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A DYNAMIC ANALYSIS OF US EXPORT WHEAT PRICING AND MARKET SHARES

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The economics of a higher loan rate to support US wheat prices is analysed. Utilising optimal control theory, a dynamic wheat trade model is developed. The basic premise underlying the model is that the United States finds itself having transient monopoly power in the wheat market. An expression for the optimal pricing policy which maximises the present value of expected profits over the indefinite future is derived. Results from both the theoretical and empirical models demonstrate that the US wheat pricing strategy depends on its costs relative to competitors' costs, the discount rate and the competitors' response function. The main policy implication of the analysis is for the dominant wheat exporting country constantly to seek to lower costs relative to competitors and to maintain a price exceeding unit cost without encouraging competitors' expansion.

The United States is the dominant supplier of wheat to the world marketplace. Over the past 20 years, US exports have made up about 40 per cent of the world wheat trade while the nearest competitor, Canada, has supplied less than 20 per cent of the market.

The US government's farm program, with high support and release prices for wheat, is designed to raise farm incomes through raising prices. However, this pricing policy also has an impact on competitors in the international market. The US loan rates have acted as a floor for world wheat prices (Dunmore 1985; Wilson 1986) and as a result there has been increasing competition from the European Community, Canada, Australia and Argentina. The US share of world wheat trade has been falling since the late 1970s. Consequently, US stocks of wheat have grown in recent years while those of the competitors have remained low (Mitchell and Duncan 1987). These trends may have been the outcome of quasi-optimal pricing.

Partly in response to the declining market share, a major provision of the 1985 farm bill was a lowering of the loan rate. As discussed by Gardner (1985), an alternative option would involve using a higher loan rate and production controls to support prices. In this paper, the economics of this alternative are analysed. The dominant position of the United States suggests the short-run response of rival competitors to price signals may be slow. The United States could, therefore, raise the loan rate higher than its present level and increase profits even though some market share would be eroded. However, these short-run profits

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may be short-lived as high loan rates will induce the expansion of competitors' exports and reduce long-run profits. To account for expanded exports from other countries, it is argued that the United States should pursue a dynamic pricing strategy that would gradually lower loan rates in future years.

The objective in this paper is to determine an optimal pricing strategy for US wheat. The current market structure (that is, world demand and competitive fringe supply from other exporters) is taken as given and the current US price support program is evaluated *vis-à-vis* the optimal pricing strategy.[1] With the aid of a dominant country model, both the short-run static and long-run dynamic optimal price policies are investigated. With the dynamic model it is shown that the optimal wheat price is bounded above by the short-run profit maximising price and below by the competitors' unit cost of production.

The Model

The US wheat trade is traditionally modelled by an excess demand curve. Demand curves are represented geometrically as two-dimensional schedules relating quantities demanded to the price charged (see Paarlberg, Webb, Morey and Sharples 1984). But what does the price dimension mean in a dynamic long-run framework? If the price axis reflects the current price only, the quantity variable (presumably some weighted average of quantities demanded in various periods) is not fully determined since future import demand depends on future prices as well as the current price. If the price represents that charged in every period, this is unrealistic, for even the most cautious oligopolists revise their prices from time to time. If price is viewed merely as an average of current and future levels, nothing can be concluded about the actual price in any specific period, including the current period for which decisions must be taken.

The notion of a dynamic long-run demand curve can only be handled by relating the firm's profit maximisation decision to the rate at which the quantity sold changes over time. It is well known that maximising immediate profits is often not the optimal strategy if the planning horizon extends beyond the present. A policy for achieving the highest overall profit may dictate the sacrifice of some current short-term gain. It is, therefore, necessary to develop a dynamic model in which the high short-term profits associated with the pursuit of monopoly pricing have to be balanced against the loss of long-term profits upon entry of other suppliers attracted by the high price.

The basic assumption of the model developed in this paper is that the United States finds itself having transient monopoly power in the wheat market. The problem is to determine the optimal pricing behaviour for a wheat exporting country which enjoys, at each instant of time, potential monopoly power with respect to its current importing customers but which could not indefinitely maintain a price above the market price without losing all its customers.

Long-run profit maximisation requires that the consequences of today's pricing decisions be weighted far into an uncertain future.

[1] For a discussion of the structure of the world wheat market see Schmitz, McCalla, Mitchell and Carter (1981), Wilson (1986) or Mitchell and Duncan (1987).

Exporters making such decisions may reach divergent conclusions concerning probabilities and contingencies under identical objective functions. To avoid hypotheses that explain everything (for example, non-price preferences and institutional factors) but predict nothing, this paper emphasises market variations linked in a systematic and compelling way to observable structural variables.[2]

Optimal Wheat Price in the Short Run

First, consider a static model which determines the short-run profit maximising wheat policy for the United States. Assume that the United States dominates the export supply of wheat and that additional supplies are provided by a competitive fringe made up of the European Community, Canada, Australia, and Argentina. Let:

p = price of wheat per bushel;
c = average total cost of producing one bushel of wheat in the United States;
$f(p)$ = market demand curve which is downward sloping and twice differentiable;
q = the level of wheat sales by the competitive fringe;
Q = the level of wheat sales by the United States; and
$\pi(p)$ = US profit function which is assumed to be a smooth concave function of price along the initial demand curve.

In equilibrium, $Q+q=f(p)$. Rearranging terms,

(1) $Q=f(p)-q$

In addition the following relationship holds:

(2) $\pi(p)=(p-c)Q$

From (1) and (2), it is easy to obtain:

(3) $\pi(p)=(p-c)[f(p)-q]$

If the dominant country (the United States) uses p as a decision variable, the profit maximising price is then obtained by differentiating expression (3) with respect to p and setting the result equal to zero as follows:

(4) $\partial\pi(p)/\partial p=(p-c)f'(p)+f(p)-q=0$

where the prime denotes $\partial/\partial p$. Solving for the maximising price p^* yields:

(5) $p^*=[q-f(p^*)+cf'(p^*)]/f'(p^*)=\{[q-f(p^*)]/f'(p^*)\}+c$

Thus, p^* is set above the unit cost by the United States.

The response function of the competitive fringe, q, can be defined as being an increasing function of the wheat price above the limit price, p_L. The limit price is that at which the net entry of the competitive fringe is zero. If $p_L>c$, then (p_L-c) measures the cost advantage enjoyed by the United States. From equation (4) and assuming $p_L=c$, it may be noted

[2] Non-price factors are ultimately reflected in the price of wheat and, therefore, are captured in the model proposed in this paper.

that at the point where $p=c$, the total demand is supplied by the fringe and the US market share is zero. The dominant country will prevent this extreme result by ensuring that $p>c$ and thus $p>p_L$.

Let the response function be represented as:

$$(6) \quad q=h(p-p_L) \qquad \partial q/\partial p>0$$

The United States can thus determine its market share, S, as:

$$(7) \quad S=Q/f(p)=[f(p)-q]/f(p)$$

Substituting for q from equation (6) yields:

$$(8) \quad S=[f(p)-h(p-p_L)]/f(p)$$

Differentiating (8), the following result can be established:

$$(9) \quad \partial S/\partial p=\frac{[f'(p)-h'(p-p_L)]f(p)-f'(p)[f(p)-h(p-p_L)]}{f(p)^2}$$
$$=\frac{-h'(p-p_L)f(p)+f'(p)h(p-p_L)}{f(p)^2}<0$$

The erosion of market share will be larger, the larger the market response of the competitive fringe, $h'(p)$. This result indicates that the United States will lose its market share by maximising short-run profits and raising the price of wheat. This would most likely require the use of production controls in the United States in order to prevent stocks from accumulating. These controls could be adjusted so as to reduce both production and existing stocks. As long as the demand for US wheat exports is not perfectly elastic, importers will pay some of the cost of supporting US farm income (Johnson, Womack, Meyers, Young and Brandt 1985). On the other hand, if the United States is going to maintain prices above the no-program level for political reasons, production controls can be part of a second-best program package because they help offset target price incentives to overproduce (Schnittker 1985). In addition, these controls may also be used as an efficient stabilisation instrument given inflexible support prices (Wright 1985).

Optimal Wheat Price Over Time

The model presented in this paper overcomes the difficulties associated with the dynamic price behaviour of wheat which cannot be accommodated in static analyses such as the excess demand approach. Defining the state variable, $q(t)$, as the sales from the competitive fringe and $\dot{q}(t)$ as the rate of change in the level of these sales, assume that $\dot{q}(t)$ is directly proportional to the difference between the current price, $p(t)$ (the control variable), and the limit price, p_L. In other words, the state variable, $q(t)$, reflects the structure of the market as it is shaped by the US pricing policy. It is worth pointing out that the value of $q(t)$ influences the dominant wheat exporter's pricing policy at any moment in time (that is, structure affects conduct). The choice of a pricing policy in turn affects $\dot{q}(t)$ and hence has important feedback effects on market structure.

This model is, in essence, related to the excess demand approach in an oligopolistic framework. The specific functional dependence of sales on time is determined by the nature of the entry phenomenon. It is assumed that the level of the dominant country's sales can be decomposed into additive univariate functions of price and time, respectively. In other words, the sole decision variable is price. An expression for the price policy during the pre-entry stage that maximises the present value of expected profits over the indefinite future is derived. The model developed here is superior to the excess demand model because it extends the latter to an oligopolistic structure and it provides an appropriate framework for including the time dimension in the analysis of market shares. Using this model an otherwise short-run static model of the excess demand becomes a fully dynamic long-run model.

Let the residual demand curve facing the United States be:

$$Q[p(t), t] = f[p(t)] - q(t)$$

where:

$f[p(t)]$ = initial demand schedule; and

$q(t)$ = the level of wheat sales by the competitive fringe at any point in time.

The residual demand curve $Q[p(t), t]$ at any specific instant, as assumed above in the short-run case, is found by subtracting the output of the competitive fringe from the total market demand. This equation indicates that the net effect of changes in the level of rival wheat sales is to shift the US residual demand curve to the left. This would be the case if the rival exporter's short-run supply curves were completely inelastic. The rate of change in the level of rival wheat sales, $\dot{q}(t)$, is naturally determined by their expected rate of return. If potential entrants view the current wheat price as a proxy for future price, the rate of entry will be a monotonically non-decreasing function of current price.

Utilising the mathematical techniques of optimal control theory, one can apply Gaskins' (1971) limit-pricing model to the problem and explore the optimal wheat pricing policy through time.[3] To formulate the problem in the framework of optimal control theory, the law of motion for the system, an objective functional, and the initial conditions need to be specified.

The law of motion can be specified as the following differential equation:

$$(10) \quad \dot{q}(t) = k[p(t) - p_L]$$

where k is a constant 'response' coefficient and $\dot{q}$ is dq/dt, the rate of change in the level of rival wheat sales.

The US objective functional is then:

$$(11) \quad V = \int_0^{\infty} [p(t) - c][f(p) - q(t)]e^{-rt}dt$$

where r is the US discount rate. The initial condition is given by $q(0) = q_0$.

[3] See Rausser and Hochman (1979) for further applications of optimal control theory to agriculture.

The maximum principle can be considered as an extension of the method of Lagrange multipliers to the dynamic optimisation (control) problem. This problem is one of maximisation subject to constraints, where the expression to be maximised is the objective functional:

$$(12)\quad \max V = \int_0^{\infty} [p(t)-c][f(p)-q(t)]e^{-rt}dt$$

and the constraints are the n differential equations and the initial conditions:

$$(13)\quad \dot{q}(t) = k[p(t)-p_L] \qquad q(0)=q_0$$

Thus the problem may be posed as maximising (12) subject to (13). Proceeding in a way analogous to that in static problems, a row vector of new variables $[\lambda(t)]$ can be added to the problem.

The next step is to define a Lagrangian function, called the Hamiltonian, which equals the objective function plus the inner product of the Lagrange multiplier vector and constraints. Yet again, by analogy to the static case, a saddle point of the Hamiltonian would yield the solution. However, in this case the saddle point is in the space of functions. The control trajectory $[p(t)]$ then solves the control problem.

The present value Hamiltonian can, therefore, be formed as follows:

$$(14)\quad H = [p(t)-c][f(p)-q(t)]e^{-rt} + \lambda(t)k[p(t)-p_L]$$

where $\lambda(t) = \partial V/\partial q(t)$, which is interpreted as the shadow price of an additional bushel of wheat supplied by the fringe at any point in time. This interpretation of $\lambda(t)$ stems from the fact that the objective functional has the dimension of an economic value, that is, profit and the state variable have the dimension of an economic quantity. As a result, $\lambda(t)$ must have the dimension of a price — a shadow price. The interpretation of $\lambda(t)$ is obviously the dynamic analogue to the interpretation of the Lagrange multipliers of static economising problems (Intrilligator 1971).

Using Pontryagin's maximum principle, the necessary conditions for the optimal path may be derived (Pontryagin, Boltyanskii, Gamkrelidze and Mishenko 1962). These conditions are:

$$(15)\quad \dot{q}(t) = k[p(t)-p_L] \qquad q(0)=q_0$$

$$(16)\quad \dot{\lambda}(t) = [p(t)-c]e^{-rt} \qquad \lim_{t\to\infty} \lambda(t) = 0$$

where:

$$(17)\quad \lambda(t) = \frac{\{q(t)-f(p)-[p(t)-c]f'(p)\}e^{-rt}}{k}$$

For a stationary market demand curve and by eliminating $\lambda(t)$ from these equations, the simultaneous differential equations can be obtained:

$$(18)\quad \dot{q}(t) = k[p(t)-p_L] \qquad q(0)=q_0$$

$$(19) \quad \dot{p}(t) = \frac{k(p_L - c) + r[q - f(p) - (p-c)f'(p)]}{-2f'(p) - (p-c)f''(p)}$$

These two equations generate a family of trajectories in the q–p plane.

Setting equations (18) and (19) equal to zero and solving for q^*, which is the equilibrium level of fringe output, the following will result:

$$(20) \quad q^*(t) = (p_L - c)f'(p_L) + f(p_L) - [k(p_L - c)]/r$$

Defining the US market share as before will yield:

$$(21) \quad S^*(t) = \frac{[k(p_L - c)/r] - [f'(p_L)(p_L - c)]}{f(p_L)}$$

Differentiating (21), the following result will be established:

$$(22) \quad \partial S^*/\partial c < 0$$

This relationship means the larger the unit cost of production for the United States, the smaller its market share. From equation (21), two more equivalent conditions will follow:

$$(23) \quad \partial S^*/\partial(p_L - c) > 0 \quad \text{and} \quad \partial q^*/\partial(p_L - c) < 0$$

That is, the larger the cost advantage enjoyed by the United States, the larger its market share and hence the smaller the fringe market share. On the basis of these conditions, the US market share is an increasing function of its cost advantage. The United States prices itself out of the market in the long run if it has no cost advantage. This result, however, is not valid once a growth of the export wheat market is assumed. Then, given that the increases in market demand are monopolised by the United States, the long-run price of wheat will be above the average cost of production. But if these increases are shared out according to the current market shares, the result can be shown to be analogous to the static demand model assumed above.

Furthermore, the following conditions can also be obtained:

$$(24) \quad \partial S^*/\partial r \leq 0 \text{ which is equivalent to } \partial q^*/\partial r \geq 0$$

$$(25) \quad \partial S^*/\partial k \geq 0 \text{ which is equivalent to } \partial q^*/\partial k \leq 0$$

Condition (24) indicates that as the US discount rate increases, the United States will sacrifice a portion of its long-run market share. A higher discount rate indicates that future profits become relatively less important. Condition (25) indicates that the more rapidly the fringe responds to price signals, the larger will be the US long-run market share. This result is counter-intuitive and would seem to question the efficacy of any wheat price support program designed to increase the responsiveness of the fringe.[4]

There is yet one final and important observation which can be established. With the static model, maximum profits were given by equation (4) which is repeated here:

[4] Another pertinent result, which will be obtained empirically in the next section of the paper, is that the optimal price trajectory will be lowered as the responsiveness of the fringe to price signals is increased.

(26) $(p-c)f'(p)+f(p)-q=0$

It may also be noted that maximisation of the Hamiltonian at every point along the optimal path implies that:

(27) $-\lambda(t)=\{f'(p)[p(t)-c]+f(p)-q(t)\}e^{-rt}/k$

where $\lambda(t)<0$, rewriting (27) as:

(28) $[p(t)-c]f'(p)+f(p)-q(t)+ke^{rt}\lambda(t)=0$

Equations (4) and (28) are both satisfied if and only if $p>p(t)$. This condition indicates that the short-run profit maximising price always exceeds the long-run optimal price. With the static model it was shown previously that $p_L \leq p$. Hence, the price bounds for US wheat are given by $p_L \leq p(t)<p$.

Empirical Model

It is possible to generate the optimal price trajectory from the basic model. The trajectory is a solution to the original optimisation problem, that is, maximising (12) subject to (13). The necessary conditions for the optimal price trajectory are equations (15), (16) and (17). Letting $\theta(t)=\lambda(t)e^{-rt}$, the above necessary conditions for the optimal path will be the solutions to the following system of ordinary linear differential equations involving constant coefficients a, b, c, k, p_L and r:

$$(29) \quad \dot{\theta}(t)=\left(r+\frac{k}{2b}\right)\theta(t)+\left(-\frac{1}{2b}\right)q(t)+\left(\frac{a-bc}{2b}\right)$$

$$(30) \quad \dot{q}(t)=\left(\frac{k^2}{2b}\right)\theta(t)+\left(-\frac{k}{2b}\right)q(t)+\left[\frac{k(a-2bp_L+bc)}{2b}\right]$$

Using the laws of the simultaneous linear differential equations in Gaskins (1970) and Pontryagin (1962), the general solution of this system may be found. It is equation (30) which is of interest.

The solution for this equation is:

$$(31) \quad q(t)=\left[\frac{k}{c}(c-p_L)+b(c-2p_L)+a\right]\left[1-\exp\left(\frac{r}{2}-\sqrt{\frac{r^2+2kr/b}{2}}\right)t\right]$$

Differentiating (31) with respect to time will yield the possible price trajectories for finite time horizon:

$$(32) \quad p(t)=p_L-\left\{\left(\frac{r}{2}-\sqrt{\frac{r^2+2kr/b}{2}}\right)\left[\frac{k}{r}(c-p_L)+b(c-2p_L)+a\right]\exp\left(\frac{r}{2}-\sqrt{\frac{r^2+2kr/b}{2}}\right)t\right\}/k$$

Given a, b, c, k, p_L and r, (31) and (32) can easily be estimated.

The dynamic economising problem in general is that of optimisation over a particular time horizon. Therefore, it is the choice of time paths for the control variable, p, via a set of differential equations which is of interest rather than the exact values of wheat prices as such. The figures and tables represent the particular feasible price trajectories that maximise the objective functional among the set of all such trajectories.

As an example, to generate price trajectories, the world excess demand function for wheat estimated by Sharples and Paarlberg (1982) was used, that is, $q_D = 142 \cdot 8 - 0 \cdot 284p$. Letting c = US\$2.00 per bushel and p = US\$2.50 per bushel and noting the fact that $a = 142 \cdot 8$ and $b = 0 \cdot 284$, $p(t)$ and $q(t)$ can be estimated.

The empirical results demonstrate that over a relatively long period of time, for example, $t = 10$ years, the optimal price trajectory is a decaying exponential (see Figure 1) while the fringe response function is a monotonically increasing function of time (see Figure 2).

These results, therefore, re-establish the theoretical finding that the short-run profit maximising price always exceeds the long-run optimal price. They indicate that a limit pricing strategy would return low short-run profits. However, different values of r and k will yield different boundary values for equations (31) and (32). Concentrating exclusively on (32) and considering the alternative specifications in Table 1, the following important conclusions can be drawn: as the discount rate increases, the US optimal price level at any point in time will increase as well. From Table 1, it is also apparent that an increase in fringe response to price changes (that is, k) will decrease the US optimal price level. In order to see the impact of changes in the cost advantage on the US optimal price level, let $c = p$ = US\$2.50 per bushel. These results,

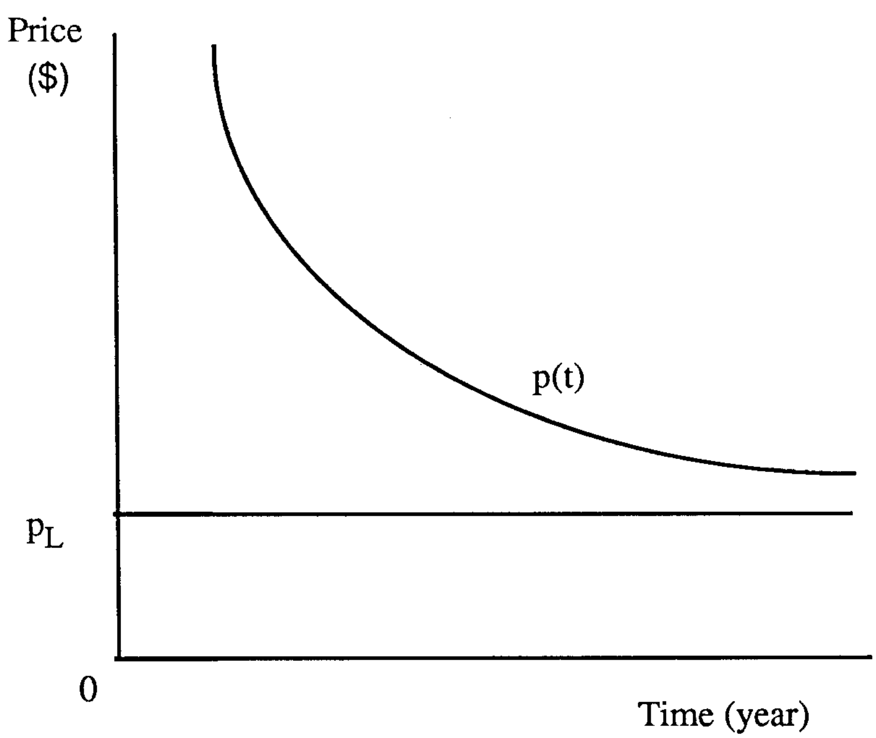

FIGURE 1—Optimal Price Trajectory for US Wheat Prices: Ten-Year Horizon.

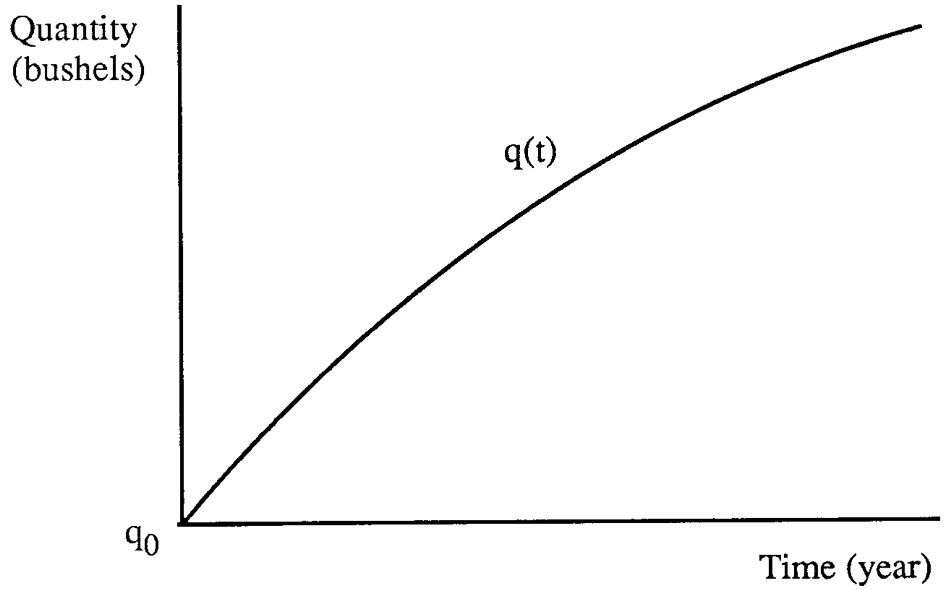

FIGURE 2—Fringe Response Function.

TABLE 1

Estimated Impacts of Discount Rate Changes and Fringe Response to Price Changes on the US Optimal Price Level[a]

Scenario	Discount rate	Response coefficient	Estimated current price	Estimated limit price	Estimated optimal price[b]
1	0·05	0·50	52·00	10·00	42·00
2	0·05	0·75	42·00	6·60	35·40
3	0·05	1·00	37·00	4·90	32·10
4	0·05	1·50	29·00	3·50	25·50
5	0·05	2·00	25·00	3·00	22·00
6	0·075	0·50	61·00	9·20	51·80
7	0·075	0·75	51·00	5·60	45·40
8	0·075	1·00	44·00	4·20	39·80
9	0·075	1·50	36·00	3·10	32·90
10	0·075	2·00	30·00	2·80	27·20
11	0·10	0·50	69·00	8·20	60·80
12	0·10	0·75	57·00	5·00	52·00
13	0·10	1·00	50·00	3·70	46·30
14	0·10	1·50	41·00	2·90	38·10
15	0·10	2·00	35·00	2·70	32·30

[a] Prices are all in US$ per bushel.
[b] These are boundary values.

shown in Table 2, indicate that a decrease in the cost advantage enjoyed by the United States will increase the US optimal price level.

It is worth noting that the empirical results confirm the theoretical propositions that the cost advantage, the discount rate and the responsiveness of the fringe are the main factors to be considered by the

TABLE 2

Estimated Impacts of Changes in the Cost Advantage on the US Optimal Price Level[a]

Scenario	Average cost	Discount rate	Response coefficient	Estimated current price	Estimated limit price	Estimated optimal price[b]
1	2·00	0·10	0·50	69·00	8·20	60·80
2	2·50	0·10	0·50	70·00	8·30	61·70

[a] Prices and costs are all in US$ per bushel.
[b] These are boundary values.

United States for a dynamic pricing strategy in a limit price setting (see conditions (23), (24) and (25)).

Finally, to test the impact of time on the price trajectory, the time span was extended from 10 to 50 years. The results of this scenario demonstrate that lengthening the time horizon will shrink the optimal price path (see Figure 3).

Concluding Remarks

Optimal pricing strategies for the United States as a dominant wheat exporter have been discussed. The paper extends the standard excess demand approach to modelling US wheat trade by generalising the latter to an oligopolistic structure and presents a framework for including the time dimension in an analysis of wheat market shares.

It was shown that a high loan rate will generate high short-run profits but will induce an increase in fringe supply. The demand for US wheat

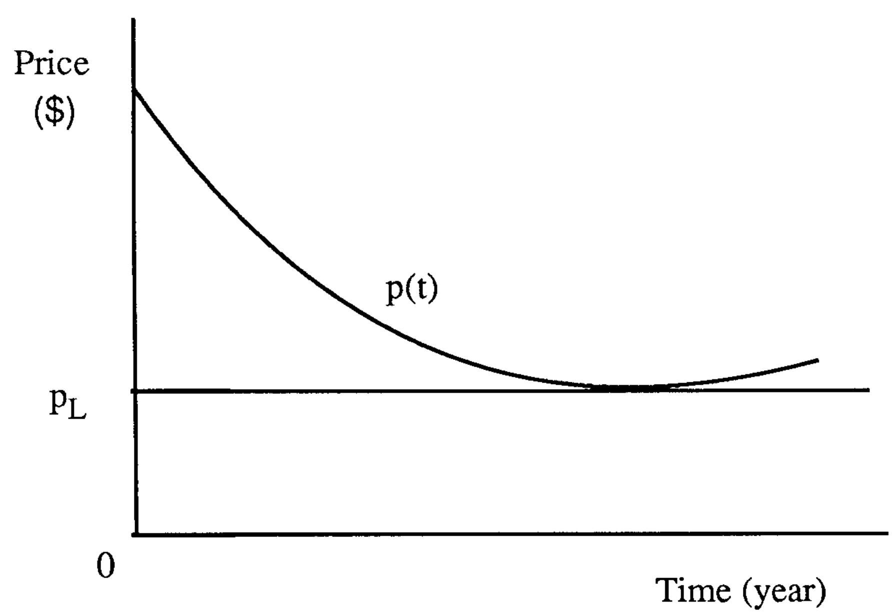

FIGURE 3—Optimal Price Trajectory for US Wheat Prices: Fifty-Year Horizon.

exports will subsequently fall and thus long-run profits will be lower than otherwise would be the case. Alternatively, a limit pricing strategy, with a low loan rate, would return low short-run profits and discourage fringe supply.

Which pricing strategy the United States should choose over time depends on its cost advantage, the discount rate, and the responsiveness of the fringe. The optimal strategy for the United States is to maintain persistently a price exceeding unit cost without encouraging competitors' expansion. An increase in US cost advantage will induce a decrease in the optimal price level, other things being equal. This implies that the United States will respond to an increase in its relative cost advantage by lowering its price to secure a larger future share of the market.

The discount rate is a measure of the weight attached to current versus future profits by the United States. A rise in the discount rate puts more emphasis on current profits, hence justifying a rise in current price and a sacrifice of some future profits. The optimal price is lowered as rivals respond more quickly to a profit stimulus. An increase in the response of the competitive fringe to price signals will mean a larger volume of future profits sacrificed for the marginal current profit dollar. Obviously, to restore the optimal balance between current and future profits, the United States must lower current price.

It can be concluded that an increase in the US discount rate or an increase in the response of rivals to price signals will cause the optimal price trajectory to approach the limit price more rapidly. It will, therefore, be rational to drive out competitors if a sufficient cost advantage exists. A polar case of this strategy is for the United States to incur initial short-run losses to enlarge its market share.

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