The Economics of Land-Zoning

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Abstract

Land-use restrictions are frequently applied to separate polluting from non-polluting activities. In contrast to the existing literature, we incorporate spatial and intertemporal aspects of the problem simultaneously and determine the border of the zones endogenously. The results, based on a two-stage optimization method, show that non-spatially differentiated Pigouvian taxes on the final emissions are able to establish the socially optimal outcome. Second-best instruments alone, such as a spatially differentiated tax on inputs or outputs are not able to support the socially optimal outcome and need to be complemented by land-zoning or land-use taxes. We compare the efficiency of different spatial environmental policies such as land-use taxes or land zoning. The necessary changes required to transform a spatially optimal, yet static, environmental policy into an intertemporally and spatially optimal environmental policy are presented and discussed.

Key words: land-use taxes, zoning, land allocation, water pollution, optimal control.

JEL Classification: C61, H23, Q24, R38 and R52

1 Introduction

Economists have long attempted to explain the value of existing spatial structure and the development pattern of a city or region. Economic theory has identified market failures, such as externalities, which call for the intervention of authorities at the local, regional or national level in the form of land-use control (Anas, Arnott and Small, 1998). This is motivated by incompatible land uses (e. g., noise, odor, traffic, or contamination of bodies of water); preservation of open space (e. g., parks and greenbelts); and the notion that land-owners' property taxes should cover the cost of public goods provided in their area (O'Sullivan, 1993). The most common tool to achieve these ends is land-use zoning which has been frequently applied to separate polluting from non-polluting activities. Incorporating space in economic analysis not only introduces transportation cost but also the process of diffusion or dispersion of the pollutant. As a result monetary damage may decrease with distance from the source of the pollution, or the vulnerability of the environment may differ from one location to another.

In the existing literature, zones are often exogenously determined. However, economic analysis focuses on the economic consequences of a given set of zoning policies on variables of interest, e. g., property values, pollution or traffic congestion. With to spatial pollution, the separation of two conflicting land uses is taken as given, and a single zone for each type of land use is assumed. To account for pollution externalities, spatially differentiated Pigouvian taxes were proposed at an equivalent rate of the marginal damage from emissions at the location (Tietenberg, 1995), (Hochman and Ofek, 1979) and (Hochman, Pines and Zilberman, 1977). Exogenous zoning, however, explains neither the zoning process itself nor the optimal location and size of the zone. Endogenous zoning takes these aspects into account by analyzing the influence of various pressure groups on the outcome of the zoning process (Pogodzinski and Sass, 1994) and (Epple, Romer and Filimon, 1988). However, the second aspect of endogenous zoning, the optimal location and size of the zone, has received little attention in the literature with respect to pollution control.

Within an endogenous zoning framework, Tomasi and Weise (1994) demonstrate that spatial differentiated Pigouvian taxes on emissions are generally not sufficient to ensure that a competitive equilibrium is Pareto efficient. Taxes need to be complemented by land-use restrictions. Hochman and Rausser (1997) confirm the case for multiple zones (Tomasi and Weise, 1994), where the number and border of zones were determined endogenously.

As with the papers by Tomasi and Weise (1994) and Hochman and Rausser (1997), this paper considers land-use restrictions as an endogenous instrument for controlling pollution. However, we take into account that pollutants may accumulate over time. e. g., **e. g., acidic depositions in form of acid rain resulting from the emissions of SO_2 and NO_x,** or the contamination of water with nitrate or phosphorus. This extension is motivated by the fact that even an optimal spatial allocation of land and optimal use of inputs support a long-run equilibrium only by chance; thus, policies addressing them need to be designed within a spatial intertemporal framework. We not only analyze land use policies in the context of land zoning but also elaborate on land-use taxes as a more efficient instrument to achieve the optimal allocation of land. Despite this, neither land use taxes nor land zoning alone is sufficient to establish a socially optimal outcome. A non-spatially differentiated Pigouvian tax on final emissions is the only single instrument to achieve a socially optimal outcome. Second-best instruments, such as taxes on input or output, need to be complemented by land use taxes or land zoning to achieve a socially optimal outcome. Only if the marginal final emissions stay constant with changes in input or output, spatially differentiated Pigouvian taxes on input or output are efficient and need not be complemented by a land control policy.

A very essential part of this paper consists of the novel approach in presenting a two-stage solution to a spatial-intertemporal optimal control problem. The novelty is obtaining a two-stage solution, allowing to derive the qualitative characteristics of the solution better and more easily, than a single-stage solution. The first stage consists of the optimal spatial allocation, and the second stage is comprised of the intertemporal optimization of the solution from the first stage. The two stages are linked by the common shadow price, allowing a relationship to form between optimal short-run and long-run input demand and a land demand function. Most importantly, it allows the necessary changes needed to transform a spatially optimal, yet static, environmental policy analysis to an intertemporally and spatially optimal environmental policy.

2 Land Allocation over Space and Time

2.1 Conceptual Framework

We assume that the region emerges on a line starting at point 0 which corresponds to the urban center, and terminates at point $\bar{\alpha}$, the region's border. The entire economic transaction of the region takes place at the urban center where the spatial extension has collapsed on a point because it is considerable small relative to the region's area. The land in the region has one potential economic use: as a location for different industrial activities. We assume that labor supply is perfectly elastic, and that the different industrial activities operate in a competitive system, where the market share is relatively small, so that product and factor prices are given exogenously. The industrial activities, however, cause pollutants to accumulate in surface water (coast, lake, or river) and/or ground water, leading to final emissions at the urban center. We concentrate on final emissions instead of emissions at the source because some of the emissions of firms will be lost (absorbed, decomposed, volatilized) before they reach the area of concern. The priority of consumers living in the city compared those living outside the city is based on our assumption that far more consumers live in the city than in the region. Hence, we concentrate on consumers in the city. The left and center part of Figure 1 illustrate two possible geographical schemes where the urban center and the nearby surface waters map on the starting point ($\alpha = 0$) and the region's border maps on the terminal point ($\alpha = \bar{\alpha}$) of the one-dimensional space line of the right part of Figure 1.[1]

Figure 1

[1] For illustrative purposes, space is presented in two dimensions in Figure 1, whereas the model analysed in this paper is based on a one-dimensional space model.

Given this situation, we determine the optimal allocation of different industries over space and time. To this end, we assume that a regional planner will maximize the present discounted net benefits of different industries and take into account the monetary losses resulting from the contamination of surface water and ground water.

2.2 The Regional Decision Model

To keep the regional decision model simple and tractable, we concentrate, without loss of generality, on the case where there are only two industries. For example, one can consider the mining industry, tanning industry, paper mills, fruit-growing industry, viniculture, or agriculture. Each industry produces a homogeneous output based on the production function $f(x_i(t,\alpha),\alpha;\beta_i)$, $i=1,2$ where $x_i(t,\alpha)$, $i=1,2$ indicates industry i's vector of inputs employed at calendar time t at location α. We assume that $f(\cdot)$ is twice differentiable in α and x_i, $i=1,2$, and is strictly concave in x_i, $i=1,2$ with $f_{x_i}>0$, $i=1,2$, where a subscript with respect to a variable indicates the partial derivative of the function. The location variable α has a direct impact on the output. The most obvious case is given by agriculture or viniculture. The parameter β_i, $i=1,2$ represents an index of the input productivity. It is assumed that a higher index of productivity, embodied in a more advanced technology, incurs a higher fixed cost[2] denoted by k_i, $i=1,2$. The share of land employed by industry i, $i=1,2$ at location α and time t is denoted by the non-negative term, $\delta_i(t,\alpha)$, $i=1,2$, with $\sum_{i=1}^{2}\delta_i(t,\alpha)\leq 1$. The output and input prices at location α are denoted by $p_i(\alpha)$ and $w_i(\alpha)$, $i=1,2$, respectively. The prices $p_i(\alpha)$ and $w_i(\alpha)$, $i=1,2$, only differ from $p_i(0)$ and $w_i(0)$, $i=1,2$ by the transportation cost. Given this setup, the maximization of net benefits, from the two industries located within the rural industrial region, corresponds to the maximization of the quasirent at the regional level. The regional planner's decision problem (R), is therefore, given by the following

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One way to interpret the assumption of exogenously determined annualized fixed costs is to interpret these as rental costs of the fixed capital inputs .

maximization problem:

$$\max_{x_i(t,\alpha),\delta_i(t,\alpha)} \int_0^T \exp^{-rt} \int_0^{\bar{\alpha}} \sum_{i=1}^{2} \delta_i(t,\alpha)\Big[p_i(\alpha) f(x_i(t,\alpha),\alpha;\beta_i) - k_i - w_i(\alpha) x_i(t,\alpha)\Big] g(\alpha)\,d\alpha \\ - m(s(t))\,dt \tag{R}$$

subject to

$$\frac{ds(t)}{dt} = z(t) - \xi s(t)$$

$$s(0) = s_0 \qquad x_i(t,\alpha) > 0, \quad \delta_i(t,\alpha) \geq 0, i = 1,2, \quad \text{and} \quad 1 - \sum_{i=1}^{2} \delta_i(t,\alpha) \geq 0,$$

where $z(t)$ denotes the aggregate pollution generated in the region and is given by $z(t) = \int_0^{\bar{\alpha}} \sum_{i=1}^{2} \delta_i(t,\alpha)\phi(x_i(t,\alpha),\alpha;\gamma_i) g(\alpha)\,d\alpha$. Analogous to the production function, we introduce the function $\phi(x_i(t,\alpha),\alpha;\gamma_i)$, $i = 1,2$ which presents the final emissions of a pollutant into the body of water at the urban center. The final emissions take into account the firms' emissions of a pollutant, together with their fate, while the pollutant travels into the urban center via water. We assume that $\phi(\cdot)$ is twice differentiable in α and x_i, $i = 1,2$, and is strictly convex in x_i, $i = 1,2$ with $\phi_{x_i} > 0$, $i = 1,2$. The incorporation of a dispersion function, which considers the fate and transport of the pollutant within the region, seems to be more appropriate. However, as mentioned before, we emphasize the economic analysis of the relationship between city and region. The parameter γ_i, $i = 1,2$ presents an index which captures the amount of final emissions in relation to the amount of inputs employed (for instance, a land classification system reflecting the water contamination potential of each location). The function $g(\alpha)$ denotes a density function, with respect to the location α with $\int_0^{\bar{\alpha}} g(\alpha)\,d\alpha = 1$. In the case where α is interpreted as distance, $g(\alpha)$ is uniformly distributed; in the case where α stands for soil quality, $g(\alpha)$ reflects the distribution of soil quality over the region. Parameter r indicates the social discount rate, function $s(t)$ is the amount of the pollutant in the body of water, and function $m(s(t))$ captures the monetary damages resulting from the contamination of the body of water.[3] Finally, we indicate that T denotes the regional planner's end of the planning horizon, and ξ denotes the natural decay of the pollutant in the water.

Utilizing Pontryagin's Maximum Principle, the Hamiltonian H of the regional planner's decision problem is given by

$$H \equiv \int_0^{\bar{\alpha}} \sum_{i=1}^{2} \delta_i(t,\alpha)\Big(p_i(\alpha) f(x_i(t,\alpha),\alpha;\beta_i) - k_i - w_i(\alpha) x_i(t,\alpha)\Big) g(\alpha)\, d\alpha \qquad (1)$$

$$-m(s(t)) - \lambda\Big(\int_0^{\bar{\alpha}} \sum_{i=1}^{2} \delta_i(t,\alpha)\phi(x_i(t,\alpha),\alpha;\gamma_i) g(\alpha)\, d\alpha - \xi s(t)\Big). \qquad (2)$$

To facilitate the interpretation of the costate variable λ, it has been multiplied by minus one. In this way λ has a positive value. To simplify the notation, the arguments t and α of the variables/functions $x_i, \delta_i, \lambda, p_i, w_i, g$, $i = 1,2$ will be suppressed, unless it is required for an unambiguous notation. Taking into account the constraints on the control variables leads to the Lagrangian $\mathcal{L}$ given by $\mathcal{L} \equiv H + \sum_{i=1}^{2}(\zeta_i x_i + \eta_i \delta_i) + \chi(1 - \sum_{i=1}^{2} \delta_i)$ where ζ_i, η_i, χ, $i = 1,2$ denote Lagrange multipliers. A solution of problem (H) has to satisfy

[3] We assume that the consumers' utility function is quasilinear with respect to traded goods and externality. Thus, the optimal level of externality is independent of the consumers' expenditures, and it is possible to derive a utility function which depends only on the externality $s(t)$ (Mas-Colell, Whinston and Green, 1995). To discuss the results of our model in a practical setting, we propose that the derived utility function be represented by the damage function $m(s(t))$. Additionally, we assume that there is no cost to public funds, and lump-sum transfers are available to redistribute income so that Pigouvian taxes are not distortionary (Sandmo, 1995). The assumptions made, with respect to the quasilinearity of the utility function, the existence of costless public funds, and the previously defined final emission function instead of a dispersion function, help to keep the model simple. It also allows us to concentrate our analysis on the optimal land allocation and its implication for regional/environmental policies taking account of space and time.

the following necessary conditions:

$$\mathcal{L}_{x_i} \equiv \delta_i\Big(p_i f_{x_i}(x_i,\alpha;\beta_i) - w_i - \lambda\phi_{x_i}(x_i,\alpha;\gamma_i)\Big)g \qquad \begin{pmatrix} \leq 0; & x_i = 0, i = 1,2 \\ = 0; & x_i > 0, i = 1,2 \end{pmatrix} \quad (3)$$

$$\mathcal{L}_{\delta_i} \equiv \Big(p_i f(x_i,\alpha;\beta_i) - k_i - w_i x_i - \lambda\phi(x_i,\alpha;\gamma_i)\Big)g + \eta_i - \chi = 0, \quad i = 1,2 \quad (4)$$

$$\frac{d\lambda(t)}{dt} = r\lambda + \mathcal{H}_s = \lambda(r+\xi) - m'(s(t)) \quad (5)$$

$$\frac{ds(t)}{dt} = z(t) - \xi s(t) \qquad\qquad s(0) = s_0. \quad (6)$$

The analytical solution of necessary conditions (3) - (6) is complicated; therefore, we propose a two-stage approach to solve this optimal control problem. Moreover, the discussion of necessary conditions (3) - (6) is simplified and discussed in later section s.

3 Two-Stage Approach for the Optimal Control Problem

To obtain an analytical solution for problem (R) more easily, we break it apart. In the first stage we solve the optimal spatial allocation by determining the optimal values of $x_i(\alpha), \delta_i(\alpha)$, $i = 1,2$ over the entire range of α, given a restriction on the value of z, i. e., the amount of the final emissions of a pollutant. The value function of the spatial allocation problem $V(x_i^*(\alpha), \delta_i^*(\alpha); z)$, evaluated at the optimal values, reflects the value of the solution for the spatial allocation problem of the first stage, given a prespecified value of the final emissions z. In the second stage, we solve the *intertemporal* allocation problem of the already-optimized allocation problem. For this purpose, we employ $V(\cdot)$ as a function of the parameter z. Basically, the right-hand side value of the first-stage restriction becomes the decision variable in the second stage. The simultaneous solution of the two stages yields the optimal trajectories of $x_i(t,\alpha), \delta_i(t,\alpha), s(t)$, $i = 1,2$, and the associated costate variable in the second stage.

The two-stage approach for an optimal control problem can, in essence, be characterized by the fact that the decision problem occurs at the microlevel, while this decision affects a state variable at the macrolevel. For this particular reason, the state and costate variables in problem (R) depend only on t, but not on α. However, setting up the problem in a single stage could lead to necessary conditions which are difficult to solve. Therefore, we propose to split the problem into an intertwined spatial and intertemporal allocation problem.

3.1 The Optimal Land Allocation over Space

In the first stage the solution of the regional planner's decision problem is given by the value function defined as:

$$V(z) \equiv \max_{x_i(\alpha),\delta_i(\alpha)} \int_0^{a} \sum_{i=1}^{2} \delta_i(\alpha) \Big[p_i(\alpha) f(x_i(\alpha), \alpha; \beta_i) - k_i - w_i(\alpha) x_i(\alpha) \Big] g(\alpha)\, d\alpha \qquad (R1)$$

subject to

$$z = \int_0^{a} \sum_{i=1}^{2} \delta_i(\alpha) \phi(x_i(\alpha), \alpha; \gamma_i) g(\alpha)\, d\alpha$$

$$x_i(\alpha) > 0, \quad \delta_i(\alpha) > 0, i = 1,2, \quad \text{and} \quad 1 - \sum_{i=1}^{2} \delta_i(\alpha) \geq 0.$$

As before, the argument α of the variables/functions $x_i, \delta_i, p_i, w_i, g$, $i = 1,2$ and the Lagrange multiplier μ, to be introduced later, will be suppressed to simplify the notation, unless it is required for an unambiguous notation.

The Lagrangian for the spatial allocation problem ($R1$) is given by

$$\mathcal{L} \equiv \int_0^{a} \sum_{i=1}^{2} \delta_i \Big(p_i f(x_i, \alpha; \beta_i) - k_i - w_i x_i \Big) g\, d\alpha + \mu \Big(z - \int_0^{a} \sum_{i=1}^{2} \delta_i \phi(x_i, \alpha; \gamma_i) g\, d\alpha \Big) \qquad (7)$$
$$+ \sum_{i=1}^{2} (\zeta_i x_i + \eta_i \delta_i) + \chi (1 - \sum_{i=1}^{2} \delta_i).$$

A solution of problem $(R1)$ has to satisfy the following necessary conditions

$$\mathcal{L}_{x_i} \equiv \delta_i\Big(p_i f_{x_i}(x_i,\alpha;\beta_i) - v_i - \mu\phi_{x_i}(x_i,\alpha;\gamma_i)\Big)g \qquad \begin{pmatrix} \leq 0; & x_i = 0, i = 1,2 \\ = 0; & x_i > 0, i = 1,2 \end{pmatrix} \tag{8}$$

$$\mathcal{L}_{\delta_i} \equiv \Big(p_i f(x_i,\alpha;\beta_i) - k_i - v_i x_i - \mu\phi(x_i,\alpha;\gamma_i)\Big)g \; + \eta_i - \chi = 0, \quad i = 1,2 \tag{9}$$

$$z - \int_0^{\bar{\alpha}} \sum_{i=1}^{2} \delta_i(\alpha)\phi(x_i(\alpha),\alpha;\gamma_i)g(\alpha)\, d\alpha = 0. \tag{10}$$

The Lagrange multiplier μ is interpreted as the shadow cost of the prespecified level of the final emissions z from the entire region, i. e., the range of α, from 0 to $\bar{\alpha}$. Thus, μ, in contrast to variables x_i and δ_i, is constant over α because it is evaluated not at a particular α but over the entire range. The necessary condition (8) indicates that the value of the marginal product, with respect to x_i, equals the sum of the marginal cost of x_i, $i = 1,2$, and the marginal cost of pollution. The next necessary condition, equation (9), shows that the location should be assigned to the industrial activity with the highest quasirent throughout the entire region.[4] In other words, the industry with the higher quasirent has the higher industrial bid rent function. The maximal quasirent for industry i, Π_i^*, however, changes over α; thus, the optimally located industry varies throughout the region.

To analyze changes in Π_i^*, equation (9) is differentiated with respect to α. By utilizing equation (9) and the fact that μ is constant over α, one obtains

$$\Pi_{i_\alpha}^* \equiv \quad \Big[p_i'(\alpha)f(x_i,\alpha;\beta_i) - v_i'(\alpha)x_i\Big] + \Big[p_i(\alpha)f_\alpha(x_i,\alpha;\beta_i) - \mu\phi_\alpha(x_i,\alpha;\gamma_i)\Big] \gtreqless 0. \tag{11}$$

[Transportation Effect] [Site Effect]

Equation (11) shows that the quasirent for each industry changes over space as a result of two factors: (1) the marginal transport cost (the terms in the first square brackets) and

[4] In this paper the term quasirent refers to the socially optimal quasirent.

(2) the value of the marginal product of the site minus the marginal pollution cost of the site (the terms in the second square brackets). Since $p_i'(\alpha) < 0$ and $w_i'(\alpha) \geq 0$, $i = 1,2$, the transportation cost will always increase with α.[5] The sign of the second term in square brackets cannot be determined without further specifying the derivatives of f_α and ϕ_α. If we look at the tanning industry or paper mills, it seems reasonable to assume that $f_\alpha = 0$, i. e., the production is location-insensitive. However, looking at α as an index of soil quality suggests that the production is location-sensitive, $f_\alpha > 0$. As the industry operates farther away from the urban center, or as the soil quality increases, most likely $\phi_\alpha < 0$. Therefore, in the case of location-insensitive production, and recalling that μ is constant over α, the transportation cost will increase with α, while the cost of pollution will decrease. Thus, depending on the magnitude of these diametric effects, the quasirent of the industry may decrease or increase with α. In the case of location-sensitive production and soil quality, the net benefits of a change in location are positive due to the most likely case that $\phi_\alpha < 0$.

Up to now we have not defined the complete properties of the functions $f(\cdot)$ and $\phi(\cdot)$. In particular, we did not order the relative production and pollution intensities of the two industries. Therefore, many land-use patterns may evolve. To structure the problem, we assume that there are pollution-intensive and pollution-extensive industries. In the case of pollution-intensive production, which captures the case of location-insensitive and location-sensitive production, we assume that the transportation effect is dominated by the site effect. In this case, the social quasirent is upward sloping with a change in α, e. g., $\Pi_{i_\alpha}^* > 0$, $i = 1, 2$. The evolving land-use pattern can be established by evaluating the inequality $\Pi_1^*(0) \gtreqless \Pi_2^*(0)$ and $\Pi_1^*(\bar{\alpha}) \gtreqless \Pi_2^*(\bar{\alpha})$. In the case where the order of the inequality of $\Pi_1^*(\alpha)$ and $\Pi_2^*(\alpha)$ is reversed, once evaluated at $\alpha = 0$ and at $\alpha = \bar{\alpha}$, we know that the social quasirent functions intersect.[6] Thus, we have established that it is

[5] It could be assumed that labor is perfectly mobile throughout the rural industrial region implying that $w_i'(\alpha) = 0$, $i = 1, 2$, or it could be assumed that labor is not the dominant tradeable input such that $w_i'(\alpha) => 0$, $i = 1, 2$.

[6] The number of crossings of the quadrents determines endogenously the optimal number of zones within the region. For simplicity of the exposition, we only consider the case where the quadrents of the two industrial activities intersect once. However, for an analytical treatment of this question within a static

optimal to diversify the land use. The switching point in land-use is given by α^* where $\Pi_1^*(\alpha^*) = \Pi_2^*(\alpha^*)$ holds. In the case where the social quasirent function does not intersect, we see that it is optimal not to diversify the land use. An example of diversification is given in Figure 2. The high final emissions of industry one lead, close to the urban center, to a higher social quasirent of industry two. At location $\alpha = \bar{\alpha}$, however, the order of the inequality of $\Pi_1^*(\alpha)$ and $\Pi_2^*(\alpha)$ is reversed.

Figure 2

Figure 2 also illustrates the case where final emissions are so high that the quasirents for both activities ($\Pi_1^{\prime *}$) and ($\Pi_2^{\prime *}$) close to the city are negative. Thus, a buffer zone between the urban center and the industrial sector, from $\alpha = 0$ to $\alpha = \alpha_0$, is established. This buffer zone depicts the case of locally unwanted land use (LULU).

In the case where both industries are pollution extensive, we assume that the site effect (pollution) is dominated by the transportation effect for the location-insensitive and sensitive production, e. g., $\Pi_{i_\alpha}^* < 0$, $i = 1, 2$. This situation coincides with the standard case in location theory where the quasirent decreases with distance from the urban center. As in the previous paragraph, the evaluation of the order of inequality Π_1^* and Π_2^* at the locations $\alpha = 0$ and $\alpha = \bar{\alpha}$ allows determination of whether a mono land-use pattern, or a diversified land-use pattern, evolves. An example of diversified land-use pattern is given in Figure 3. It also depicts the case where the transportation effect is so strong that the social quasirents for both industries, $\Pi_1^{\prime *}$ and $\Pi_2^{\prime *}$, turn negative beyond location α_0. This situation leads to distantly unwanted land use (DULU) for the zone α_0 to $\bar{\alpha}$.

Figure 3

Finally, we consider the case where one industry is pollution intensive and the other is pollution extensive. This classification incorporates the case of location-insensitive and sensitive production. The optimal allocation of land is determined, once again, by evaluating the order of Π_1^* and Π_2^* at locations $\alpha = 0$ and $\alpha = \bar{\alpha}$. An example of this

framework, see a recent paper by (Hochman and Rausser, 1997).

situation is given in Figure 4, where the social quasirent for industry one is upward sloping and, for industry two, downward sloping. Figure 4 also illustrates the case where industry one is pollution intensive, $\Pi_1'^*$, and industry two is pollution extensive with very high transportation cost, $\Pi_2'^*$. As a result, the social quasirent for both industries may be negative between α_0 and α_0', leading to the case of regional unwanted land use (RULU).

Figure 4

The decision variable, δ_i, $i = 1, 2$, can only take on interior values of its domain if (9) vanishes over a positive interval of space $[\alpha_1, \alpha_2]$ of positive length. Hence, the evaluation of equation (11), which is the derivative of equation (9) with respect to α, allows determination of whether there exists a solution where both industrial activities coexist at the same location. According to (11), the marginal transportation cost has to cancel out the marginal net benefits of the site effect over some positive interval of space, to support an interior solution of δ. It seems unlikely that this will be the case for some positive interval of space $[\alpha_1, \alpha_2]$; however, it cannot be discarded theoretically. Only in the case of location-sensitive production with constant transportation cost (marginal transportation costs are zero) can an interior solution be ruled out theoretically, since the marginal net benefit of the site effect is strictly positive over the entire range of α.

3.2 Optimal Spatial Environmental Policy

The different industries will not take into account the pollution of water. To correct this negative production externality, a market intervention is required. However, Pigouvian taxes on the emissions of pollutants were identified as insufficient in previous literature (Henderson, 1977) and (Hochman and Ofek, 1979). Tomasi and Weise (1994), as well as Hochman and Ofek (1979), demonstrated that correct spatial tax must equal the spatial aggregate of marginal damages contributed by each location. For this purpose, Hochman and Ofek (1979) introduced a dispersion function[7] which links the amount of the pollutant at location α_i to the emissions of the pollutant by a particular industry at location α_j.

[7] In the paper by Tomasi and Weise (1994), this equation is called a transition equation.

However, in this paper the damages caused by pollution occur exclusively in the city and not within the region where the industries are located. Therefore, we only need to keep track of the location where the pollution is generated.

Proposition 1 *The optimal non-spatially differentiated Pigouvian tax τ_p on the final emissions of a particular industry located at α is constant. It is given by $\tau_p = \mu$ and establishes the socially optimal allocation of land and the socially optimal utilization of inputs.*

Proof: Facing the industry with the Pigouvian tax τ_p on final emissions, given by ϕ, leads to an optimal private behavior by industry, which coincides with the optimal social behavior of industry. This can be verified directly by looking at equations (8) and (9).

Proposition 1 shows that non-spatially differentiated Pigouvian taxes on final emissions are sufficient to support the optimal social outcome and need not be complemented by an additional instrument, correcting the market outcome on top of the Pigouvian taxes. In the case where it is not possible to observe the individual immisions of each industry located at a particular location, the regional planner has to resort to second-best instruments, such as a tax on input or output.

Proposition 2 *The spatially optimal differentiated input tax $\bar{\tau}_i(\alpha)$, $i = 1,2$ is given by $\bar{\tau}_i(\alpha) = \mu\phi_{x_i}(x_i^*, \alpha; \gamma_i)$, $i = 1,2$, and the spatially optimal differentiated output tax $\hat{\tau}_i(\alpha)$, $i = 1,2$ is given by $\hat{\tau}_i(\alpha) = \mu\phi_{x_i}(x_i^*, \alpha; \gamma_i) / f_{x_i}(x_i^*, \alpha; \beta_i)$, $i = 1,2$.*

Proof: As seen from equation (8), the input tax $\bar{\tau}_i(\alpha)$, $i = 1,2$ establishes the spatially optimal differentiated input choice, x_i, $i = 1,2$, of the industry located at location α, coinciding with the optimal choice of the regional planner. Similarly, the spatial optimum of the output tax can be verified.

However, an input tax requires that the regional planner cannot only observe the total amount of input in the region, but she/he can also distinguish between the input which is employed at each location α. Otherwise, the regional planner will be unable to separate markets and levy the same input with different taxes. Provided that the regional planner cannot separate the input market but can separate the output market, she/he may impose an output tax, a tax on f, which induces the industry to choose the optimal amount of x_i, $i = 1, 2$, from a regional perspective. However, neither the input tax nor the output tax is sufficient to achieve a regional optimum, since these taxes establish only the first-order condition (8), but not equation (9). Therefore, the chosen tax needs to be complemented by an instrument which establishes the efficient allocation of land from the regional planner's perspective as required in equation (9). The optimal allocation of land can be achieved either by zoning or by the imposition of a land-use tax.

Proposition 3 *Provided that the quasirents of two industrial activities are not identical over a positive length of space, both zoning regulations and land-use taxes are able to establish the efficient allocation of land from the regional planner's perspective but differ with respect to their distributional effects.*

Proof: The optimal border of the zone, denoted by α^*, is determined by equation (9). It can be enforced per decree law by authorities who have jurisdiction over the region. Instead of an ordinance, the regional authorities could also impose a spatially differentiated land-use tax σ given by $\sigma_i(\alpha) \equiv \mu\phi(x_i^*, \alpha; \beta_i)$, $i = 1, 2$. In this way the decisions of the industry and the regional planner, with respect to the allocation of the land, are both based on necessary condition (9).

Neither land-use taxes nor zoning alone are sufficient to support a Pareto-efficient outcome. Both require an accompanying tax to establish the optimal intensity of the industry which, in turn, requires knowledge of the functions f and ϕ. However, both instruments differ with respect to their distributional effect. Zoning does not impose cost on industry, whereas land-use taxes do, by following the polluters' pay principle. Therefore, zoning is politically more viable than land-use taxes unless they are implemented through tradeable permits which are initially distributed free of charge.

As shown in the paper by Tomasi and Weise (1994), it is possible to identify conditions where a tax on input or output is sufficient to achieve the regional optimum without a zoning ordinance or the imposition of a land-use tax.

Proposition 4 *If the final emission function is linear in input or output, then an input tax or output tax, respectively, is sufficient to obtain the optimal allocation over space,and no zoning ordinance or land-use tax is required.*

Proof: For an input tax $\bar{\tau}_i$, $i = 1,2$ to satisfy the first-order conditions (8) and (9), the following equations have to hold

$$\frac{\partial(\bar{\tau}_i(x_i,\alpha)\, x_i)}{\partial x_i} = \mu\phi_{x_i}(x_i,\alpha;\gamma_i), \quad i = 1,2 \tag{12}$$

$$\bar{\tau}_i(x_i,\alpha)\, x_i = \mu\phi(x_i,\alpha;\gamma_i), \quad i = 1,2. \tag{13}$$

Calculating the derivative in equation (12) yields $\frac{\partial\tau_i}{\partial x_i}x_i + \tau_i = \lambda\phi_{x_i}$, $i = 1,2$. A solution to this equation is given by: $\bar{\tau}_i(\alpha) = \mu\phi_{x_i}(x_i,\alpha)$, with $\frac{\partial\tau_i}{\partial x_i} = 0$, $i = 1,2$. The integration of the solution over x_i, $i = 1,2$ confirms that equation (13) will also be satisfied. Hence, we can conclude that the final emission function is linear in the input, given by $\phi(x_i,\alpha) = \frac{\tau_i(\alpha)}{\mu}x_i$. A simple example of the input tax can be given by $\bar{\tau}_i = \mu c_i\,\alpha$, where c_i is constant and $\phi = c_i\alpha x_i$, $i = 1,2$. Exactly along the same line of arguments, the validity of proposition 4, with respect to the linearity of the output in the final emission function and the output tax, can be verified.

Proposition 4 turns attention to the special situation where marginal final emissions are constant, with respect to the input or output, and only depend on space. Thus, besides the influence of space, final emissions are determined only by the aggregate of inputs or outputs in the region. Similar results were obtained by Tomasi and Weise (1994) and Hochman and Rausser (1997); however, their results were in terms of the linearity of the emissions in the transition equation and dispersion function, respectively. Yet, the application of an input or output tax, despite whether or not land-use restrictions have to be imposed, requires that markets can be separated spatially by levying the same input or output with different taxes according to the location where the input is used or where the output is produced. When it is not possible to separate markets, the regional planner could use a uniform input or output tax for the entire region. Thus, the efficiency of the input or output tax decreases by the extent the tax varies over the region, i. e., the magnitude of $\phi_{x,\alpha}$ alone, since μ is constant over α. The magnitude itself is an empirical question. The loss of efficiency may be compensated by the reduction of the implementation and administration cost of the uniform tax, compared to the spatially differentiated tax. However, the optimal land allocation can be achieved, independent of the type of the tax, by either land zoning or the imposition of a land-use tax.[6]

As verified by equation (8), the optimal input use will be reduced with the introduction of any of the above proposed taxes. The optimal allocation of the land, however, changes only with the introduction of a tax if $\Pi_i^*(\alpha)$ and $\Pi_j^*(\alpha)$, $i, j = 1, 2$, $i \neq j$ intersect, either before or after the introduction of a tax. In the case where the quasirents of two industries do not intersect, the imposition of a tax has no influence on the optimal land allocation simply because it is already optimally allocated.

3.3 The Optimal Land Allocation over Space and Time

Value function V from the first stage is now employed in the second stage of the regional planner's decision problem: the intertemporal optimal allocation of land. It is given by:

$$\max_{s(t)} \int_0^T \Big(V(z(t)) - m(s(t)\Big) e^{-rt}\, dt + e^{-rT} m(s(T)), \tag{12}$$

subject to

$$\dot{s}(t) = z(t) - \xi s(t), \qquad s(0) = s_0, \qquad z(t) \in Z,$$

[6] Although the regional planner cannot observe the actual values of all arguments of the land-use tax, she/he knows their optimal values and, therefore, a land-use tax is still efficient.

where a dot over a variable denotes the operator $\frac{d}{dt}$. The right-hand side value of the first stage problem z becomes the decision variable in the second stage. It still denotes the final emissions of the entire region into the body of water, however, it now depends on t. The terminal value function is given by $m(s(T))$.[9] The set Z presents the interval $[0, \bar{z}]$, where the upper limit of the set corresponds to the highest possible final emissions. Argument t of all the dynamic variables is dropped to simplify notations whenever possible, without introducing an ambiguous notation. Hence, the Hamiltonian in the second stage $\mathcal{H}$ reads as $\mathcal{H} \equiv V(z) - m(s) - \psi(z - \xi s)$. Note again that a negative sign in front of the costate variable ψ has been introduced to facilitate its interpretation. The necessary conditions[10] for an interior solution $0 < z < \bar{z}$ of problem $(H2)$ are given by

$$\mathcal{H}_z = V_z - \psi = 0 \qquad \Rightarrow \qquad \mu(t) = \psi(t) \tag{14}$$

$$\dot{\psi} = \psi r + \mathcal{H}_s = \psi(r + \xi) + m'(s(t)) \tag{15}$$

$$\dot{s} = z - \xi s\ , \qquad s(0) = s_0, \tag{16}$$

where we made use of the dynamic envelop theorem to obtain the result of equation (14). The transversality condition requires that

$$\psi(T) = m'(s(T)). \tag{17}$$

Hence, the particular solution of differential equation (15) yields

$$\psi(t) = m'(s(T))e^{(r+\xi)(t-T)} + \int_t^T m'(s(\tau))\, d\tau, \tag{18}$$

which states that shadow costs at time t correspond to the integral of the marginal damage from time t to the end of the planning horizon, where the marginal damage at time T is discounted with the social discount rate and the natural decay rate by the remaining

[9] In order to avoid double counting the terminal value function is only included in the intertemporal allocation but not in the spatial allocation problem.

[10] See theorems 1 and 3 in Seierstad and Sydsæter (1987).

time $(T - t)$.

3.4 Impact of Alternative Policies in the Long-Run

Finally, we would like to consider the case $T \to \infty$ to study the comparative statics of the long-run solution to problem $(P2)$. Therefore, we reconsider problem $(P2)$ with $T \to \infty$, and we assume that there exists a steady-state value $\lim_{t\to\infty} s(t) = s^{\infty}$. Necessary condition (14) remains valid, but equations (15) and (16) now define the steady state, and it reads as $\dot{\psi} = \dot{s} = 0$. From equation (14), we obtain, by the using the implicit function theorem, that $\frac{ds}{d\psi} = \frac{1}{V_{ss}} < 0$, where we made use of the fact that $V_{ss} < 0$, as demonstrated in the appendix.

The transversality condition requires that $\lim_{t\to\infty} \psi(t)$ is bounded (Caputo, 1992). A linearization of the canonical system of differential equations around the steady-state values of ψ and s results in

$$\begin{pmatrix} \dot{\psi} \\ \dot{s} \end{pmatrix} = \begin{pmatrix} r+\xi & -m''(s) \\ \frac{1}{V_{ss}} & -\xi \end{pmatrix} \begin{pmatrix} \psi - \psi^{\infty} \\ s - s^{\infty} \end{pmatrix}. \qquad (19)$$

Since the trace of the Jacobian matrix $\bar{J} \equiv \begin{pmatrix} r+\xi & -m''(s) \\ \frac{1}{V_{ss}} & -\xi \end{pmatrix}$ is equal to $r > 0$ and the determinant is equal to $-\xi(r+\xi) + \frac{m''(s)}{V_{ss}} < 0$, one can conclude that the roots are real and have opposite signs. Hence, the steady-state equilibrium can be characterized locally by a saddle point.

Additionally, the entries of $\bar{J}$ allow one to draw the phase diagram in the (s, ψ) space. Figure 5 shows the stable path leading to the steady state is upward sloping while the unstable path is downward sloping. Hence, along the optimal path, the shadow cost and the amount of the pollutant in the water are positively correlated. Therefore, any water restoration policy, where $s_0 > s^{\infty}$, can be characterized by decreasing shadow cost and decreasing amount of the pollutant in the water.

Figure 5

Water restoration policy

By Figure 5 and equation (14), we conclude that $\mu(t)$ has to decrease along the optimal path. This particular link allows us to determine the optimal spatial land demand function and optimal spatial input demand function along the optimal time path, thus, deriving the optimal relationship between short-run and long-run land and input demand functions. In particular, this link allows us to relate the optimal spatial water restoration policy with the optimal intertemporal water restoration policy.It can be shown that x_1 and x_2 decrease with an increase in μ. For instance, this would be the case if a spatially or non-spatially differentiated, yet static, tax were imposed. However, a long-run optimal water restoration policy requires that μ decreases over time, leading to an increase of production intensity along the optimal path. Therefore, an intertemporally and spatially optimal water restoration policy, for $s_0 > s^{\infty}$, can be characterized by choosing the levels of inputs initially below their steady-state values. As time passes, the levels of inputs increase until their steady-state values are reached. The intertemporally and spatially optimal water restoration policy can be partially implemented by imposing taxes to reduce the inputs as discussed in section 3.2. To this end, $\mu(t)$ has to be replaced by $\psi(t)$ in defining the different taxes. The equivalence of μ and ψ is given by equation 14.

The remaining part of an optimal water restoration policy, the optimal allocation of land, will also change over time. A decrease of shadow cost along the optimal path results in an increase in the quasirent for both industries. In the case where the quasirents of different industries intersect, this increase will also lead to a different optimal allocation of land. The change in the optimal land-use pattern depends on the change in magnitude and on the difference of the slope of the quasirent functions at every location α. The following example, shown in Figure 6, illustrates the influence of these two components. Assume that the quasirent for industry one changes from Π_1^* to $\Pi_1'^*$ and for industry two, from Π_2^* to $\Pi_2'^*$. Hence, the borderline between the two industries, previously at α^*, moves to

α^{**}, and industry two moves closer to the urban center. In general terms, for quasirent functions where at least one has a positive slope we can conclude that (1) the borderline shifts towards the urban center if the increase in the quasirent is more pronounced in the industry where the slope of the quasirent is higher (the case illustrated in Figure 6), and (2) the borderline shifts away from the urban center if the increase in the quasirent is more pronounced in the industry, where the slope of the quasirent is lower. In the case where the quasirent functions both have negative slope the results have to be reversed. The borderline shifts towards the urban center if the increase in the quasirent is more pronounced in the industry where the slope of the quasirent is lower (in absolute terms), and (2) the borderline shifts away from the urban center if the increase in the quasirent is more pronounced in the industry, where the slope of the quasirent is higher (in absolute terms).

Figure 6

In the case where quasirents of the different industries do not intersect along the entire optimal path, the optimal land-use pattern does not change at all. However, the quasirent of the different industries might intersect along some part of the optimal path, leading to a change in the optimal land-use pattern during this time and constancy otherwise. The implementation of the intertemporally and spatially optimal allocation of land can be achieved either by zoning regulations or by land-use taxes. In the long-run, land-use taxes seem to be more efficient since they provide incentives for R & D activities to reduce pollution. Moreover, it seems politically viable to adjust them following the optimal intertemporal path compared to land zoning regulations.

4 Summary and Conclusions

This paper analyzes the socially optimal spatial structure of a region taking into account pollution generated by two distinct types of industries. In particular, we determine the optimal location and size of two different zones as a policy to control surface water and ground water pollution. The problem is solved within a spatial and intertemporal framework based on a two-stage approach. In the first stage, the socially optimal spatial allocation of land is determined. In the second stage the already socially optimal spatial allocation is optimized over time. This sequential procedure greatly enhances the analytical tractability of the solution, allowing for the qualitative characteristics of the solution to be more easily derived.

The results of the first stage show that the socially optimal allocation over space is determined by the transportation cost and the environmental damage resulting from pollution at a particular location. The transportation cost leads to a decrease in the socially optimal quasirent of land with distance from the urban center. This effect, however, may be overcompensated by an increase in the socially optimal quasirent with distance from the urban center, due to the decrease in damage caused by pollution. Conditions are established where land is abandoned at the far end of the region (DULU), where there is a green belt in the middle of the region (RULU), or where there is a buffer strip between the urban center and industrial activities (LULU). To support the socially optimal outcome, non-spatially differentiated taxes on the final emissions are proposed as a first-best policy instrument. When individual final emissions cannot be observed, second-best instruments, such as spatially differentiated input or output taxes, are proposed. In contrast to Pigouvian taxes on final emissions, these taxes have to be complemented by land-use policies, such as land zoning or a land-use tax, which are both able to establish the socially optimal allocation of land. However, land-use taxes seem to be a more promising instrument in the long run since they provide incentives to reduce pollution and can be adjusted over time. In the case where marginal final emissions, with respect to the input or output are constant, no additional land-use policy is required. Spatially differentiated input or output taxes are sufficient to support the socially optimal outcome. However, despite whether or not these taxes need to be complemented by land-use policies, they require that the input or output market be separated spatially. When it is not possible to separate markets, taxes can be designed uniformly over space. Compared to a spatially differentiated tax, associated loss of efficiency may be compensated in total, or at least in part, by lower administration and control cost of a uniform tax.

Socially optimal intertemporal pollution is determined in the second stage of the optimization problem. Therefore, changes in the shadow price of a pollutant over time determines the changes in the socially optimal allocation of land and utilization of inputs over time. Thus, the shadow price of the pollutant establishes the link between the short and long-run land demand and input demand functions. Moreover, the optimal intertemporal change of the shadow price determines the required changes necessary to transform a spatially, yet static, optimal water restoration policy into an intertemporally and spatially optimal water policy.

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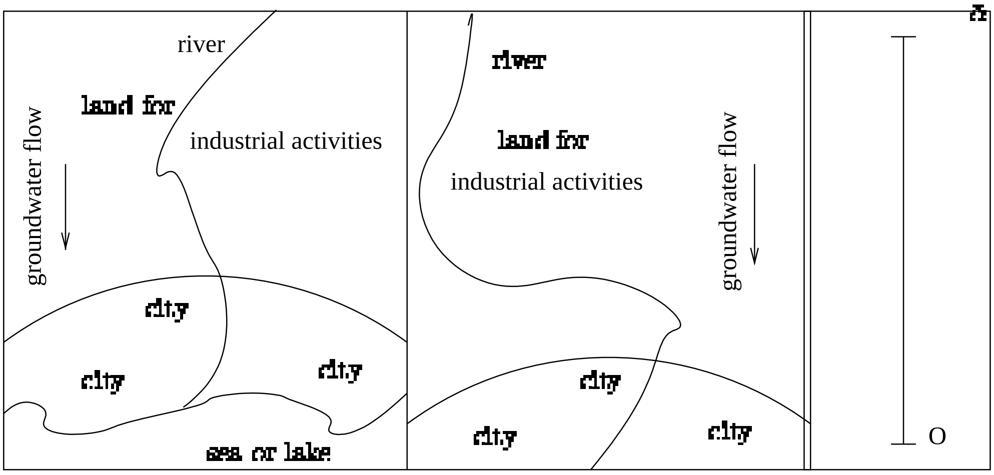

Figure 1: Possible spatial settings of the pollution incidents

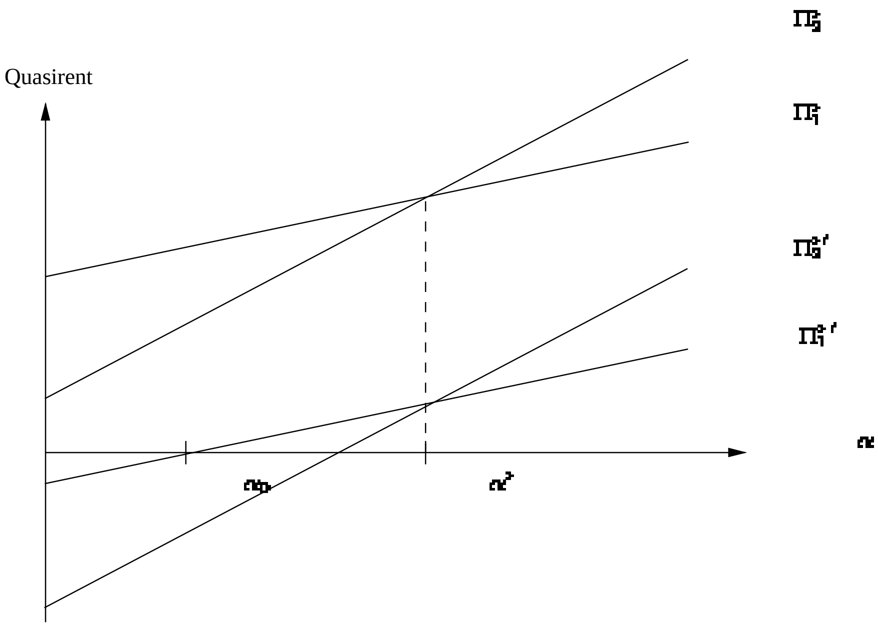

Figure 2: The quasirent/bid rent function as a function of α where both industries are pollution intensive

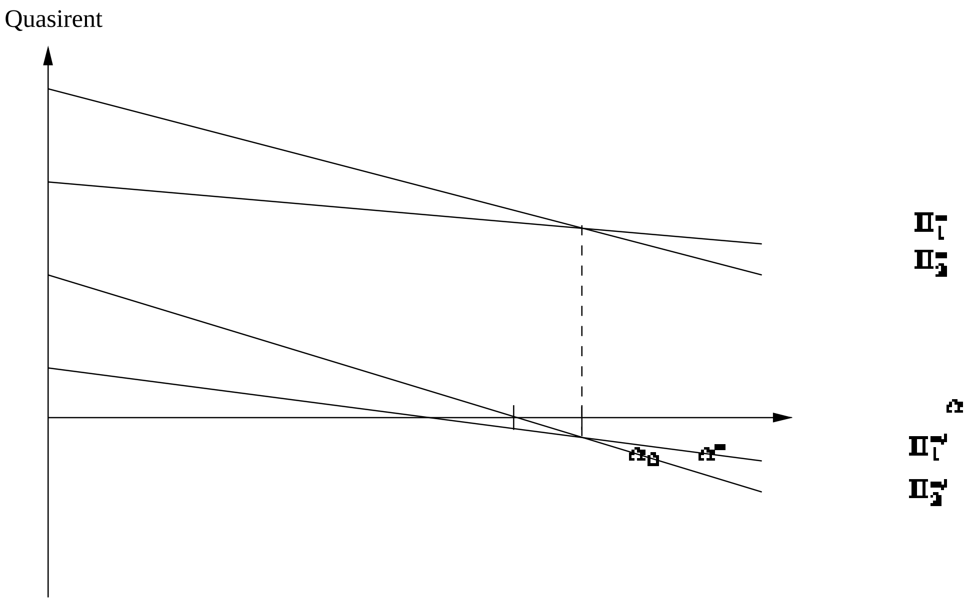

Figure 3: The quasirent/bid rent function as a function of α where both industries are pollution extensive

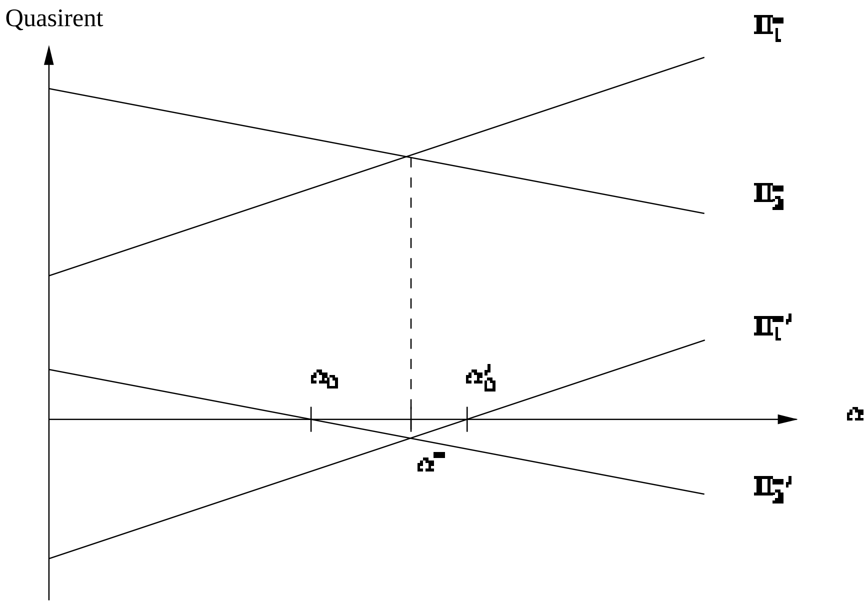

Figure 4: The quasirent/bid rent function as a function of α where industry one is pollution intensive and industry two is pollution extensive

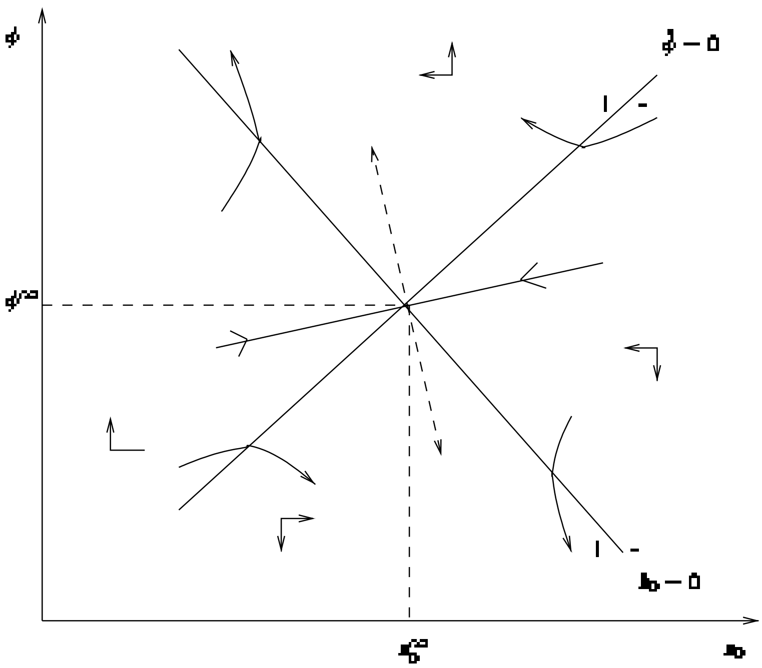

Figure 5: The phase diagram in the (s, ψ) space

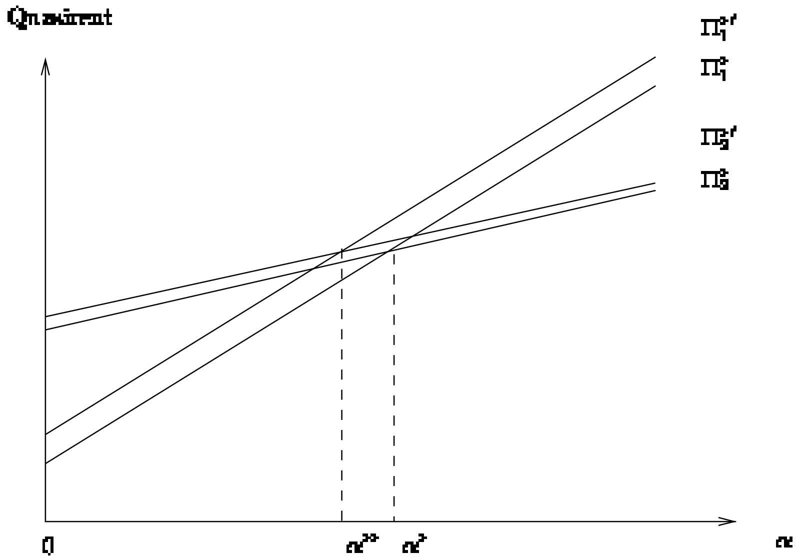

Figure 6: Changes in the land-use pattern for industry one and two resulting from a decrease in the shadow price along the optimal path