A Discrete Space Urban Model with Environmental Amenities

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Abstract

This paper analyzes the effects of providing environmental amenities associated with open space in a discrete space urban model. The discrete space model assumes distinct neighborhoods in which developable land is homogeneous within a neighborhood but heterogeneous across neighborhoods. We solve for equilibrium allocation of development, prices and welfare given a pattern of open space provision. We also analyze the optimal provision of open space across neighborhoods. In equilibrium, housing density and price in a neighborhood is increasing in the amount of open space provided in that neighborhood. Whether housing density and prices in other neighborhoods increases or decreases depends on whether the push from reduced availability of developable land in the neighborhood with increased open space, or the pull of the local amenity value in that neighborhood, is stronger.

1. Introduction

Metropolitan areas around the country are experiencing rapid growth and large-scale conversion of undeveloped to developed land. Residents of many of these metropolitan areas are concerned about rapid growth, urban sprawl and the resulting loss of open space and environmental amenities. Some local and regional governments, as well as private land trusts, have instituted policies to acquire land or conservation easements for the express purpose of preserving some undeveloped land within or at the fringe of the metropolitan area. For example, in the November 2001 election, 86 of 115 state and local open space spending measures were passed by voters, providing more than \$1.2 billion in public funds for open space protection efforts (Hollis and Fulton 2002).

There are at least two important effects of conserving open space in a metropolitan area. First, open space designation restricts the supply of land available for development, which other things equal, tends to increase the value of remaining developable land and increase development density on that land. Second, open space often generates local amenities that make nearby areas more attractive, thereby changing the spatial pattern of demand for development. Open space designation may result in shifts of demand between different locales within a given metropolitan area or it may shift overall demand by encouraging immigration to (or emigration from) the metropolitan area.

In this paper, we analyze the effect of designating open space that provides environmental amenities on the spatial pattern of development, the density of development, property values and welfare in a discrete space urban economics model. The discrete space urban model assumes that developable land is homogenous within a neighborhood but heterogeneous across different neighborhoods. Neighborhoods can differ with respect to the area available for development, the amount of open space, and access to employment. Provision of open space reduces the area available for development and increases environmental amenities. The government provides open space by purchasing land with money raised from property taxes. The government is required to have a balanced budget. Households maximize their utility by choosing where to live (which neighborhood), and how much of their disposable income after taxes to spend on housing (area) versus all other goods. We solve for the equilibrium numbers of households living in each neighborhood, the amount of housing consumed in each neighborhood, and property values in each neighborhood, as a function of open space provision. We then analyze the effect of different patterns of open space provision across different neighborhoods and show the pattern of open space provision that maximizes household welfare.

Modeling the urban area as a discrete set of neighborhoods allows us to more fully develop the notion of local environmental amenities and include multiple environmental amenities in an analytical framework. This approach contrasts with most of the urban economics literature, which has utilized a continuous space approach. The continuous space urban economics models build from the monocentric city model developed by Alonzo (1964), Mills (1967, 1972), and Muth (1969). Other than distance to the central business district where all employment is located, locations are identical (i.e., development takes place on a featureless plane). Locations close to the central business district are more desirable because of lower commuting cost. In equilibrium, property values decline as distance from the center business district increases.

The standard model has been elaborated in a number of ways (See Anas et al. (1998) and Huriot and Thisse (2000) for overviews of recent developments in the theoretical urban economics literature). The most important extension of the standard urban economics model, in the context of the present paper, involves inclusion of spatial amenities (or disamenities). While there is a substantial empirical literature that estimates the effect of environmental amenities on nearby property values (reviewed below), there are few models that analyze the spatial pattern of environmental amenities on equilibrium property values across an urban area. Polinsky and Shavell (1976) and Bruecker et al. (1999) include an environmental amenity characterized solely by its distance to the central business district. Several other papers have analyzed similar models for the effect of locating a facility that generates negative externalities on equilibrium property values in an urban area (Nelson 1979, Griffin 1991). Mills (1981), Nelson (1985), and Marshall and Homans (2001) analyze the effects of greenbelt policies that form a ring of open space around a city. Wu (2001) and Wu and Plantinga (2001) analyze more general spatial models of environmental amenities that include the location, shape and size of the amenity. In another related study, McMillan and McDonald (1993) analyze the effect of zoning on land values.

There is a large empirical literature that estimates the effect of amenities or disamenities on nearby property values. The empirical work does not attempt to describe the over-all impact of environmental amenities on the equilibrium pattern of development or land prices. This literature is useful, though, for understanding the local premiums attached to various environmental amenities. The hedonic property price model has been applied to the value of living near open space (Vaughn 1981, Acharya and Bennett 2001, Shultz and King 2001), urban parks and recreation areas (Kitchen and Hendon 1967, Weicher and Zeibst 1973, Hammer et al. 1974), and greenbelt land (Correll et al. 1978). These studies find a positive value to living near parks and open space. It is important to note that hedonic property price models typically look at the effect of marginal changes in environmental quality holding development patterns and prices constant and do not attempt to solve for the value of non-marginal changes that may result in changes in property values, other prices, or development patterns. Several studies have also used contingent valuation to estimate the value of open space (Parks and Schorr (1997), Breffle et al. (1998), Vossler et al. (2003)).

This paper is organized as follows. In the next section of the paper we lay out the basic discrete space urban model with open space. We describe the model and define equilibrium. We define the planner's problem that solves for the optimal amount and location of open space and property tax to maximize household welfare. Then, we consider a case with homothetic preferences and solve analytically for equilibrium. Using numerical simulation we determine the optimum allocation and property tax and solve for the effect of changes in parameters on the equilibrium choice variables. For example, by changing the size of the area available for development, the distance to the employment center, the amenities, as well as the preferences of an amenity in a neighborhood of residence to amenities in other neighborhoods we determine the direction and magnitude in corresponding changes in the prices of housing, the amount of housing (area) consumed, and the distribution of the households across the neighborhoods which vary in their open space and distance to the employment center.

One desirable feature of the discrete urban model is that the model is analytically solvable for the J - neighborhood equilibrium allocation and prices. The model provides a theoretical framework for analyzing environmental amenities across space. Also the model can be extended to include income differentiation, transportation networks, other amenities, multiple local governments, and other important features.

2. A Discrete Space Urban Model

2.1. The Model

Consider a city that consists of J neighborhoods. The area of each neighborhood, $j = 1, 2, \ldots, J$, is denoted by $A_j \in \mathbb{R}_+$ and $A = \{A_1, A_2, \ldots, A_J\}$ is a vector of the areas of all the neighborhoods in the city. The land in each neighborhood is used for residential housing and open space both measured in terms of the area they occupy. Open space in neighborhood j is denoted by a_j, $0 \le a_j \le A_j$. Let $a = \{a_1, a_2, \ldots, a_J\}$ represent the vector of open space areas across all of the neighborhoods in the city. The area available for housing in neighborhood j is $A_j - a_j$.

There are I identical households that reside in the city. Each household, $i = 1, 2, \ldots, I$, chooses which neighborhood to live in. Let I_j be the number of households living in neighborhood j. Households also choose how much to consume of housing and a non-housing consumption good. Let h_j^i be the amount (area) of housing consumed by household i living in neighborhood j. Let c_j^i be the amount of the non-housing consumption good consumed by household i living in neighborhood j. The price of housing in neighborhood j is p_j. The price of the non-housing consumption good is p_c. Each household receives non-property income of y. In addition, each household is endowed with $\sum_{j=1}^{J} \frac{A_j}{I}$ units of land, from which the household earns rent.

Open space generates environmental amenities. We assume that the amenity value of open space is highest in the neighborhood in which the open space is located. In general, though, environmental amenities for a household living in neighborhood j can be a function of entire vector of open space (a).

Households commute to the central business district (CBD) to work. Let d_j represent the commuting distance between neighborhood j and the CBD.

There is a municipal government that collects a property tax τ on the consumption of housing. The government uses tax revenue to provide a vector of open space, a, to households. The government is assumed to be required to balance its budget.

Household preferences are defined over consumption of housing (h_j^i), consumption of the non-housing consumption good (c_j^i), open space amenities (a), and the commuting distance (d_j). Households maximize their utility by choosing which neighborhood to live in and how much of their budget to allocate to housing and non-housing consumption, subject to their budget constraint, given housing prices, the tax rate and the pattern of open space provision.

2.2. Household Equilibrium

The city is a small open economy in which national markets set the price of a consumption good p_c and the wage rate determining y. However, the housing market is city specific and the housing prices p_j for all neighborhoods $j = 1, ..., J$ are determined endogenously.

Definition 1 *Given τ, a, p_c, and y, allocation $(c_j^i, h_j^i)_{i=1}^{I}$, $(I_j)_{j=1}^{J}$ and prices $(p_j)_{j=1}^{J}$ constitute an equilibrium for the above economy if:*

1) *Households Maximize Utility*

$\cdot$ *given prices $p_c, (p_j)_{j=1}^{J}$, for each $i = 1, 2, ..., I$, (c_j^i, h_j^i) solves*

$$V^i = \max\{V_j^i\}_{j\in J} = \max\{\max_{c_j^i, h_j^i \geq 0} u(c_j^i, h_j^i, d_j, a)\}_{j=1}^{J}$$

$$\textit{subject to} \quad p_c c_j^i + (1+\tau)p_j h_j^i \leq y + \sum_{j=1}^{J} p_j \frac{A_j}{I} \qquad \textit{for all} \quad j = 1, ..., J$$

2) *Resource Feasibility*

$$\sum_{j=1}^{J} I_j = I \tag{1}$$

$$\sum_{i=1}^{I_j} h_j^i + a_j = A_j \quad for\ all\ j = 1, ..., J \tag{2}$$

3) *Balanced Government Budget*

$$\sum_{j=1}^{J} p_j a_j = \sum_{j=1}^{J} \tau p_j \left[\sum_{i=1}^{I_j} h_j^i \right] \tag{3}$$

Households maximize preferences by first choosing an optimal consumption bundle for each neighborhood j and then choosing the maximum indirect utility function among all neighborhoods $j = 1, ..., J$. Households choose to live in a neighborhood with the highest indirect utility and they consume a bundle of goods corresponding to that neighborhood. In equilibrium, a household's indirect utility from residing in any neighborhood with positive population is equal to the indirect utility from residing in any other neighborhood, because otherwise that household would have an incentive to move to a neighborhood with a higher utility. A households' objective function is subject to a budget constraint that simply states that what they spend on a non-housing good, housing, and property tax does not exceed their income from labor and land rentals.

Resource feasibility constraint 1 says that in equilibrium, the sum of the number of households residing in each neighborhood is equal to the total number of households in the economy. Resource feasibility constraint 2 says that in equilibrium, the total area in each neighborhood is equal to the total housing area plus the area of an open space in that neighborhood. Constraint 3 says that the government has to balance its budget, that is its spending on open space equals the tax revenue that it collects from the households.

Proposition 1 *Suppose that $u(.)$ is a continuous utility function representing a monotone, strictly convex preference relation $\precsim$ defined on the consumption set $X = \mathbb{R}_+^2$, where consumption and housing goods $\left(c_j^i, h_j^i\right) \in X$ are normal goods. Further suppose that the utility function is strictly increasing in open space and that the marginal utility of open space in the neighborhood of residence is greater than or equal to marginal utility of open space in any other neighborhood. Then an increase in open space in neighborhood j, a_j, will result in:*

1 *an increase in the after tax equilibrium housing price in neighborhood j, $(1+\tau)p_j(a,\tau)$,*

2 *an increase in the household density in developed area in neighborhood j, $\frac{I_j(a,\tau)}{A_j - a_j}$.*

Proof. By assumption, housing is a normal good. Therefore, by the Slutsky equation, h^i_j is decreasing in its (after-tax) price, $(1+\tau)p_j$, for $j = 1, 2, \ldots J$. To prove the proposition then, it is sufficient to show that h^i_j, which is directly related to housing density and inversely related to price, must decline with an increase in open space in neighborhood j.

Suppose $(\bar{c}^i_j, \bar{h}^i_j)^I_{i=1}$, $(\bar{I}_j)^J_{j=1}$ and prices $(\bar{p}_j)^J_{j=1}$constitute an equilibrium given open space vector a. Consider some neighborhood j. All households in this economy are identical. Given strictly convex preferences this implies that $\bar{h}^i_j = \bar{h}_j$ for all households i living in neighborhood j. Then, the sum of housing consumption in neighborhood j is $\bar{I}_j\bar{h}_j$. Suppose an environmental amenity a_j in that neighborhood j increases to $a_j + \epsilon$. Then, by the feasibility constraint 2, $\bar{I}_j\bar{h}_j$ has to decrease by ϵ because the total area of that neighborhood remains the same. This implies that either the amount of housing consumed by each household in that neighborhood h_j decreases and/or the total number of households who choose to live in that neighborhood I_j decreases.

Suppose that $\bar{h}_j$ does not decline but either remains constant or increases with an increase in open space in neighborhood j. This must mean then that $\bar{I}_j$ falls. From resource constraint 1, this means the number of households in at least one other neighborhood, k, must rise. Since developable land in neighborhood k is constant, an increase in I_k means that per household amount of land, h_k, must fall. This occurs if and only if after tax housing price in neighborhood k rises.

Recall that in equilibrium utility function for each household evaluated at the optimal choice must be equal across all neighborhood, that is $u\left(\bar{c}_j, \bar{h}_j, a, d_j\right) = u\left(\bar{c}_k, \bar{h}_k, a, d_k\right)$ for all $j, k = 1, ..., J$, where $a = \{a_{1,}a_2, ..., a_J\}$ is a vector of open space in all neighborhoods. Housing consumption in neighborhood j increased by $\triangle\, h_j$ and its price $(1+\tau)p_j$ decreased, housing consumption in neighborhood k decreased by $\triangle\, h_k$ and its price $(1+\tau)p_k$ increased, exogenous price of consumption remained constant, and open space in neighborhood j increased by ϵ. Also, by assumption the utility function is strictly increasing in open space and marginal utility of open space in own neighborhood j is greater than or equal to marginal utility of open space in any other neighborhood k. Thus, we know that $u\left(\bar{c}_j, \bar{h}_j + \triangle\, h_j, a, d_j\right) > u\left(\bar{c}_k, \bar{h}_k - \triangle\, h_k, a, d_k\right)$, which is a contradiction.

Therefore, $\bar{h}_j = h^i_j$ must decline with an increase in open space in neighborhood j. ∎

Proposition 1 shows that the housing price and density increase in the neighborhood with an increase in open space. However the movement of households to and from the neighborhood with an increase open space is indeterminate Households can either move to that neighborhood because they enjoy open space amenities or they can move out because, by proposition 1, housing prices and household density in that neighborhood increase. Similarly, the response of housing prices and density in other neighborhoods is indeterminate We will address this question in sections 3 and 4 using an analytic solution for a specific case and simulation results.

2.3. Planner's Problem

The government collects a property tax τ on housing value and uses the tax revenues to finance the provision of open space amenities $(a_1, a_2, ..., a_J)$ to households. With a benevolent government, a planner will select the optimum amount and location of open space in the city and the property tax to maximize households' utility. The planner's optimal choice is constrained by the area availability and the government's budget constraint. Since households are identical and utility is equal in all neighborhoods with positive population in equilibrium, the planner's problem is to maximize utility for a representative household in a representative neighborhood.

Definition 2 *Allocation $\{a_1, a_2..., a_J, \tau\}$ is optimal if it solves the following planner's problem:*

$$\max_{a_1,a_2...,a_J,\tau} \sum_{j=1}^{J} I_j^* u(c_j^{i*}, h_j^{i*}, d_j, a_1, a_2, ..., a_J)$$

$$\text{subject to} \quad \sum_{j=1}^{J} p_j^* a_j = \sum_{j=1}^{J} \tau p_j^* \left(\sum_{i=1}^{I_j} h_j^{i*} \right)$$

$$0 \leq a_j \leq A_j \quad \text{for all } j = 1, ..., J$$

$$0 \leq \tau \leq 1$$

Where $\{p_j^*\}_{j=1}^{J}$ and $\{c_j^{i*}, h_j^{i*}, I_j^*\}_{j=1}^{J}$ are equilibrium prices and quantities respectively, and $u(c_j^{i*}, h_j^{i*}, d_j, a_1, a_2..., a_J) = V_j^*$ is an indirect utility function in neighborhood j.

Solution of the planner's problem requires imposing more structure, as described in the next section.

3. Analytic Solution

In this section we further study the model introduced in the previous section. By assuming households' utility function we solve for equilibrium allocation and prices, optimal planner's allocation and do comparative statics analysis.

3.1. Equilibrium

Let a household's utility function take the following form:

$$u(c_j, h_j) = \ln c_j + \ln h_j + \theta_{j1} a_1 + \theta_{j2} a_2 + ... + \theta_{jJ} a_J - d_j$$

The parameter $\theta_{jz} \in \mathbb{R}$ for all $z = 1, 2, ..., J$ shows how much a household residing in neighborhood j values open space in neighborhood z. We assume $\theta_{jj} \geq \theta_{jz} \geq 0$ for $z \neq j$, that is local amenity value θ_{jj} is always greater or equal to regional amenity value θ_{jz}. We normalize the price of a non-housing good to one, ($p_c = 1$). Each household chooses which neighborhood to live in, how much of housing and a non-housing good to consume subject to the budget constraint, given the prices, open space in each neighborhood, the property tax, and the distance from each neighborhood to the CBD. Then a household i's problem for each neighborhood j is:

$$V_j^i = \max_{c_j^i, h_j^i \geq 0} \ln c_j^i + \ln h_j^i + \theta_{j1} a_1 + \theta_{j2} a_2 + ... + \theta_{jJ} a_J - d_j$$

$$\text{subject to } c_j^i + (1+\tau) p_j h_j^i \leq y + \sum_{z=1}^{J} p_z \frac{A_z}{I} \text{ for all } j = 1, ..., J.$$

The first order necessary conditions for an interior solution are:

$\frac{\partial L}{\partial c_j^i} = \frac{1}{c_j^i} - \lambda_j^i = 0$ for all j

$\frac{\partial L}{\partial h_j^i} = \frac{1}{h_j^i} - \lambda_j^i (1+\tau) p_j = 0$ for all j

$\frac{\partial L}{\partial \lambda_j^i} = y + \sum_{z=1}^{J} p_z \frac{A_z}{I} - c_j^i - (1+\tau) p_j h_j^i = 0$ for all j,

where λ_j^i is the Lagrange multiplier on the budget constraint for household i living in neighborhood j. Note that we will obtain an interior solution because preferences satisfy the Inada conditions. The sufficient conditions are also satisfied because of concavity of the utility function (and a linear budget constraint). Solving the first order conditions for the demand functions in prices yields:

$$c_j^{i*}(p, y) = \frac{y + \sum_{z=1}^{J} p_z \frac{A_z}{I}}{2} \quad \textit{for all } j \tag{4}$$

$$h_j^{i*}(p, y) = \frac{y + \sum_{z=1}^{J} p_z \frac{A_z}{I}}{2(1+\tau) p_j} \quad \textit{for all } j \tag{5}$$

where $p = \{p_1, p_2, ..., p_J\}$ is a vector of housing prices by neighborhood.

Recall that in equilibrium, a household's indirect utility from residing in any neighborhood is equal to the indirect utility from residing in any other neighborhood, because otherwise that household would have an incentive to move to a neighborhood with a higher utility, thus

$$V_1^i(p, y) = V_2^i(p, y) = ... = V_J^i(p, y) \quad \textit{for all } i. \tag{6}$$

Using the fact of equal indirect utilities across neighborhoods along with the household demand functions and the resource feasibility conditions $I = \sum_{j=1}^{J} I_j$ and $A_j = I_j h_j^{i*}(p, y) + a_j$

for all j, we can solve for an equilibrium. Equilibrium allocation and prices for all $j = 1, 2, ..., J$ are:

$$
\begin{aligned}
p_j^* &= I\ \Psi_j && (7)\\
h_j^{i*} &= \frac{y + \sum_{z=1}^{J} A_z \Psi_z}{2(1+\tau) I \Psi_j} \quad for\ all\ i = 1, ..., I && (8)\\
c_j^{i*} &= \frac{y + \sum_{z=1}^{J} A_z \Psi_z}{2} \quad for\ all\ i = 1, ..., I && (9)\\
I_j^* &= (A_j - a_j) \frac{2(1+\tau) I \Psi_j}{y + \sum_{z=1}^{J} A_z \Psi_z} && (10)
\end{aligned}
$$

where $\Psi_j(\theta_j, A_j, d_j, a, \tau) = \frac{y e^{k_j}}{\sum_{z=1}^{J} A'_z \exp(k_z)}$, $A'_j = 2(1+\tau)(A_j - a_j) - A_j$, and $k_j = \theta_{j1} a_1 + \theta_{j2} a_2 + ... + \theta_{jJ} a_J - d_j$.

3.2. *Planner's Problem*

Using the equilibrium allocation and prices specified in equations 7 through 10, an indirect utility function for a household i in neighborhood j becomes

$$V_j^{i*} = u(c_j^{i*}, h_j^{i*}, a_1, a_2 ..., a_J, d_j) = \ln c_j^{i*} + \ln h_j^{i*} + \theta_{j1} a_1 + \theta_{j2} a_2 + ... + \theta_{jJ} a_J - d_j \quad (11)$$

Recall that all households are identical and their indirect utility function is identical for all neighborhoods, that is $V_j^{i*} = V^*$ for all $j = 1, ..., J$ and $i = 1, ..., I$, then the equally weighted sum of all indirect utility functions is $\sum_{j=1}^{J} \sum_{i=1}^{I_j} V_j^{i*} = IV^*$, where I is a total number of households in the city. With that, define the planner's problem as follows:

$$
\begin{aligned}
&\max_{a_1, a_2 ..., a_J, \tau} V^* && (12)\\
&\text{subject to} \quad \sum_{j=1}^{J} p_j^* a_j = \sum_{j=1}^{J} \tau p_j^* I_j^* h_j^{i*} && (13)\\
&0 \le a_j \le A_j \quad \text{for all } j = 1, ..., J && (14)\\
&0 \le \tau \le 1 && (15)
\end{aligned}
$$

where $\{p_j^*\}_{j=1}^{J}$ and $\{c_j^{i*}, h_j^{i*}\}_{i=1}^{I}, \{I_j^*\}_{j=1}^{J}$ are the equilibrium prices and quantities respectively.

Solving the government's budget constraint 13 for the property tax τ as a function of open space $a_1, a_2, ..., a_J$ yields:

$$\tau = \frac{\sum_{z=1}^{J} a_z e^{k_z}}{\sum_{z=1}^{J} (A_z - a_z) e^{k_z}} \tag{16}$$

Using the property tax τ equation 16 in equilibrium equations 7 to 10 we get the following equilibrium allocations and prices for all $j = 1, 2, ..., J$:

$$p_j = \frac{Iye^{k_j}}{\sum_{z=1}^{J} A_z e^{k_z}} \tag{17}$$

$$h_j^i = \frac{\sum_{z=1}^{J} (A_z - a_z) e^{k_z}}{Ie^{k_j}} \quad for\ all\ i = 1, ..., I \tag{18}$$

$$c_j^i = y \quad for\ all\ i = 1, ..., I \tag{19}$$

$$I_j = (A_j - a_j) \frac{Ie^{k_j}}{\sum_{z=1}^{J} (A_z - a_z) e^{k_z}} \tag{20}$$

where $k_j = \theta_{j1} a_1 + \theta_{j2} a_2 + ... + \theta_{jJ} a_J - d_j$.

From here, we can determine open space effects on total prices and allocations. For example, total price in neighborhood j, which is $(1 + \tau) p_j = \frac{Iye^{k_j}}{\sum_{z=1}^{J} (A_z - a_z) e^{k_z}}$, is increasing in open space a_j in that neighborhood (by proposition 1):

$$\frac{\partial (1 + \tau) p_j}{\partial a_j} = \frac{Iye^{k_j} \left(e^{k_j} + \sum_{z=1}^{J} (\theta_{jj} - \theta_{zj}) (A_z - a_z) e^{k_z} \right)}{\left(\sum_{z=1}^{J} (A_z - a_z) e^{k_z} \right)^2} > 0 \tag{21}$$

provided the households prefer open space in their own neighborhood to open space in any other neighborhood.

Since the amount of housing consumed by households is the inverse of the total price, the amount of housing consumed by each household in neighborhood j decreases when open space in this neighborhood increases. Consumption of a non-housing good is constant and equal to y. Thus, income generated by non-property income is spent on the consumption good and income generated by land rentals is spent on housing.

The price effect in some other neighborhood k from an increase of an open space in neighborhood j depends on two effects. Increasing open space in a neighborhood makes it more attractive for households and creates a pull toward that neighborhood. At the same time, increasing open space in a neighborhood decreases the supply of land available for housing pushing households away from the neighborhood. If the pull from local amenities is sufficiently strong, other neighborhoods may become less attractive and prices can fall. On the other hand, if the decrease in supply of available land is the stronger effect, then other neighborhoods may witness an increase in demand and prices will increase. We will investigate these two effects in the next section by the means of simulation.

Proceeding to solving the planner's problem for the optimal open space allocation $(a_1, ..., a_J)$ using the property tax τ that we solved for using the planner's budget constraint (equation 16), we get $a_j = A_j - \frac{1}{\theta_{jj}}$ for all $j = 1, ..., J$ as an optimal allocation when households value an open space only locally in their neighborhood ($\theta_{jj} > 0, \theta_{jz} = 0$ for all $j \neq z$) and provided the property tax τ equation 16 satisfies $0 \leq \tau \leq 1$. When households value open space in all neighborhoods, solving for optimal open space allocation $(a_1, ..., a_J)$ reduces to solving the following system of first order conditions to the planner's problem again, provided that property tax τ equation 16 satisfies $0 \leq \tau \leq 1$:

$$\sum_{z=1}^{J} \theta_{z1} (A_z - a_z) \exp(k_z) = \exp(k_1)$$
$$\sum_{z=1}^{J} \theta_{z2} (A_z - a_z) \exp(k_z) = \exp(k_2)$$
$$\vdots$$
$$\sum_{z=1}^{J} \theta_{zJ} (A_z - a_z) \exp(k_z) = \exp(k_J)$$

To solve the above system of equations and demonstrate further results we proceed with a numerical simulation.

4. Simulation

In this section we use a simple two-neighborhood numerical simulation to illustrate the results derived in previous sections. The neighborhoods are located different distances from the CBD. One neighborhood (the "city") is close to the CBD located 0.1 units away. The other neighborhood (the "suburb") is located 0.5 units away. Each neighborhood has 100 units of area. We assume that each household has a utility function as given in section 3.1, for $j = 1, 2$. There are 100 households who live in the metropolitan area. In what follows, we vary the amenity value of open space. We analyze cases where amenities from open space are relatively unimportant and cases where open space contributes greatly to utility.

In the first set of simulations, open space is located in one of the two neighborhoods. We systematically vary the size of the open space. We begin with a case where the local

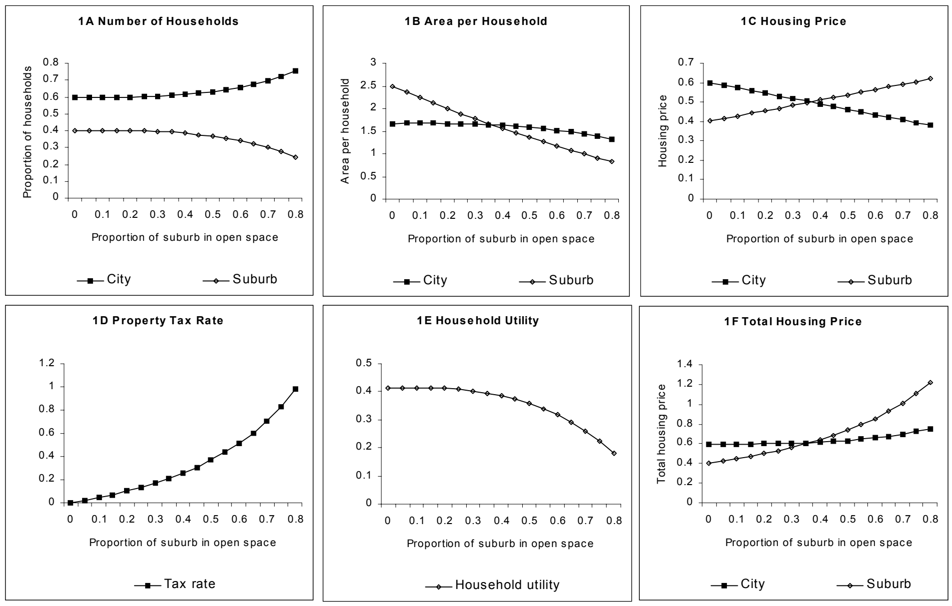

Figure 1: Effects of putting an open space in a suburb when a local amenity value is low

amenity value of the open space is low, $\theta_{jj} = 0.011$, and there is no amenity value for open space not contained in the neighborhood, $\theta_{jk} = 0.0$ for $j \neq k$. Figure 1 shows the effect of putting open space of varying size in the suburban neighborhood. With no open space, living in the city is more attractive than living in the suburb, other things equal, because of lower commuting costs. In this case, housing prices are higher, there are a greater number of households, and each household chooses less area per household in the city than in the suburb. With no open space, the tax rate is zero because there is no need to raise revenue to buy land for the open space (and there is no other government expenditure in this simple model).

As the land area in the suburb devoted to the open space is increased there is some pull of people towards the suburban neighborhood because of the amenity value of the open space. However, there is also a decline in area available for housing in the suburb. With low amenity value for the open space, the area effect dominates and the number of households in the suburban neighborhood declines (Figure 1A). Because of the contraction of land area available for housing as open space area is increased, area per household falls, though it falls much more rapidly for suburban than urban households (Figure 1B). Housing prices net of taxes rise in the suburbs because of the local amenity value of the open space while housing prices net of taxes fall in the city (Figure 1C). However, the full price of housing (housing price plus tax) rises in both the city and the suburbs as open space area increases because the overall supply of available land for development in the metropolitan area declines (Figure

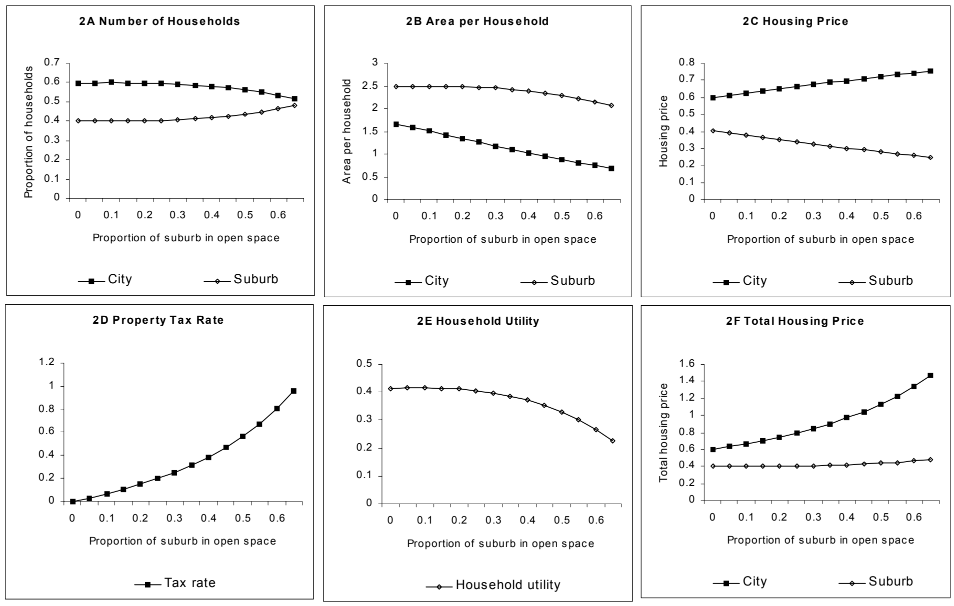

Figure 2: Effects of putting an open space in a city when a local amenity value is low

1F). The tax rate starts to rise dramatically as the proportion of the suburban neighborhood devoted to open space land rises above 50% (Figure 1D). Because of the low amenity value of the open space, which fails to offset the negative effects of crowding from lower supply of developable area, household utility, which is identical in both neighborhoods, declines with increases in open space area (Figure 1E).

A similar pattern emerges when an open space is located in the city (see Figure 2). In this case, there is a decline in the number of households living in the city (Figure 2A). Density increases rapidly in the city while it increases slightly in the suburb (Figure 2B). Housing prices net of taxes rise in the city while they fall in the suburb (Figure 2C). The full price of housing rises in both neighborhoods because of the increase in taxes as open space area rises (Figure 2F). Again, utility of households declines with increasing open space area (Figure 2E).

Figures 3 and 4 repeat the exercise of locating open space in the suburb (Figure 3) and in the city (Figure 4) but this time with a high local amenity value for the open space ($\theta_{jj} = 0.022$). In this case, the pull of the local amenity outweighs the effect of declining area in the neighborhood with the open space. Population shifts toward the neighborhood with the open space as open space size increases (Figures 3A and 4A). Because of the strong pull created by local open space amenities, the area per household falls rapidly in the neighborhood with the open space but actually rises in the other neighborhood because of the decline in population (Figures 3B and 4B). The change in housing prices net of taxes shows a similar pattern as

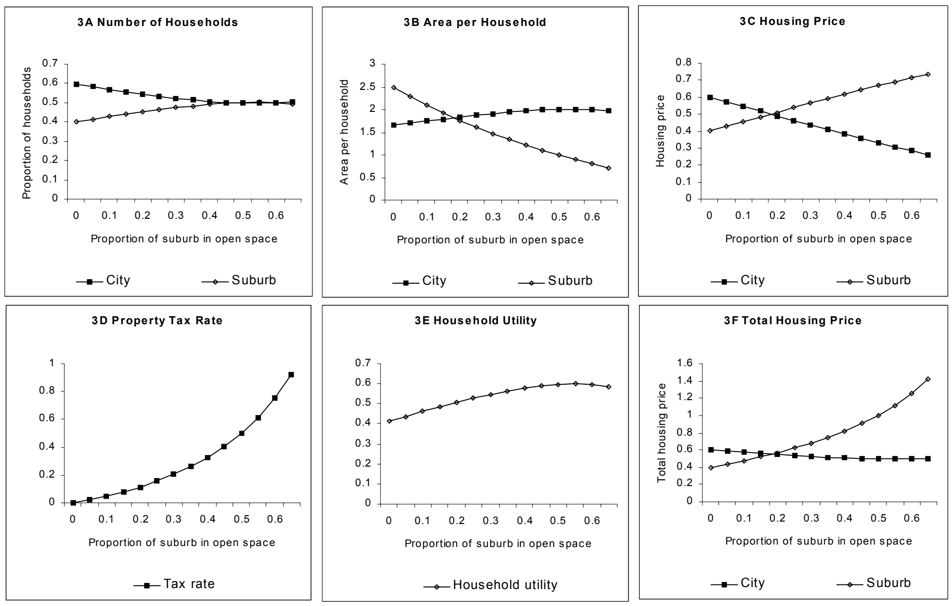

Figure 3: Effects of putting an open space in a suburb when a local amenity value is high

before except that the change in prices is more dramatic. House prices net of taxes in the neighborhood with the open space increase rapidly with an increase in open space area, while housing prices in the neighborhood without the open space decline rapidly (Figure 3C and 4C). Because of the large value created by the open space, household utility increases with open space size (Figure 3E and 4E). Those living in the neighborhood with the open space enjoy the open space amenities. Those living in the neighborhood without the open space gain from less crowding and lower housing prices (Figure 3F and 4F).

Next, we consider the effect of locating open space in the suburb or the city for the case where there is a large local amenity effect ($\theta_{jj} = 0.022$) but where the regional amenity effect of the open space on the other neighborhood is positive and smaller than the local amenity effect ($\theta_{jz} = 0.011$). Figure 5 shows the effect of locating open space in the suburb. Figure 6 shows the effect of locating open space in the city. Even though the open space has a large local amenity, the pattern of household location follows a pattern similar to that shown in Figures 1 and 2 when local amenity value was low (Figures 5A and 5B). What really matters in household location is the relative attractiveness of the two neighborhoods. In terms of relative attractiveness, a situation with high local amenity value and positive but smaller regional amenity value can be identical to a case with low local amenity value and zero regional amenity value. Housing price, area per household, and tax rate also show a similar pattern to what was seen in Figures 1 and 2. The one major difference between Figure 5 and 6 from Figures 1 and 2 is that utility is rising with open space area when there

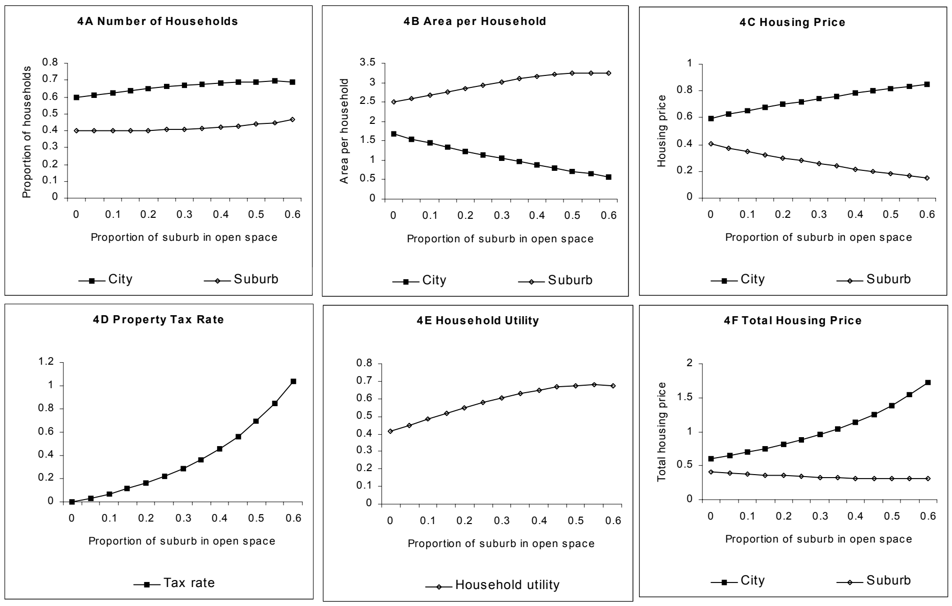

Figure 4: Effects of putting an open space in a city when a local amenity value is high

is a high local amenity value and a positive regional amenity value. Comparing Figures 5 and 6, we can also see that if open space can only be located in one neighborhood it is better to locate the open space in the suburb than in the city. Space in the city neighborhood is devoted to housing because city dwellers can benefit from the shorter distance to CBD and from the regional amenity of the suburban open space.

Finally, we consider the planner's problem of deciding on the optimal amount and location of open space across the metropolitan area. In Table 1, we show the solution to the planner's problem of optimizing the choice of open space size in each neighborhood where the objective of the planner is to maximize household utility. We consider the same three cases as discussed with reference to Figures 1 – 6. When the local amenity value of open space is small and there is no regional amenity value of open space, optimal solution is to choose identically sized small open space in each neighborhood. When the local amenity value of open space is large and there is no regional amenity value of open space, optimal solution is to choose identically sized large open space in each neighborhood. In fact, utility is rising in open space size. Beyond 50% open space area, however, it is no longer possible to pay for more open space through a property tax limited such that property taxes do not exceed the value of the property. When there is a positive regional amenity value from open space, the optimal solution involves shifting open space somewhat toward the suburbs. Slightly more open space area per dollar can be purchased if open space is bought in the suburbs than in the city and doing so allows more people to live in the city where commuting costs are lower.

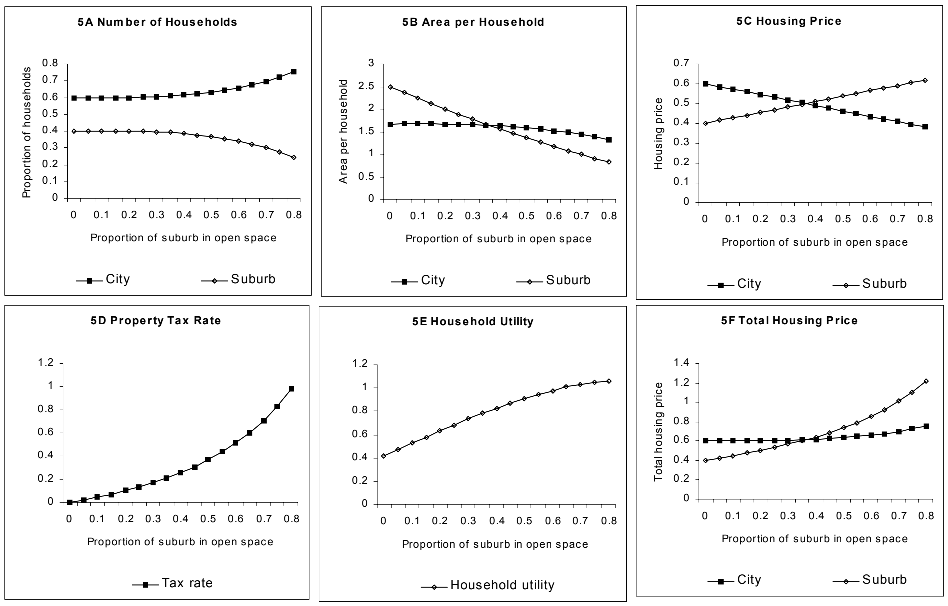

Figure 5: Effects of putting an open space in a suburb

However, fewer people gain the larger local amenity value. Having some positive value to city dwellers from suburban open space, though, is enough to tilt the optimal location of open space towards the suburban neighborhood.

	Case 1		Case 2		Case 3	
Attribute	Suburb	City	Suburb	City	Suburb	City
Open space area (% neighborhood)	0.09	0.09	0.50	0.50	0.57	0.44
Housing price	0.40	0.60	0.40	0.60	0.43	0.57
Households (% total)	0.40	0.60	0.40	0.60	0.37	0.63
Household area	2.27	1.52	1.25	0.84	1.16	0.89
Household utility	0.42		0.82		1.38	
Tax rate	0.10		1		1	

Case 1: open space value θ_{jj} is small; θ_{jk} is zero.
Case 2: open space θ_{jj} is large; θ_{jk} is zero.
Case 3: open space θ_{jj} is large; θ_{jk} is small.

Table 1: Planner's problem optimization results for different amenity values

This simulation illustrates the effects that designating open space have on housing prices, the number of households, population density and household utility. The simulation nicely illustrates the two effects of increasing open space size within a neighborhood: the pull

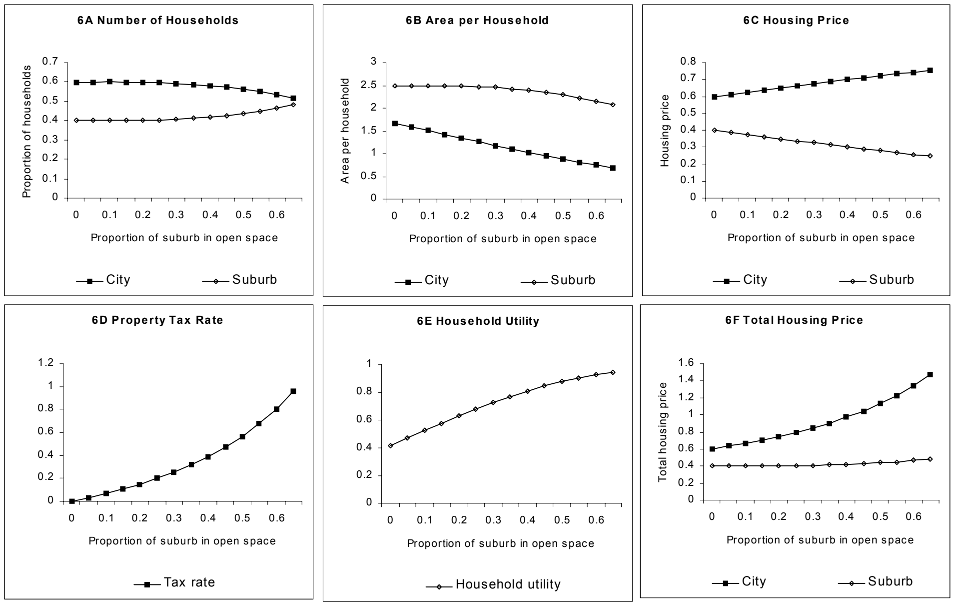

Figure 6: Effects of putting an open space in a city

toward the neighborhood created by the local amenity value of the open space, and the push away from the neighborhood from the reduced supply of developable land. When the local amenity value of open space is low or not much greater than the regional amenity value of open space, the distribution of households will move away from a neighborhood as the amount of open space in the neighborhood increases. High local amenity value will attract households as well as increase density and housing prices in the neighborhood.

5. Discussion

In this paper we analyzed the effect of open space designation in a discrete space urban model. The discrete space model allowed us to consider provision of multiple environmental amenities in an array of possible spatial patterns. This contrasts with the approach of many urban models that are constrained to characterize amenities by their distance to the CBD. We find that provision of open space within a neighborhood necessarily increases property values and housing density on non-open space land within the neighborhood. This result occurs for two reasons. First, open space provides local environmental amenities that make the neighborhood more desirable. Second, open space takes up space and therefore reduces the supply of available developable land. Overall population density within the neighborhood may increase or decrease with additional open space depending upon the pull from open space amenities versus the push from higher housing prices. An increase in open space in one neighborhood will affect housing prices and density in other neighborhoods as well because the relative attractiveness of different neighborhoods will change.

We also addressed the optimal provision of open space across neighborhoods (the planner's problem). When neighborhoods are of equal size and there is only local amenity value from open space, it is optimal to provide the same amount of open space in all neighborhoods. However, allowing for cross-neighborhood amenity values from open space will tend to make optimal provision unequal across neighborhoods, even when the cross-neighborhood amenity effect parameters are symmetric. For example, in the simulation results, we found that it was optimal to provide more open space in the suburban neighborhood when the cross-neighborhood amenity effect was not equal to zero.

At present there is a large gap between the highly stylized general equilibrium spatial models of much of urban economics, and the largely empirical partial equilibrium models of the value of amenities of environmental economics. One important goal for future research on the value of environmental amenities in metropolitan areas is to close this gap. Doing so will require further advances in general equilibrium spatial modeling approaches and increased understanding of the full impacts of amenities across space in urban settings.

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