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Presence of Stochastic Errors in the Input Demands: Are Dual and Primal Estimations Equivalent?

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***Paper prepared for presentation at the American Agricultural Economics Association
Annual Meeting, Montreal, Canada, July 27-30, 2003***

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1 - Introduction

Duality is a very useful approach in production theory because it represents a direct and natural way to elaborate and analyze an economic problem. But the dual representation does not exist independently of the primal formulation. Although the primal approach has the advantage of an immediate and intuitive interpretation, duality can be analytically more convenient in some complex problems. But the debate about which approach best serves economists is still alive.

Duality held great promise when it was popularized over 30 years ago. According to Just (2000), to capture the empirical benefits of duality both primal and dual implications of their estimates must be compared to other empirical studies, regardless of whether the estimated relationships have been derived by primal or dual approaches.

This study is an attempt to illustrate the possibility that both dual and primal formulations produce good results when some empirical complications are added to the model. A data set based on some representative agent behavior is created through Monte Carlo simulation and used to estimate econometrically the primal and dual functions associated to the technology chosen. The objectives of this investigation are: first, the empirical verification of the properties of the OLS estimator for primal and dual formulations under a Hicks-Neutral technical change when there are stochastic errors in the output and input demands. Second, the policy implications when the presence of such errors is not taken into account by the policy makers.

¹ The author would like to acknowledge Prof. Brian Roe for his valuable contributions and suggestions for this study, and to CAPES Foundation (Brazil) for the financial support.

The paper is organized as follows. The next section discusses the literature review about duality and the main issues of considering the transmission of shocks through the production process. Section three exposes the data source and describes the main features of the model to be evaluated. Section four discusses the main results obtained in three different sections. The conclusions are in the section five.

2 - Literature Review

According to Pope (1982) and Taylor (1989), some of the advantages of the dual are: easy applicability, flexibility in measurement, less data requirements, and convenient to analyze more problems than the primal. The drawbacks are that not all problems can be studied using dual approach, for example, multi-product joint production, models under risk aversion and uncertainty, nonlinear and dynamic models.

A profit-maximizing (dual) formulation in the presence of errors or distortions can result in a wrong choice of the resources allocation. Taylor (1989) says that a critical assumption for this event is that the Hotelling's lemma does not hold. In this manner, the Hotelling's results from a standard profit function cannot be obtained through a profit function in presence of stochastic errors in output and input demands. The only exception is the case where the production function is quadratic in inputs².

As noted by Pope & Just (2001b), the issues in identifying errors in supply/demand systems may include: a) stochastic representation of the production functions, b) correlations of regressors with errors in factor demands, c) corrections and

² It is a substantial problem when there is transmission of the optimization errors from the input choice for the output. See Pope and Just (2001a) for details and mathematical propositions.

verification of simultaneous equation bias, and d) consistent representation of stochastic elements with a dual system.

Mundlak & Hoch (1965) mention that the estimation of the Cobb-Douglas production function can be inconsistent if the input demands are not independent of the error term specified in the production function. Depending on the degree of “transmittability” of the production shocks to the input demand functions, the estimates are not consistent. In our investigation, we will consider a multiplicative shock to the output. For instance, we can consider that the final level of output depends upon weather, which is a common occurrence in agricultural settings. It is not difficult to imagine a situation in which weather events affect only the final output, where input decisions are made at the outset of the growing season. Therefore, the production error is explained by weather occurrences during the growing season. However, managers may respond to early season shocks by altering input decision, hence entangling the production error with input decisions. Depending on the farmer’s ability to react, the effect of such shock can affect the ability of the researcher to consistently identify the underlying economic structure of the problem.

Following Pope & Just (2001b), if the decision maker does not know the stochastic variations when the input decisions are made, then production disturbances cannot affect input decisions. But errors in input decisions can either affect output or not, depending if they affect the actual amounts of inputs used or if they only represent errors in measurement. This implies that the transmission of errors among production, factor and supply is not symmetric.

It is normal to think that these influences are fully or partially transmitted to input demands. Then, when it occurs, the standard econometric estimates of the production and the dual functions are not consistent, as showed by many studies such as Mundlak & Hoch (1965), McElroy (1987), Pope & Just (1996), Moschini (2001), Pope & Just (2001a) and Pope & Just (2001b).

Just & Pope (1996) and Moschini (2001) showed that the presence of stochastic errors in the input making decision together with stochastic shocks in the output can generate nonlinear errors-in-variables that will produce inconsistent estimates under conventional econometric procedures.

Pope & Just (2001a) demonstrated that *ad hoc* addition of disturbances to supply and demand specifications derived under certainty could destroy integrability. The main problem is that the econometric methods assume independent errors, but errors in optimization impose dependence. Therefore, if actual outputs depend on actual inputs that differ from expected inputs, in the Mundlak & Hoch's (1965) terms, the input errors are transmitted to the final outputs.

According to Mundlak & Hoch (1965), in case of full transmission of the output shocks to the input allocation, the OLS estimator is inconsistent and can be upward biased in case of decreasing returns to scale. A simple instrumental variable (IV) approach provides consistent estimates. In case of partial transmission, OLS and IV estimators are inconsistent and can be biased up or down. For the no transmission case, which will be the situation considered in our study, OLS is consistent and unbiased.

Following Mundlak (1996), the case of no transmission when estimated by OLS produces unbiased, consistent and efficient estimates, which was the same conclusion obtained by Zellner et al. (1966) for a Cobb-Douglas production function.

3 - Data and the Theoretical Model

The Standard Profit Maximization

The initial assumptions are related to the agent problem and to the technology to be adopted. The problem here is based on a profit-maximizing agent. It can be considered a representative farmer that maximizes its profits. Considering a decreasing return to scale Cobb-Douglas technology, which is often used in empirical studies in agricultural economics, this farmer maximizes its profits subject to the available Cobb-Douglas technology. This technology is composed, basically, by two inputs: capital and labor. Then, the farmer's maximization problem is represented by:

$$\begin{aligned} \text{Max}_{x_1, x_2} . \Pi(w_1, w_2, p) &= py - w_1x_1 - w_2x_2 \\ \text{s.t. } f(x_1, x_2) &= x_1^a x_2^b \leq y \end{aligned}$$

Where w_i is the price of the i -th input; x_i is the quantity used of the i -th input; y is the production level; p is the output price; a and b are positive parameters of the labor (x_1) and capital (x_2) inputs, respectively.

Since this profit function satisfies all the theoretical properties of a profit function, such as nonnegativity, nondecreasing in p , nonincreasing in w_i , convexity and continuity, and positive linear homogeneity, it is possible to recover the underlying production function from this profit function specification. This means that we can apply the dual theory to recover the underlying technology.

The critical question in this study is: Which method (primal or dual or both) can better recover the underlying technology?

To answer this question we need to make some assumptions about the initially known technology that will be employed in this study. First of all, the input prices are considered exogenous to the problem. Second, we need to fix the parameters **a** and **b** in such way that we can guarantee a decreasing return to scale technology. We choose **a** = **0.7** and **b** = **0.2** to satisfy these criteria.

Once we have the estimated primal function, the production function can be easily recovered. The same happens with the dual function when it is possible to apply the Hotelling's Lemma to the estimated profit function to get the input demands³, which can also be used to recover the production function.

We generate 60 observation-data sets (monthly data for 5 years) for input prices, input demands, expected output, and also a production shock (as weather) that can affect the final output in each period. The wages vary uniformly with average of 0.6 monetary units; the capital rent varies uniformly with average of 0.5 monetary units; the expected output price varies uniformly with average of 2 monetary units. A random lognormal shock is also generated, which determines the actual or observed output. To generate the input demands for labor and capital, it is necessary to solve the farmer's profit maximization problem to get the expressions of the input demands that maximizes profit under given technology.

³ But in presence of some errors in the input demands the typical Hotelling's Lemma applied to the profit function does not yield input demand and output supplies in the conventional ways (Just & Pope, 2001a). The main consequence for the econometric estimation would be inconsistent estimator from OLS technique. Theoretical proofs for such important comments are not the main concern of this paper.

After having substituted the production function into the objective function, the first order conditions for the problem above are given by:

$$w_1 = apx_1^{a-1}x_2^b \quad (1)$$

$$w_2 = bpx_1^ax_2^{b-1} \quad (2)$$

Then, solving this system of equations we can get the following equations:

$$x_1^*(w_1, w_2, p) = \left(\frac{b}{w_2}\right)^{\frac{b}{(1-a-b)}} \left(\frac{a}{w_1}\right)^{\frac{1-b}{(1-a-b)}} p^{\frac{1}{(1-a-b)}} \quad (3)$$

$$x_2^*(w_1, w_2, p) = \left(\frac{b}{w_2}\right)^{\frac{1-a}{(1-a-b)}} \left(\frac{a}{w_1}\right)^{\frac{a}{(1-a-b)}} p^{\frac{1}{(1-a-b)}} \quad (4)$$

These expressions are the optimal input demands for the profit-maximizing farmer. The maximum profit is given by the following expression:

$$\Pi^*(w_1, w_2, p) = p[(x_1^a(x_1^*, w_2, p))(x_2^b(x_2^*, w_2, p))] - w_1x_1^* - w_2x_2^* \quad (5)$$

Based on the assumptions done before and using the expressions above, input demands and expected output are calculated given generated data for output price, input prices and random shocks. Random shocks were included as a multiplicative random production shock, such as weather conditions and variations. So we can say that $\tilde{y} = y \cdot \exp(\epsilon)$, where ϵ is a random normal shock and \tilde{y} is the observed output. 500 trials are generated using Monte Carlo procedures, providing 500 data sets of 60 observations that were used to estimate the primal and dual coefficients through the average of the regression coefficients.

In order to capture the technological changes over time, it is also assumed that the model has a Hicks-Neutral technical change component. This technical change is considered as a disembodied or investment-neutral technical change, implying that the output levels increase without new capital investments. According to Chambers (1988), we have that a production function is Hicks-Neutral if only if it can be written as:

$$y = f(\phi(x_1, x_2), t) \quad (6)$$

Since the Cobb-Douglas technology is linearly homogeneous and homothetic in labor and capital, then the expression (6) implies that:

$$\frac{\partial}{\partial t} \frac{\partial f(x_1, x_2, t) / \partial x_1}{\partial f(x_1, x_2, t) / \partial x_2} = 0 \quad (7)$$

The expression (7) says that technical changes do not change the ratio of marginal products employed in the production process. Therefore, changes in the technology will not change the marginal rate of technical substitution.

Due to the homotheticity of the underlying Cobb-Douglas technology used in this study and to the Hicks-Neutrality assumption, the technology will be cost neutral and also profit neutral at the same time. This implies that technical changes do not change either the optimal input ratios and the profit maximizing input ratios.

The Modified (Expected) Profit Maximization

The complication to be included in our analysis is the possibility that there are stochastic errors not only in the production function, due to climatic and pest factors that are outside of the producer's control, but also in the input demands for capital and labor due to the farmers make decision errors. Then, we could go a step further in comparison to what Pope & Just (1996) did considering that the input demands were deterministic.

The major discussions and details about the implications and consequences on having these stochastic errors in the model can be seen in Moschini (2001). Both papers discussed the problems on using the “ex-post” and “ex-ante” cost functions since the expected output level that is relevant for the cost minimization is not observable.

Our analysis has two different sources of errors: the primal error due to the stochastic production function, and input demand errors. According to Moschini (2001), the main consequence of these errors is that the estimating equations on duality belong to the class of nonlinear errors-in-variables models, which produces inconsistent estimates.

Although the farmers know about the presence of the input errors, they cannot avoid them. It means that the farmers choose the inputs such as $x = f(\tilde{x}, e)$, where \tilde{x} denotes a vector of unobserved input levels and e is the random uniform input errors. Moschini (2001) developed and suggested an approach based on the expected profit maximization problem that yields consistent estimates of the parameters of the underlying technology because it removes the errors-in-variables problem.

According to Mundlak & Hoch (1965) and Pope & Just (2001b), we are assuming that there is no transmission from the stochastic errors from the output to input and vice-versa. Therefore, the OLS estimators are expected to be consistent, unbiased and efficient. In the Appendix we show a simple case where the OLS is not consistent simply because the errors were not correctly captured by the economic agent, which could be due to differences in the quality of raw material or to differences in managerial ability, as suggested by Brown & Walker (1995).

Therefore, the primal and dual models to be estimated in our study under technical change and errors in input demands are:

$$\ln y^*(x_1^*, x_2^*, t, \varepsilon) = k + a \ln x_1^*(w_1, w_2, p, e) + b \ln x_2^*(w_1, w_2, p, e) + \eta + \varepsilon \quad (8)$$

$$\ln \Pi^*(w_1, w_2, p, t, u) = \alpha + \phi \ln p + \mu \ln w_1 + \phi \ln w_2 + \beta t + u \quad (9)$$

Where in (9) we have that \mathbf{u} includes both \mathbf{e} and ε from equation (8), and \mathbf{k} and α are constant terms. Both \mathbf{e} and ε have constant variance, are not autocorrelated, and are independent by construction. The estimates of \mathbf{a} and \mathbf{b} can be recovered multiplying ϕ and μ by the negative of the inverse of ϕ .

Equation (8) will be estimated by two different procedures: Ordinary Least Squares (OLS) and Instrumental Variables⁴ (IV) methods, and the results will be compared.

4 – Results and Discussion

a) The Standard Profit Maximization Problem

The econometric procedure is twofold. First, we estimate the primal problem through an ordinary least square estimation (OLS) of the production function using the natural log of the observed output (\tilde{y}) as dependent variable against the natural logs of the labor (x_1^*), capital (x_2^*) and shocks. Second, the dual problem is estimated through the natural log of the profit function (5) as dependent variable against the natural logs of the wages (w_1), capital rents (w_2), and input demands given by (3) and (4).

The estimated regressions for the primal and dual are in Table 1. It can be seen that the coefficients \mathbf{a} and \mathbf{b} estimated are very similar to those assumed in the Monte Carlo generation of the data. In both estimates only \mathbf{a} was significantly different than zero

⁴ Output and input prices will be used as instruments in this case.

at 1 %. To get the coefficients from the dual estimation it was necessary to multiply the coefficient from labor and capital by: $-1/\text{coefficient of } p$.

TABLE 1 –Estimates of the primal and dual from the standard profit maximization problem under a decreasing return to scale Cobb-Douglas technology.

Coefficients	Primal	t-value	Dual	t-value.	True Values
a	0.692	3.53	0.700	3.57	0.7
b	0.208	1.04	0.201	1.01	0.2

Using 500 draws. The t-value is used to test if the parameters are statistically different than zero.

The estimations did not present any significant variation when the number of draws was reduced from 500 to 200, 100, 80, 50 and 20. Therefore, the estimated coefficients were consistent with the known parameters, as expected.

It was tested if the estimated coefficients were statistically equal to the known coefficients **a** and **b**. In Table 2, the results showed that we couldn't reject the null hypothesis that **a** estimates from primal and dual are equal to 0.7, since the calculated t-values were: -0.03 and 0.003, respectively. The same hypothesis test showed that the estimates of **b** are statistically equal to 0.2, where the calculated t-values were: 0.04 and 0.05, respectively for the primal and the dual estimates.

TABLE 2 –Estimates of the primal and dual from the standard profit maximization problem under a decreasing return to scale Cobb-Douglas technology.

Coefficients	Primal	t-value	Dual	t-value	True Values
a	0.692	-0.03	0.700	0.003	0.7
b	0.208	0.04	0.201	0.05	0.2

Using 500 draws. The t-value is used to test if the parameters are statistically different than the true values $a = 0.7$ and $b = 0.2$.

The performance of both approaches was similar if we compare the numbers obtained from Table 1 and the t-tests described above. For this reason, it was done an extra verification of the estimates through a 95 % confidence interval for the estimates that can be seen in Table 3.

TABLE 3 – 95 % Confidence interval for the estimates of the primal and dual.

Coefficients	Primal		Dual	
	Lower CI	Upper CI	Lower CI	Upper CI
a	0.313	1.055	0.640	0.760
b	-0.153	0.596	0.140	0.217

Using 500 draws.

Even though the true parameters were inside of the confidence interval in both primal and dual estimations, Table 3 shows that the dual estimates were better in terms of efficiency, since the estimates distribution is concentrated in a very narrow interval. Therefore, the interval of the estimates for the primal was larger than that for the dual.

b) The Modified (Expected) Profit Maximization Problem

The results of the primal and dual formulations under stochastic errors in the input demands can be seen in Tables 4, and 5 below, which shows the average bias in estimated parameters and sample size with respect to the true parameters $a = 0.7$ and $b = 0.2$.

These tables show that the average bias decreases as the sample size increases, though the confidence intervals appear to become larger with additional draws. This suggests that the estimator is consistent but inefficient. IV estimator seems to have smaller bias, but less efficiency in comparison to the OLS.

The dual estimation appears to converge to small levels of bias more rapidly than does the primal approach, though the absolute level of bias appears larger for the smallest sample size. Dual estimates also have narrower confidence intervals than those from the primal, suggesting that dual estimates provide less bias and more efficiency.

TABLE 4 –Average bias for primal and dual estimations

Average Bias (OLS)					
Coefficients	T = 50	T=100	T = 200	T=500	T=550
a	-0.02063	-0.01352	-0.00915	-0.00148	-0.00116
b	0.02032	0.01328	0.00959	0.00141	0.00068
Average Bias (IV)					
Coefficients	T = 50	T=100	T = 200	T=500	T=550
a	-0.01452	-0.01064	-0.00586	-0.00154	-0.00159
b	0.01382	0.01007	0.00536	0.00102	-0.00072
Average Bias (Dual)					
Coefficients	T = 50	T=100	T = 200	T=500	T=550
a	-0.00198	-0.00236	-0.00446	-0.00073	-0.00018
b	0.00150	0.00136	-0.00058	0.00014	0.00018

c) Policy Implications

What would happen if an economic agent (or an econometrician) does not consider possible errors in the input choices? What would be the main implications for policy makers when trying to improve the productive sector and such errors are not taking into account?

To answer these questions, we can suppose that the government is willing to increase the agricultural production through a subsidy policy on the price of the capital

used in the sector. Therefore, the expected production and profits should increase in the sector.

TABLE 5 –Confidence intervals for the estimates of the primal and dual

95 % Confidence Interval (OLS)					
Coefficients	T = 50	T=100	T = 200	T=500	T=550
a	0.58	0.57	0.57	0.57	0.57
	0.76	0.80	0.82	0.83	0.82
b	0.12	0.06	0.07	0.06	0.06
	0.32	0.32	0.33	0.33	0.33
95 % Confidence Interval (IV)					
Coefficients	T = 50	T=100	T = 200	T=500	T=550
a	0.52	0.48	0.49	0.48	0.49
	0.83	0.87	0.88	0.88	0.88
b	0.03	-0.005	-0.001	-0.008	-0.005
	0.34	0.42	0.42	0.42	0.42
95 % Confidence Interval (Dual)					
Coefficients	T = 50	T=100	T = 200	T=500	T=550
a	0.63	0.58	0.61	0.61	0.61
	0.77	0.77	0.78	0.78	0.80
b	0.16	0.16	0.15	0.15	0.15
	0.23	0.24	0.24	0.25	0.25

But this task is not so simple because the problems that can be affecting the decisions on production, as we discussed before, can also result in some errors in the resource allocation, and/or errors in the measurement of main inputs used in the production process by the policy makers, or by the econometrician that is modeling the agricultural sector to evaluate the impact of the subsidy policy over the economy.

To exemplify the implications on not considering the presence of errors in the input demands, we can suppose a 10 % subsidy on the capital price (w_2) in our Cobb-Douglas production function, which implies that the random price of capital generated before the subsidy is reduced in 10 %. Table 6 shows the production and profit values for each sample size, and the actual values for production and profits as well when computing the errors in the input demands.

From Table 7 we can see the size of the average bias on production and profits with subsidy in the price of capital, when the error in input demand is not considered by the policy maker or by the farmer in the making decision process. We also can see that the estimate bias does not vanish as the sample size is increased. In the Appendix it can be seen in a more formal way why the OLS does not provide good estimates when the existent input errors are not considered in the estimation in case of measurement type of errors⁵.

In the presence of stochastic errors in the input demands, not considering such errors can result in less production and profits, which can be illustrated by Taylor (1989), where the optimum level of input to be used is not employed, resulting in a solution far away from the profit-maximizing levels of output and input. Therefore, any kind of policy created to improve the total production of a particular sector has to consider the issues so far discussed to avoid such bias problems and inefficiency before be applied to the real world.

⁵ The case of errors in optimization is not formally showed in this study.

TABLE 6 –Average production and profits with 10 % of subsidy on the price of capital used (in thousands).

Levels of	Average Amount and Sample Size Without Errors in Input				
	T = 50	T=100	T = 200	T=500	T=550
Production	26.633	27.122	27.064	26.170	26.123
Profits	50.742	51.517	50.989	48.994	48.909
Levels of	Average Amount and Sample Size With Errors in Input				
	T = 50	T=100	T = 200	T=500	T=550
Production	53.360	55.085	55.291	53.382	53.281
Profits	96.427	99.359	98.780	94.801	94.652

Therefore, it is not a surprise that the profits and outputs with errors in inputs are greater than those without such errors. It happens because the way that the model was defined, equations (8) and (9), with additive error terms in the log specification of the model⁶, and also by strong monotonicity of the production function ($\frac{\partial y}{\partial e} > 0$).

TABLE 7 – Bias for production and profits with 10 % of subsidy on the price of capital used (in thousands).

Levels of	Bias and Sample Size				
	T = 50	T=100	T = 200	T=500	T=550
Production	-26.728	-27.962	-28.227	-27.211	-27.158
Profits	-45.685	-47.841	-47.790	-45.806	-45.743

⁶ We also used negative errors in the input demands under a log specification and the results for output and profits were exactly the opposite from those in Table 6. For instance, for T=500, the average profits with errors was 43.6 and the average profits without errors was 73.8.

5 - Conclusions

The present study was an attempt to verify the properties of duality in empirical work, with the inclusion of Hicks-Neutral technical changes and stochastic errors in the input demands in a profit maximization problem, using some synthetic data from a Monte Carlo simulation.

The approach used requires several assumptions about the production, each of which has substantial precedent in the literature. Theoretically, it was expected that the results from primal and dual formulations were similar, since the dual function is directly linked with the primal function. Empirically, the dual estimation would be more appealing, since the quantity of information necessary is less restrictive than those from the primal function, as for the dual estimation we would need only data for profit, input prices and output levels, as also noted by Young (1982). The empirical estimations showed that both primal and dual did a very good job on recovering the true parameters of the underlying Cobb-Douglas technology. The dual estimation was better with respect to both the breadth of the confidence interval and bias.

According to Zellner et al. (1966), the least squares are consistent to Cobb-Douglas technology model under the assumption that production shocks are unknown at the time of decisions so that the input demands must be based on expected output. Actually, as we did not consider any causal relationship among errors from the input demands and errors from output, since they were generated by Monte Carlo simulation, they were totally independent with any “transmittability” problem, as mentioned by Mundlak & Hoch (1965).

Just & Pope (1979) consider that the specification of the production function as employed in this study has some problems concerned to the multiplicative shock used. They pointed out that this specification is too restrictive and implies that changes in inputs are directly related to changes in the variance⁷ of the output. These authors also conclude that this restriction contradicts other empirical studies, and that the estimates are not so useful in evaluating policies. They consider that the best specification should include at least two components, one explaining the effect of input on expected output, and another explaining the effects of the inputs on the output variance.

Our study presents just a simplified version of the possible influence of the stochastic errors in the input demands and production on the profit maximization solution. Our objective was to verify the validity on using OLS procedure to recover the technology parameters under the hypothesis of no existence of transmission among the errors in input and production. But there were transmissions from input to output that, in practice, imply that other classes of estimators should be employed to be able to get unbiased, consistent and efficient estimates. As Pope & Just (2001b) suggested, depending on the source of the error faced, a different approach to estimation is required and, given the potential of multiple errors sources, a new way of modeling producer behavior is suggested. Therefore, this study needs a further extension to evaluate the reasons for the good performance of the OLS procedure in both primal and dual estimations.

Some possible empirical implications were also discussed using a hypothetical implementation of subsidy policy on the price of the capital in the agriculture. Farmers or policy makers that do not take into account the presence of stochastic errors in the input

⁷ See Just and Pope (1979) for more details about how this relationship can be inferred.

demands can base their decisions on the wrong direction, far away from the efficient resource allocation, with substantial biases due to the wrong or any perception of such errors.

Summing up, this study just stressed the importance of future investigations with more realistic and sophisticated econometric and mathematical tools, where different model formulations can be tested at the same time to verify the asymptotic properties of their estimates. A possible future investigation may consider the possibility to incorporate, at the same time, the potential transmission of production errors to input demands and vice-versa. It would suggest a combination of the principles discussed in Pope & Just (2001b) and Mundlak & Hoch (1965). Another possibility would be the inclusion of non-neutral Hicks technical change, since some empirical studies rejected the Hicks neutrality of technical changes in the production, as pointed out by Shumway (1995).

APPENDIX:

The Inconsistency of OLS Estimation of the Primal with Stochastic Errors in the Inputs

Consider that we can observe the production and the input used in a production function as:

$$y_i^* = y_i + u_i \quad \text{and} \quad x_i^* = x_i + v_i$$

The small letters can represent the logarithmic form of variables Y (production level) and X (input level).

We also assume that:

- 1) $E(u_i) = E(v_i) = 0$;
- 2) $\text{Var}(u_i) = \sigma_u^2$ and $\text{Var}(v_i) = \sigma_v^2$;
- 3) $\text{Cov}(u_i, v_i) = \text{Cov}(v_i, v_i) = 0$ for $i \neq j$

4) $\text{Cov}(u_i, v_j) = 0$ for $\forall i, j$, which implies that the production errors are independent from input errors.

The problem emerge when we try to estimate the production function using only the observable variables \mathbf{y}^* and \mathbf{x}^* . Then, if the true production function is given by $y_i = \beta_1 + \beta_2 x_i$, we have:

$$(y_i^* - u_i) = \beta_1 + \beta_2 (x_i^* - v_i) \quad \text{which implies that:}$$

$$y_i^* = \beta_1 + \beta_2 x_i^* + e_i \quad \text{where} \quad e_i = u_i - \beta_2 v_i$$

The error term e_i has zero mean and constant variance, but its covariance with input level (\mathbf{x}) is not zero:

$$\text{Cov}(x_i^*, e_i) = E[(x_i^* - E(x_i^*))(e_i - E(e_i))] =$$

$$E(v_i e_i) = E[v_i (u_i - \beta_2 v_i)] = E(v_i u_i) - E(\beta_2 v_i^2) = -\beta_2 \sigma_v^2$$

Then if we apply OLS to estimate the primal, we will get biased and inconsistent estimates, as we have that:

$$p \lim b_2 = p \lim \frac{\left[T^{-1} \sum_{i=1}^T (y_i^* - \bar{y}^*)(x_i^* - \bar{x}^*) \right]}{\left[T^{-1} \sum_{i=1}^T (x_i^* - \bar{x}^*)^2 \right]} = p \lim \beta_2 + \frac{p \lim \left[T^{-1} \sum_{i=1}^T (x_i^* - \bar{x}^*) e_i \right]}{p \lim \left[T^{-1} \sum_{i=1}^T (x_i^* - \bar{x}^*)^2 \right]} =$$

$$\beta_2 - \frac{\beta_2 \sigma_v^2}{\sigma_{x^*}^2} = \beta_2 \left(1 - \frac{\sigma_v^2}{\sigma_{x^*}^2} \right)$$

The OLS estimator is not only biased, as the expression above shows, but also inconsistent, since the bias does not converge in probability to zero. It is interesting to note that the bias of the estimator \mathbf{b}_2 is proportional to the ratio of the variances of input errors (\mathbf{v}_i) and the observed input level (\mathbf{x}^*).

It is clear that if the errors are only in the final output, the OLS estimators are consistent and unbiased because $p \lim b_2 = \beta_2$. But it is only true if we are considering that the error in the

production is not transmitted to the input demands, no transmission case as considered by Mundlak & Hoch (1965).

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