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# Incorporating Uncertainty in Integrated Assessment Modeling

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# Incorporating Uncertainty in Integrated Assessment Modeling

## Introduction and Background

Integrated assessment research is becoming a common method used to examine many resource and environmental issues. In many applications, models from several disciplines are linked and used to provide a single point estimate result. However, this result is only one of a range of possible outcomes and may not reflect the mean i.e. expected, model outcome (Morgan and Henrion 1990) or the degree of uncertainty and potential range of model outcomes. In many cases the empirical validation of results from these models is difficult if not impossible, thus any insights into model uncertainty or the range of potential model outcomes can be valuable for decision analysis and policy. Poor information for policy decisions can result in a number of costs. These could be measured in several ways, for example the amount of insurance a person would willingly purchase to offset the same event occurring with different degrees of uncertainty; or the relative cost of outcomes that occur with and without information (McCarthy *et al* 2001). Investigation and characterization of the uncertainty associated with model outcomes could improve policy decisions; provide better understanding of the current state of knowledge and its limitations and implications (Morgan 1978); improve the quality of decisions by describing the full range of outcome uncertainty; provide additional insight into those factors that are responsible for creating uncertainty and guide the future design and refinements of existing models among other benefits. For these, and other reasons, improvements in the integration of economic and biophysical models and methods to quantify their uncertainties were identified as high priority areas for action in a recent Intergovernmental Panel on Climate Change report (McCarthy *et al*. 2001). This is an area that is poorly treated or incompletely addressed in many studies (Rotmans and Dowlatabadi 1998).

This paper examines how uncertainty analysis can be used to examine parameter uncertainty; determine the expected value of model outcomes rather than a single point estimate and provide estimates of model uncertainty and the range of possible model outcomes for a non-linear integrated economic and biophysical model. The second section considers sources of uncertainty in many modeling and estimating efforts. The third section examines methods and procedures that can be used to characterize uncertainty and develop estimates of expected model outcomes. The fourth section is an empirical application to an integrated assessment model that examines the marginal costs of soil carbon sequestration within the dry-land crop region of Montana. The final section presents the results discussion and conclusions. We show that in the case of a non-linear integrated assessment model, the initial point estimate model outcome does not represent the expected model outcome. In addition we use the analysis to develop bounds for the comparison of likely model outcomes and show that the initial point estimate model outcome lies within the $25^{th}$ and $75^{th}$ percentiles. In many cases the range of model outcomes suggested by our analysis could have conflicting policy implications that are not apparent from a single model run.

**Sources of Uncertainty**

Data collection, model structure and estimation procedures are characterized by different sources and degrees of uncertainty that affect model outcomes. In general, uncertainty arises from, spatial or temporal choices about which there is imperfect information or data in addition to inherent variability of physical processes and estimation error, model structure or form and information translation and interpretation (McCarthy *et al* 2001; Katz 2001; Adams, Cook and Corner 2000). In addition, the appropriate scale of analysis can be uncertain resulting in models

constructed at an inappropriate scale being used for policy analysis (Antle, Capalbo and Mooney 1999). Several authors have characterized sources of uncertainty; for example Morgan and Henrion (1990); Rowe (1994) and Adams, Cook and Corner (2000) among others. In integrated assessment modeling, economic data describing costs, prices and production technology are used to construct economic models that are coupled with biophysical models populated with relevant soil, climate, water and other data. Often results from one model are used as inputs to the other. In these complex model designs, there are many sources of uncertainty.

**Uncertainty analysis**

Several papers have recognized that uncertainty is an important factor to consider when analyzing model outcomes. For example, Hellerstein (1995) examines how data and model scales influence study outcomes; Alston and Chalfont (1991) examine uncertainty arising from errors in model specification; while Dorfman, Kling and Sexton (1990) use a Monte-Carlo analysis to provide confidence intervals to accompany elasticity point estimates. Although several economic models have explored some sources of uncertainty, few have examined the role of parameter uncertainty (a notable exception is Abler, Rodriguez and Shortle (1999)). In contrast, the biophysical sciences have examined the impact of parameter uncertainty on many biophysical phenomena, for example: water quality (Bobba, Singh and Bengtsson 1996; Haan and Zhang 1996; Yu, Yang and Chen 2001); crop growth (Aggrawal 1995; Wallach *et al*. 2002) and soils models (Barkman and Alvetig 2001) among many others.

A number of techniques have been proposed to describe the influence of parameter uncertainties on model outcomes; for example, sensitivity analysis, scenario analysis and Monte Carlo analysis (Katz 2001; Morgan and Henrion 1990). In sensitivity analysis, a single model

input is varied and used to provide a simple linear approximation of the effect of this change on model output. Sensitivity analysis is a very common approach used to characterize uncertainty. It is simple to execute and not computationally intensive. In addition the analysis can be tailored toward scenarios identified as likely or having an important influence on model outcomes. A disadvantage of this approach is that it reflects modeler's biases toward parameters or factors that they think are important. These factors may not have the largest influence on model outcomes and thus this technique could overlook important sources of uncertainty. Under a scenario analysis, a new set of inputs are chosen for the model based on future likely events for example best case and worst case or in some analyses several parameters can be changed and their combined effects evaluated using a combinatorial scenario or decision tree (Katz 2001; Morgan and Henrion 1990). A combinatorial scenario or decision tree approach allows practitioners to examine the influence of jointly changing several parameters within a model and can provide a richer source of information about the influence of parameter uncertainty than simple sensitivity analysis. If the model has few parameters this technique can provide a comprehensive depiction of model uncertainties. However, if the model has a large number of parameters this technique can become computationally intractable because the number of computations is exponentially related to the number of parameters perturbed (Morgan and Henrion 1990). A third technique, Monte Carlo analysis, develops posterior distributions of model outcomes by manipulating inputs whose prior distributions are known or can be approximated. This technique can be used to calculate confidence intervals for model outputs (Smith 1973); identify the expected, i.e. mean, model outcome and identify parameters whose values have a significant influence on model outcomes. The technique provides a complete description of uncertainty arising from variations in all model parameters even when non-linearities are present in the model and in

cases where parameter distributions are not clearly known (Haan and Zhang 1996). This technique can be computationally intensive, but the number of computations required can be reduced using a variety of sampling schemes (for example, Latin hypercube sampling). The output distribution can be regarded as the model prediction error, representing the combined effects of all the parameter uncertainties (Bobba, Singh and Bengtsson 1996; Aggarwal 1995). This technique can account for probabilistic interdependencies between parameters and can be used to develop statistical representations of uncertainty arising from the value of parameter estimates. Finally, this technique is linear in the number of parameters described as uncertain and can be used to examine uncertainty in large models. Integrated assessment models are complex with many different parameters and thus, the Monte Carlo technique is a good choice for uncertainty analysis with these models.

**Expected value of model output**

Consider a model represented by a function *f* that maps a vector of parameter values ($\hat{\boldsymbol{\beta}}$) within a given model structure to create an output value y. In the case where parameter values represent the expected value of each $\hat{\beta}_i$, the model output is $y_0$ (1).

$$(1) \qquad y_0 = f(E[\hat{\boldsymbol{\beta}}]) \qquad \text{where, } E(\hat{\beta}_i) = \beta_i \text{ for } i = 1, \ldots\ldots, n.$$

If the parameter estimates used as inputs to the model change, the value of y will also change and the effect of uncertainty in $\hat{\boldsymbol{\beta}}$ on y can be estimated by examining the effect of all possible

combinations of $\hat{\beta}_i$ on y. The distribution of y is a function of the input parameters and the model structure.

Deviations in the value of output y from $y_0$ can be expressed in terms of deviations of parameters from their initial expected values using a Taylor Series expansion. Using the first two terms and taking the expected value of the deviation we find:

$$E[y - y_0] \approx \sum_{i=1}^{n} E[\beta_i - \hat{\beta}_i]\left[\frac{\partial y}{\partial \beta_i}\right]_{E[\hat{\boldsymbol{\beta}}]} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} E[\beta_i - \hat{\beta}_i][\beta_j - \hat{\beta}_j]\left[\frac{\partial_2 y}{\partial \beta_i \partial \beta_j}\right]_{E[\hat{\boldsymbol{\beta}}]}$$

We know that $E[\hat{\beta}_i] = \beta_i$ and so $E[\beta_i - \hat{\beta}_i] = 0$ and the first term drops out, leaving (2)

$$(2) \qquad E[y - y_0] \approx \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} Cov[\beta_i, \beta_j]\left[\frac{\partial_2 y}{\partial \beta_i \partial \beta_j}\right]_{E[\hat{\boldsymbol{\beta}}]}$$

If the function is linear, $\frac{\partial_2 y}{\partial \beta_i \partial \beta_j} = 0$, and $E[y] = y_0$. However if the function is non-linear, the second derivative terms will be non zero and $E[y] = \bar{y} \neq y_0$, that is, the mean output value is not equal to $y_0$, produced by input parameters evaluated at their expected value. This illustrates that *the expected value of model output cannot be obtained by evaluating the model with all parameters set to their expected values if the model is non-linear*.

Information about E[y] can be developed by perturbing parameter values to obtain a distribution function for y and using this to generate a value for $\bar{y}$ and its confidence intervals. In addition the analysis can be used to identify parameter values that cause large deviations in y from $\bar{y}$ and identify sources of model sensitivity to parameter values. This option is not explored within this paper but is planned for future analyses.

An approximate distribution for y can be generated by selecting *a priori* distributions on parameter values $\hat{\beta}$ and then drawing sets of parameters at random to generate a distribution of

independent random outcomes for y. Two important factors need to be considered. First how many independent outcomes need to be generated to provide an estimate of E[y] with desired error and confidence; and second the appropriate distribution for each input parameter.

The sample size is a function of the output distribution. This is not known *a priori* (Bobba, Singh and Bengtsson 1996) but can be calculated using standard statistical techniques (Morgan and Henrion 1990; Abler, Rodriguez and Shortle 1999; Bobba, Singh and Bengtsson 1996). For a model with a single output of interest Morgan and Henrion (1990) suggest (3) for estimating the sample size ($n$) required to estimate a confidence interval for E[y] with given error bounds ($\varepsilon$) and confidence level ($\alpha$).

$$n > \left( \frac{2Z_{\alpha} S}{\varepsilon} \right)^2, \tag{3}$$

Where $Z_{\alpha}$ is a normal deviate, and $S$ is the estimated standard deviation of the output of interest. A procedure for estimating sample size when the model provides information about multiple outputs is described in Abler, Rodriguez and Shortle (1999).

The appropriate distribution for each random variable is not always known *a priori*. In many cases, a probability distribution is assumed for each variable using "best judgement", past experience, goodness-of-fit tests and theoretical expectations among other means (Hann and Zhang 1996; Bobba, Singh and Bengtsson 1996; Abler, Rodriguez and Shortle 1999). In their analysis, Haan *et al*. (1998) found that good estimates of the mean and variance of input parameters are more important than the correct distributional form.

**Empirical Model**

In this section we describe the structure and empirical application of an integrated economic and biophysical model. First we describe the "original" base model, used in previous analyses to provide point estimate outcomes. Then we describe how the model structure can be changed to provide an estimate of the expected model outcome and additional information such as confidence intervals.

*"Original" base model*

Our base model is an econometric process model that can be coupled with site-specific biophysical models to estimate the economic and biophysical consequences of changes in agricultural production practices. The conceptual underpinnings and previous empirical applications of this model are presented in Antle and Capalbo (2001) and Antle *et al* (2003). Site-specific agricultural production data are used to estimate econometric production models of output supply, cost and input demand. Parameter estimates from these models are then used to parameterize a non-linear simulation model representing producer choices as a sequence of discrete and continuous land use and production decisions. The simulation model contains field specific data describing field location and input and output prices that vary by location. It is initialized by selecting an initial crop type, tillage practice and other management variables for each field based on random draws from sample distributions estimated by the data. Expected yields, output and production costs are simulated for each field using parameter estimates from the econometric models. Many alternative economic and biophysical scenarios can be incorporated into the model by changing input and parameter values. For each scenario, the model is run for 5 decision periods, each spanning 4 years, representing a production cycle. Within each period, crop production decisions are based on a comparison of the possible net

returns from each crop system. At the end of these periods, the observed mean frequencies of crop production over the period are calculated as well as any changes in the biophysical outputs of interest.

In this paper the model is used to estimate producers' land use and management decisions in response to a range of market prices offered for carbon (C) credits. Atmospheric concentrations of greenhouse gases can be reduced by sequestering C in agricultural soils. Additional C can be sequestered by increasing cropping intensity, e.g. switching from a crop-fallow system to a continuous cropping system; and/or reducing tillage intensity e.g. switching from conventional to no-till. If we assume that the goal of each individual producer is to maximize their profits from production, they will be engaged currently in land use and management practices that yield the highest net returns and will change practices to those that sequester additional C, only if they are provided with an incentive that compensates them for any economic losses. There are many ways to structure these incentives (Antle and Mooney 2001). We consider a market-based payment for each additional tonne of C that producers sequester as a result of a management change.

A personal interview survey was used to collect field level production data from a sample of more than 1200 fields in the dry-land grain producing area of Montana. The sample is statistically representative of the USDA's Major Land Resource Areas (MLRA's) in the region. Crop-fallow and continuous cropping are the predominant systems in the region and used to produce, winter wheat, spring wheat and barley. Log-linear supply and cost functions for these three crops are estimated using three stage least squares, while fallow costs are estimated using ordinary least squares. A total of ninety-six parameters are estimated econometrically and used to parameterize the simulation model.

Soil C accumulation rates expected under each cropping system are estimated using the Century ecosystem model (Parton *et al*. 1994; Paustian *et al*. 1996) and used as inputs into the simulation model. Century is a generalized biogeochemical ecosystem model that simulates C (i.e. biomass), nitrogen and other nutrient dynamics. It includes sub-models for soil biogeochemistry, growth and yield sub-models for crop, grass, forest and savanna vegetation and simple water and heat balance. It employs a monthly time step and the main input requirements (in addition to management variables) are monthly precipitation and temperature, soil physical properties (e.g. texture, soil depth) and atmospheric nitrogen inputs.

The three MLRAs represented in the production data were each stratified into two sub-zones (sub-MLRAs), based on precipitation differences suggested by historical climate data, giving a total of six sub-MLRAs (figure 1). The suffix "high" denotes sub-zones with higher rainfall while the suffix "low" denotes sub-zones with lower rainfall. Soils and climate data were collected for the six sub-MLRAs, and used as inputs into Century in addition to information about crop rotations, fertilization and tillage practices. Century calculates average changes in soil C stocks within each sub-MLRA over a 20-year period from winter wheat, spring wheat and barley under continuous and crop-fallow systems. Carbon stocks for grass, a non-crop land use, are also calculated to reflect areas used for conservation or livestock grazing. The soil C rates for each change in production system are assumed to be the same within each sub-MLRA, but differ across the sub-MLRAs.

The simulation model is run for a sequence of ten carbon payments ranging from $10/tonne C to $100/tonne C in $10 increments. The model determines how producers change their crop choices in response to these payments and the corresponding change in total soil C

sequestered. This formulation is then used to generate marginal cost curves for C sequestration within each sub-MLRA.

*Uncertainty Extension*

The base model described above only provides point estimate marginal cost curves that represent a single model outcome generated from a set of expected parameter values. As discussed earlier this result does not represent the expected model outcome given that the simulation model is non-linear. Neither can this formulation provide any additional insight into the range of possible model outcomes. In this section we describe how we can use a Monte Carlo technique to generate an estimate of the expected model outcome and develop data that can be used to examine the consequences of parameter uncertainty for the quantity of C predicted by the model.

The variance co-variance matrix from each econometric model is used with a multivariate normal random number generator to generate perturbed parameter estimates for each of the underlying econometric models. One advantage of the econometric production model formulation is that parameters are based on econometrically estimated models thus we know their *a priori* distributions for the analysis. The simulation model is then initialized multiple times and at each initialization selects new values for all parameters from the set of perturbed parameters. The predicted C amounts at every price level, within each sub-MLRA are saved to form separate output distributions i.e. within a single sub-MLRA we save 10 distributions representing individual distributions of carbon quantities predicted at each of the price levels offered to producers within that area.

The sample size required to estimate the mean amount of C sequestered at each price level ($p$) within each sub-MLRA ($a$), $E[y_{ap}]$, is calculated using (3). The standard deviation of

soil C levels (S) is estimated by calculating the standard deviation from 500 model runs. The allowable error ($\varepsilon$) is 3 percent and the confidence interval for the expected mean carbon quantity at each price level within each sub-MLRA is 95 percent. The sample size ($n$) selected for the analysis is $\text{Max}\{n_{ap}\}$. The simulation model is run $\text{Max}\{n_{sp}\}$ times to generate a distribution of model outcomes reflecting the range of variability in parameter estimates. The mean of the distribution is the expected value of carbon sequestered within each area at each price level, $E[y_{ap}]$.

**Expected model outcomes and possible variability**

The Monte Carlo framework and econometric production model described above are used to estimate the expected model outcome and compare this against the previously calculated point estimate result. The marginal cost of soil C sequestration is examined under each of the alternative model formulations and percentiles are used to provide additional information about the potential range of model outcomes that could be observed.

The model was run 500 times to develop an estimate of the standard deviation of C sequestration within each area and price level (table 1). In general, the standard deviation of C sequestered decreases as price increases. As the price offered for C increases more producers agree to sequester soil C, increasing the denominator in the expression for S and lowering the standard deviation. The sample sizes needed to estimate the mean model outcome with 3% error and 95% confidence within each sub-MLRA at each price level are calculated in accordance with (3) and are also shown in table 1. Within each sub-MLRA, the sample size required to estimate the mean C produced decreases as the price per tonne of C increases. In part this is a function of the smaller standard deviation in C quantities at higher C prices and the fact that 3% of the mean

C value at each price is a large number at higher prices because more C is sequestered. The largest sample size is required in sub-MLRA 52 high at a price of $10/tonne C while the smallest sample size is required in Sub-MLRA 53 high at a price of $100/tonne C. Thus Max$\{n_{sp}\}$ = 4,873 or approximately 5,000 model runs.

The original parameter estimates $\hat{\beta}_i$ are perturbed 5,000 times to generate new parameter estimates for the simulation model. Their maximum and minimum values over 5,000 perturbations in addition to their expected values used in the original simulation model are presented in table 2. In total 96 parameter estimates were perturbed for the analysis. In some cases, their values deviate from their initial estimated value more than 40 times. Some parameters received values that cross between positive and negative ranges.

The expected value of the number of tonnes C sequestered is calculated by averaging the outcome from 5,000 runs within each area at each C price (table 3). As expected, the quantity of C sequestered increases as the price offered per unit of C increases because more producers are interested in sequestering C at higher C prices. The expected carbon values from the Monte Carlo analysis ($E[y_{ap}]$) range between 0.71 MMT (million metric tons) at $10/tonne C in sub-MLRA 53 high to 4.16 MMT at $100/tonne in sub-MLRA 52 high. In contrast the carbon values produced by a single model run using the estimated parameter values range between 0.66 MMT at $10/tonne in sub-MLRA 53 high to 4.32 MMT at a price of $100/tonne C in sub-MLRA 52 high. At low carbon prices, the model's expected value is less than the original point estimate while at higher prices the models expected carbon quantity is lower than predicted from the point estimate model. This pattern is followed within each of the six sub-MLRA areas. The expected carbon quantities under a Monte Carlo simulation analysis do differ from the nominal carbon values generated from a model with all parameters set to their expected values supporting the

earlier theoretical result showing that the expected value of model output cannot be obtained by evaluating the model with all parameters set to their expected values if the model is non-linear. These results suggest that the point estimate model underestimates the quantity of C sequestered at low prices and over estimates at high prices. If the price of carbon is relatively low as suggested by many entities, the point estimate model underestimate the competitiveness of Montana agriculture within a market for C.

A wide range of outcomes are possible as a result of perturbing the parameter estimates by their variance co-variance matrix (figure 2). The marginal cost curve developed using the point estimate model crosses the expected curve estimated using 5,000 model runs (figure 2) illustrating the different predictions from the two model formulations. At prices of $10 and $20/tonne C, the expected marginal cost curve suggests that producers within each region will sequester an additional 3 percent to 27 percent more carbon than suggested by the point estimate curve. The distribution of model outcomes at each price within each sub-MLRA are non-normal and thus we chose to use percentiles rather than confidence intervals to describe model outcomes. In all areas the point estimate model outcomes lie within the $25^{th}$ to $75^{th}$ percentiles that encompass 50% of the model outcomes (figure 1). The width of each percentile range varies by area. For example, model outcomes in area 52 low have a wide span of values while outcomes in 53 high are narrowly clustered. The C quantities sequestered can vary by as much as 300 percent and this variation tends to be much larger at lower C prices than at high prices. These values show that the response of producers to a market payment per tonne of additional C sequestered could be very variable although the range of variability is not constant across all areas (figure 2). These additional analyses demonstrate that the potential for producers to generate income from C sequestration activities can differ significantly.

## Conclusions

In this paper we developed two model formulations, the first resulting in a single point estimate outcome for C sequestration and the second a Monte Carlo analysis that allowed us to estimate the expected model outcomes and generate additional information about the degree of uncertainty or potential variability in possible outcomes. We applied both models to a case study the examined how producers' crop production choices would change in response to payments for soil C sequestration within the dry-land crop region of Montana.

The results show that there are differences between the point estimate outcomes from a single model run using expected parameter values and the expected model outcome generated from 5,000 model runs. This supports the earlier statement that a single model run using parameters set at their expected values is not the expected value of a non-linear model. In addition the Monte Carlo model was able to generate additional information concerning the complete range of model outcomes.

In this empirical example, the point estimate model tended to under predict the quantity of C that could be sequestered at low C prices. Although the current market for C credits is thin, estimates suggests that C prices are likely to be at the lower range of those examined in this study. Further a full uncertainty analysis was shown to provide estimates of the degree of variability that could occur in response to this policy.

These findings suggest that uncertainty analysis is a powerful and useful tool for policy analyses that examine complex problems with economic and biophysical interrelationships. In particular this technique can offer greater insight into the possible distribution of outcomes as well as their mean, providing a much richer source of information for policy analysis.

Several extensions to this work are planned for future research. First the current analysis only considers uncertainty arising from the economic parameter estimates, in future work the biophysical parameters will be added to the analysis. In addition we are estimating which parameter values and their magnitudes cause the model to predict either very high or very low carbon values. Information about what causes these events can be useful both for future model development and for directing further data collection.

Table 1. Estimated standard deviation (500 runs) and sample size by area and carbon price

| Price per tonne ($) | Sub-MLRA 52H | | Sub-MLRA 52L | | Sub-MLRA 53H | | Sub-MLRA 53L | | Sub-MLRA 58H | | Sub-MLRA 58L | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Standard Deviation | Sample Size | Standard Deviation | Sample Size | Standard Deviation | Sample Size | Standard Deviation | Sample Size | Standard Deviation | Sample Size | Standard Deviation | Sample Size |
| 10 | 941,970 | 4,873 | 924,002 | 2,855 | 187,215 | 1,263 | 333,370 | 2,950 | 525,842 | 1,032 | 335,095 | 1,135 |
| 20 | 970,294 | 3,580 | 907,287 | 2,232 | 177,333 | 936 | 338,749 | 2,420 | 498,153 | 764 | 317,021 | 854 |
| 30 | 960,123 | 2,561 | 871,802 | 1,716 | 164,739 | 692 | 335,501 | 1,950 | 470,726 | 583 | 294,757 | 640 |
| 40 | 913,356 | 1,794 | 825,177 | 1,315 | 150,694 | 511 | 324,233 | 1,538 | 439,576 | 449 | 268,401 | 471 |
| 50 | 842,804 | 1,245 | 771,674 | 1,009 | 136,376 | 380 | 308,779 | 1,210 | 411,645 | 358 | 242,537 | 351 |
| 60 | 761,681 | 863 | 712,949 | 770 | 120,836 | 276 | 291,094 | 955 | 385,798 | 291 | 220,441 | 269 |
| 70 | 674,981 | 594 | 652,240 | 586 | 107,700 | 208 | 273,914 | 769 | 363,946 | 245 | 205,120 | 220 |
| 80 | 591,616 | 413 | 593,166 | 449 | 97,062 | 162 | 258,268 | 635 | 348,044 | 214 | 194,334 | 189 |
| 90 | 512,256 | 287 | 542,571 | 353 | 88,676 | 131 | 243,761 | 532 | 338,061 | 194 | 186,706 | 168 |
| 100 | 446,950 | 206 | 496,214 | 281 | 81,335 | 108 | 230,878 | 455 | 335,581 | 185 | 184,629 | 160 |

Table 2. Original parameter estimates and maximum and minimum value of perturbed parameters (5000 perturbations)

| | Winter Wheat | | | Spring Wheat | | | Barley | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\hat{\beta}$ | Max | Min | $\hat{\beta}$ | Max | Min | $\hat{\beta}$ | Max | Min |
| **Supply Function** | | | | | | | | | |
| Intercept | 2.5025 | 4.2470 | 0.3212 | 3.0409 | 4.1890 | 1.9273 | 3.8509 | 5.1335 | 2.4156 |
| Land | 0.9886 | 1.2060 | 0.8079 | 1.0013 | 1.0912 | 0.9038 | 0.9081 | 1.0915 | 0.6754 |
| Fallow Dum. | 0.3266 | 0.7143 | -0.0607 | 0.2240 | 0.3509 | 0.0797 | 0.0424 | 0.2741 | -0.1782 |
| Fertilizer Price | -0.3421 | 0.1956 | -0.8460 | -0.1378 | 0.1373 | -0.4253 | -0.2821 | 0.1489 | -0.7769 |
| Herbicide Price | -0.0150 | 0.1242 | -0.1513 | -0.0171 | 0.0696 | -0.1064 | -0.0329 | 0.0747 | -0.1523 |
| 52-high Dum. | 0.0205 | 0.4202 | -0.3547 | 0.0571 | 0.3141 | -0.2048 | -0.0571 | 0.3155 | -0.4031 |
| 53-low Dum. | -0.0639 | 1.2925 | -1.1395 | -0.2312 | 0.0026 | -0.4466 | -0.4935 | -0.0226 | -0.9397 |
| 53-high Dum. | -0.3206 | 0.5278 | -1.0073 | -0.3681 | -0.1498 | -0.6138 | -0.6746 | -0.2318 | -1.1676 |
| 58-low Dum. | -0.3288 | 0.2467 | -0.9081 | -0.3746 | -0.0551 | -0.6547 | -0.3119 | 0.2759 | -0.9549 |
| 58-high Dum. | -0.1981 | 0.1677 | -0.5634 | -0.3745 | -0.1305 | -0.6811 | -0.4093 | -0.0480 | -0.7883 |
| **Machinery Cost** | | | | | | | | | |
| Intercept | 0.4710 | 7.2013 | -7.6847 | 1.0931 | 8.5545 | -8.4708 | 2.2450 | 5.3255 | -0.8219 |
| Land | 1.1371 | 1.3186 | 0.9734 | 1.1097 | 1.2348 | 0.9881 | 1.0922 | 1.3058 | 0.8831 |
| Fallow | 0.0074 | 0.3273 | -0.3842 | -0.1138 | 0.0695 | -0.2827 | 0.0189 | 0.3005 | -0.2230 |
| Crop price | 1.1033 | 6.7187 | -3.6820 | 0.6623 | 7.3059 | -4.3115 | 0.1938 | 4.1446 | -3.4978 |
| Fertilizer price | -0.0364 | 0.4109 | -0.5665 | -0.1416 | 0.2489 | -0.5643 | 0.0664 | 0.4873 | -0.4936 |
| Herbicide price | 0.0410 | 0.1916 | -0.1036 | 0.0024 | 0.1221 | -0.1254 | -0.0077 | 0.0993 | -0.1251 |
| 52-high Dum. | 0.1401 | 0.5058 | -0.2429 | 0.1122 | 0.3708 | -0.2136 | 0.2435 | 0.5883 | -0.1487 |
| 53-low Dum. | 0.5814 | 1.6530 | -0.5264 | 0.0420 | 0.4488 | -0.3110 | -0.0373 | 0.4838 | -0.5929 |
| 53-high Dum. | 0.2032 | 0.9721 | -0.5434 | -0.1808 | 0.2527 | -0.5510 | 0.0779 | 0.5935 | -0.5317 |
| 58-low Dum. | -0.0007 | 0.7421 | -0.5482 | -0.0588 | 0.5304 | -0.4612 | -0.1887 | 0.5483 | -0.9947 |
| 58-high Dum. | 0.0647 | 0.4357 | -0.2911 | 0.0973 | 0.6387 | -0.2862 | 0.0884 | 0.5757 | -0.3722 |
| **Cost Function** | | | | | | | | | |
| Intercept | 0.6627 | 3.9007 | -3.2707 | -0.6187 | 1.7255 | -2.8261 | -2.5908 | 1.7389 | -5.9969 |
| Output | 1.0129 | 1.4514 | 0.6091 | 1.1150 | 1.4086 | 0.8434 | 1.2922 | 1.6816 | 0.7903 |
| Ferlilizer price | 0.8606 | 0.9340 | 0.7945 | 0.8296 | 0.8946 | 0.7668 | 0.8553 | 0.9227 | 0.7990 |
| Fallow Dummy | -0.5951 | 0.0656 | -1.3388 | -0.5494 | -0.1800 | -0.9187 | -0.4150 | 0.1733 | -0.8250 |
| 52-high Dum. | -0.2395 | 0.4176 | -0.9536 | -0.2011 | 0.3768 | -0.7994 | -0.0795 | 0.6992 | -0.8517 |
| 53-low Dum. | 0.0454 | 2.6763 | -2.0361 | -0.0546 | 0.5625 | -0.6510 | 0.0940 | 1.2926 | -0.9042 |
| 53-high Dum. | 0.1421 | 1.5811 | -1.3726 | 0.0188 | 0.7415 | -0.6258 | 0.1162 | 1.0196 | -0.9498 |
| 58-low Dum. | 0.0482 | 1.2504 | -1.3025 | 0.0802 | 0.9180 | -0.7145 | 0.5515 | 1.3653 | -1.2148 |
| 58-high Dum. | 0.0401 | 0.7780 | -0.6344 | 0.3153 | 1.1426 | -0.4149 | 0.1776 | 0.9384 | -0.6792 |
| **All Crops** | | | | | | | | | |
| **Cost Mechanical Fallow** | | | | | | | | | |
| Intercept | 1.3972 | 2.1319 | 0.5170 | | | | | | |
| Land | 1.0786 | 1.2636 | 0.9134 | | | | | | |
| **Cost Chemical Fallow** | | | | | | | | | |
| Intercept | -0.2346 | 2.7129 | -3.6150 | | | | | | |
| Price | 0.0560 | 1.0879 | -0.9733 | | | | | | |
| Land | 1.3417 | 1.6900 | 0.9509 | | | | | | |
| Chemical quantity | -1.1121 | -0.4518 | -1.7745 | | | | | | |

Table 3. Original point estimate value and expected carbon value

| Price per tonne ($) | Sub-MLRA 52H | | Sub-MLRA 52L | | Sub-MLRA 53H | | Sub-MLRA 53L | | Sub-MLRA 58H | | Sub-MLRA 58L | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Point estimate MMT C | Expected value MMT C | Point estimate MMT C | Expected value MMT C | Point estimate MMT C | Expected value MMT C | Point estimate MMT C | Expected value MMT C | Point estimate MMT C | Expected value MMT C | Point estimate MMT C | Expected value MMT C |
| 10 | 1.42 | 1.79 | 1.81 | 2.30 | 0.66 | 0.71 | 0.73 | 0.82 | 1.91 | 2.21 | 1.14 | 1.34 |
| 20 | 1.87 | 2.16 | 2.17 | 2.56 | 0.78 | 0.78 | 0.89 | 0.92 | 2.26 | 2.43 | 1.38 | 1.46 |
| 30 | 2.37 | 2.53 | 2.50 | 2.81 | 0.85 | 0.84 | 1.01 | 1.02 | 2.50 | 2.62 | 1.51 | 1.57 |
| 40 | 2.77 | 2.88 | 2.85 | 3.04 | 0.93 | 0.89 | 1.10 | 1.11 | 2.72 | 2.78 | 1.63 | 1.66 |
| 50 | 3.20 | 3.19 | 3.17 | 3.25 | 0.99 | 0.94 | 1.24 | 1.19 | 2.91 | 2.92 | 1.79 | 1.74 |
| 60 | 3.58 | 3.47 | 3.32 | 3.44 | 1.01 | 0.97 | 1.34 | 1.26 | 3.05 | 3.02 | 1.83 | 1.81 |
| 70 | 3.89 | 3.70 | 3.49 | 3.60 | 1.04 | 1.00 | 1.43 | 1.32 | 3.17 | 3.11 | 1.87 | 1.86 |
| 80 | 4.05 | 3.89 | 3.73 | 3.74 | 1.05 | 1.02 | 1.47 | 1.37 | 3.24 | 3.18 | 1.94 | 1.90 |
| 90 | 4.19 | 4.04 | 3.90 | 3.86 | 1.06 | 1.03 | 1.53 | 1.41 | 3.26 | 3.24 | 1.96 | 1.93 |
| 100 | 4.32 | 4.16 | 3.96 | 3.96 | 1.07 | 1.04 | 1.54 | 1.44 | 3.31 | 3.29 | 1.99 | 1.96 |

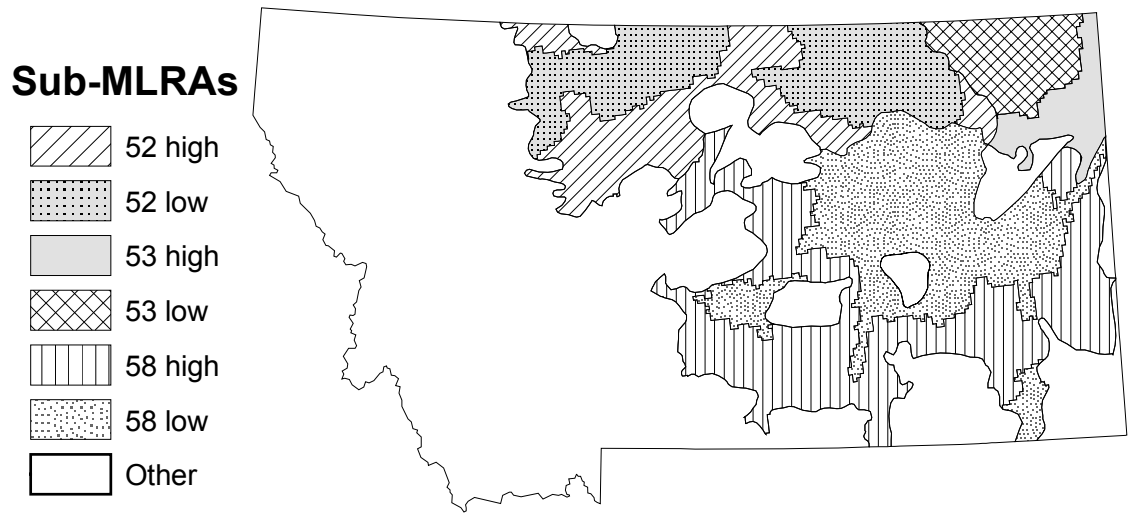


Figure 1. Six sub-MLRA regions in Montana – suffix “high” denotes areas of teach MLRA with higher rainfall, suffix “low” denotes areas with lower rainfall.

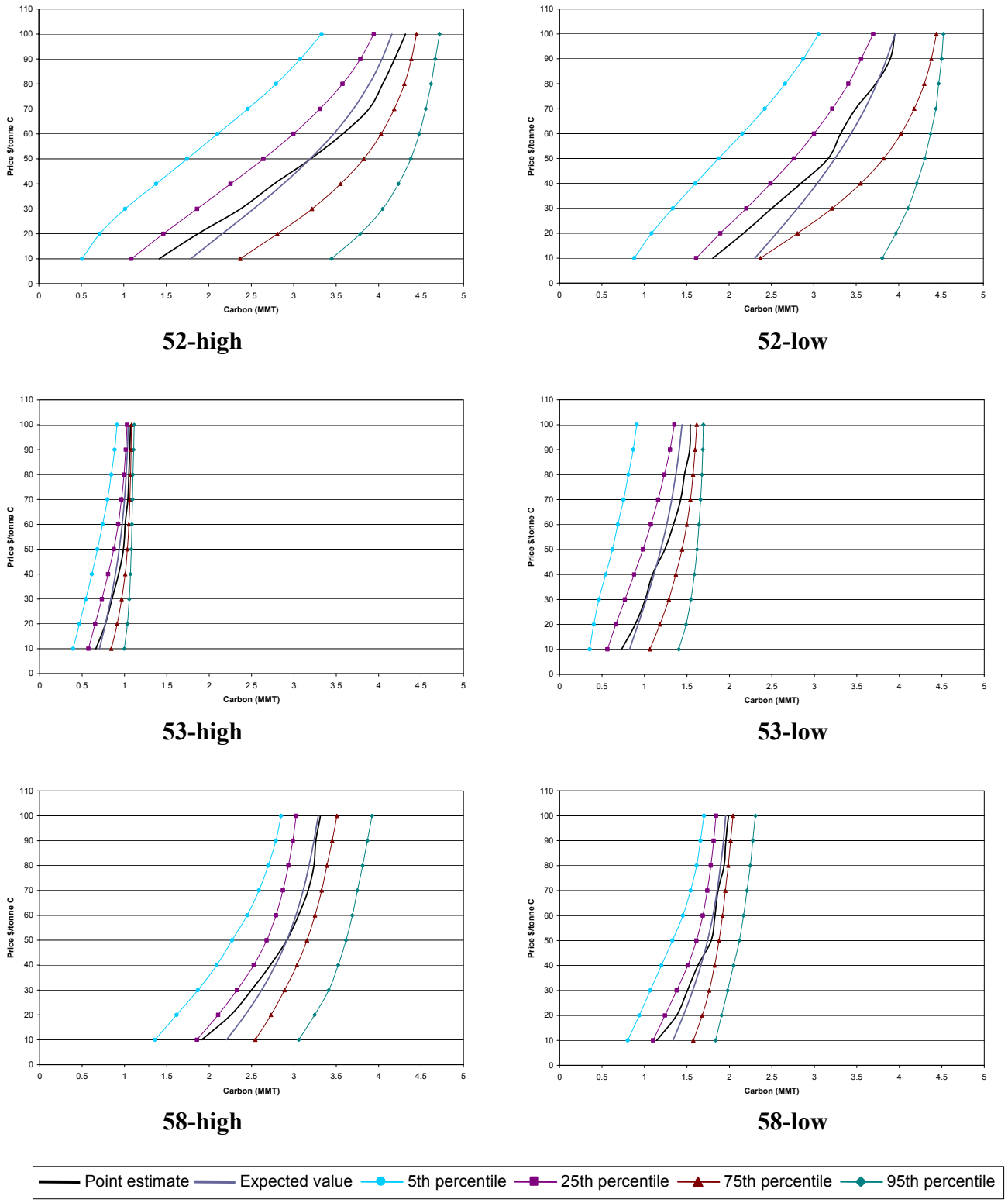


Figure 2. Marginal cost of soil Carbon sequestration from original point estimate, expected value and selected percentiles.

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