costs an appropriate letter, say c, the parametric budgeting method outlined above can be used.

question of the sensibility of farmers' rules of thumb I am not convinced by Dr. Candler's example of, to quote, "with quite reasonable assumptions a farmer could double his income if he put 90% (instead of 20%) of his land into wheat and oats." Dr. Candler then goes on to say that "there are good agronomic reasons why cropping should not be pushed to this extreme." It seems to me that one cannot have reasonable assumptions, get an answer to the problem and then dismiss the answer for "good agronomic reasons." Because of the mechanistic nature of the simplex solution linear programming could in inexperienced hands yield nonsensical results which then have to be rejected on criteria external to the problem. The solution, as Dr. Candler points out, is to incorporate the agronomic restrictions into the problem. Because it is a more powerful technique linear programming can handle all the restrictions imposed on the plan whereas a budgeteer could not hope to keep track of the restrictions if they are numerous. Linear programming will, I think, make its greatest contribution to practical farm problems by providing bench marks which will narrow the field of alternatives about which the budgeteer will be concerned. However, at the individual farm level the use of simple budget procedures has yet to be exploited in extension in Australia.

I found Dr. Candler's use of parametric programming most informative. The technique seems to me to admit of more general budget solutions by assigning various values to some of the key parameters. This obviates the necessity of reworking the budget each time, say, price changes. The only query I have on parametric budgeting has to do with the linearity assumptions in certain of the functions. For example, in the revenue equation $R = wp - 5w - 367$, Dr. Candler assumes the selling costs constant at 5d. per lb. of wool and the running costs per wether including the cost of flock maintenance constant at 367d. There may be cases in selling costs where unit costs per lb. are not constant, this would be the situation with freight costs on rail transport. Similarly, I suggest there may be non-linearities in the running costs per wether. Dipping costs per wether, for example, would tend to decline with the number of sheep dipped.

These linearity assumptions may result in a loss in flexibility in budgeting since the budgeteer using conventional budget procedures could admit non-linearities into the budget. There is no doubt, however, that parametric budgeting does permit a more general solution which results in economies of time and computation for the budgeteer.

Mr. Defries: I am doubtful of the use of elaborate mathematical tools in production economics. The farmer relies on his own intuitive judgment and agricultural economists substitute their own economic viewpoint for what is a problem involving sociology, psychology as well as economics.

Dr. Schapper: I would like to pursue the question of the difficulties of getting acceptance of parametric budgeting since we are replacing intuition by more formal methods.

Mr. Gruen: Linear programming may not give different results from those obtained by trial and error. In industry linear programming has resulted in gains of 1%–5%; in practice gains of this order may not

be important in small-scale enterprises. Linear programming is essentially concerned with short run problems and perhaps in longer run problems marginal analysis may come into its own.

Mr. Rowe: In the United Kingdom attempts have been made to integrate the linear programming and budgeting procedures with linear programming providing standards for extension workers using budget procedures at the individual farm level.

Dr. Candler: Linear programming does provide a rational basis for resource valuation and these valuations are key pieces of information in farm management decisions. The linearity assumption seems to serve fairly well in many cases. Should non-linearities exist on the production side I would expect these to come through into the revenue equation. If there are non-linearities in some of the cost functions then, as I have suggested in a footnote in the paper, the introduction of further parameters would allow for them.

I agree with those comments that results must be presented in a form suitable for farmers and the diagrams are intended for this purpose.