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An Economic Analysis of Product Differentiation under Latent Separability^{©, ¶}

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Abstract: This paper develops an analysis of markets for differentiated products. It relies on the concept of latent separability for consumer preferences. As proposed by Blundell and Robin, latent separability assumes that purchased products are allocated in the production of latent intermediate utility-yielding goods. Product differentiation can arise when each product makes a different contribution to the production of the latent goods. Latent separability is particularly attractive in the investigation of markets for branded products where the number of brands is large. It allows focusing on the demand for a smaller number of latent goods. Our approach is based on a quadratic almost ideal demand system (Q-AIDS), which provides a flexible representation of consumer behavior. Its usefulness is illustrated in an empirical analysis of markets for carbonated soft drinks (CSD). First, the econometric analysis accounts for the endogeneity of prices for differentiated brands. Second, it provides an empirical evaluation of the number of relevant latent goods. Third, it shows how latent separability improves the efficiency of parameter estimates. Finally, it generates estimates of shadow prices of the latent goods, information that gives useful insights into the economics of differentiated products.

An Economic Analysis of Product Differentiation under Latent Separability

1 Introduction

Industrial organization and marketing science literature have seen much focus on product differentiation. This is particularly relevant in the study of branded products. In this context, own-price and cross-price elasticities of demand provide useful information on strategic positioning of brands. And it can generate useful insights into price-cost margins and the assessment of the effects of hypothetical mergers and acquisitions (e.g., Nevo, 2001). As a result, in recent years research on brand-level demand system has become a critical issue. Two types of demand specifications have been used quite extensively. First, consumer behavior can be investigated through the estimation of demand specifications such as the Rotterdam model (Theil, 1975), the translog model (Christensen, Jorgenson, and Lau, 1975), or the Almost Ideal Demand System (Deaton and Muellbauer, 1980a)). Second, discrete choice model specification have been used (e.g., Nevo, 2001). However, estimating the demand for branded products becomes problematic when the number of brands becomes (even moderately) large. Because of collinearity problems, it becomes difficult to estimate reliably the relevant price elasticities. This has stimulated a search for alternative approaches. An appealing alternative is to focus not on the products themselves, but rather on how the products are perceived by consumers. This appears quite attractive in the context of differentiated products: it is precisely the variations in consumer perceptions that allow firms to differentiate their products or brands. Nevo (2001) has developed this argument using a discrete choice model representing consumer choice over product characteristics. However, one significant issue remains: identifying and measuring product characteristics can sometimes be difficult. This suggests a need to refine empirical linkages between the demand for differentiated products and consumer behavior.

This paper explores how the concept of latent separability can be used to analyze the

economics of product differentiation. As proposed by Blundell and Robin (2000), latent separability assumes that purchased products are allocated in the production of latent intermediate utility-yielding goods. Product differentiation can arise when each product makes a different contribution to the production of the latent goods. Latent separability is particularly attractive in the investigation of markets for branded products where the number of brands is large. It allows focusing on the demand for a smaller number of latent goods. Our approach is based on a quadratic almost ideal demand system (Q-AIDS), which provides a flexible representation of consumer behavior. Its usefulness is illustrated in an empirical analysis of markets for carbonated soft drinks (CSD). The analysis is based on quarterly IRI (Information Resources Inc.)-Infoscan scanner data of supermarket sales of CSD from 1988-Q1 to 1992-Q4. Our research involves several innovations. First, to the best of our knowledge this is the first application of the quadratic almost ideal demand system (Q-AIDS) to brand level demand analysis. Second, our econometric analysis accounts for the endogeneity of prices for differentiated brands. Third, it provides an empirical evaluation of the number of relevant latent goods. Fourth, it shows how latent separability improves the efficiency of price elasticity estimates. Finally, it generates estimates of shadow prices of the latent goods, information that gives useful insights into the economics of differentiated products.

The article is organized as follows. After formally defining latent separability, we review its implications for consumer behavior. This involves identifying and testing for the relevant number of latent goods. Also, identification and estimation issues under latent separability are discussed. After providing an overview of the data used in this analysis, we present our empirical model, followed by the econometric results. Finally, the last section contains concluding remarks.

2 Preliminaries

The empirical analysis of consumer behavior has been the subject of much research. On the one hand, much progress has been made in the specification and estimation of consumer

demand well grounded in neoclassical theory of consumers (e.g., Theil, 1985; Christensen, Jorgenson, and Lau, 1975; Deaton and Muellbauer, 1980b). For example, the Almost Ideal Demand System (AIDS, Deaton and Muellbauer, 1980a) has been commonly used as a practical and flexible specification of consumer demands. On the other hand, discrete choice demand models have recently become more popular (e.g., Anderson et al., 1992) because of their ease of use in differentiated product markets. They provide a convenient framework to investigate empirically the demand for product characteristics (e.g., Nevo, 2000).

Two major problems have arisen in the empirical investigation of consumer demand for differentiated products. First, the number of relevant products is often large, implying a need to estimate a large number of own-price and cross-price elasticities. Second, multicollinearity problems can make it difficult to obtain reliable estimates of price elasticities. Two solutions have been advanced to overcome these problems: (1) using multi-stage budgeting under weak separability assumptions (e.g. Hausman, Leonard and Zona, 1994); (2) discrete choice specifications reflecting the implicit demand for product characteristics (e.g. Nevo 2001).

Multistage budgeting under weak separability is a convenient way of avoiding dimensionality and multicollinearity problem. In this approach brands in a market are grouped into different segments. In a first stage, the consumer allocates its income among groups. In a second stage, the consumer makes decisions for brands within each group. The second stage allocation can then be estimated for each segment separately conditional on group expenditure. This can reduce greatly the complications of estimating large dimensional demand system and associated multicollinearity problems. Yet, the choice of market segments can be difficult. It is often subjective and can be a source of disagreements between researchers (e.g., Cotterill and Haller, 1997).

Discrete choice models avoid the dimensionality problem by specifying demand in terms of characteristics. To the extent that product differentiation takes place by rearranging existing characteristics, the number of characteristics would not increase with the number of brands. This can allow analyzing a large number of brands. Yet, the choice of characteristics

space can also be subjective and somewhat arbitrary. Different researchers may not agree on a single set of product characteristics that define a market. Also, in some markets products may not have well defined characteristics (see Hausman, 1994 for a discussion in defining characteristics in the French wine market).

This suggests the need for an alternative approach that would be empirically tractable even if some of the product characteristics are difficult to assess or measure. As mentioned in the introduction, the principal motivation of this paper is to devise an approach that addresses the shortcomings of current literature on differentiated products. For that purpose, we propose to use the concept of latent separability in consumer demand.

Blundell and Robin (BR) developed the concept of latent separability as an alternative to weak separability. Latent separability assumes that purchased products are allocated in the production of latent intermediate utility-yielding goods. The idea behind separability in consumer preferences is that there exists certain "natural" groupings of related products that reflect the budgeting decisions of consumers. Weak separability assumes that consumer preferences can be expressed in terms of preferences for commodities within each group. Under latent separability, preferences are separable into different groups based on their latent characteristics. Latent separability is equivalent to weak separability in latent rather than purchased products. In this paper, following BR, we show how the concept of latent separability can be used: (1) to assess empirically the number of relevant latent goods motivating consumer decisions; (2) to improve significantly the precision of the estimates of demand parameters. One attractive feature of the approach is that it does not require explicit data measuring the latent space. Also, we innovate (beyond BR) in several ways. First, we employ the concept of latent separability in brand-level demand analysis. The previous and only other application of this concept was in Blundell and Robin (2000) on demand for broadly defined household products (such as: wines, clothing, gas, etc.). Second, this is apparently the first paper to use the quadratic almost ideal demand system (Q-AIDS, a rank-3 demand specification; see Banks et al., 1997) in brand-level demand analysis. This

provides a flexible approach to analyze strategic interactions between brands. Third, in contrast to BR, we account for price endogeneity in our econometric investigation of latent separability.

3 The Concept of Latent Separability

Blundell and Robin (BR) proposed the concept of latent separability as a flexible way of aggregating goods in consumption analysis. Let $q \in R_n^+$ denote a vector measuring household consumption of n commodities. Household preferences are represented by the household utility function $U(q)$. Following BR, latent separability holds if the household utility function can be written as $U(q) = \max_{\tilde{q}} \{U_0(U_1(\tilde{q}_1), \dots, U_m(\tilde{q}_m)) : q = \sum_{k=1}^m \tilde{q}_k, \tilde{q}_k \in R_+^n, k = 1, \dots, m\}$, where $U_0 : R^m \rightarrow R$ and $U_k : R_+^n \rightarrow R, k = 1, \dots, m < n$, are each non-satiated and quasi-concave functions. Thus, under latent separability, the commodity bundle $q \in R_+^n$ is allocated among m latent groups $\{(\tilde{q}_1, \dots, \tilde{q}_k) : q = \sum_{k=1}^m \tilde{q}_k\}$ to produce intermediate utilities $U_k(\tilde{q}_k), k = 1, \dots, m < n$. These intermediate utilities are interpreted as m latent goods. In that sense, latent separability is equivalent to weak separability in the latent space. Homothetic latent separability holds if, in addition, the sub-utility functions $U_k : R_+^n \rightarrow R, k = 1, \dots, m$, are each homothetic. Note that, although latent separability and the theory of characteristics are seemingly close, they are conceptually different. Indeed, in characteristics theory, each subutility would be $U_K(q)$ and depend on total consumption q (see BR).

3.1 Rank of a Demand System and Latent Separability

In this paper, we present the concept of latent separability under the Q-AIDS-class of demand specification. Following BR, let $e(p, u)$ be the household expenditure function, where $p \in R_{++}^n$ is the $(n \times 1)$ price vector of the $(n \times 1)$ vector of consumption goods $q \in R_+^n$, and u is a reference utility level. Under the almost ideal class of demand systems, $\ln e(p, u) = \ln a(p) + c(p)[d(b) + u^{-1}]^{-1}$, where $\ln a(p) = \alpha_0 + \alpha^T \ln p + 1/2 * (\ln p)^T \Gamma (\ln p)$, $\ln c(p) = \beta^T \ln p$ and $d(p) = \tau^T \ln p$ (Banks, Blundell and Lewbell, 1997). Let k_n denote the $(n \times 1)$ vector $[k \dots k]'$,

where k is a scalar. The parameters $(\alpha, \beta, \tau, \Gamma)$ satisfy the restrictions: $\alpha^T \mathbf{1}_n = 1$, $\beta^T \mathbf{1}_n = 0$, $\tau^T \mathbf{1} = 0$, $\Gamma \mathbf{1}_n = \mathbf{0}_n$ (homogeneity and adding up restrictions), and $\Gamma^T = \Gamma$ (symmetry). Letting $M > 0$ be household income, the corresponding Marshallian expenditure share $w = (p_1 q_1^*/M, \dots, p_n q_n^*/M)^T$ are:

$$(1) \quad w = \alpha + \Gamma \ln p + \beta [\ln M - \ln a(p)] + \frac{\tau}{c(p)} [\ln M - \ln a(p)]^2$$

Now consider homothetic latent separability. Under homothetic separability, the expenditure function $e(p, u)$ takes the form $e(p, u) = \tilde{e}(b^1(p), \dots, b^m(p), u)$, where $b^k(p)$ is a linear homogenous aggregate price function for the k -th latent group, $k = 1, \dots, m < n$ (BR, p. 61). Let $b = (b^1, \dots, b^m)^T$. Under the almost ideal class of demand systems (such as Q-AIDS), the expenditure function can be written as $\ln \tilde{e}(b, u) = \ln \tilde{a}(b) + \tilde{c}(b) [\tilde{d}(b) + u^{-1}]^{-1}$, where $\ln \tilde{a} = \tilde{\alpha}_0 + \tilde{\alpha}^T \ln b + \frac{1}{2} (\ln b)^T \tilde{\Gamma} (\ln b)$, and $\tilde{d}(p) = \tilde{\tau}^T \ln b$. The parameters $(\tilde{\alpha}, \tilde{\beta}, \tilde{\tau}, \tilde{\Gamma})$ satisfy the restrictions: $\tilde{\alpha}^T \mathbf{1}_m = 1$, $\tilde{\beta}^T \mathbf{1}_m = 0$, $\tilde{\tau}^T \mathbf{1}_m = 0$, $\tilde{\Gamma} \mathbf{1}_m = \mathbf{0}_m$ (homogeneity/adding up), and $\tilde{\Gamma}^T = \tilde{\Gamma}$ (symmetry). And the group price aggregates $b(p)$ take the form: $\ln b(p) = \Pi \ln p$, where Π is a $(m \times n)$ matrix satisfying: $\Pi \mathbf{1}_n = \mathbf{1}_m$. As noted by BR, a priori restrictions are needed to identify the parameters in Π . As a result, it will be useful to partition the Π matrix as follows: $\Pi = [\Lambda, \Psi]$ where Λ and Ψ are matrices of dimensions $(m \times m)$ and $m \times (n - m)$, respectively. As identifying restrictions, BR suggests using the "exclusivity restrictions" where Λ is restricted to be a diagonal matrix, imposing the existence of at least one exclusive commodity per latent group. Under latent separability, the expenditure shares (1) become:

$$(2) \quad w = \Pi^T [\tilde{\alpha} + \tilde{\Gamma} \ln b + \tilde{\beta} [\ln M - \ln \tilde{a}(b)]] + \frac{\tilde{\tau}}{\tilde{c}(b)} [\ln M - \ln \tilde{a}(b)]^2$$

Comparing (1) and (2), it follows that homothetic latent separability implies that $\alpha = \Pi^T \tilde{\alpha}$, $\beta = \Pi^T \tilde{\beta}$, $\tau = \Pi^T \tilde{\tau}$, and $\Gamma = \Pi^T \tilde{\Gamma} \Pi$. Let $B = (\alpha, \beta, \tau)^T$ a $(K \times n)$ matrix, and $\Theta = \begin{bmatrix} B \\ \Gamma \end{bmatrix}$ a $(K + n) \times n$ matrix. Let $\tilde{B} = (\tilde{\alpha}, \tilde{\beta}, \tilde{\tau})^T$ a $(K \times m)$ matrix.¹ Homothetic latent separability implies that $B = \tilde{B} \Pi$, and $\Gamma = \Pi^T \tilde{\Gamma} \Pi$.

For any matrix X , we denote by \overline{X} the X matrix without its first row, by $X|$ the

X matrix without its last column, and by \underline{X} by the X matrix without its last row. The homogeneity/adding up restrictions imply that $B1_n = \begin{bmatrix} 1 \\ 0_{K-1} \end{bmatrix}$, $\tilde{B}1_m = \begin{bmatrix} 1 \\ 0_{K-1} \end{bmatrix}$, $\Gamma 1_n = 0_n$, and $\tilde{\Gamma}1_m = 0_m$. Thus, in general, $rank(\overline{B}) \leq \min(n-1, K-1)$, $rank(\underline{\Gamma}) \leq n-1$, and $rank(\overline{\Theta}) \equiv rank(\overline{\begin{bmatrix} B \\ \Gamma \end{bmatrix}}) \leq n-1$. And under homothetic latent separability with $m < n$, it follows that $rank(\overline{B}) \leq \min(m-1, K-1)$, $rank(\underline{\Gamma}) \leq m-1$, and $rank(\overline{\Theta}) \leq m-1$. This shows that homothetic latent separability reduces the rank of the $\overline{\Theta}$ matrix. When the matrix $\overline{\Theta}$ is “full-rank”, this last result generates a basis to investigate the number m of latent variables: $m = rank(\overline{\Theta}) + 1$. This is Proposition 4.i in BR (p. 71).

The main question is: how to use this result in the empirical investigation of consumption behavior? In general, latent homothetic separability implies that $\Theta = \begin{bmatrix} B \\ \Gamma \end{bmatrix} = \begin{bmatrix} \tilde{B}\Pi \\ \Pi^T\tilde{\Gamma}\Pi \end{bmatrix}$, where $\tilde{B}1_m = \begin{bmatrix} 1 \\ 0_{K-1} \end{bmatrix}$, $\tilde{\Gamma}1_m = 0_m$, $\tilde{\Gamma} = \tilde{\Gamma}^T$, $\Pi 1_n = 1_m$. Without a loss of information, Θ can be written in general as:

$$(3) \quad \underline{\Theta} \equiv \begin{bmatrix} B \\ \Gamma \end{bmatrix}$$

where $\underline{\Gamma} = \underline{\Gamma}^T$

Under latent homothetic separability, Θ can be written as:

$$(4) \quad \underline{\Theta} = \begin{bmatrix} \tilde{B}\Pi \\ \Pi^T\tilde{\Gamma}\Pi \end{bmatrix} = \begin{bmatrix} \begin{bmatrix} \tilde{B} & \begin{pmatrix} 1 \\ 0_{K-1} \end{pmatrix} - \tilde{B}1_{m-1} \end{bmatrix} \Pi \\ \Pi^T \begin{bmatrix} \tilde{\Gamma} & -\tilde{\Gamma}1_{m-1} \\ -1_{m-1}^T\tilde{\Gamma} & 1_{m-1}^T\tilde{\Gamma}1_{m-1} \end{bmatrix} \Pi \end{bmatrix}$$

where $\tilde{\Gamma} = \tilde{\Gamma}^T$.

And under BR’s exclusivity assumption (where Λ is a diagonal matrix), (4) becomes

$$(5) \quad \underline{\Theta} = \begin{bmatrix} \begin{bmatrix} \tilde{B} & \begin{pmatrix} 1 \\ 0_{K-1} \end{pmatrix} - \tilde{B}1_{m-1} \end{bmatrix} [\Lambda \ \Psi] \\ \begin{bmatrix} \Lambda \\ \Psi^T \end{bmatrix} \begin{bmatrix} \tilde{\Gamma} & -\tilde{\Gamma}1_{m-1} \\ -1_{m-1}^T\tilde{\Gamma} & 1_{m-1}^T\tilde{\Gamma}1_{m-1} \end{bmatrix} [\Lambda \ \Psi] \end{bmatrix}$$

Expression (3) means that, in general, Θ contains $(n-1)[K+n/2]$ free parameters. And from (5), under m -latent homothetic separability and BR exclusivity (where Λ is a diago-

nal matrix), Θ contains $[(m-1)[K+m/2] + m(n-m)]$ free parameters. Thus, m -latent homothetic separability imposes $(n-m)[K+(n-m-1)/2]$ restrictions on the parameters.

3.2 Rank Test

From the discussion above, estimating demand parameters in the latent space first requires determining the rank of the Π matrix. And the rank of the Π depends on the rank of the Θ matrix. In this section, following Cragg and Donald (1997), we evaluate two tests of the rank of the Θ matrix.

Before implementation of the test statistics, reorder the n commodities such that the first $(m-1)$ columns of $\overline{\Theta} = \begin{bmatrix} \overline{B} \\ \overline{\Gamma} \end{bmatrix}$ are linearly independent. Assuming that $\text{rank}(\overline{\Theta}) \geq m-1$, this can be done taking the columns selected by the first $(m-1)$ steps of a LU decomposition of $\overline{\Theta}$ with complete pivoting. Consider the partition $B| = \begin{bmatrix} b^T \\ \overline{B} \end{bmatrix} = \begin{bmatrix} b_1^T & b_2^T \\ B_1 & B_2 \end{bmatrix}$, where the $1 \times (n-1)$ vector $b^T = [b_1^T \quad b_2^T] = \alpha^T|$ is the first row of $B|$, $\overline{B} = [B_1 \quad B_2]$ is a $(K-1) \times (n-1)$ matrix, b_1^T and b_2^T are vectors of dimensions $1 \times (m-1)$ and $1 \times (n-m)$, respectively, and B_1 and B_2 are matrices of dimensions $(K-1) \times (m-1)$ and $(K-1) \times (n-m)$, respectively. Let $\Pi| = [\Lambda \quad \Psi|] = \begin{bmatrix} \Lambda_1 & \Psi_1 \\ \Lambda_2 & \Psi_2 \end{bmatrix}$, where Λ_1 , Λ_2 , Ψ_1 and Ψ_2 are of dimensions $(m-1) \times (m-1)$, $1 \times (m-1)$, $(m-1) \times (n-m)$ and $1 \times (n-m)$, respectively. Assume that the matrix $[\Lambda_1 - 1_{m-1}\Lambda_2]$ is *non-singular* (note that this assumption is required throughout the analysis). Define $\Phi \equiv [\Lambda_1 - 1_{m-1}\Lambda_2]^{-1}[\Psi_1 - 1_{m-1}\Psi_2]$. It follows from (3c) that $\overline{B}| = \widetilde{B}\Pi| = \widetilde{B}[\Lambda \quad \Psi|]$ can be written as:

$$(6) \quad \overline{B}| = [B_1 \quad B_2] = [B_1 \quad B_1\Phi]$$

Next, consider the partition $\underline{\Gamma}| = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix}$, where Γ_{11} , Γ_{12} , Γ_{21} and Γ_{22} are matrices of dimensions $(m-1) \times (m-1)$, $(m-1) \times (n-m)$, $(n-m) \times (m-1)$ and $(n-m) \times (n-m)$, respectively. It follows from (5) that $\overline{\Gamma}| = \Pi|^T \widetilde{\Gamma} \Pi| = \begin{bmatrix} \Lambda \widetilde{\Gamma} \Lambda & \Lambda \widetilde{\Gamma} \Psi| \\ \Psi|^T \widetilde{\Gamma} \Lambda & \Psi|^T \widetilde{\Gamma} \Psi| \end{bmatrix}$ can be written as:

$$(7) \quad \underline{\Gamma}| = \begin{bmatrix} \Gamma_{11} & \Gamma_{11}\Phi \\ \Phi^T \Gamma_{11} & \Phi^T \Gamma_{11}\Phi \end{bmatrix}$$

Combining equations (6) and (7) gives:

$$(8) \quad \begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix} = \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix} \Phi$$

where $\begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix}$ is a $(K+n-2) \times (n-m)$ matrix, $\begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix}$ is a $(K+n-2) \times (m-1)$ matrix, and Φ is a $(m-1) \times (n-m)$ matrix. Expression (8) can be formulated as a regression model

$$(9) \quad \text{vec} \begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix} = \begin{bmatrix} I_{n-m} \otimes \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix} \end{bmatrix} \text{vec}(\Phi) + u$$

with $(K+n-2)(n-m)$ observations, $(m-1)(n-m)$ parameters in Φ , and where u is an error term. Denote by Ω the variance of $\text{vec} \begin{bmatrix} B_2 & B_1 \\ \Gamma_{12} & \Gamma_{11} \\ \Gamma_{22} & \Gamma_{21} \end{bmatrix}$. Noting that (9) can be alternatively

written as: $\text{vec} \begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix} = [\Phi^T \otimes I_{K+n-2}] \text{vec} \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix} + u$, it follows that the variance of u is:

$$(10) \quad V = \begin{bmatrix} I_{(K+n-2)(n-m)} & -\Phi^T \otimes I_{K+n-2} \end{bmatrix} \Omega \begin{bmatrix} I_{(K+n-2)(n-m)} \\ -\Phi^T \otimes I_{K+n-2} \end{bmatrix}$$

The minimum-distance estimator of Φ in (9) is given by (Theil, p. 279):

$$(11) \quad \text{vec}(\Phi^e) = \{ [I_{n-m} \otimes \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix}]^T] V^+ [I_{n-m} \otimes \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix}] \}^{-1} [I_{n-m} \otimes \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix}]^T] V^+ \text{vec} \begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix}$$

where V^+ denotes the Moore-Penrose generalized inverse of V , allowing for the matrix V to be singular under the symmetry restrictions. Note that there are $(n-m)(n-m-1)/2$ symmetry restrictions on Γ_{22} . As a result, the matrix V is necessarily singular, with $\text{rank}(V) \leq (K+n-2)(n-m) - (n-m)(n-m-1)/2$.² And the number of overidentifying restrictions for Φ in (9) is: $[\text{rank}(V) - (m-1)(n-m)] \leq (n-m)[K + (n-m-1)/2]$. In the case where V is full rank (with $\text{rank}(V) = (K+n-2)(n-m) - (n-m)(n-m-1)/2$), the number of overidentifying restrictions for Φ in (9) becomes: $(n-m)[K + (n-m-1)/2]$. Note

that this is the same number of restrictions implied by m -latent homothetic separability as identified above. This means that, in situations where the structural parameters are exactly identified, the estimation of Φ in (9) would be efficient: it would impose all the theoretical restrictions implied by m -latent homothetic separability.

As noted above, under some regularity conditions, m -latent homothetic separability implies that $m = \text{rank}(\overline{\Theta}) + 1$, where $\overline{\Theta} = \begin{bmatrix} \overline{B} \\ \overline{\Gamma} \end{bmatrix}$. Thus, testing for m -latent homothetic separability can be conducted by testing the rank of the $\overline{\Theta}$ matrix. To do this, reorder the rows and columns of the $\overline{\Theta}$ matrix to obtain the $(K+n-2) \times (n-1)$ matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ such that the absolute value of the determinant of the $(m-1) \times (m-1)$ sub-matrix A_{11} is maximized. This can be done taking the rows and columns selected by the first $(m-1)$ steps of a LU decomposition of the $\overline{\Theta}$ matrix with complete pivoting. Note that $\text{rank}(\overline{\Theta}) = \text{rank}(A)$ being equal to $(m-1)$ is equivalent to $\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = 0$, where A_{11} , A_{12} , A_{21} and A_{22} are respectively of dimensions $(m-1) \times (m-1)$, $(m-1) \times (n-m)$, $(K+n-m-1) \times (m-1)$, and $(K+n-m-1) \times (n-m)$, and $\begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$ is a $(K+n-2) \times (n-m)$ matrix of rank $(n-m)$. With $\det(A_{11}) \neq 0$, this can be written as $[A_{22} - A_{21}A_{11}^{-1}A_{12}]V_2 = 0$ for all V_2 . It follows that $\text{rank}(\overline{\Theta}) = m-1$ is equivalent to $[A_{22} - A_{21}A_{11}^{-1}A_{12}] = 0$. Consider the case where the parameters in $\overline{\Theta}$ have been estimated, using a consistent and asymptotically efficient estimation method. Denote the corresponding estimate of A by \tilde{A} , with W being the asymptotic variance of $\text{vec}(\tilde{A})$. Note that the asymptotic variance of $\text{vec}(\tilde{A}_{22} - \tilde{A}_{21}\tilde{A}_{11}^{-1}\tilde{A}_{12})$ can be written as $[SWST]$, where $S = [-A_{12}^T(A_{11}^T)^{-1} \quad I_{n-m}] \otimes [-A_{21}(A_{11})^{-1} \quad I_{K+n-m-1}]$ (Cragg and Donald, 1996). Under the null hypothesis $H_0 : \text{rank}(\overline{\Theta}) = m-1$ (corresponding to m -latent homothetic separability), the estimate $\text{vec}(\tilde{A}_{22} - \tilde{A}_{21}\tilde{A}_{11}^{-1}\tilde{A}_{12})$ is asymptotically distributed $N(0, W)$. Then, consider the test statistic as:

$$(12) \quad T_1 = \text{vec}(\tilde{A}_{22} - \tilde{A}_{21}\tilde{A}_{11}^{-1}\tilde{A}_{12})^T [\tilde{S}\tilde{W}\tilde{S}^T]^+ \text{vec}(\tilde{A}_{22} - \tilde{A}_{21}\tilde{A}_{11}^{-1}\tilde{A}_{12})$$

where \tilde{S} and \tilde{W} are consistent estimates of S and W , and $[\tilde{S}\tilde{W}\tilde{S}^T]^+$ denotes the Moore-Penrose generalized inverse of $[\tilde{S}\tilde{W}\tilde{S}^T]$. This allows the variance matrix to be singular due to symmetry restrictions on parts of the $(K+n-m-1) \times (n-m)$ A_{22} matrix. Let s denote the

number of symmetry restrictions in A_{22} . We expect $rank[\widetilde{S}\widetilde{W}\widetilde{S}^T] = (K+n-m-1)(n-m)-s$. Then, under the null hypothesis of m -latent separability, T_1 has an asymptotic chi-square distribution with $[(K+n-m-1)(n-m)-s]$ degrees of freedom (Cragg and Donald, 1996).

Alternatively, (6)-(7) can provide another way of testing for m -latent homothetic separability. Given $\overline{\Theta} = \begin{bmatrix} \overline{B} \\ \overline{\Gamma} \end{bmatrix} = \begin{bmatrix} B_1 & B_2 \\ \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} = \begin{bmatrix} B_1 & B_1\Phi \\ \Gamma_{11} & \Gamma_{11}\Phi \\ \Phi^T\Gamma_{11} & \Phi^T\Gamma_{11}\Phi \end{bmatrix}$ where the first $(m-1)$ columns of $\overline{\Theta}$ are linearly independent, note that Φ being a matrix of dimension $(m-1) \times (n-m)$ implies $rank(\overline{\Theta}) \leq m-1$. But we have seen that $rank(\overline{\Theta}) = m-1$ is equivalent to m -latent homothetic separability. This suggests considering the test statistic

$$(13) \quad T_2 = \left\{ vec \begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix} - [I_{n-m} \otimes \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix} vec(\Phi^e)] \right\}^T V^+ \left\{ vec \begin{bmatrix} B_2 \\ \Gamma_{12} \\ \Gamma_{22} \end{bmatrix} - [I_{n-m} \otimes \begin{bmatrix} B_1 \\ \Gamma_{11} \\ \Gamma_{21} \end{bmatrix} vec(\Phi^e)] \right\}$$

where $vec(\Phi^e)$ is given in (11). When $rank(V) = (K+n-2)(n-m) - (n-m)(n-m-1)/2$ and under the null hypothesis of m -latent separability, the test statistic T_2 has an asymptotic chi-square distribution, with $(n-m)[K + (n-m-1)/2]$ degrees of freedom (Cragg and Donald, 1997). This test is equivalent to testing the null hypothesis that $rank(\overline{\Theta}) = m-1$ (hence m -latent homothetic separability) under the maintained hypothesis that the Γ matrix is symmetric.

The two tests T_1 and T_2 are asymptotically equivalent (Cragg and Donald, 1996). They provide alternative ways to investigate whether particular consumption data are consistent with m -latent separability. We want to stress that this examination of the number of latently separable groups avoids the pitfalls of arbitrary segmentation of groups (as in Hausman, Leonard and Zona, 1994), and arbitrary choice of characteristics (as in Nevo, 2001).

3.3 Identification of Parameters in the Latent Space

We have shown above how the parameters Φ can be estimated based on estimates of (B, Γ) (see 11). It remains to show that estimating B, Γ , and Φ identifies the structural

parameters $(\tilde{B}, \tilde{\Gamma}, \Lambda, \Psi)$ representing the latent space. Consider the partitioned matrix $\tilde{B} \equiv \begin{bmatrix} \tilde{b}^T \\ \tilde{B} \end{bmatrix} = \begin{bmatrix} \tilde{b}^T & 1 - \tilde{b}^T |_{1_{m-1}} \\ \tilde{B} & -\tilde{B} |_{1_{m-1}} \end{bmatrix}$, where $\tilde{b}^T = \begin{bmatrix} \tilde{b}^T & 1 - \tilde{b}^T |_{1_{m-1}} \end{bmatrix}$ is the first row of \tilde{B} , and $\tilde{B} = \begin{bmatrix} \tilde{B} & -\tilde{B} |_{1_{m-1}} \end{bmatrix}$ is a $(K-1) \times (m-1)$ matrix, with $\tilde{b}^T |$ and $\tilde{B} |$ being of dimensions $1 \times (m-1)$ and $(K-1) \times (m-1)$, respectively. From the first row in (5), the first row of $B |$ can be written as

$$b^T = \begin{bmatrix} \tilde{b}^T & 1 - \tilde{b}^T |_{1_{m-1}} \end{bmatrix} [\Lambda \quad \Psi] = \begin{bmatrix} \tilde{b}^T & 1 - \tilde{b}^T |_{1_{m-1}} \end{bmatrix} \begin{bmatrix} \Lambda_1 & \Psi_1 \\ \Lambda_2 & \Psi_2 \end{bmatrix}.$$

This yields

$$b^T \equiv [b_1^T \quad b_2^T] = \tilde{b}^T | [\Lambda_1 - 1_{m-1} \Lambda_2 \quad \Psi_1 - 1_{m-1} \Psi_2] + [\Lambda_2 \quad \Psi_2],$$

where b_1^T and b_2^T are of dimensions $1 \times (m-1)$ and $1 \times (n-m)$, respectively. This implies:

$$(14) \quad b_1^T = \tilde{b}^T | [\Lambda_1 - 1_{m-1} \Lambda_2] + \Lambda_2$$

and, given $\Phi \equiv [\Lambda_1 - 1_{m-1} \Lambda_2]^{-1} [\Psi_1 - 1_{m-1} \Psi_2]$,

$$(15) \quad b_2^T = [b_1^T - \Lambda_2] \Phi + \Psi_2$$

Equation (15) is a system of $(n-m)$ linear equations with $(n-1)$ unknowns: $(m-1)$ unknowns in Λ_2 , and $(n-m)$ unknowns in Ψ_2 . It requires $(m-1)$ a priori restrictions to identify the parameters (Λ_2, Ψ_2) . Note that these $(m-1)$ restrictions can be obtained from the BR exclusivity assumption (which assumes $\Lambda_2 = 0$). This shows that the identification analysis presented by BR is a special case. In the presence of at least $(m-1)$ identifying restrictions, equation (15) can then be solved for (Λ_2, Ψ_2) . (For example, in the BR case, $\Lambda_2 = 0$ implies that $\Psi_2 = b_2^T - b_1^T \Phi$). To illustrate, note that (15) can be written as: $[b_2 - \Phi^T b_1] = [-\Phi^T \quad I_{n-m}] \begin{bmatrix} \Lambda_2^T \\ \Psi_2^T \end{bmatrix}$. Denote the (at least $(m-1)$) identifying restrictions by: $R_1 \begin{bmatrix} \Lambda_2 \\ \Psi_2^T \end{bmatrix} = r_1$. Then, in the case of exact identification, the solution to the system of equations is $\begin{bmatrix} \Lambda_2 \\ \Psi_2^T \end{bmatrix} = \begin{bmatrix} [-\Phi^T & I_{n-m}] \\ R_1 \end{bmatrix}^{-1} \begin{bmatrix} b_2 - \Phi^T b_1 \\ r_1 \end{bmatrix}$, where $\begin{bmatrix} [-\Phi^T & I_{n-m}] \\ R_1 \end{bmatrix}$ is a $(n-1) \times (n-1)$ non-singular matrix.

Next, note that $\Phi \equiv [\Lambda_1 - 1_{m-1} \Lambda_2]^{-1} [\Psi_1 - 1_{m-1} \Psi_2]$ can be written as $[\Lambda_1 - 1_{m-1} \Lambda_2] \Phi = [\Psi_1 - 1_{m-1} \Psi_2]$, or using (15) $\Lambda_1 \Phi = \Psi_1 + 1_{m-1} [\Lambda_2 \Phi - \Psi_2] = \Psi_1 + 1_{m-1} [\Lambda_2 \Phi + (b_1^T - \Lambda_2) \Phi - b_2^T]$.

This yields

$$(16) \quad \Lambda_1 \Phi = \Psi_1 + 1_{m-1} [b_1^T \Phi - b_2^T]$$

Equation (16) is a system of $(m-1)(n-m)$ linear equations with $(m-1)(n-1)$ unknowns: $(m-1)^2$ unknowns in Λ_1 , and $(m-1)(n-m)$ unknowns in Ψ_1 . It requires $(m-1)^2$ a priori restrictions to identify the parameters (Λ_1, Ψ_1) . Note that these $(m-1)^2$ restrictions can be obtained from the BR exclusivity assumption which assumes that Λ_1 is diagonal (generating $(m-1)(m-2)$ restrictions) and that the first column of Ψ_1 is zero (generating $(m-1)$ restrictions). Thus, in the presence of exactly identifying restrictions, the estimation of the structural parameters $(\Lambda_1, \Lambda_2, \Psi_1, \Psi_2)$ can be obtained from (15) and (16).

Next, note that \tilde{b}^T can be obtained from (14):

$$(17) \quad \tilde{b}^T = [b_1^T - \Lambda_2][\Lambda_1 - 1_{m-1}\Lambda_2]^{-1}$$

And noting from (5) that $\overline{B|} \equiv [B_1, B_2] = \widetilde{B}[\Lambda, \Psi|]$, it follows that:

$$(18) \quad \widetilde{B|} = B_1[\Lambda_1 - 1_{m-1}\Lambda_2]^{-1}$$

Finally, noting from (5) that $\underline{\Gamma|} = \begin{bmatrix} \Gamma_{11} & \Gamma_{12} \\ \Gamma_{21} & \Gamma_{22} \end{bmatrix} = \Pi^T \tilde{\Gamma} \Pi = \begin{bmatrix} \Lambda \tilde{\Gamma} \Lambda & \Lambda \tilde{\Gamma} \Psi| \\ \Psi| \tilde{\Gamma} \Lambda & \Psi| \tilde{\Gamma} \Psi| \end{bmatrix}$, it follows that:

$$(19) \quad \tilde{\Gamma} = [\Lambda_1^T - \Lambda_2^T 1_{m-1}^T]^{-1} \Gamma_{11} [\Lambda_1 - 1_{m-1}\Lambda_2]^{-1}$$

Thus, (14), (18) and (19) provide an estimate of $\widetilde{B|}$ and $\underline{\Gamma|}$.

4 Description of Data

Our empirical analysis focuses on the Carbonated Soft Drink (CSD) industry. In 1998, CSD accounted for 49% of total US beverage volume sales, generating over \$54 billion in revenues, and with 56.1 gallons per capita consumption. In contrast, the second largest beverage category (beer) accounted for only 19.4% of sales volume, with 22.1 gallons per

capita being consumed. CSD demand provides an excellent example of differentiated products where taste, packaging and brand-based advertisement influence consumers' perception and contribute to product differentiation across brands.

IRI-Infoscan data used in this analysis contain detailed brand level information on supermarket CSD sales, merchandising and price discount information from 46 major metropolitan marketing areas within the continental US. A total of 920 quarterly observations on 46 cities over 20 quarters (from 1988-Q1 to 1992-Q4) are used. It covers 16 CSD brands, including 9 regular brands and 7 diet brands.

The following CSD brands are included in the data set: 1/ regular brands - Coke, Pepsi, 7-Up, Mountain Dew, Sprite, RC Cola, Dr. Pepper, Private label, and an aggregate All-Other brand; and 2/ diet brands - diet Coke, diet Pepsi, diet 7-Up, diet Sprite, diet Dr. Pepper, diet Private Label, and diet All-Other brand. Data for diet Mt. Dew and diet RC Cola were unavailable for most of the study period. Since their sales account for less than 0.01% of the CSD market, they were dropped from the analysis.

Detailed descriptive statistics of the brand and metropolitan area (city) level variables used in this study are presented in table 1. For regular brands, Dr. Pepper is the most expensive (average price = \$3.97/gal) and Private label the least expensive (average price = \$2.34/gal). In terms of share of consumer expenditures, regular Coke has the highest share (13%) and RC Cola the lowest share (1%). For diet brands, Diet Dr. Pepper is the most expensive (average price = \$4.12/gal) and private label is the least expensive (average price = \$2.36/gal). In terms of market share diet All-Other has highest share (31%) and diet Private label has the lowest market share (0.69%). More detailed descriptions of other variables are presented in the empirical section of the paper.

5 Empirical Model Specification

Our empirical analysis of consumer behavior is based on the specification (1). This generates 16 demand equations. Given the singularity of the covariance matrix (due to the adding-

up restriction that the sum of shares is 1), we drop one demand equation, generating 15 equations to estimate. Explanatory variables include prices, total expenditures on CSD, and socio-demographic variables. We account for expenditures endogeneity, allowing total expenditures to be correlated with unobservable factors affecting demand. And going beyond BR, we also account for price endogeneity. Indeed, under differentiated products, firms have a great latitude in formulating prices. To the extent that pricing decisions are made based on information not available to investigators, this raises the possibility that prices are correlated with unobservable factors affecting consumer behavior. Dhar, Chavas and Gould (2003) investigated this issue and found strong evidence that such correlations exist and that ignoring it generates significant endogeneity bias. On that basis, our econometric analysis accounts for both expenditures and price endogeneity. To control for such endogeneity, we specified and estimated reduced form price and expenditure equations. So, the empirical model consists of 15 brand demand equations, 16 price equations and 1 expenditure equation. In this section we describe our model in details.

5.1 Demand Specification

Banks, Blundell and Lewbel (1997) have argued that income effects (i.e., the shape of Engel curves) can be highly nonlinear, implying that rank-2 demand systems (such as AIDS) can be inappropriate. Preliminary analysis of brand level expenditure given total expenditure showed evidence of highly non-linear Engel curves for CSD brand demands. This suggests a need to switch from a rank-2 specification to a rank-3 demand system. Rank-3 systems allow for more complex Engel relationships. As proved in Gorman (1981), rank-3 is the maximum possible rank for any demand system that is linear in functions of income. As proposed by Banks et al. (1997), the Q-AIDS specification is currently the best available exactly aggregable demand system to belong to the rank-3 class. Unlike the standard AIDS model (Deaton and Muelbauer, 1980b) and the exactly aggregable Translog model of Jorgenson et al. (1982), the Q-AIDS model permits goods to be luxuries at some income level and

necessities at others. On that basis, we rely on the Q-AIDS specification:

$$(20) \quad w_{ilt} = \alpha_i + \sum_{j=1}^N \gamma_{ij} \ln(p_{jlt}) + \beta_i \ln\left(\frac{M_{lt}}{P_{lt}}\right) + \frac{\tau_i}{N} \ln\left(\frac{M_{lt}}{P_{lt}}\right)^2 \prod_{i=1}^N p_{ilt}^{\beta_i}$$

where $p = (p_1, \dots, p_N)'$ is a $(N \times 1)$ vector of prices for x , and $w_{ilt} = (p_{ilt}x_{ilt}/M_{lt})$ is the budget share for the i^{th} commodity consumed in the l^{th} city at time t . The term P is a price index expressed as: $\ln(P_{lt}) = \delta + \sum_{m=1}^N \alpha_m \ln(p_{mlt}) + \frac{1}{2} \sum_{m=1}^N \sum_{j=1}^N \gamma_{mj} \ln(p_{mlt}) \ln(p_{jlt})$.

5.1.1 Demographic Translating of the AIDS Model

The above AIDS specification (20) can be modified to incorporate the effects of socio-demographic variables $(Z_{1lt}, \dots, Z_{Klt})$ on consumption behavior, where Z_{klt} is the k^{th} socio-demographic variable in the l^{th} city at time t , $k = 1, \dots, K$. This can be done by specifying the parameters α to depend on Z . This allows demographic effects to affect behavior as well as elasticity estimates. Under demographic translating, we assume that α_i takes the following form: $\alpha_{ilt} = \alpha_{0i} + \sum_{k=1}^K \lambda_{ik} Z_{klt}$, $i = 1, \dots, N$.

As a result, our AIDS model incorporates a set of regional dummy variables along with selected socio-demographic variables. In previous studies using multi-market scanner data, Cotterill, Franklin and Ma and Hausman, Leonard and Zona use city specific dummy variables to control for city specific fixed effects for each brand. Here we control for regional differences by including nine regional dummy variables. To maintain theoretical consistency of the AIDS model, the following restrictions based on adding up restrictions are applied to demographic translating parameter α_{0i} :

$$(21) \quad \alpha_{0i} = \sum_{r=1}^9 d_{ir} D_r, \quad \sum_{r=1}^9 d_{ir} = 1, \quad i = 1, \dots, N.$$

where d_{ir} is the parameter for the i^{th} brand associated with the regional dummy variable D_r for the r^{th} region. Note that as a result, our demand equations do not have intercept terms.

Our AIDS specification also incorporates six demand shifters capturing the effects of demographics across marketing areas. These variables include: percentage of Hispanic pop-

ulation, median household size, median household age, percent of household earning less than \$10,000, percentage of household earning more the \$50,000. To capture the effect of any city specific variation in outlet types used to purchase soft drinks, we also use data on the ratio of supermarket sales to total grocery sales as a demand shifter in the share equation.

5.2 Specifications of the Reduced Form Price and Expenditure Functions

As mentioned above, reduced form price and expenditure equations are also estimated to control for endogeneity bias. For products like CSD, raw material cost is only a small fraction of retail price. Conversely, merchandising and packaging costs tend to be a larger portion of the retail price. As a result most recent studies of differentiated products modeled price as a function of supply and demand shifters, assuming these shifters are exogenous to the price formation mechanism (e.g., Cotterill, Franklin and Ma; Cotterill, Putsis and Dhar; and Kadiyali, Vilcassim and Chintagunta). Our reduced form price specification is similar in spirit and we specify the price functions with marketing and other product characteristics as explanatory variables:

$$(22) \quad p_{ilt} = \theta_{i0} + \theta_{i1}UPV_{ilt} + \theta_{i2}MCH_{ilt} + \theta_{i3}PRD_{ilt} + \theta_{i4}CR_{lt}^4$$

where UPV_{ilt} is the unit volume of the i^{th} product in the l^{th} city at time t and represents the average size of the purchase. For example, if a consumer purchases only one gallon bottles of a brand, then unit volume for that brand will be just one. Conversely, if this consumer buys a half-gallon bottle then the unit volume will be 2. This variable is used to capture packaging-related cost variations, as smaller package size per volume implies higher costs to produce, to distribute and to shelve. The variable MCH_{ilt} measures percentage of a CSD brand i sold in a city l through any types of merchandising (e.g., buy one get one free, cross promotions with other products, etc.). This variable captures merchandising costs of selling a brand. For example, if a brand is sold through promotion such as: ‘buy one get one free’, then the cost of providing the second unit will be reflected in this variable. The variable PRD_{ilt} is the

percent price reduction of brand i and is used to capture any costs associated with specific price reductions (e.g., aisle end displays, freestanding newspaper inserts). Simply lowering the shelf price with no aisle end display or local newspaper advertisement telling consumer the brand is ‘on special’ does not effectively communicate the price change to consumers. Finally the variable $CR4_{lt}$ measures the four firm concentration ratios of supermarkets in city l . This variable captures any market power effect on price formation. In earlier studies, it is found that supermarket concentration is a significant variable in explaining retail price variations across regions. Regions with higher supermarket concentration tend to have higher price (Cotterill, Dhar and Franklin).

The reduced form expenditure function is specified as:

$$(23) \quad M_{lt} = \eta TR_t + \sum_{r=1}^9 \delta_r D_r + \phi_1 INC_{lt} + \phi_2 INC_{lt}^2, \quad t = 1, \dots, 20$$

where TR_t in (23) is a linear trend, capturing any time specific unobservable effect on consumer soft-drink expenditure. The D_r ’s are the regional dummy variables defined above and capture region specific variations in per capita expenditure. The variable INC_{lt} is the median household income in city l and is used to capture the effect of income differences on CSD purchases.

We assume the demand shifters and the variables in the reduced form price and expenditure specification are exogenous. In general the reduced form specifications (i.e. equation (22) and (23)) are always identified. The issue of parameter identification is rather complex in non-linear structural model. We checked the order condition for identification that would apply to a linearized version of the demand equations and found them to be satisfied. Finally, we did not uncover numerical difficulties in implementing the full information maximum likelihood estimation. As pointed out by Mittelhammer, Judge and Miller (p.474-475), we interpret this as evidence that each of the demand equations is identified.

6 Empirical Results

A system of 32 equations was estimated by full information maximum likelihood (FIML) under normality assumptions. We estimate a total of 467 parameters. Of them 263 are significant at the 5% level. Our brand level analysis is based on these estimated demand parameters. Of the 375 parameters in Q-AIDS specification 205 are significant at the 5% level of significance.

We first test for the rank of the $\overline{\Theta}$ matrix in our demand system using test statistics T_1 (equation 12) and T_2 (equation 13). The two tests being asymptotically equivalent, we present only the results from the T_2 test in table 2. The T_2 test procedure is implemented in the following manner. For rank of 1 to rank of 14, T_2 test statistic was generated with complete pivoting. From any rank the lowest T_2 test statistic is selected as final test statistics. At the 5% level of significance we reject the null hypothesis that the rank of the $\overline{\Theta}$ matrix is less than or equal to 7. However, we fail to reject the same hypothesis with $\text{rank}[\overline{\Theta}] = 8$. Given $m = \overline{\Theta} + 1$, we conclude that there are 9 significant latent characteristics defining the CSD market. So, subsequent analysis using latent separability assumes $m = 9$. Under BR exclusivity assumptions, complete pivoting suggests the following brands to be exclusive in the 9 latent groups: diet Pepsi, Diet All-Other, regular Coke, regular Dr. Pepper, Regular Mt. Dew, regular Pepsi, regular R.C. Cola, regular Private label, and regular All-Other. They include 2 diet drinks and 7 regular brands. This result is consistent with the common notion of extensive segmentation in the CSD market. Our estimated latent groupings encompasses all the unique brands in the market. They include: only-cola drinks (e.g. diet Pepsi, regular Coke, regular Pepsi, regular R.C. Cola), teen oriented cola drink (e.g. regular Dr. Pepper), teen oriented clear drink (e.g. regular Mt. Dew), budget CSDs (e.g. regular Private label), and combinations of orange, cherry and other flavored CSDs (e.g. diet and regular All-Others). Our analysis also suggest that except for the diet characteristics most of the diet drinks can be expressed as extension of regular drinks in latent characteristics; in other word diet drinks are brand extensions of regular drinks.

Under CR exclusivity assumption, our analysis provides a framework to estimate the shadow prices of the latent goods. Table 3 presents the estimated Π matrix. And table 4 presents estimated means and standard errors of the price aggregator (e.g. $b^k(p)$) These estimates can also be interpreted as the price of the latent variables. As expected these prices are all positive and significant. The highest price is on latent characteristics 3 (1.15) and regular Coke is the unique variable associated with this variable. And the lowest latent variable price is associated with regular All-Other (0.998).

Based on $\text{rank}[\overline{\Theta}]$ of 8 and using equation (8) estimate restricted parameter estimates of the demand system under homothetic latent separability. Restricted estimates can be expressed as:

$$(24) \quad B_R = B_U - VR'(RVR')^{-1}RB_U, \text{ where } R = [I_{(K+n-2)(n-m)} \quad -\Phi^T \otimes I_{K+n-2}]$$

and B_R and B_U are the restricted and unrestricted parameters estimates. The variance of the B_R is expressed as:

$$(25) \quad V_R = [V_U - V_U R'(RV_U R')^{-1}RV_U]$$

To calculate efficiency gain from imposing latent separability restrictions we use absolute percentage deviation (APD) between restricted (B_R) and unrestricted (B_U) variance estimates. The APD is defined as:

$$(26) \quad APD_{UR} = \frac{[100 * |\vartheta_U - \vartheta_R|]}{[0.5 * |\vartheta_U + \vartheta_R|]}$$

where ϑ_U and ϑ_R are respectively the unrestricted and restricted variance estimates. We find large efficiency gains from imposing latent separability. On average, the variance APD gain is 363%.³ At the the same time the mean absolute differences between restricted and unrestricted parameter estimates is 0.04%. This implies that, while parameter estimates do not change much under latent separability restrictions, the precision in their estimates improves a lot. As latent separability restrictions reduce significantly the adverse effects of multicollinearity on the estimates of demand elasticities.

Table 5 presents restricted and unrestricted expenditure estimates. From the estimated standard errors it is clear that there are large and significant efficiency gains. In the case of expenditure elasticity average *APD* gain is 119%. Again, to compare the restricted and unrestricted estimates, we estimate mean absolute differences (0.084) and standard errors (0.3). In terms of unrestricted expenditure elasticity, Diet Dr. Pepper gives the highest elasticity (2.18) and regular Dr. Pepper the lowest (0.41). And from restricted expenditure elasticities, regular Private label has the highest elasticity (1.81) and regular Dr. Pepper the lowest (0.34).

In table 6 and 7 we present our estimated price elasticities. We find significant efficiency gain: the average variance *APD* gain is 110%. And restricted and unrestricted parameter estimates exhibit a mean difference of 0.01. Again, while the estimates of price effects do not much under latent separability restrictions, the precision in their estimates improves a lot. As expected we find own price elasticities of all the brands to be negative and significant at the 5% level. From the restricted estimates, of the estimated 240 cross price elasticities, 84 are significant at the 5% level of significance. In terms of cross price elasticities between regular and diet brands, we do find strong evidence of significant cross price effects between diet and non-diet drinks. Note that such significant relationships arise in situations of latent separability. This can be seen as an improved characterization of consumer behavior going beyond more traditional weak separability assumptions (e.g., diet versus non-diet brands).

7 Concluding Remarks

This paper explored relationships between latent characteristics and models of consumer behavior. Following Blundell and Robin (BR), we develop methodologies to apply the concept of latent separability using brand-level demand specifications. The application has the potential to solve some of the significant shortcomings of consumer demand specifications. First, it minimizes the dimensionality problem associated price elasticities with a large number of brands. Second, it decreases multicollinearity problems.

In the empirical section of this paper we estimate the number of latent variables in the CSD market using a Q-AIDS demand specification. This is done by testing for the rank of a parameter matrix. It identifies the presence of 9 significant latent variables underlying the CSD market. The analysis also generates the exclusive brands associated with the 9 latent variables. This selection of exclusive brands seems reasonable based on anecdotal and other a priori information. We also show how using the concept of latent separability dramatically improves efficiency of the parameter estimates. We achieve significant efficiency gain in estimated elasticities.

While this paper illustrates how latent separability can generate useful insights in demand analysis and in the economics of differentiated products. Future research is needed to examine further the economic implications of latent variables for strategic firm behavior.

8 References

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Notes

¹Note that this can handle the case where other factors (besides prices and income) affect consumption behavior. Let $z = (1, z_1, z_2, \dots)^T$ be a column vector representing these factors (e.g., socio-demographic or strategic variables). They can be introduced in the model by allowing (α, β, τ) to become $(\alpha z, \beta z, \tau z)$, and $(\tilde{\alpha}, \tilde{\beta}, \tilde{\tau})$ to become $(\tilde{\alpha} z, \tilde{\beta} z, \tilde{\tau} z)$, except that $\alpha, \beta, \tau, \tilde{\alpha}, \tilde{\beta}$ and $\tilde{\tau}$ are now matrices (instead of vectors obtained when $z = 1$). This lets consumption behavior depend on the factors z . In this situation, our analysis still holds: it simply increases K , the number of rows of either $B \equiv (\alpha, \beta, \tau)^T$ or $\tilde{B} \equiv (\tilde{\alpha}, \tilde{\beta}, \tilde{\tau})^T$.

²In BR to avoid singularity of the matrix they suggest the use of specific design matrix to get rid of elements of matrix causing singularity. Our approach is much simpler to implement, although there might be a slight loss of mathematic precision.

³Details of the variance estimates are not presented in the paper due space constraints. These estimates can be obtained from the authors on request.

Table 1. Descriptive Statistics of Variables Used in the Econometric Analysis

Brands	Price		Expenditure Share		Unit per Volume		% Price Reduction		% Merchandising	
	Mean	Std. err	Mean	Std. err	Mean	Std. err	Mean	std. err	Mean	std. err
Diet Pepsi	3.77	0.36	6.64	1.93	2.18	0.33	26.88	6.58	80.88	8.10
Diet Coke	3.76	0.32	9.98	1.85	2.20	0.42	26.67	6.73	81.45	7.78
Diet 7-Up	3.85	0.41	1.46	0.57	2.52	0.36	25.67	6.95	64.11	14.00
Diet Sprite	3.71	0.37	0.87	0.30	2.44	0.34	26.36	7.29	69.15	12.69
Diet Dr. Pepper	4.12	0.57	0.78	0.66	2.44	0.33	24.19	7.13	55.10	17.08
Diet Private Label	2.36	0.32	0.69	0.54	5.70	2.89	21.35	9.10	47.64	20.30
Diet All-Other	3.72	0.30	31.27	3.36	2.93	0.50	26.01	5.73	69.04	8.34
Reg. 7-Up	3.74	0.40	2.35	1.11	2.49	0.27	25.93	6.99	69.18	12.98
Reg. Coke	3.71	0.31	12.52	4.86	2.20	0.36	27.45	6.83	84.04	7.14
Reg. Dr. Pepper	3.97	0.47	2.08	2.04	2.29	0.32	24.71	7.12	63.80	17.86
Reg. Mt. Dew	3.88	0.41	1.63	1.10	2.23	0.35	25.70	6.61	72.50	13.02
Reg. Pepsi	3.65	0.37	11.50	3.16	2.19	0.35	27.11	6.66	84.08	7.46
Reg. RC Cola	3.33	0.45	0.89	0.73	2.46	0.38	22.15	7.48	63.73	21.16
Reg. Sprite	3.63	0.33	2.01	0.75	2.29	0.30	27.47	7.03	80.11	9.07
Reg. Private Label	2.34	0.27	3.90	2.45	5.61	2.22	21.33	6.91	54.59	18.12
Reg. All-Other	3.56	0.40	11.41	3.11	3.60	0.90	23.62	4.96	58.90	9.90

Mean Values of the other Explanatory Variables

Variables	Units	Mean	std.err
Median Age (<i>Demand Shift Variable - [Z_{it}]</i>)	Years	33.2	2.4
Median HH Size (<i>Demand Shift Variable - [Z_{it}]</i>)	#	2.6	0.1
% of HH less than \$10k Income (<i>Demand Shift Variable - [Z_{it}]</i>)	%	15	3.3
% of HH more than \$50k Income (<i>Demand Shift Variable - [Z_{it}]</i>)	%	24.2	6.5
Supermarket to Grocery Sales ratio (<i>Demand Shift Variable - [Z_{it}]</i>)	%	75.8	5.7
Percentage of Hispanic Population (<i>Demand Shift Variable - [Z_{it}]</i>)	%	7.2	9.6
Concentration Ratio (<i>Price Function: CR⁴_{it}</i>)	%	64.7	13.1
Per Capita Expenditure (<i>M_{it}</i>)	\$	5.91	1.22
Median Income (<i>Expenditure Function: INC_{it}</i>)	\$	32,353	7,130

Table 2: Rank Test

Rank	Test statistics	Degrees of Freedom	P-Value
1	2655.08	413	0.0000
2	2115.48	377	0.0000
3	1467.00	342	0.0000
4	1067.55	308	0.0000
5	720.36	275	0.0000
6	399.06	243	0.0000
7	228.40	212	0.2093
8	122.42	182	0.9998
9	84.87	153	1.0000
10	42.47	125	1.0000
11	27.34	98	1.0000
12	18.28	72	1.0000
13	5.25	47	1.0000
14	1.08	23	1.0000

Table 3: Estimated II Matrix

Brands	<u>[BR1]</u>	<u>[BR7]</u>	<u>[BR9]</u>	<u>[BR10]</u>	<u>[BR11]</u>	<u>[BR12]</u>	<u>[BR13]</u>	<u>[BR15]</u>	<u>[BR16]</u>	[BR2]	[BR3]	[BR4]	[BR5]	[BR6]	[BR8]	[BR13]	[BR14]	Agg
Diet Pepsi	3.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.20	0.00	0.25	0.00	0.25	0.61	-0.59	-0.83	-1.32
<u>[BR1]</u>										<i>0.19</i>	<i>0.14</i>	<i>0.09</i>	<i>0.18</i>	<i>0.18</i>	<i>0.14</i>	<i>0.43</i>	<i>0.16</i>	<i>0.60</i>
Diet All-Other	0.00	-2.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.56	0.37	0.43	0.37	0.43	0.58	0.66	0.91	0.39
<u>[BR7]</u>										<i>0.07</i>	<i>0.06</i>	<i>0.04</i>	<i>0.07</i>	<i>0.09</i>	<i>0.06</i>	<i>0.19</i>	<i>0.08</i>	<i>0.26</i>
Reg. Coke	0.00	0.00	6.43	0.00	0.00	0.00	0.00	0.00	0.00	-1.16	-0.41	-0.68	-0.41	-0.68	-0.83	-0.89	-0.95	-0.50
<u>[BR9]</u>										<i>0.06</i>	<i>0.05</i>	<i>0.04</i>	<i>0.06</i>	<i>0.07</i>	<i>0.05</i>	<i>0.15</i>	<i>0.05</i>	<i>0.20</i>
Reg. Dr. Pepper	0.00	0.00	0.00	3.62	0.00	0.00	0.00	0.00	0.00	-0.35	0.01	0.60	0.01	0.60	-0.25	-0.03	-1.13	-1.46
<u>[BR10]</u>										<i>0.13</i>	<i>0.09</i>	<i>0.07</i>	<i>0.13</i>	<i>0.15</i>	<i>0.10</i>	<i>0.34</i>	<i>0.14</i>	<i>0.47</i>
Reg. Mt. Dew	0.00	0.00	0.00	0.00	1.47	0.00	0.00	0.00	0.00	-0.36	0.10	-0.27	0.10	-0.27	-0.10	-0.37	0.26	0.27
<u>[BR11]</u>										<i>0.05</i>	<i>0.04</i>	<i>0.03</i>	<i>0.04</i>	<i>0.05</i>	<i>0.04</i>	<i>0.12</i>	<i>0.05</i>	<i>0.16</i>
Reg. Pepsi	0.00	0.00	0.00	0.00	0.00	2.24	0.00	0.00	0.00	-0.35	-0.11	-0.33	-0.11	-0.33	-0.45	-0.13	-0.23	0.36
<u>[BR12]</u>										<i>0.05</i>	<i>0.04</i>	<i>0.03</i>	<i>0.05</i>	<i>0.06</i>	<i>0.04</i>	<i>0.12</i>	<i>0.05</i>	<i>0.17</i>
Reg. RC Cola	0.00	0.00	0.00	0.00	0.00	0.00	0.47	0.00	0.00	0.08	0.04	0.15	0.04	0.15	0.02	0.14	0.09	0.02
<u>[BR13]</u>										<i>0.08</i>	<i>0.06</i>	<i>0.04</i>	<i>0.07</i>	<i>0.09</i>	<i>0.07</i>	<i>0.19</i>	<i>0.09</i>	<i>0.27</i>
Reg. Private Label	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.84	0.00	-0.13	-0.08	0.05	-0.08	0.05	-0.02	-0.18	0.12	0.40
<u>[BR15]</u>										<i>0.27</i>	<i>0.20</i>	<i>0.14</i>	<i>0.26</i>	<i>0.29</i>	<i>0.21</i>	<i>0.65</i>	<i>0.26</i>	<i>0.90</i>
Reg. All-Other	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.40	0.00	0.01	0.10	0.01	0.10	0.01	-0.07	0.15	0.41
<u>[BR16]</u>										<i>0.33</i>	<i>0.25</i>	<i>0.17</i>	<i>0.32</i>	<i>0.37</i>	<i>0.26</i>	<i>0.81</i>	<i>0.33</i>	<i>1.13</i>

* Reg.: Implies non-diet.

*Underlined numbers are the estimates and the italicized numbers are the standard errors.

*% Price changes are by column and % quantity changes are by rows; highlighted numbers are significant at 5% level

Table 4: Estimated latent Price Aggregators

Latent Variable	Unique Brands	Mean	std.Err
1.00	Diet Pepsi	1.024	0.24
2.00	Diet All-Other	1.007	0.21
3.00	Reg. Coke	1.151	0.72
4.00	Reg. Dr. Pepper	1.061	0.38
5.00	Reg. Mt. Dew	1.001	0.16
6.00	Reg. Pepsi	1.023	0.28
7.00	Reg. RC Cola	0.999	0.08
8.00	Reg. Private Label	0.999	0.10
9.00	Reg. All-Other	0.998	0.08

Table 5: Expenditure Elasticity

Brands	Unrestricted	Restricted	APD
Diet Pepsi	1.10	1.05	
	<u>0.09</u>	<u>0.01</u>	<i>178.17</i>
Diet Coke	0.91	0.75	
	<u>0.08</u>	<u>0.01</u>	<i>166.05</i>
Diet 7-Up	0.54	0.73	
	<u>0.15</u>	<u>0.01</u>	<i>178.49</i>
Diet Sprite	0.87	0.95	
	<u>0.13</u>	<u>0.32</u>	<i>86.09</i>
Diet Dr. Pepper	2.18	1.49	
	<u>0.38</u>	<u>0.31</u>	<i>19.90</i>
Diet Private Label	1.13	0.45	
	<u>0.28</u>	<u>0.21</u>	<i>28.65</i>
Diet All-Other	0.90	0.91	
	<u>0.04</u>	<u>0.00</u>	<i>187.84</i>
Reg. 7-Up	0.90	0.76	
	<u>0.12</u>	<u>0.04</u>	<i>104.46</i>
Reg. Coke	1.21	1.23	
	<u>0.11</u>	<u>0.00</u>	<i>197.01</i>
Reg. Dr. Pepper	0.41	0.34	
	<u>0.32</u>	<u>0.03</u>	<i>162.36</i>
Reg. Mt. Dew	1.36	1.18	
	<u>0.25</u>	<u>0.05</u>	<i>139.64</i>
Reg. Pepsi	1.16	1.18	
	<u>0.11</u>	<u>0.02</u>	<i>135.89</i>
Reg. RC Cola	1.12	0.73	
	<u>0.39</u>	<u>0.32</u>	<i>18.11</i>
Reg. Sprite	1.09	1.58	
	<u>0.13</u>	<u>0.06</u>	<i>79.74</i>
Reg. Private Label	1.75	1.81	
	<u>0.25</u>	<u>0.03</u>	<i>161.02</i>
Reg. All-Other	0.68	0.85	
	<u>0.11</u>	<u>0.05</u>	<i>72.22</i>

* Reg.: Implies non-diet.

*Underlined numbers are the estimates and the italicized numbers are the standard errors.

*Highlighted numbers are significant at 5

Table 6: Elasticity Matrix - Unrestricted

Brands	[BR1]	[BR2]	[BR3]	[BR4]	[BR5]	[BR6]	[BR7]	[BR8]	[BR9]	[BR10]	[BR11]	[BR12]	[BR13]	[BR14]	[BR15]	[BR16]
Diet Pepsi	-2.83	1.04	0.21	0.17	0.11	0.13	-1.60	0.49	1.28	0.81	0.06	-1.74	0.09	0.26	0.03	0.39
<i>[BR1]</i>	<u>0.34</u>	<u>0.22</u>	<u>0.55</u>	<u>0.58</u>	<u>0.83</u>	<u>0.95</u>	<u>0.12</u>	<u>0.48</u>	<u>0.21</u>	<u>0.61</u>	<u>0.76</u>	<u>0.26</u>	<u>1.19</u>	<u>0.52</u>	<u>0.29</u>	<u>0.14</u>
Diet Coke	0.71	-3.36	0.10	0.00	0.17	-0.26	-1.69	0.41	0.44	0.32	-0.14	1.25	0.45	0.03	0.69	-0.02
<i>[BR2]</i>	<u>0.33</u>	<u>0.44</u>	<u>0.77</u>	<u>0.87</u>	<u>1.27</u>	<u>1.41</u>	<u>0.15</u>	<u>0.59</u>	<u>0.31</u>	<u>0.75</u>	<u>0.97</u>	<u>0.30</u>	<u>1.48</u>	<u>0.72</u>	<u>0.43</u>	<u>0.19</u>
Diet 7-Up	0.98	0.68	-2.53	0.01	-0.26	-0.28	0.35	-0.71	1.10	0.61	0.39	0.22	-0.32	-0.13	-0.10	-0.54
<i>[BR3]</i>	<u>0.12</u>	<u>0.11</u>	<u>0.42</u>	<u>0.34</u>	<u>0.44</u>	<u>0.43</u>	<u>0.05</u>	<u>0.28</u>	<u>0.08</u>	<u>0.24</u>	<u>0.35</u>	<u>0.10</u>	<u>0.53</u>	<u>0.25</u>	<u>0.12</u>	<u>0.06</u>
Diet Sprite	1.34	-0.01	0.01	-4.61	0.24	0.51	5.26	0.39	-0.11	-0.26	-1.73	-0.42	-0.65	-1.28	-0.33	0.77
<i>[BR4]</i>	<u>0.08</u>	<u>0.08</u>	<u>0.20</u>	<u>0.46</u>	<u>0.30</u>	<u>0.33</u>	<u>0.03</u>	<u>0.15</u>	<u>0.06</u>	<u>0.17</u>	<u>0.20</u>	<u>0.06</u>	<u>0.30</u>	<u>0.20</u>	<u>0.08</u>	<u>0.03</u>
Diet Dr. Pepper	0.84	2.10	-0.51	0.25	-4.65	-0.63	1.48	-0.43	1.67	-0.63	0.21	0.45	0.02	0.66	-0.98	-2.01
<i>[BR5]</i>	<u>0.10</u>	<u>0.10</u>	<u>0.23</u>	<u>0.27</u>	<u>0.60</u>	<u>0.46</u>	<u>0.05</u>	<u>0.17</u>	<u>0.08</u>	<u>0.33</u>	<u>0.33</u>	<u>0.09</u>	<u>0.49</u>	<u>0.25</u>	<u>0.14</u>	<u>0.06</u>
Diet Private Label	1.23	-3.71	-0.61	0.64	-0.70	-4.84	4.01	-0.08	1.61	1.64	-2.59	-1.51	-0.56	1.06	0.30	2.97
<i>[BR6]</i>	<u>0.10</u>	<u>0.10</u>	<u>0.20</u>	<u>0.26</u>	<u>0.41</u>	<u>0.77</u>	<u>0.04</u>	<u>0.16</u>	<u>0.08</u>	<u>0.24</u>	<u>0.31</u>	<u>0.09</u>	<u>0.49</u>	<u>0.25</u>	<u>0.14</u>	<u>0.05</u>
Diet All-Other	-0.33	-0.54	0.01	0.15	0.05	0.09	-2.69	0.27	0.91	0.05	-0.11	0.84	-0.08	0.26	0.24	-0.01
<i>[BR7]</i>	<u>0.56</u>	<u>0.46</u>	<u>1.01</u>	<u>1.20</u>	<u>1.81</u>	<u>1.97</u>	<u>0.36</u>	<u>0.95</u>	<u>0.51</u>	<u>1.43</u>	<u>1.88</u>	<u>0.60</u>	<u>2.61</u>	<u>1.13</u>	<u>0.74</u>	<u>0.32</u>
Reg. 7-Up	1.41	1.72	-0.45	0.14	-0.13	-0.02	3.61	-3.27	-1.38	-0.50	0.88	-1.85	-0.03	-0.20	-0.43	-0.39
<i>[BR8]</i>	<u>0.17</u>	<u>0.14</u>	<u>0.46</u>	<u>0.40</u>	<u>0.53</u>	<u>0.53</u>	<u>0.07</u>	<u>0.37</u>	<u>0.12</u>	<u>0.36</u>	<u>0.49</u>	<u>0.14</u>	<u>0.71</u>	<u>0.32</u>	<u>0.18</u>	<u>0.09</u>
Reg. Coke	0.67	0.32	0.12	-0.01	0.11	0.09	2.18	-0.27	-5.50	0.32	0.58	0.44	0.06	-0.23	-0.11	0.02
<i>[BR9]</i>	<u>0.39</u>	<u>0.40</u>	<u>0.72</u>	<u>0.83</u>	<u>1.34</u>	<u>1.47</u>	<u>0.20</u>	<u>0.63</u>	<u>0.53</u>	<u>1.13</u>	<u>1.32</u>	<u>0.42</u>	<u>1.70</u>	<u>0.75</u>	<u>0.59</u>	<u>0.31</u>
Reg. Dr. Pepper	2.62	1.57	0.43	-0.10	-0.22	0.55	0.90	-0.55	2.03	-4.78	-1.59	0.10	-0.35	0.07	-1.58	0.48
<i>[BR10]</i>	<u>0.19</u>	<u>0.16</u>	<u>0.34</u>	<u>0.41</u>	<u>0.89</u>	<u>0.71</u>	<u>0.10</u>	<u>0.32</u>	<u>0.19</u>	<u>1.03</u>	<u>0.72</u>	<u>0.22</u>	<u>1.00</u>	<u>0.35</u>	<u>0.37</u>	<u>0.17</u>
Reg. Mt. Dew	0.23	-0.90	0.34	-0.93	0.10	-1.11	-2.29	1.26	4.40	-2.05	-7.77	5.51	1.22	-1.24	2.02	-0.16
<i>[BR11]</i>	<u>0.19</u>	<u>0.16</u>	<u>0.38</u>	<u>0.38</u>	<u>0.70</u>	<u>0.73</u>	<u>0.10</u>	<u>0.34</u>	<u>0.17</u>	<u>0.56</u>	<u>1.06</u>	<u>0.22</u>	<u>1.10</u>	<u>0.40</u>	<u>0.24</u>	<u>0.12</u>
Reg. Pepsi	-1.01	1.06	0.02	-0.03	0.04	-0.09	2.20	-0.39	0.48	0.00	0.78	-4.30	0.13	0.26	-0.67	0.36
<i>[BR12]</i>	<u>0.46</u>	<u>0.35</u>	<u>0.77</u>	<u>0.83</u>	<u>1.36</u>	<u>1.56</u>	<u>0.22</u>	<u>0.68</u>	<u>0.39</u>	<u>1.22</u>	<u>1.56</u>	<u>0.62</u>	<u>2.03</u>	<u>0.72</u>	<u>0.66</u>	<u>0.34</u>
Reg. RC Cola	0.69	5.02	-0.53	-0.64	0.02	-0.44	-2.88	-0.08	0.78	-0.83	2.24	1.71	-10.05	-0.24	-0.06	4.17
<i>[BR13]</i>	<u>0.16</u>	<u>0.13</u>	<u>0.33</u>	<u>0.31</u>	<u>0.57</u>	<u>0.63</u>	<u>0.07</u>	<u>0.27</u>	<u>0.12</u>	<u>0.43</u>	<u>0.60</u>	<u>0.16</u>	<u>1.43</u>	<u>0.30</u>	<u>0.20</u>	<u>0.12</u>
Reg. Sprite	0.84	0.14	-0.10	-0.55	0.26	0.36	4.03	-0.24	-1.43	0.06	-1.00	1.48	-0.11	-5.07	-0.35	0.59
<i>[BR14]</i>	<u>0.16</u>	<u>0.15</u>	<u>0.34</u>	<u>0.46</u>	<u>0.63</u>	<u>0.73</u>	<u>0.07</u>	<u>0.28</u>	<u>0.12</u>	<u>0.34</u>	<u>0.49</u>	<u>0.13</u>	<u>0.68</u>	<u>0.55</u>	<u>0.18</u>	<u>0.08</u>
Reg. Private Label	0.00	1.69	-0.06	-0.08	-0.19	0.05	1.62	-0.28	-0.41	-0.87	0.84	-2.04	-0.02	-0.19	-2.88	1.06
<i>[BR15]</i>	<u>0.17</u>	<u>0.17</u>	<u>0.31</u>	<u>0.34</u>	<u>0.68</u>	<u>0.80</u>	<u>0.09</u>	<u>0.30</u>	<u>0.18</u>	<u>0.69</u>	<u>0.58</u>	<u>0.22</u>	<u>0.89</u>	<u>0.35</u>	<u>0.56</u>	<u>0.17</u>
Reg. All-Other	0.26	0.00	-0.07	0.06	-0.13	0.18	0.04	-0.08	0.09	0.08	-0.01	0.41	0.33	0.11	0.41	-2.37
<i>[BR16]</i>	<u>0.24</u>	<u>0.22</u>	<u>0.46</u>	<u>0.43</u>	<u>0.84</u>	<u>0.87</u>	<u>0.12</u>	<u>0.43</u>	<u>0.29</u>	<u>0.93</u>	<u>0.86</u>	<u>0.34</u>	<u>1.59</u>	<u>0.45</u>	<u>0.50</u>	<u>0.31</u>

* Reg.: Implies non-diet.

*Underlined numbers are the estimates and the italicized numbers are the standard errors.

*% Price changes are by column and % quantity changes are by rows; highlighted numbers are significant at 5% level

Table 7: Price Elasticity with Latent Separability Restrictions

Brands	[BR1]	[BR2]	[BR3]	[BR4]	[BR5]	[BR6]	[BR7]	[BR8]	[BR9]	[BR10]	[BR11]	[BR12]	[BR13]	[BR14]	[BR15]	[BR16]
Diet Pepsi [**]	-3.20	0.62	0.13	0.23	0.14	0.10	-1.05	0.32	1.42	0.51	0.19	-1.63	0.34	0.60	0.13	0.09
<i>[BR1]</i>	<u>0.02</u>	<u>0.01</u>	<u>0.79</u>	<u>1.42</u>	<u>0.32</u>	<u>1.86</u>	<u>0.00</u>	<u>0.24</u>	<u>0.02</u>	<u>0.06</u>	<u>0.12</u>	<u>0.01</u>	<u>0.48</u>	<u>0.05</u>	<u>0.04</u>	<u>0.21</u>
Diet Coke	0.43	-2.88	0.04	0.12	0.11	-0.02	-1.44	0.24	0.31	0.16	-0.04	1.60	0.55	0.11	0.61	-0.67
<i>[BR2]</i>	<u>0.01</u>	<u>0.10</u>	<u>0.27</u>	<u>0.39</u>	<u>0.63</u>	<u>0.66</u>	<u>0.00</u>	<u>0.10</u>	<u>0.00</u>	<u>0.05</u>	<u>0.11</u>	<u>0.00</u>	<u>0.33</u>	<u>0.12</u>	<u>0.06</u>	<u>0.12</u>
Diet 7-Up	0.60	0.27	-1.13	-0.08	-0.11	-0.05	0.01	-0.17	0.73	-0.34	0.09	-0.04	-0.58	-0.01	-0.17	0.25
<i>[BR3]</i>	<u>0.17</u>	<u>0.04</u>	<u>0.26</u>	<u>0.06</u>	<u>0.39</u>	<u>0.03</u>	<u>0.01</u>	<u>0.09</u>	<u>0.02</u>	<u>0.02</u>	<u>0.73</u>	<u>0.08</u>	<u>0.54</u>	<u>0.15</u>	<u>0.08</u>	<u>0.19</u>
Diet Sprite	1.75	1.40	-0.13	-1.84	-0.20	-0.66	4.25	-0.62	-0.60	-0.58	-1.77	-0.18	-0.31	-1.45	0.22	-0.23
<i>[BR4]</i>	<u>0.19</u>	<u>0.03</u>	<u>0.04</u>	<u>0.37</u>	<u>0.56</u>	<u>0.17</u>	<u>0.02</u>	<u>0.29</u>	<u>0.12</u>	<u>0.08</u>	<u>0.12</u>	<u>0.01</u>	<u>0.32</u>	<u>0.05</u>	<u>0.07</u>	<u>0.17</u>
Diet Dr. Pepper	1.20	1.39	-0.21	-0.23	-1.46	0.10	0.77	-0.35	1.55	-1.41	-0.59	-0.34	-0.82	-0.49	-1.52	0.91
<i>[BR5]</i>	<u>0.04</u>	<u>0.05</u>	<u>0.21</u>	<u>0.50</u>	<u>0.57</u>	<u>0.19</u>	<u>0.04</u>	<u>0.20</u>	<u>0.13</u>	<u>0.27</u>	<u>0.20</u>	<u>0.01</u>	<u>1.00</u>	<u>0.03</u>	<u>0.39</u>	<u>0.27</u>
Diet Private Label	1.04	-0.26	-0.11	-0.83	0.12	-2.29	2.50	-0.28	1.85	1.14	-1.39	-0.95	-0.34	-1.07	1.79	-1.39
<i>[BR6]</i>	<u>0.19</u>	<u>0.05</u>	<u>0.01</u>	<u>0.14</u>	<u>0.17</u>	<u>0.65</u>	<u>0.02</u>	<u>0.26</u>	<u>0.12</u>	<u>0.07</u>	<u>0.12</u>	<u>0.01</u>	<u>0.27</u>	<u>0.01</u>	<u>0.08</u>	<u>0.13</u>
Diet All-Other [**]	-0.21	-0.47	0.00	0.12	0.02	0.05	-2.89	0.32	0.95	0.14	-0.12	0.78	-0.23	0.24	0.26	0.15
<i>[BR7]</i>	<u>0.01</u>	<u>0.00</u>	<u>0.27</u>	<u>0.69</u>	<u>1.69</u>	<u>0.92</u>	<u>0.00</u>	<u>0.08</u>	<u>0.02</u>	<u>0.07</u>	<u>0.25</u>	<u>0.01</u>	<u>0.60</u>	<u>0.04</u>	<u>0.03</u>	<u>0.13</u>
Reg. 7-Up	0.92	1.03	-0.11	-0.23	-0.11	-0.08	4.25	-1.95	-1.52	-0.70	0.82	-2.02	0.20	-0.30	-0.78	-0.20
<i>[BR8]</i>	<u>0.09</u>	<u>0.02</u>	<u>0.14</u>	<u>0.78</u>	<u>0.60</u>	<u>0.88</u>	<u>0.01</u>	<u>0.04</u>	<u>0.01</u>	<u>0.01</u>	<u>0.29</u>	<u>0.04</u>	<u>0.52</u>	<u>0.15</u>	<u>0.06</u>	<u>0.07</u>
Reg. Coke [**]	0.74	0.20	0.08	-0.04	0.10	0.10	2.27	-0.30	-5.60	0.31	0.58	0.45	0.13	-0.31	0.01	0.05
<i>[BR9]</i>	<u>0.04</u>	<u>0.01</u>	<u>0.18</u>	<u>1.77</u>	<u>2.08</u>	<u>2.11</u>	<u>0.01</u>	<u>0.05</u>	<u>0.01</u>	<u>0.06</u>	<u>0.04</u>	<u>0.01</u>	<u>0.21</u>	<u>0.07</u>	<u>0.04</u>	<u>0.25</u>
Reg. Dr. Pepper [**]	1.68	0.82	-0.24	-0.24	-0.52	0.38	2.23	-0.78	1.99	-4.56	-1.13	0.36	0.76	-0.33	-1.56	0.81
<i>[BR10]</i>	<u>0.02</u>	<u>0.01</u>	<u>0.03</u>	<u>0.17</u>	<u>0.71</u>	<u>0.22</u>	<u>0.00</u>	<u>0.01</u>	<u>0.01</u>	<u>0.20</u>	<u>0.04</u>	<u>0.00</u>	<u>0.12</u>	<u>0.08</u>	<u>0.04</u>	<u>0.06</u>
Reg. Mt. Dew [**]	0.76	-0.27	0.08	-0.95	-0.28	-0.60	-2.44	1.18	4.49	-1.46	-7.66	5.82	0.54	-2.01	1.74	-0.12
<i>[BR11]</i>	<u>0.03</u>	<u>0.02</u>	<u>0.81</u>	<u>0.20</u>	<u>0.41</u>	<u>0.29</u>	<u>0.01</u>	<u>0.20</u>	<u>0.01</u>	<u>0.03</u>	<u>0.02</u>	<u>0.01</u>	<u>0.04</u>	<u>0.09</u>	<u>0.11</u>	<u>0.12</u>
Reg. Pepsi [**]	-0.95	1.35	-0.01	-0.02	-0.02	-0.06	2.03	-0.42	0.49	0.05	0.82	-4.32	0.14	0.24	-0.67	0.18
<i>[BR12]</i>	<u>0.01</u>	<u>0.00</u>	<u>0.61</u>	<u>0.13</u>	<u>0.09</u>	<u>0.17</u>	<u>0.00</u>	<u>0.21</u>	<u>0.01</u>	<u>0.01</u>	<u>0.06</u>	<u>0.01</u>	<u>0.39</u>	<u>0.01</u>	<u>0.06</u>	<u>0.08</u>
Reg. RC Cola [**]	2.55	6.12	-0.95	-0.30	-0.71	-0.26	-7.95	0.54	1.93	1.75	0.99	1.88	-11.63	0.25	-0.53	5.61
<i>[BR13]</i>	<u>0.07</u>	<u>0.03</u>	<u>0.33</u>	<u>0.29</u>	<u>1.16</u>	<u>0.36</u>	<u>0.02</u>	<u>0.20</u>	<u>0.01</u>	<u>0.05</u>	<u>0.01</u>	<u>0.03</u>	<u>0.51</u>	<u>0.03</u>	<u>0.12</u>	<u>0.13</u>
Reg. Sprite	1.95	0.47	-0.02	-0.63	-0.19	-0.38	3.52	-0.38	-1.96	-0.36	-1.63	1.34	0.10	-2.59	-0.13	-0.69
<i>[BR14]</i>	<u>0.01</u>	<u>0.02</u>	<u>0.20</u>	<u>0.11</u>	<u>0.08</u>	<u>0.02</u>	<u>0.00</u>	<u>0.13</u>	<u>0.01</u>	<u>0.07</u>	<u>0.11</u>	<u>0.00</u>	<u>0.07</u>	<u>0.15</u>	<u>0.01</u>	<u>0.07</u>
Reg. Private Label [**]	0.17	1.45	-0.08	0.04	-0.31	0.31	1.80	-0.50	-0.02	-0.87	0.72	-2.05	-0.13	-0.07	-2.90	0.62
<i>[BR15]</i>	<u>0.03</u>	<u>0.02</u>	<u>0.20</u>	<u>0.31</u>	<u>1.97</u>	<u>0.44</u>	<u>0.00</u>	<u>0.10</u>	<u>0.01</u>	<u>0.08</u>	<u>0.26</u>	<u>0.02</u>	<u>0.52</u>	<u>0.02</u>	<u>0.01</u>	<u>0.14</u>
Reg. All-Other	0.07	-0.60	0.03	-0.02	0.07	-0.09	0.43	-0.04	0.10	0.14	-0.01	0.21	0.44	-0.11	0.25	-1.72
<i>[BR16]</i>	<u>0.35</u>	<u>0.13</u>	<u>1.44</u>	<u>2.30</u>	<u>3.93</u>	<u>2.11</u>	<u>0.05</u>	<u>0.33</u>	<u>0.23</u>	<u>0.35</u>	<u>0.82</u>	<u>0.07</u>	<u>1.64</u>	<u>0.39</u>	<u>0.40</u>	<u>0.45</u>

* Reg.: Implies non-diet.

*Underlined numbers are the estimates and the italicized numbers are the standard errors.

*% Price changes are by column and % quantity changes are by rows; highlighted numbers are significant at 5% level