HEDGING WITH INDIVIDUAL AND INDEX-BASED CONTRACTS

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Abstract

We examine the optimal hedging strategy with an individual insurance policy, sold at an unfair price, and a fair contract based on an index, which is imperfectly correlated with the individual loss. The tradeoff between transaction costs and basis risk is first analyzed in the expected utility framework in order to highlight the role of the agent's attitude toward risk, and then in the linear mean-variance model to stress the importance of the degree of correlation between the individual loss and the index.

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1. Introduction

Insurance and financial markets offer individual and index-based contracts to producers who want to manage their risks. For example, farmers can choose forward contracts and/or futures contracts to deal with commodity price risk. They can also cover crop yield shortfalls using individual yield insurance, area yield insurance contracts or, more recently, weather derivatives.[1] While the former provide indemnities based on the realized individual loss, the latter offer payoffs that depend on the realization of an index which is correlated with, but not equal to, the individual loss. Suggested disadvantages and advantages of index-based contracts are well known; the imperfect correlation between individual and index-based losses creates an imperfect loss protection, but this type of basis risk lowers problems of asymmetric information (moral hazard and adverse selection) thanks to the removal of the direct link between individual losses and indemnities (Chambers 1989, Quiggin 1994). In addition, administrative costs are substantially reduced because there is no individual claim settlement (Mahul 1999).

Most models on optimal hedging/insurance of a single risk assume that only one type of hedging instruments is at the producer's disposal. Surprisingly, there is only a few papers that examine the hedging strategy with both individual and index-based contracts and they essentially provide a descriptive investigation. An interesting exception is Frechette (2000). Using a linear mean-variance model, he examines the demand for hedging and the value of hedging opportunities for hedgers facing spatial basis risk and he analyzes the incremental value of a second futures market to assess the cost of basis risk.

The primary contribution of this paper is to provide a theoretical analysis of the tradeoff between transactions costs and basis risk by modeling the simultaneous demand for individual and index-based contracts. We consider a producer facing a single source of risk. He can manage this risk using two instruments; an individual insurance policy sold at an actuarially unfair price and an index-based insurance contract sold at a fair price. The key elements in this tradeoff are captured within two frameworks; the expected utility model and the linear mean-variance model.

The problem is first examined in the expected utility framework. The producer faces a random loss, and the no-loss outcome may be reached with a positive probability. The individual contract is geared to losses borne by the producer. Under the index-based contract,

[1] The U.S. insurance program does not allow the producers to purchase joint individual yield and area yield crop insurance policies. However, this hedging strategy may have been possible using area yield crop insurance futures and options offered on the CBOT until recently.

the occurrence of a loss is observable but the severity of this loss cannot be observed. The (unconditional) correlation between the individual loss and the index is thus characterized through the no-loss state; the higher the probability of the no-loss state, the higher the (unconditional) correlation between the individual loss and the index. Optimal individual and index-based contracts are first designed in this framework. We show that an optimal individual contract displays full (marginal) coverage above a deductible and an optimal index-based contract pays a fixed indemnity in the event of a loss. The optimal individual insurance deductible and the lump-sum payment (when a loss occurs) are first examined and then the impact of a change in the probability of incurring a loss is analyzed.

The optimal hedging strategy is reconsidered in the linear mean-variance model when individual coinsurance (forward) and index-based coinsurance (futures) contracts are available. This framework allows us to focus on the impact of the degree of correlation between the individual loss and the index on the hedging decisions. The optimal futures and forward hedge ratios are first explicitly characterized. They are broken down into three components; a pure hedge ratio, a speculative component and a cross-hedge component. The introduction of unbiased futures contracts into the hedging strategy decreases the demand for forward contracts. These two hedging contracts are thus substitutes. The effectiveness of the dual hedge depends, critically, on the correlation between the individual loss and the index. Under unbiased futures markets, the optimal futures hedge ratio is increasing and convex in the correlation coefficient, while the optimal forward hedge ratio is decreasing and concave in this coefficient.

Finally, the linear mean-variance hedging strategy is illustrated when individual yield crop insurance and area yield crop insurance contracts are available for representative wheat farmers located in France. The efficient frontier of the individual yield insurance policy and that of a combination of individual and area yield insurance contracts are computed. As expected, the area yield insurance contract is shown to be market enhancing; its optimal combination with an individual yield crop insurance policy allows the producer to reduce the yield variance for a given expected yield. This reduction depends, among other things, on the degree of correlation and on the variance of individual yield.

2. The Model within the Expected Utility Framework

2.1. The setting

The agent's preferences are represented by a von Neumann-Morgenstern utility function $u(.)$ that is assumed to be three-times differentiable, with $u' > 0$ and $u'' < 0$. The risk-averse agent is endowed with a nonrandom initial wealth w_0 and faces a risk of incurring a positive loss $\tilde{y}$ with probability $q \in (0,1)$. The loss $\tilde{y}$ is a random variable with a cumulative distribution function $F(.)$ defined over the support $(0, y_{\max}]$, with $0 < y_{\max} < w_0$. To protect against the loss, he can purchase two forms of insurance.

Under the first policy, the insurance company can observe the occurrence of a loss but not its severity. As a consequence, this contract pays an indemnity in the event that a loss occurs, but which is not conditioned on the size of the realized loss. Obviously, no indemnity is paid in the no-loss state. The contract is thus restricted to the form $[K, Q]$ where $K \geq 0$ is a lump-sum reimbursement paid in the loss-state and Q is the associated premium. This fixed-reimbursement insurance policy is assumed to be sold at a fair price, i.e., $Q = qK$. This contract thus provides an imperfect coverage because, conditional on the occurrence of a loss, the indemnity is independent of the severity of the loss.

The agent can also purchase an insurance policy that provides an indemnity based on the individual loss; it is described by a couple $[I(.), P]$ where $I(y)$ is the payoff when the loss is y and P is the premium. The indemnity schedule is assumed non-negative and not higher than the loss:

(1) $$0 \leq I(y) \leq y.$$

The insurance premium is assumed to be proportional to the expected indemnity:

(2) $$P = mqEI(\tilde{y}),$$

where E is the expectation operator conditional on the occurrence of a loss and m is the loading factor. This tariff is sustained by a competitive insurance market with risk-neutral insurers and transaction costs (e.g., costs of gathering data about individual losses) that are proportional to claims. The individual insurance policy is sold at an unfair price, $m > 1$. It can be easily shown that an optimal insurance contract design displays full (marginal) coverage above a deductible $D \geq 0$:

(3) $$I^*(y) = \max(y - D, 0).$$

An optimal insurance strategy with both individual and fixed-reimbursement contract is obtained by finding the lump-sum reimbursement K and the deductible D that maximize the policyholder's expected utility of final wealth. Formally, this problem is

(4) $$\underset{D \geq 0, K \geq 0}{Max}\ V \equiv qEu(w_0 - \tilde{y} + \max(\tilde{y} - D, 0) + K - P - Q) + (1-q)u(w_0 - P - Q)$$

subject to conditions (2) and $Q = qK$.

2.2. Optimal levels of coverage

The objective function V can be rewritten as

(5) $$V \equiv (1-q)u(w_1) + q[1-F(D)]u(w_2) + q\int_0^D u(w_0 - y - P + K - Q)dF(y),$$

where $w_1 = w_0 - P - Q$, $w_2 = w_0 - D + K - P - Q$ and $w_3 = w_0 - y + K - P - Q$.

The first-order conditions of the maximization problem (4) are

(6) $$\frac{\partial V}{\partial D} = -P_D'(1-q)u'(w_1) - (1+P_D')q[1-F(D)]u'(w_2) - P_D'q\int_0^D u'(w_3)dF(y) = 0,$$

where $P_D' \equiv \partial P/\partial d = -mq[1-F(D)]$, and

(7) $$\frac{\partial V}{\partial K} = q(1-q)[Eu'(w_0 - \tilde{y} + \max(\tilde{y} - D, 0) + K - P - Q) - u'(w_0 - P - Q)] = 0.$$

Observe that $V_{KK} < 0$ under risk aversion. The other second-order conditions of this maximization problem are assumed to hold; $V_{DD} < 0$ and $\Delta \equiv V_{DD}V_{KK} - V_{DK}^2 > 0$. It is first noteworthy that

(8) $$\begin{aligned}\left.\frac{\partial V}{\partial D}\right|_{D=0} &= mq(1-q)u'(w_1) - (1-mq)qu'(w_0 + K - P - Q) \\ &= mq(1-q)[u'(w_1) - u'(w_0 + K - P - Q)] + (m-1)qu'(w_0 + K - P - Q).\end{aligned}$$

The first right-hand side (RHS) term in (8) is non-negative because $w_1 \le w_0 + K - P - Q$ and $u'' < 0$. The second RHS term is positive because $m > 1$. This implies that the optimal deductible is positive, i.e., $D^* > 0$. Therefore, the presence of a second insurance policy does not affect the well-known result of the optimality of partial insurance under unfair pricing.

We now examine the optimal lump-sum reimbursement in the loss states. Using the concept of the precautionary (equivalent) premium defined by Kimball (1990), the first-order condition (7) can be rewritten as

(9) $$u'(w_0 - E[\tilde{y} - \max(\tilde{y} - D, 0)] + K - P - Q - \Psi(\overline{w}, \tilde{z})) = u'(w_0 - P - Q),$$

where Ψ is the precautionary (equivalent) premium, with $\overline{w} \equiv w_0 - E[\tilde{y} + \max(\tilde{y} - D, 0)] + K - P - Q$ and $\tilde{z} = -\tilde{y} + \max(\tilde{y} - D, 0)$. Given the assumption of the strict concavity, $u'' < 0$, it follows

(10) $$K^* = E[\tilde{y} - \max(\tilde{y} - D, 0)] + \Psi(\overline{w}, \tilde{z}).$$

Observe that the optimal fixed reimbursement is the solution of the implicit equation (10) because it appears in both sides of this equation through the wealth term $\overline{w}$. From Kimball (1990), we know that the precautionary premium Ψ is positive if and only if the policyholder exhibits prudence, i.e., his marginal utility function is convex, $u''' > 0$. Prudence is justified by some fairly solid economic rationale. It is a necessary condition for decreasing absolute risk aversion. A prudent agent is more sensitive to low realizations of wealth. He is thus encouraged to select a lower marginal payoff in order to shift wealth from states providing low marginal utility to states providing high marginal utility. The role of prudence in the design of optimal insurance contracts has been stressed Gollier (1996), Mahul (2000) and Eeckhoudt, Mahul and Moran (2003). The optimal fixed payment thus satisfies $K^* > E[\tilde{y} - \max(\tilde{y} - D, 0)]$ if and only if $u''' > 0$. This finding generalizes the result derived by Eeckhoudt, Mahul and Moran (2003, Proposition 1) to the case where individual insurance is available in addition to fixed-reimbursement insurance. We also have

(11)
$$\begin{aligned}\frac{1}{q(1-q)}\frac{\partial V}{\partial K}\bigg|_{K=D} &= Eu'\left(w_0 - \tilde{y} + \max(\tilde{y} - D, 0) + D - P - Q\right) - u'\left(w_0 - P - Q\right) \\ &= \int_0^D u'\left(w_0 - y + D - P - Q\right) dF(y) - F(D)u'\left(w_0 - P - Q\right) < 0.\end{aligned}$$

This implies that $K^* < D$. This discussion is summarized in the following proposition.

Proposition 1.

Under actuarially unfair individual insurance, the optimal deductible is positive, $D^* > 0$.

If the fixed-reimbursement insurance policy is actuarially fair and the policyholder exhibits prudence ($u''' > 0$ *), then the optimal lump-sum payment satisfies:* $E\min[\tilde{y}, D^*] < K^* < D^*$.

As an illustration, consider that there exist an index x, a trigger $\hat{x}$ and a deterministic function $g(.)$ such that $y = 0$ for all $x \le \hat{x}$ and $y = g(x) > 0$ for all $x > \hat{x}$. The probability of incurring a loss is thus $q \equiv \text{Prob}[\tilde{x} > \hat{x}]$. Under the individual insurance policy the trigger $\hat{x}$ and the function $g(.)$ are known and the realized index x is observable (at some costs); the indemnity is thus based on the individual loss. Under the fixed-reimbursement insurance contract, the insurer only knows whether the realized index is higher or lower than the trigger, i.e., he only knows whether the policyholder does face a loss or he does not. The optimal insurance deductible and lump-sum payments are drawn on Figure 1.

[INSERT FIGURE 1 HERE]

2.3. Changes in the probability of loss

Within the expected utility model, the (unconditional) degree of correlation between the index-based payoff (the lump-sum payment) and the individual loss can be measured through the probability of incurring a loss q.[2] The higher the probability of the no-loss state, $(1-q)$, the stronger the (unconditional) correlation between the individual loss and the fixed reimbursement. We examine how a change in this probability of loss will affect the optimal deductible under the individual insurance contract and the optimal lump-sum payment under the fixed-reimbursement insurance policy. Assuming the second-order conditions hold and applying the Cramer's rule yield

(12) $$\text{sign}\left[\frac{dD^*}{dq}\right] = \text{sign}\left[V_{DK}V_{Kq} - V_{KK}V_{Dq}\right]$$

(13) $$\text{sign}\left[\frac{dK^*}{dq}\right] = \text{sign}\left[V_{DK}V_{Dq} - V_{DD}V_{Kq}\right].$$

We can derive the following results (see the Appendix).

First $V_{Kq} > (<) 0$ if and only if the policyholder's utility function u satisfies decreasing (increasing) absolute prudence (DAP, IAP), while $V_{Kq} = 0$ if and only if u satisfies constant absolute risk aversion (CARA). In other words, when the level of deductibility is fixed and under actuarially fair pricing, the policyholder responds to an increase in the probability of loss by increasing (remaining unchanged, decreasing) his fixed reimbursement if and only if his preferences satisfy DAP (CARA, IAP).

Second, V_{Dq} can be decomposed into three effects; the *wealth effect* is negative (null) under decreasing (constant) absolute risk aversion, the *risk effect* is negative, and the *premium effect* is positive. In addition, the sum of the risk effect and the premium effect is negative (positive) if the loading factor m is the individual insurance premium is "sufficiently" high (low). This implies that V_{Dq} is negative (positive) if u satisfies non-increasing risk aversion and the individual insurance premium is "sufficiently" unfair ("not too" unfair). In other words, when the lump-sum payment is not a decision variable, an increase in the probability of loss leads the policyholder with preferences satisfying DARA or CARA to reduce (increase) his optimal deductible, should the insurance premium be "sufficiently" unfair ("not too" unfair). Such an ambiguous effect is not surprising. The impact of increases in risk on the optimal level of deductibility is shown to be usually indeterminate, except in some very specific cases (Eeckhoudt, Gollier and Schlesinger 1991).

[2] Obviously, the degree of correlation conditional on the occurrence of the loss is zero.

Third, $V_{DK} > 0$ is shown to be positive under DARA or CARA. This means that when the level of deductibility is the only decision variable, the policyholder responds to an increase in the fixed-reimbursement payoff by increasing his deductible under DARA or CARA.

Finally, it should be noticed that decreasing absolute prudence is a sufficient condition for absolute risk aversion to be decreasing Gollier (2001, Proposition 21). In other words, DAP implies DARA.

This discussion leads to the following proposition.

Proposition 2.

Define $V_{Dq} \equiv \partial^2 Eu / \partial D \partial q$.

(i) *Suppose the policyholder's preferences exhibit constant absolute risk aversion. An increase in the probability of loss induces him to increase (reduce) his optimal deductible under the individual insurance policy and his optimal lump-sum payment if and only if* V_{Dq} *is positive (negative).*

(ii) *Suppose the policyholder's preferences exhibit decreasing absolute prudence (DAP) and* $V_{Dq} \geq 0$. *An increase in the probability of loss induces his to increase his optimal deductible and his optimal lump-sum payment.*

Proposition 2 shows that under CARA, the effect of a change in the probability of loss on the optimal deductible and fixed payment only depends on the sign of the partial derivative V_{Dq}. Both insurance contracts are substitutes as the probability of loss changes. When the producer's preferences exhibit DAP (and hence DARA), the individual insurance contract and the fixed-reimbursement policy are substitute if $V_{Dq} \geq 0$; the policyholder responds to an increase in the probability of loss by reducing his individual coverage (increase in the deductible) and increasing his index-based coverage (increase in the fixed payment in the loss states). However, the two contracts may be complements when $V_{Dq} < 0$; under DARA, an increase in the probability of loss may decrease (decrease) the optimal fixed reimbursement and increase (decrease) the optimal insurance deductible.

3. The Model within the Linear Mean-Variance Framework

3.1. Optimal hedging decisions

The optimal combination of individual insurance and index-based insurance contracts is reconsidered in the linear mean-variance model in order to highlight the impact of the degree of correlation on the optimal hedging strategy.

The produce can buy or sell futures contracts and/or he can purchase a coinsurance policy. The notation is defined as follows: w_0 is his initial wealth subject to a loss $y \in (0, y_{\max})$, with $0 < y_{\max} < w_0$; x is the futures prices at the end of the period, that is correlated with, but not identical to, the individual loss y; x_f is the futures price at the beginning of the period; a is the futures quantity purchased (if positive) or sold (if negative); $P = mE\tilde{y}$ is the insurance premium per unit of coverage, with $m > 0$; b is the level of coinsurance. In order to highlight the tradeoff between the degree of correlation and the cost of insurance, we assume that trading futures contracts on financial markets are not subject to transaction costs. In addition, we assume that the producer cannot sell insurance and he cannot be over-insured, i.e., $b \in [0,1]$.

The producer's random end-of-period wealth using both futures and coinsurance can be written as

(14) $$\tilde{w} = w_0 - \tilde{y} + a\left(\tilde{x} - x_f\right) + b\left(\tilde{y} - P\right).$$

The mean and variance of his final wealth are respectively

(15) $$E\tilde{w} = w_0 - E\tilde{y} + a\left(E\tilde{x} - x_f\right) + b\left(E\tilde{y} - P\right),$$

(16) $$s^2(\tilde{w}) = (1-b)^2 s_y^2 + a^2 s_x^2 - 2a(1-b)s_x s_y r ,$$

where $s_x^2 = Var(\tilde{x})$, $s_y^2 = Var(\tilde{y})$ and $r = \mathrm{cov}(\tilde{x}, \tilde{y})/(s_x s_y)$, with $r \in (0,1)$.

Under the linear mean-variance framework, the producer's objective function is

(17) $$\underset{a,b}{Max} V \equiv E\tilde{w} - l\, s^2(\tilde{w}),$$

where l is his level of risk aversion. This problem yields the first-order conditions for interior solutions

(18) $$\left(E\tilde{x} - x_f\right) - 2 l\, s_x \left[a^* s_x - (1-b^*) s_y r\right] = 0 ,$$

(19) $$\left(E\tilde{y} - P\right) - 2 l\, s_y \left[-(1-b^*) s_y + a^* s_x r\right] = 0 .$$

They can be rewritten as

(20) $$a^{*} = \frac{E\tilde{x} - x_f}{2 l s_x^2} + \left(1 - b^{*}\right) r s_y / s_x ,$$

(21) $$b^{*} = 1 + \frac{E\tilde{y} - P}{2 l s_y^2} - a^{*} r s_x / s_y .$$

The optimal future hedge x^{*} can be divided into a speculative component (first RHS term in (20)) and a cross-hedge component (second RHS term in (20)). The optimal level of coinsurance b^{*} can be divided into a pure hedge component equal to one, a speculative component and a cross-hedge component. Observe that introducing futures contracts induces the producer to reduce his level of coinsurance, should the futures market is unbiased ($x_f = E\tilde{x}$) or exhibits normal backwardation ($x_f < E\tilde{x}$).

Rearranging equations (20) and (21) yields[3]

(22) $$a^{*} = \frac{1}{\left(1 - r^2\right) 2 l s_x} \left[\frac{E\tilde{x} - x_f}{s_x} - \left(E\tilde{y} - P\right) \frac{r}{s_y} \right],$$

(23) $$b^{*} = 1 + \frac{1}{\left(1 - r^2\right) 2 l s_y} \left[\frac{E\tilde{y} - P}{s_y} - \left(E\tilde{x} - x_f\right) \frac{r}{s_x} \right].$$

From (23), the producer will purchase coinsurance, $b^{*} > 0$, if and only if the cost of insurance is not too high:

(24) $$P < \left[\left(1 - r^2\right) 2 l s_y - \left(E\tilde{x} - x_f\right) \frac{r}{s_x} \right] s_y + E\tilde{y} .$$

3.2. Changes in the degree of correlation

Suppose first that the futures contracts are biased ($x_f \neq E\tilde{x}$). Defining $A \equiv \frac{x_f - E\tilde{x}}{s_x} \Big/ \frac{P - E\tilde{y}}{s_y}$,

the optimal hedging levels (for interior solutions) can be rewritten as

(25) $$a^{*} = \frac{1}{\left(1 - r^2\right) 2 l s_x} \left[r - A \right] \frac{P - E\tilde{y}}{s_y} ,$$

(26) $$b^{*} = 1 + \frac{1}{\left(1 - r^2\right) 2 l s_y} \left[r - A^{-1} \right] \frac{x_f - E\tilde{x}}{s_x} .$$

After some technical manipulations, it follows that

(27) $$\frac{da^{*}}{dr} = \frac{P - E\tilde{y}}{2 l s_x s_y} \frac{2 r}{\left(1 - r^2\right)^2} \left[\frac{1 + r^2}{2 r} - A \right],$$

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[3] Frechette (2000, equations (13) and (14)) derives similar equations in a slightly different model.

(28) $$\frac{db^*}{dr} = \frac{x_f - E\tilde{x}}{2 l s_x s_y} \frac{2r}{\left(1 - r^2\right)^2} \left[\frac{1 + r^2}{2r} - A^{-1} \right].$$

Observe that $f(r) = \left(1 + r^2\right)/2r$ is a positive and decreasing function that is higher than one for all $r \in (0,1)$. Interpreting these two equations yields the following proposition.

Proposition 3.

Define $A \equiv \dfrac{x_f - E\tilde{x}}{s_x} \Big/ \dfrac{P - E\tilde{y}}{s_y}$ *and* $f(r) = \left(1 + r^2\right)/2r$ *, and suppose* $b^* > 0$.

(i) *A marginal increase in the degree of correlation* r *leads to an increase (decrease) of the optimal futures hedge* a^* *if an only if* $A < (>) f(r)$.

(ii) *When the futures market exhibits contango,* $x_f > E\tilde{x}$ *, a marginal increase in* r *leads to an increase (decrease) in the coinsurance level* b^* *if and only if* $A^{-1} < (>) f(r)$. *When the futures market exhibits normal backwardation,* $x_f < E\tilde{x}$ *, a marginal increase in* r *leads to a decrease in the coinsurance level* b^* .

If the futures market exhibits normal backwardation, then the producer responds to a marginal increase in r by increasing his futures hedge a^* and decreasing his coinsurance level b^*; the two hedging contracts are substitutes as the degree of correlation changes. When the futures market exhibits contango, the parameter A is positive and it may be either higher or lower than one. If $0 < A \leq 1$, $da^*/dr > 0$ while db^*/dr can be either positive or negative. If $A > 1$, $db^*/dr > 0$ while da^*/dr can be either positive or negative. Hence, the two hedging contracts can be either substitutes or complements as r changes.

Consider now that the futures market is unbiased, $x_f = E\tilde{x}$. From equations (22) and (23), the interior solutions are given by

(29) $$a^* = \left(P - E\tilde{y}\right) \frac{r}{\left(1 - r^2\right) 2 l s_x s_y} ,$$

(30) $$b^* = 1 - \left(P - E\tilde{y}\right) \frac{1}{\left(1 - r^2\right) 2 l s_y^2} .$$

It is easy to show that a^* is increasing and convex in r while b^*, with $b^* \in (0,1)$, is decreasing and concave in r. These two hedging instruments are thus substitutes as r varies.

Because, the coinsurance contract is sold at an unfair price, $P > E\tilde{y}$, the optimal coinsurance level is always lower than one, as shown in equation (30). However, the non-negativity constraint on the coinsurance level may be binding if the insurance policy is too costly. If $(P - E\tilde{y}) \geq 2 l s_y^2$, the producer does not purchase coinsurance, whatever the degree of correlation. If $(P - E\tilde{y}) < 2 l s_y^2$, he will purchase individual coinsurance if and only if the degree of correlation is not too high: $r < \hat{r}$ with $\hat{r} \equiv \sqrt{1 - (P - E\tilde{y}) / 2 l s_y^2}$. This discussion is summarized in Proposition 3 and Figure 2.

Proposition 3.

Suppose the futures market is unbiased and define, when it exists, $\hat{r} \equiv \sqrt{1 - (P - E\tilde{y}) / 2 l s_y^2}$.

(i) *If* $(P - E\tilde{y}) \geq 2 l s_y^2$*, the producer does not purchase individual coinsurance.*

(ii) *Suppose* $(P - E\tilde{y}) < 2 l s_y^2$*. As long as* $r < \hat{r}$*, the optimal futures hedge* a^* *is positive, increasing and convex in the degree of correlation* r*, and the optimal coinsurance level* b^* *is positive, decreasing and concave in* r*. When* $r \geq \hat{r}$*, the producer does not buy individual coinsurance,* $b^* = 0$*, and the optimal futures hedge ratio is linear in the degree of correlation (with a slope equal to* s_y / s_x.

[INSERT FIGURE 2 HERE]

4. An Illustration of the Optimal Mean-Variance Hedging Strategy

The optimal hedging strategy characterized in the linear mean-variance model is illustrated for French wheat farmers. The individual and area yields are represented by the positive random variables $\tilde{Y}$ and $\tilde{X}$, respectively. The producer can purchase individual yield crop insurance (IYCI) contracts that guarantees an individual crop yield Y_f, with $Y_f < E\tilde{Y}$, and/or area yield crop insurance (AYCI) contracts that guarantee the expected area yield $E\tilde{X}$. The final gross revenue per hectare using individual yield crop insurance and/or area yield crop insurance is given by

(31) $$w = Y + a\left(E\widetilde{X} - X\right) + b\left(Y_f - Y\right),$$

where a is the AYCI quantity sold (if positive) and b is the IYCI quantity sold, with $b \in [0,1]$. Define $Y = y_{\max} - y$ and $X = x_{\max} - x$, where $x_{\max}$ is the highest realization of the random variable $\widetilde{X}$. The optimal hedge ratios, for interior solutions, are easily derived from equations (28) and (29)

(32) $$a^* = \left(E\widetilde{Y} - Y_f\right)\frac{r}{\left(1 - r^2\right)2 l s_x s_y}$$

(33) $$b^* = 1 - \left(E\widetilde{Y} - Y_f\right)\frac{1}{\left(1 - r^2\right)2 l s_y^2}.$$

If the individual crop insurance policy is only available, it is well-known that to the optimal hedge is $b^{**} = 1 - \left(E\widetilde{Y} - Y_f\right)/2 l s_y^2$. One can easily show that the difference between the optimal utility level when the two contracts are optimally traded, $V^* = Ew^* - l \operatorname{var} w^*$ where w^* is the producer's wealth expressed in (31) evaluated at (a^*, b^*) and the utility level when the individual insurance policy is only available, $V^{**} = Ew^{**} - l \operatorname{var} w^{**}$ where w^{**} is the producer's wealth evaluated at $(0, b^{**})$ is

(34) $$\Delta V \equiv V^* - V^{**} = \left(\overline{Y} - Y_f\right)^2 \frac{1}{4 l s_y^2}\left[\frac{1}{1 - r^2} - 1\right] > 0.$$

Observe that the higher the individual yield variance and/or the lower the degree of correlation and/or the lower the deviation of the yield guarantee form the expected yield, the lower ΔV, i.e., the lower the efficiency gain provided by the index-based contract.

We first consider a representative farm in Marne. This administrative subdivision, located in the northern part of France, is characterized by highly fertile soils and temperate climates. Agriculture is dominated by cereals and oilseeds produced using intensive cropping technology. The index of the area yield crop insurance contract is the average national yield. The associated local yield data and the national yield data over the period 1970-2001 were obtained from the French Farm Accountancy Data Network.[4] Local wheat yields have a mean equal to 82 quintals/hectare and a standard deviation equal to 6.02 quintals/hectare.[5] The mean and standard deviation of national wheat yields are equal to 70 quintal/hectare and 4.98 quintal/hectare, respectively. The degree of correlation between wheat yields in Marne and wheat yields in France is estimated at 0.7. This high level of correlation is due to the fact that

[4] This yield data was adjusted for secular trends to reflect 2001 production levels.
[5] One quintal per hectare corresponds to 1.49 bushels per acre.

this subdivision has a significant weight in the average national yields and that yield variations are mainly due to climatic factors that affect several departments are the same time and thus the average national yields.

Figure 3 shows the efficient frontier of the individual yield crop insurance policy (IYCY), with a yield guarantee $Y_f = 0.9 \times E\tilde{Y}$, and that of the optimal combination of the IYCI contract and the area yield crop insurance policy (IYCI+AYCI). The IYCI curve is increasing with the variance. When the variance decreases, the loss in expected yield is due to the existence of transaction costs. The (IYCI+AYCI) curve is an horizontal line as long as the yield variance is higher than 18.48 and then it decreases as the variance decreases. This horizontal line is generated by an hedging strategy requiring a short position in the unbiased AYCI contract only. The existence of transaction costs on the IYCI policy induces a moderately risk averse producer to sell only AYCI contracts. Because these contracts are unbiased and not subject to transaction costs, they allow him to reduce the yield variance without reducing the expected yield. However, the imperfect correlation between the individual yield and the index generates basis risk. The producer can reduce this basis risk by selling actuarially unfair IYCI contracts. The higher the level of risk aversion, the higher the IYCI hedge ratio and the lower the AYCI hedge ratio.

[INSERT FIGURE 3 HERE]

As expected, the AYCI contract is market enhancing; efficiently combined with the IYCI policy, it allows the producer to increase the expected yield for the same level of yield variance or, equivalently, to reduce the yield variance for a given expected yield. For example, for a yield variance equal to 18, trading AYCI contracts in addition to IYCI contracts allows the producer to increase the expected yield from 79.58 to 81.89 quintals/hectare, i.e., a gain in expected yield equal to 2.31 quintals/hectare. Conversely, for an expected yield of 80 quintals/hectare, the optimal hedging strategy using the two contracts allows the producer to divide the yield variance by almost two, from 20.75 to 10.57 (quintals/hectare)^2.

The efficient frontiers are drawn in Figure 4 for a representative farm in Eure-et-Loir where the expectation and the standard deviation of the individual yields are 72 quintals/hectare and 11.6 quintals/hectare, respectively, the degree of correlation with the national yields is 0.42 and the individual yield guarantee is $Y_f = 0.9 \times E\tilde{Y}$. As expressed in equation (34), a higher individual yield variance and a lower degree of correlation reduce the efficiency gain generates by the AYCI policy. For example, for a yield variance equal to 18, trading AYCI contracts in addition to IYCI contracts allows the producer to increase the expected yield from 67.44 to 67.70 quintals/hectare, i.e., a gain in expected yield equal to 0.23 quintal/hectare.

[INSERT FIGURE 4 HERE]

5. Conclusion

The optimal combination of individual index-based insurance contracts is examined in two conceptual frameworks.

Within the expected utility model, the imperfect coverage provided by the index-based policy is captured with a contract displaying a lump-sum payoff whenever a loss occurs. We show that the optimal insurance policy displays full (marginal) coverage above a deductible and that the optimal level of deductibility is positive when the contract is sold at an unfair price. Therefore, the presence of a fixed-reimbursement insurance policy does not affect this well-known result in the literature of insurance economics. The optimal lump-sum payment is shown to be positive and lower than the optimal individual insurance deductible. In addition, the prudent policyholder will choose a fixed payoff higher than the expected value of the minimum function between the optimal deductible and the random individual loss.. Under CARA, an increase in the probability of loss will increase (decrease) both the insurance deductible and the lump-sum payment if and only if, the level of deductibility, when it is the only hedging decision, increases (decreases) with the probability of loss.

Within the linear mean-variance model, the optimal futures hedge and the optimal coinsurance level are explicitly characterized. When the futures market is unbiased, the optimal future hedge is increasing and convex in the degree of correlation between the individual loss and the index. The coinsurance level, when positive, is a decreasing and concave function of the degree of correlation. The role of an index-based contract as a market enhancing instrument is illustrated using two representative wheat farms located in France. The efficient frontiers show the efficiency gain provided by an optimal combination of the two hedging instruments. However, this gain decreases as the individual yield variability increases and/or the degree of correlation decreases.

These two models allows us to investigate the optimal combination of an unfair individual insurance policy and a fair index-based contract in the management of individual risks. In insurance language, they bring an insight into the tradeoff between transaction costs and basis risk.

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Appendix

Sign of V_{Kq}

Because $u'' < 0$, the first-order condition (9) can be rewritten as

(A1) $$-E\left[\tilde{y} - \max\left(\tilde{y} - D, 0\right)\right] + K - \Psi(\overline{w}, \tilde{z}) = 0.$$

Differentiating (A1) with respect to the probability of occurring a loss q yields

(A2) $$\text{sign}\left(V_{Kq}\right) = \text{sign}\left(-K\Psi'(\overline{w}, \tilde{z})\right).$$

V_{Kq} is thus positive (null, negative) if and only if the producer's utility function exhibits DAP (CARA, IAP).

Sign of V_{Dq}

Differentiating the first-order condition (6) with respect to q and rearranging the terms yields

(A3)

$$V_{Dq} \equiv \left(P_q' + K\right)\left[-qP_D'\int_0^D A(w_3)u'(w_3) - q\left(1 + P_D'\right)A(w_2)u'(w_2)\left(1 - F(D)\right) - P_D'\left(1 - q\right)A(w_1)u'(w_1)\right]$$
$$-\left(1 - F(D)\right)mu'(w_1) + \left(1 - F(D)\right)u'(w_2)$$

where $P_D' = -mq\left(1 - F(D)\right)$, $P_q' \equiv \partial P/\partial q = m\int_D^{y_{\max}}(y - D)dF(y)$ and $A \equiv -u''/u'$ is the index of absolute risk aversion. The first right-hand side (RHS) term in (A3) is the wealth effect. It is negative under DARA and null under CARA. The second RHS term in (A3) is the risk effect; it is negative. The third RHS term in (A2) is the premium effect; it is positive. Because $w_1 > w_2$ and under risk aversion, we have $u'(w_2) > u'(w_1)$. Because $m > 0$, the sum of the premium effect and the risk effect is positive (negative) if the loading factor is "sufficiently" low (high).

Sign of V_{DK}

Differentiating the first-order condition (6) with respect to K and rearranging the terms gives

(A4) $$V_{DK} \equiv \left(1 - q\right)\left[+qP_D'\int_0^D A(w_3)u'(w_3) + q\left(1 + P_D'\right)A(w_2)u'(w_2)\left(1 - F(D)\right) - qP_D'A(w_1)u'(w_1)\right]$$

Because $P_D' < 0$, this implies that

(A5) $$V_{DK} > \left(1 - q\right)\left[qP_D'\int_0^D A(w_3)u'(w_3) + q\left(1 + P_D'\right)A(w_2)u'(w_2)\left(1 - F(D)\right)\right].$$

For all $y < D$, we have $w_3 > w_2$ and thus $A(w_3) < (=) A(w_2)$ under DARA (CARA). This implies that

(A6) $$V_{DK} > (1-q)A(w_2)\left[+qP_D'\int_0^D u'(w_3) + q(1+P_D')u'(w_2)(1-F(D))\right].$$

From the first-order condition (6), the RHS term into brackets is equal to $-P_D'(1-q)u'(w_1) > 0$. Consequently $V_{DK} > 0$ under DARA or CARA.

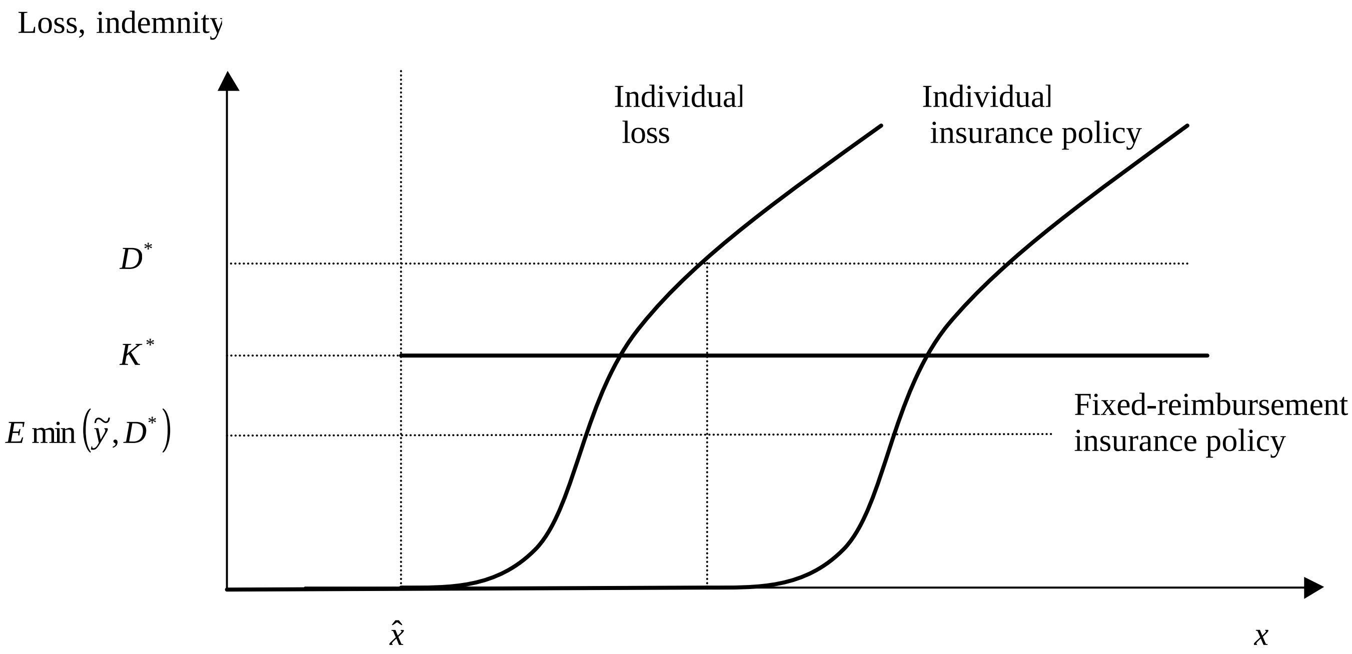

Figure 1. Optimal individual insurance deductible and optimal lump-sum payment, for a prudent policyholder.

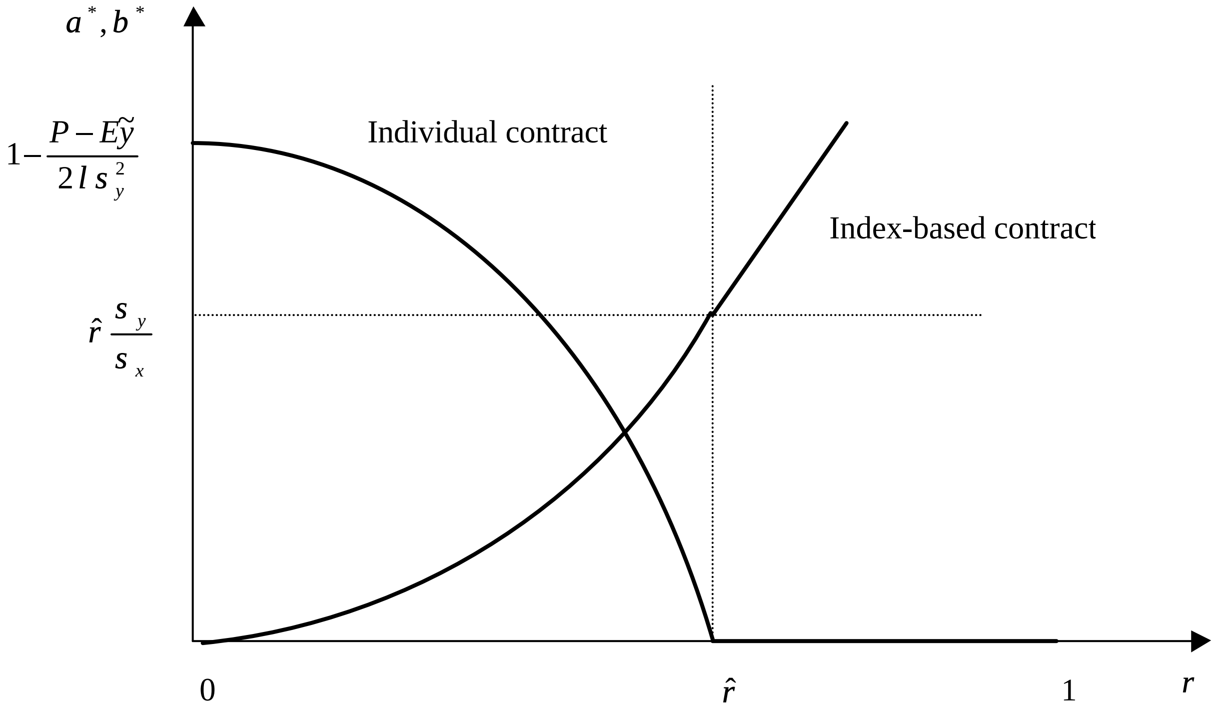

Figure 2. Optimal hedging decisions with unfair individual insurance policy and unbiased index-based contract.

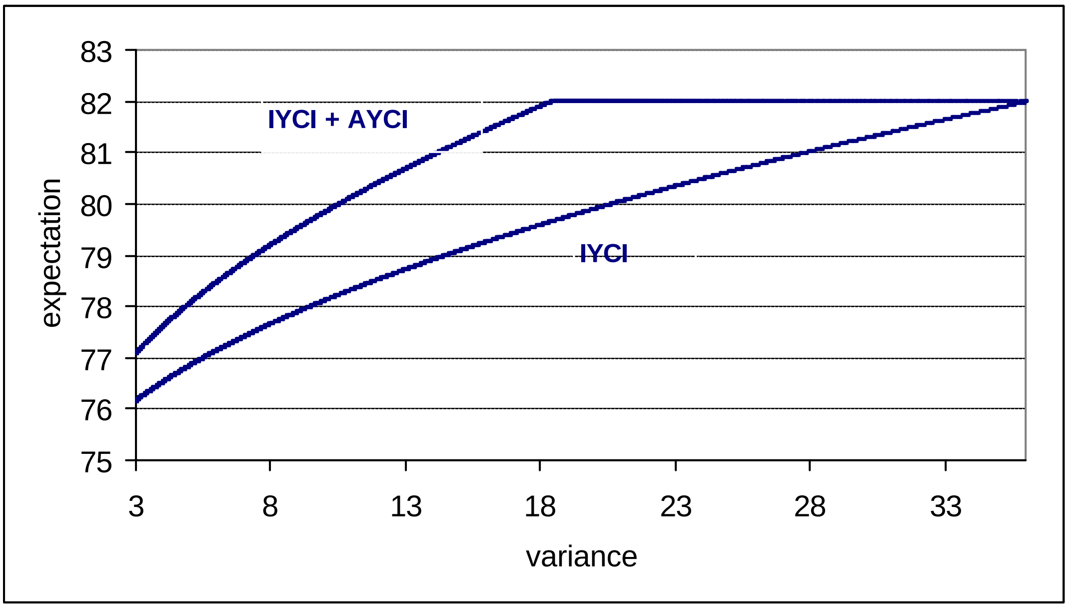

Figure 3. Efficient frontier of the individual yield crop insurance policy (IYCI) and of the combination of individual and area yield crop insurance contracts (IYCI+AYCI) for a representative wheat farm in Marne (France). Expectation is in quintals/hectare and variance is in (quintals/hectare)^2.

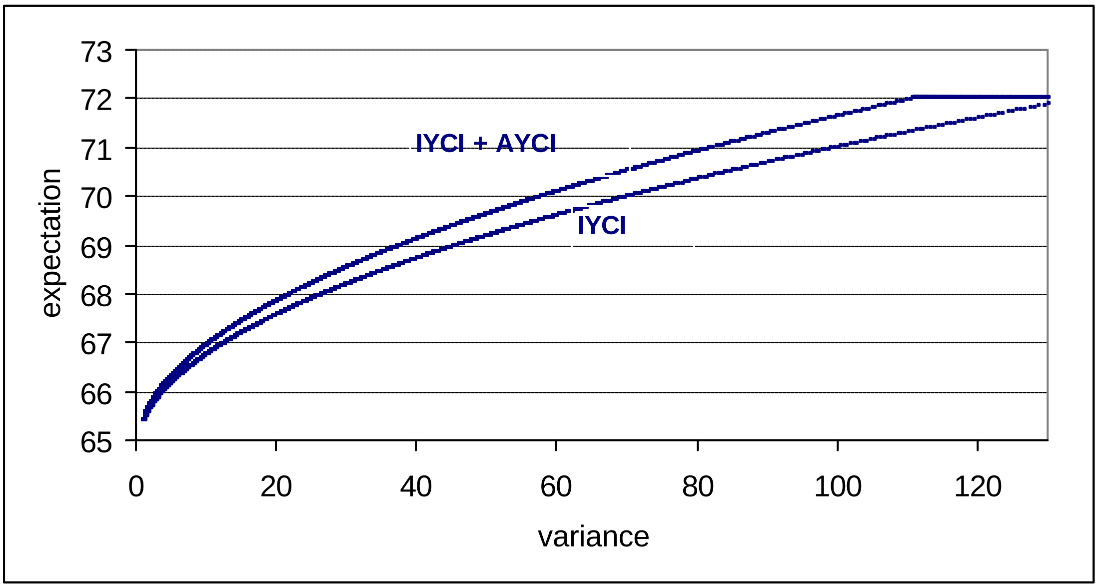

Figure 4. Efficient frontier of the individual yield crop insurance policy (IYCI) and of the combination of individual and area yield crop insurance contracts (IYCI+AYCI) for a representative wheat farm in Eure-et-Loir (France). Expectation is in quintals/hectare and variance is in (quintals/hectare)^2.