DO ECONOMIC RESTRICTIONS IMPROVE FORECASTS?

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Using several popular demand systems in conjunction with food consumption data, Kastens and Brester (KB) show that theory-constrained demand systems forecast better out-of-sample (hereafter forecast) than their unrestrained counterparts. While at first this seems to provide some justification for imposing theoretical constraints, it does not address the question of whether the forecast benefit derives from economic theory or higher degrees of freedom.

Parameter restrictions serve to enhance degrees of freedom regardless of whether the restrictions are derived from theory or not. Because models with greater degrees of freedom forecast better, in this paper we ask whether the theory-constrained models in KB forecast better because the restrictions are "true" or because their degrees of freedom are higher.[1] We wish to separate the contribution of forecast improvements due to economic theory from that of higher degrees of freedom.

We use the data from the KB study to re-estimate their models with arbitrary restrictions. These arbitrary restrictions are not derived from theory, but they increase the degrees of freedom by an identical amount as the economic restrictions. Results indicate that arbitrary restrictions, due to more degrees of freedom, do improve forecasts relative to no restrictions. However, economic restrictions improve forecasts even more, suggesting that there is valuable information contained in economic theory, and that economic theory has an important role in forecasting.

THE VALUE OF PARAMETER RESTRICTIONS

It may seem strange that theoretical restrictions would be rejected in-sample, and then reduce forecast errors out-of-sample.[2] Why would theoretical restrictions appear informative out-of-sample but not in-sample?[3] One reason, based on the concept of sample and non-sample information, is that economic restrictions improve forecasts because economic theory is informative. The other explanation, based on degrees of freedom issues, is that any restriction might improve forecasts, regardless of whether the restriction is true or not.

Sample information refers to a set of observations. Theoretical restrictions are a form of non-sample information. They represent information researchers believe to be true but may not be reflected adequately in a random sample. The three most popular restrictions; symmetry, homogeneity, and adding-up are derived from theory of the representative consumer. Their derivation rests on several assumptions that may be too restrictive. Assumptions commonly made are that all consumers possess and maximize the same utility function, the parameters of that function are time-invariant, all consumers face identical real prices, and either all specified goods must be exhaustive or a subset must be separable (Deaton and Mullbauer).

But restrictions derived from economic theory need not hold perfectly to have value. Economic restrictions convey information even if none of the above assumptions hold. If beef is a strong substitute for pork, pork should be a strong substitute for beef. The symmetry condition ensures this is the case.[4] The point is that theoretical restrictions may convey much information we know about consumers, even if their parametric representation is not perfectly accurate.

Suppose we wish to estimate a parameter vector β for use in forecasting, and the prediction errors (either in- or out-of-sample) are an increasing function of the distance between the true vector β and its estimate $\hat{\beta}$. The more information contained in $\hat{\beta}$, the smaller this distance. Information in $\hat{\beta}$ is a function of sample and non-sample information. Let the unrestricted estimate be denoted $\hat{\beta}_U$ and its theory-constrained counterpart be $\hat{\beta}_R$, where $\hat{\beta}_U$ only contains sample information and $\hat{\beta}_R$ contains sample and non-sample input. When predicting in-sample observations of the dependent variable, it is possible that sample information may dominate the non-sample information (the information in theoretical restrictions). Though the restrictions do reflect reality to some degree, their parametric representation is not exactly true, and allowing the estimation routine to search unrestricted over all possible value for β results in significantly smaller [in-sample] prediction errors than if constrained by theory.

Now, let us turn to the case where $\hat{\beta}_U$ and $\hat{\beta}_R$ are used for forecasting out-of-sample. Specifically, we focus on the case where observations from earlier dates are used to forecast future observations. It is likely that the true parameter vector β changes over time due to changing consumer preferences, model misspecification, and other complexities involved in econometrics, in ways difficult to capture even with the most advanced random coefficient estimation techniques. If this is true, then sample information from previous time periods are of less use in explaining future observations as they were in explaining in-sample observations. But the value of non-sample information via theoretical restrictions stays the same because theory is not time dependent. The amount of information in theoretical restrictions, relative to the information contained in the in-sample observations, is now greater, and the restricted estimates' forecasting ability, relative to unrestricted estimates, begins to improve.

Some evidence for this is given in Table 1 using data from KB and their form of the AIDS model. This table shows the ratio of forecast errors from an unrestricted AIDS model to an AIDS model with symmetry and homogeneity imposed. With only a one-year-ahead forecast horizon, the restricted model performed better in some cases and worse in others. Once this horizon increases, the restricted form has lower errors for all food groups. As the forecast horizon increases, the theoretically constrained model forecasts better. This may be due to the economic content of the restrictions, i.e., that the restrictions are theory-based and the theory is sound.

Restrictions do not have to be based on theory, empirical results, or even make sense to improve forecasts. Restrictions may improve forecasts simply because they increase the degrees of freedom (Brieman). As Sawa notes, even if one model is a closer approximation to the true model analytically, in small samples, models with more degrees of freedom may better represent the true data generating process. Consider again the data and AIDS model used by KB. In Table 1, an unrestricted AIDS model is compared to a parsimonious AIDS model, where the value of all

parameters except for own-price and intercept terms are set to zero. At a one-year horizon, the unrestrained AIDS model has lower forecast errors for four out of six goods, but at a 1-11 year horizon, the parsimonious AIDS model has better forecasts for four out of six goods. At longer forecast horizons, forecast improvements can be obtained simply by increasing the degrees of freedom. This finding is not isolated; it is generally accepted that models with more degrees of freedom tend to forecast better.

Consider again one forecast series from the parameter vector $\hat{\beta}_U$ and one from the vector $\hat{\beta}_R$. In this case, it is assumed that $\hat{\beta}_R$ is estimated using restrictions not based on theory, but since restrictions are imposed, the degrees of freedom are higher for $\hat{\beta}_R$ than $\hat{\beta}_U$. The mean-squared error of $\hat{\beta}_R$ from its true value β is the variance of the estimator plus the squared bias, i.e., $E(\hat{\beta}_R - \beta)^2 = V(\hat{\beta}_R) + [E(\hat{\beta}_R) - \beta]^2 = V_R + BIAS_R^2$. Forecasts from $\hat{\beta}_R$ will be more accurate than those from $\hat{\beta}_U$ if $V_R + BIAS_R^2 < V_U + BIAS_U^2$. If the restrictions are not true, it is certainly the case that $BIAS_R^2 > BIAS_U^2$. However, since degrees of freedom are higher for the restricted estimate, it may be that $V_R < V_U$, such that the mean-squared error for $\hat{\beta}_R$ is lower than $\hat{\beta}_U$, thus producing better forecasts. A case can be made for V_R being lower than V_U. With more degrees of freedom, the restricted parameter estimates are derived from more observations; thus, their variability in repeated samples should be smaller (Breiman).

The KB study found that models with economic restrictions forecast better than their unrestrained counterparts. We have just explained how this could occur. First, the economic theory used to derive those restrictions might be valuable non-sample information, i.e., the theory might be correct. Second, even if the theory is not correct, restrictions serve to increase degrees of freedom, and more degrees of freedom could result in more accurate forecasts. Which explanation is correct? This is an important question to address, because the answer will guide

economists as to whether improved forecasts can be obtained by developing better theories, using more parsimonious models, or both.

A simple method can be used to test this. This method entails estimating demand while imposing arbitrary restrictions that have no reason to be true, but increase degrees of freedom by an equal amount as economic restrictions, and then compare those forecasts to a scenario where economic restrictions are imposed. If theoretical restrictions provide better forecasts than these arbitrary restrictions, then we can say the theoretical restrictions contain non-sample information useful for forecasting. If they do not, we must conclude all forecasting improvements are due to increases in degree of freedom, and not theory. We perform this test using the exact data and estimation methods of the KB study. These methods are discussed in the next two sections.

DEMAND SYSTEMS WITH ECONOMIC AND ARBITRARY RESTRICTIONS

The Rotterdam, AIDS, and the first-difference-double-log (FDDL) models are demand systems used by Kastens and Brester (KB) for food demand analysis. Six food groups are used: meats, eggs, dairy, fats, cereals, and sweets. A seventh group "all other goods" is also constructed. Thus, there are $i = 1, \ldots, 7$ exhaustive goods, where the price and quantity of those goods are denoted p_i and q_i, respectively. Denoting per capita nominal income by X, the Rotterdam model is given by

$$(1) \quad \hat{w}_{i,t} \Delta \ln(q_{i,t}) = \alpha_i + \sum_{j=1}^{N} \gamma_{i,j} \Delta \ln(p_{j,t}) + \beta_i \left[\Delta \ln(X_t) - \sum_{j=1}^{N} \hat{w}_{j,t-1} \Delta \ln(p_{j,t}) \right] + \varepsilon_{i,t}$$

where $\hat{w}_{i,t}$ is the average expenditure share

$$(2) \quad \hat{w}_{i,t} = \frac{1}{2} w_{i,t} + \frac{1}{2} w_{i,t-1} = \frac{1}{2} \frac{p_{i,t} q_{i,t}}{X_t} + \frac{1}{2} \frac{p_{i,t-1} q_{i,t-1}}{X_{t-1}}$$

and Δ is the across-period difference operator. The version of the AIDS model used by KB is

$$(3) \quad \Delta w_{i,t} = \alpha_i + \sum_{j=1}^{N} \gamma_{i,j} \Delta \ln(p_{j,t}) + \beta_i \left[\Delta \ln(X_t) - \sum_{j=1}^{N} w_{j,t-1} \Delta \ln(p_{j,t}) \right] + \varepsilon_{i,t}$$

6

The FDDL model is

$$(4) \quad \Delta \ln(q_{i,t}) = \alpha_i + \sum_{j=1}^{N} \gamma_{i,j} \Delta \ln(p_{j,t}) + \beta_i \Delta \ln(X_t) + \varepsilon_{i,t}$$

Each of the N goods has one equation, and each equation has N+2 parameters. If estimated as a system, one equation must be dropped leaving N-1 goods.[5] The remaining system contains (N-1)(N+2) parameters where each, if not constrained, must be estimated. The number of parameters to estimate can be reduced by imposing economic restrictions. Two examples are the homogeneity and symmetry conditions. The homogeneity and symmetry conditions decrease the number of parameters to estimate by (N-1) and (N-1)(N-2)/2, respectively. The adding-up condition is not imposed because it is automatically satisfied by the data.

To determine if better forecasts using economic restrictions are the result of accurate theory, more degrees of freedom, or both, we compare models with economic restrictions to models with arbitrary (not derived from theory) restrictions. The arbitrary conditions are chosen such that they enhance degrees of freedom by the same amount as economic restrictions, and in a similar way, but are not based on theory and are randomly chosen.

For the AIDS and Rotterdam model, the symmetry condition states that $\gamma_{i,j} = \gamma_{j,i} \; \forall \; i \neq j$. The symmetry condition for the FDDL model can be stated as a linear function

$\gamma_{i,j} = \dfrac{w_j}{w_i} \gamma_{j,i} - w_j (\beta_i - \beta_j) \; \forall \; i \neq j$. For generality, the symmetry condition for all three models

is written as $\gamma_{i,j} + (a_{i,j})\gamma_{j,i} = b_{i,j} \; \forall \; i \neq j$. In the AIDS and Rotterdam models, $a_{i,j} = -1$ and $b_{i,j} = 0 \; \forall \; i$, j, and for the FDDL model $a_{i,j} = (-w_j/w_i)$ and $b_{i,j} = -w_j(\beta_i - \beta_j)$. We impose arbitrary conditions by changing the symmetry condition to $\gamma_{i,j} + (a_{i,j})\gamma_{k,r} = b_{i,j}$ where both $i = r$ and $j = k$ cannot hold, but one of them can. Since these restrictions can only be locally imposed on the FDDL model, they were imposed at the sample mean of budget shares. A computer program was written to randomly select 100 unique *symmetry-like* conditions.

For the Rotterdam and AIDS model the homogeneity condition is $\sum_{j=1}^{N} \gamma_{i,j} = 0$ and for the

FDDL model is $\sum_{j=1}^{N} \gamma_{i,j} + \beta_i = 0$. Note that this condition is a within-equation restriction and

increases degrees of freedom by N-1. To provide one set of arbitrary within-equation restrictions

that increase degrees of freedom by an identical amount, the homogeneity condition is reversed

between the AIDS/Rotterdam and the FDDL model. This "*homogeneity-like restriction*" is now

$\sum_{j=1}^{N} \gamma_{i,j} + \beta_i = 0$ for the AIDS and Rotterdam models and $\sum_{j=1}^{N} \gamma_{i,j} = 0$ for the FDDL model.

Though there are 100 symmetry-like conditions, there is only one homogeneity-like condition.

More attention is paid to generating symmetry-like restrictions because they are easier to generate

and contribute more toward increasing degrees of freedom.

MODEL ESTIMATION AND FORECASTING

All data, estimation, and forecasts are performed identical to Kastens and Brester (KB)

and are discussed thoroughly in their article. The original data and code were made available by

the authors, and we compared our estimates to those published in KB to ensure our estimation

procedure was the same. Thus, we defer most details to the KB article. Data covered the years

1924-1992, in which three model updating methods were used. The first updating method utilizes

a one-year horizon and a constant sample size of 25 observations. First, data from the years

1924-1948 are used for estimation and forecasting food quantities in 1949. Then, data from

1925-1949 are used to forecast in 1950. This continues until data from 1968-1991 are used to

forecast in 1992. This provides 44 forecasts.

The second updating method employs multiple horizons of one to eleven years. First,

data from 1924-1948 are used to forecast in the years 1949-1959. Then, data from 1935-1959

were used to forecast in 1960-1970. Finally, the third forecast series' horizons go from 1 to 22

years, always maintaining a constant sample size of 25. Each of the three updating methods is

referred to herein as a "forecast horizon." With the three updating methods and six food groups, there are eighteen forecast series to compare.

Four types of comparisons are made between economic and arbitrary restrictions, and are illustrated in Figure 1. First, the forecast errors from models with symmetry and symmetry-like conditions are compared. This assessment is made with and without homogeneity imposed on all models (providing two comparisons). Forecasts from models with the symmetry condition are compared to the average forecast of 100 models with symmetry-like conditions to ensure the results are not due to a particular selection of a single symmetry-like condition. Then, forecasts from an economic model with symmetry and homogeneity imposed are compared to the average forecast using 100 symmetry-like conditions and the one homogeneity-like conditions. Finally, an economic model with homogeneity is compared to an arbitrary model with the homogeneity-like restriction. Models with no restrictions are always included in the comparisons.

For each four comparisons, results are given in two ways. The first is identical to KB,[6] which ranks models according to which have the lowest mean-squared error across the three forecast horizons and six goods. The second method illustrates differences in the median-squared error between restriction types. Let $(e_{M,G,S})^2$ be a squared forecast error from Model M (M = AIDS, Rotterdam, or FDDL), Good G (G = meat, eggs, etc.) and forecast series S (S = one year forecast horizon; 1-11 year horizon; or 1-22 year horizon). Consider the first comparison (illustrated in Figure 1) where we compare symmetry versus symmetry-like restrictions with homogeneity always imposed. Let $(e_{M,G,S}{}^{S,H})^2$ denote the forecast error with symmetry and homogeneity and $E[(e_{M,G,S}{}^{SL,H})^2]$ be the average forecast error across the 100 symmetry-like conditions (where homogeneity is imposed on all 100 models). A variable $P((e_{M,G,S}{}^{S,H})^2 < E[(e_{M,G,S}{}^{SL,H})^2])$ is constructed which equals one if true and zero if false. For each four forecast comparisons, there are 1,908 unique values of P(.).[7]

The individual values of P(.) do not constitute a random sample. Their values are likely to be correlated across models, goods, and forecast horizons. However, P is still a good indicator

of how well models with economic restrictions perform relative to arbitrary restrictions across all settings, and so is used as guidance even though conventional statistical tests of P(.) will not be correct. It is especially useful because it provides one number comparing forecasts across different goods, models, and forecast horizons. Were the values of P(.) a random sample, we could test the null hypothesis that $P = \frac{1}{2}$, which states that the two types of restrictions forecast equally well. Specifically, it states the median-squared forecast error using the two restriction types are identical.

If P is significantly greater than $\frac{1}{2}$, the median-squared forecast error using economic restrictions is smaller. We would then conclude that economic restrictions do convey valuable information, and have a greater use than simply enhancing degrees of freedom. The test is referred to as a nonparametric sign test and the test statistic is $2T^{1/2}[P - \frac{1}{2}]$ (Mendendhall, Wackerly, and Schaeffer) where T is the number of forecasts. This test is also used to evaluate whether the median forecast error is smaller using arbitrary restrictions or no restrictions. Again, this test is an indicator; the extent to which it can be considered valid is left to the reader.

FORECAST RESULTS AND DISCUSSION

Table 2 shows the model rankings based on which have the lowest mean-squared error. A higher number corresponds to a higher ranking (lower mean-squared errors) across the three forecast horizons and six goods. As Kastens and Brester note, the FDDL is a better forecaster than the AIDS or Rotterdam models. Also, models with restrictions forecast better than their unrestricted counterparts--regardless of whether those restrictions are derived from theory or arbitrary. This finding is robust; it occurs for every model and across all four comparisons.[8] This suggests that part of the reason economic restrictions improve forecasts is because they enhance degrees of freedom. According to Table 3, forecasts using arbitrary restrictions are smaller relative to no restrictions greater than 50% of the time, though it is debatable whether this is significantly greater than 50%.

10

The most important result is that forecasts using economic restrictions are superior to those using arbitrary restrictions. Based on mean-squared error rankings shown in Table 2, economic restrictions out-forecast arbitrary restrictions and no restrictions alike.[9] Table 3 shows this result holds when forecast performance is determined by median-squared error as well. Across the three models, six goods, and three forecast horizons, forecast errors using economic restrictions are smaller compared to arbitrary restrictions 55%-66% of the time. To the extent the sign test is valid, this percentage is significantly larger than 50%. Median-squared errors using economic restrictions were significantly lower compared to no restrictions as well.

This study finds unambiguous support for the use of economic restrictions in demand systems. Even though they are typically rejected in-sample, the value of economic information is nicely demonstrated by the ability of economic restrictions to improve forecasts. Part of this improvement is due to greater degrees of freedom, as the results show restrictions can improve forecasts even when they are not true. However, much of this improvement emanates from the fact that relationships implied by economic theory are reflected in economic data.

FOOTNOTES

1) Suppose Models A and B are approximations to a true unknown functional form. Sawa shows that, although Model B may be a closer approximation in a large sample sense, or its structure may resemble the true form more, in small samples Model A may be a better approximation if its degrees of freedom are larger. Appeals to model selection criteria that contain penalty parameters are typically made based on this fact.

2) In-sample refers to the set of observations used to calculate parameter estimates. For instance, the likelihood ratio test is an in-sample statistic because the likelihood functions are calculated using the same observations in which the parameters were generated. Conversely, out-of-sample refers to a set of observations that were not used to estimate parameters. For instance, if data from 1970-1990 are used to predict prices in 1970-2000, the predictions for 1970-1990 are in-sample, and those for 1991-2000 are out-of-sample predictions, or as more commonly called, forecasts.

3) In-sample, "informative" is defined as not being rejected. If they are not rejected then they may (but do not have to) be interpreted as true. Out-of-sample, something is "informative" if it reduces forecast errors.

4) That is, assuming the budget shares of both goods are small, eliminating effects from income elasticities.

5) If there are N exhaustive goods, then $\sum_{i=1}^{N} w_i = 1$. This results in the unfortunate fact that for the AIDS and Rotterdam models, if one tries to simultaneously estimate equations for each good using conventional methods, the matrix of independent variables is singular, preventing unique parameter estimates.

6) Specifically, we use the nominal ranking method as shown in Table 5 of KB. Consider the first comparison as shown in Figure 1, where each three models are estimated with three different restriction sets (economic restrictions, arbitrary restrictions, and no restrictions). This provides nine models to rank. The model with the lowest out-of-sample-root-mean squared error for a

single good and forecast horizon is given a value of nine and the model with the highest error is given a value of one. Since there are six goods and three forecast horizons, this implies a total of 18 rankings. Letting R_i ($i = 1, …, 18$) be the model ranking for a model and single good and forecast horizon, the overall nominal ranking for that model is $\sum_i R_i/18$. Thus, a higher value indicates a better model.

7) Three models, six goods, three forecast series, and 44 forecasts per series implies 2,376 total forecasts. However, (26*3*6) 468 forecasts are redundant, leaving 1,908 unique forecasts.

8) AGS tests (Ashley, Granger, and Schmalensee) suggests these lower squared errors are almost always significantly lower. For instance, across all models, goods, and forecast horizons, the mean-squared error from unrestrained models is significantly higher than if symmetry-like conditions are imposed 94% of the time.

9) Recall this compares forecast errors using the symmetry condition to the average forecast error from 100 symmetry-like conditions. It turns out that forecasts using the symmetry condition are smaller than each 100 symmetry-like condition 90% of the time. This means that symmetry will forecast better than a randomly generated symmetry-like condition 90% of the time. Also, AGS tests reveal that models with economic restrictions have significantly lower mean-squared errors than those with arbitrary conditions 67% of the time.

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TABLE 1
FORECASTING PERFORMANCE OF AN AIDS MODEL UNDER ALTERNATIVE PARAMETER RESTRICTIONS

Forecast Horizon	Food Group					
	Meats	Eggs	Dairy	Fats	Cereals	Sweets
	Ratio of Root-Mean-Squared Forecast Errors of Unrestricted to Theoretically Constrained AIDS Demand Systems[a]					
1 Year[b]	1.25	0.96	1.24	0.98	1.37	1.23
1-11 Years[c]	2.09	1.29	1.29	1.33	1.32	1.45
	Ratio of Root-Mean-Squared Forecast Errors of Unrestriced AIDS Demand Systems to Parsimonious AIDS Systems[d]					
1 Year[b]	0.72	1.17	0.87	1.07	0.91	0.93
1-11 Years[c]	0.68	1.10	1.27	1.09	1.53	0.95

Note: See Kastens and Brester for more details on the data and model specification. All forecasts were performed identical to Kastens and Brester.
a) This model is denoted by FDLA/ALIDS in Kastens and Brester. The theoretically constrained system imposes symmetry and homogeneity, both of which were rejected using likelihood ratio tests.
b) This means the quantity of the food group in year t was forecasted using observations from the previous 25 years.
c) This means the quantity of the food group in years t through t+10 were forecasted using observations from the 25 years previous to year t.
d) The parsimonious AIDS model sets all parameters except the own-price and intercept terms to zero. This model was not used in Kastens and Brester.

TABLE 2
AVERAGE RANKINGS OF DEMAND SYSTEMS

Model	Restriction Type	Average Model Ranking	Model	Restriction Type	Average Model Ranking
Symmetry Versus Symmetry-Like Conditions With Homogeneity[a]			*Symmetry Versus Symmetry-Like Conditions Without Homogeneity[b]*		
A Higher Number Indicates a Higher Average Ranking / Lower Mean-Squared Forecast Errors					
AIDS	ECONOMIC	4.11	AIDS	ECONOMIC	4.44
AIDS	ARBITRARY[e]	3.28	AIDS	ARBITRARY	3.17
AIDS	NONE	1.44	AIDS	NONE	1.39
ROTTERDAM	ECONOMIC	5.78	ROTTERDAM	ECONOMIC	5.67
ROTTERDAM	ARBITRARY	5.33	ROTTERDAM	ARBITRARY	4.72
ROTTERDAM	NONE	3.11	ROTTERDAM	NONE	3.39
FDDL	ECONOMIC	8.56	FDDL	ECONOMIC	8.39
FDDL	ARBITRARY	7.94	FDDL	ARBITRARY	7.67
FDDL	NONE	5.44	FDDL	NONE	6.17
Symmetry and Homogeneity Versus Symmetry-Like and Homogeneity-Like Conditions [c]			*Homogeneity Versus Homogeneity-Like Conditions [d]*		
AIDS	ECONOMIC	4.39	AIDS	ECONOMIC	3.28
AIDS	ARBITRARY	2.56	AIDS	ARBITRARY	2.11
AIDS	NONE	1.50	AIDS	NONE	2.06
ROTTERDAM	ECONOMIC	5.89	ROTTERDAM	ECONOMIC	5.44
ROTTERDAM	ARBITRARY	5.17	ROTTERDAM	ARBITRARY	5.44
ROTTERDAM	NONE	3.33	ROTTERDAM	NONE	3.61
FDDL	ECONOMIC	8.94	FDDL	ECONOMIC	8.83
FDDL	ARBITRARY	7.50	FDDL	ARBITRARY	7.28
FDDL	NONE	5.72	FDDL	NONE	6.94

Note: The average model ranking is over eighteen rankings (six food types times three forecast horizons). The highest ranked model (model with the lowest out-of-sample-root-mean-squared-error) in any ranking is assigned a value of nine, and the lowest ranked is given a value of one. The reported rankings above are the average ranking for each model across all eighteen rankings. Thus, the higher the average ranking the better the model forecasts.

a) Models with both arbitrary and economic restrictions had homogeneity imposed. Models with economic restrictions also had symmetry imposed, and models with arbitrary restrictions had symmetry-like conditions imposed.

b) Homogeneity was not imposed on any model. Models with economic restrictions had symmetry imposed, and models with arbitrary restrictions had symmetry-like conditions imposed.

c) Models with economic restrictions also had symmetry and homogeneity imposed, and models with arbitrary restrictions had symmetry-like and homogeneity-like conditions imposed.

d) Models with economic restrictions had homogeneity imposed, and models with arbitrary restrictions had homogeneity-like conditions imposed.

e) Recall the models with symmetry-like restrictions are a composite model of 100 individual models, each with a unique and randomly generated symmetry-like condition.

TABLE 3
TESTS FOR SIGNIFICANT DIFFERENCES IN MEDIAN-SQUARED FORECAST ERRORS USING ECONOMIC, ARBITRARY, AND NO RESTRICTIONS

Restriction Set A	Versus Restriction Set B	Percent of Smaller Forecast Errors Using Restriction Set A Relative To Set B (t-statistic)
Symmetry and Homogeneity	Symmetry-Like and Homogeneity	0.64[a] (12.64)[b]
Symmetry	Symmetry-Like	0.64 (12.13)
Symmetry and Homogeneity	Symmetry-Like and Homogeneity-Like	0.66 (13.51)
Homogeneity	Homogeneity-Like	0.55 (4.58)
Symmetry-Like and Homogeneity	No Restrictions	0.56 (4.81)
Symmetry-Like	No Restrictions	0.50 (0.09)
Symmetry-Like and Homogeneity-Like	No Restrictions	0.51 (1.10)
Homogeneity-Like	No Restrictions	0.51 (1.19)

Note: This test is only valid to the extent it represents a random sample.
a) This is the percent of squared forecast errors using Restriction Set A which are smaller than the squared forecast errors using Restriction Set B. There are a total of 1,908 unique forecast comparisons from the three models, six food groups, and three forecast horizons.
b) This is a nonparametric sign test of the null hypothesis that the median squared forecast errors are equal across both restriction types against the alternative hypothesis they are different from zero. A significantly positive statistic indicates Restriction Set A has a lower median forecast error. Asymptotically and under the null hypothesis, the test statistic is distributed $N(0,1)$.

FIGURE 1
FOUR RESTRICTION COMPARISONS
(Models With No Restrictions Are Always Included)

1) Symmetry Versus 100 Symmetry-Like Conditions (With homogeneity imposed on all models)

2) Symmetry Versus 100 Symmetry-Like Conditions (Without homogeneity imposed on all models)

Models
1) AIDS
2) Rotterdam
3) FDDL

3) Symmetry and Homogeneity Versus 100 Symmetry-Like and 1 Homogeneity-Like Condition

4) Homogeneity versus 1 Homogeneity-Like Condition